Strong Paths Defining Connectivities in Neutrosophic Graphs

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Abstract

New setting is introduced to study strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. Forming neutrosophic paths from a sequence of consecutive vertices forming different types of paths in the terms of length and maximum values of edges forming it to get minmum number to assign to neutrosophic graphs and couple of vertices, is key type of approach to have these notions namely strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. Two numbers and two different paths are assigned to couple of vertices and to neutrosophic graphs, are obtained but now both settings lead to approach is on demand which is to compute and to find sequence of consecutive vertices forming different types of paths in the terms of length and maximum values of edges forming it to get minimum number to assign to neutrosophic graphs and couple of vertices. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then a path from x to y is called strongest path if its length is minimum. This length is called strongest number amid x and y. The maximum number amid all vertices is called strongest number of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}(NTG)$; a path from x to y is called neutrosophic strongest path if its strength is $\mu(uv)$ which is greater than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called neutrosophic strongest number amid x and y. The minimum number amid all vertices is called neutrosophic strongest number of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of strong neutrosophic graphs namely strong-path-neutrosophic graphs, strong-cycle-neutrosophic graphs, strong-complete-neutrosophic graphs, strong-star-neutrosophic graphs, strong-complete-bipartite-neutrosophic graphs, strong-complete-t-partite-neutrosophic graphs and strong-wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of strongest Number," and "Setting of Neutrosophic strongest Number," for introduced results and used classes. This approach facilitates identifying vertices which form strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The value of an edge has eligibility to define strongest number and neutrosophic strongest number but the sequence has eligibility to define strongest path and neutrosophic strongest path. Some results get more frameworks and perspective about these definitions. The way in that, sequence of consecutive vertices forming different types of paths in the terms of length and maximum values of edges forming it to get minimum number to assign to neutrosophic graphs and couple of vertices, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic path connectivity is applied in different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Strongest Number, Neutrosophic Strongest Number, Classes of Neutrosophic Graphs

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Chromatic number and neutrosophic chromatic number in **Ref.** [2], closing numbers and super-closing numbers as (dual)resolving and (dual)coloring alongside (dual)dominating in (neutrosophic)n-SuperHyperGraph in **Ref.** [3], co-degree and degree of classes of neutrosophic hypergraphs in **Ref.** [4], different types of neutrosophic chromatic number in **Ref.** [5], dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [6], independent set in neutrosophic graphs in **Ref.** [7], neutrosophic chromatic number based on connectedness in **Ref.** [8], properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref.** [9], some polynomials related to numbers in classes of neutrosophic graphs in **Ref.** [10], three types of neutrosophic alliances based on connectedness and edges in **Ref.** [11], are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [1].

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "strongest number", "neutrosophic strongest number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two vertices have key roles to assign strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. The concept of having number of neutrosophic paths inspires us to study the behavior of edges in the way that, some types of numbers, strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sequence of consecutive vertices forming different types of path in the terms of length and maximum values of edges forming it to get minimum number to assign to neutrosophic graphs and couple of vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs, in section "Setting of strongest Number," as individuals. In section "Setting of strongest Number," strongest number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely strong-path-neutrosophic graphs, strong-cycle-neutrosophic graphs, complete-neutrosophic graphs, strong-star-neutrosophic graphs, strong-complete-bipartite-neutrosophic graphs, strong-complete-t-partite-neutrosophic graphs and strong-wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of strongest Number," and "Setting of Neutrosophic strongest Number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and strong-complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

$(ii):$ μ is called **neutrosophic edge set**.

$(iii):$ $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

$(iv):$ $\sum_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

$(v):$ $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

$(vi):$ $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

$(i):$ a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

$(ii):$ **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_ix_{i+1})$;

$(iii):$ **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\ \bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1});$$

$(iv):$ a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

$(v):$ it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

$(vi):$ t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

$(vii):$ complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii):$ a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

$(ix):$ it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

$(x):$ it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 1.5. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 1.6. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_{σ} if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **cycle** and denoted by CYC where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1, σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1, σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1, σ_2}.

Remark 1.7. Using notations which is mixed with literatures, are reviewed.

1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)),\ \mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;
2. $CMT_{\sigma}, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

Definition 1.8. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) a path from x to y is called **strongest path** if its length is minimum. This length is called **strongest number** amid x and y. The maximum number amid all vertices is called **strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}(NTG)$;

(*ii*) a path from x to y is called **neutrosophic strongest path** if its strength is $\mu(uv)$ which is greater than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic strongest number** amid x and y. The minimum number amid all vertices is called **neutrosophic strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.9. In Figure (1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(CMT_{\sigma}) = 1$;

(*ii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_{\sigma}) \neq 2$;

(*iii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_{\sigma}) \neq 2$. For every given couple of vertices x and y, strongest path is existed, strongest number is one and $\mathcal{S}(CMT_{\sigma}) = 1$;

(*iv*) if n_1, n_4, n_3, n_2 is a path from n_1 to n_2, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

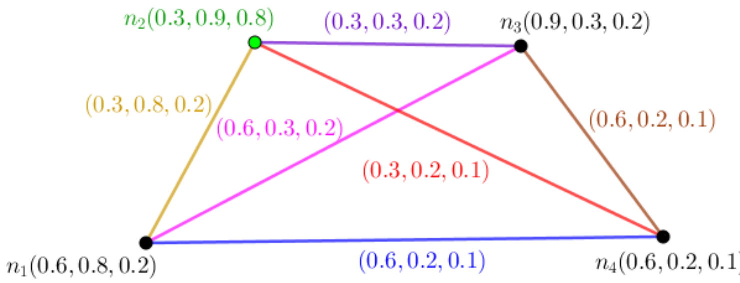

Figure 1. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

$$\begin{aligned} P_1 : n_1, n_4, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\ P_4 : n_1, n_2 &\Rightarrow (0.3, 0.8, 0.2) \\ \text{Maximum is } & (0.3, 0.8, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(v) if n_2, n_1, n_4, n_3 is a path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.6, 0.3, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_2, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1) \\ P_2 : n_2, n_4, n_3 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_2, n_1, n_3 &\Rightarrow (0.6, 0.3, 0.2) \\ P_4 : n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2) \\ \text{Maximum is } & (0.6, 0.3, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(vi) if n_3, n_2, n_1, n_4 is a path from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_3, n_2, n_1, n_4 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_3, n_1, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ P_3 : n_3, n_2, n_4 &\Rightarrow (0.3, 0.2, 0.1) \\ P_4 : n_3, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ \text{Maximum is } & (0.6, 0.2, 0.1) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$.

2 Setting of strongest Number

In this section, I provide some results in the setting of strongest Number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, strong-path-neutrosophic graph, strong-cycle-neutrosophic graph, strong-star-neutrosophic graph, strong-bipartite-neutrosophic graph, strong-t-partite-neutrosophic graph, and strong-wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Minimum path is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's a path containing all vertices and there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. But the length of the path forms strongest number. Thus

$$\mathcal{S}(CMT_\sigma) = 1.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.2. In Figure (2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(CMT_\sigma) = 1$;

(*ii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$;

(*iii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$. For every given couple of vertices x and y, strongest path is existed, strongest number is one and $\mathcal{S}(CMT_\sigma) = 1$;

(*iv*) if n_1, n_4, n_3, n_2 is a path from n_1 to n_2, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned}
P_1 : n_1, n_4, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_2 : n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_3 : n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_4 : n_1, n_2 &\Rightarrow (0.3, 0.8, 0.2)\\
\text{Maximum is } &(0.3, 0.8, 0.2)
\end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

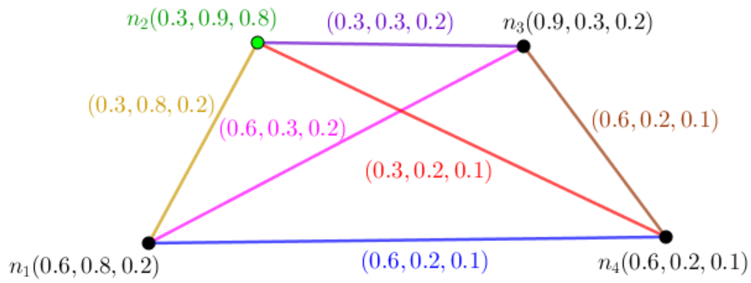

Figure 2. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

(v) if n_2, n_1, n_4, n_3 is a path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.6, 0.3, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_2, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1) \\ P_2 : n_2, n_4, n_3 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_2, n_1, n_3 &\Rightarrow (0.6, 0.3, 0.2) \\ P_4 : n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2) \\ \text{Maximum is } &(0.6, 0.3, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(vi) if n_3, n_2, n_1, n_4 is a path from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_3, n_2, n_1, n_4 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_3, n_1, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ P_3 : n_3, n_2, n_4 &\Rightarrow (0.3, 0.2, 0.1) \\ P_4 : n_3, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ \text{Maximum is } &(0.6, 0.2, 0.1) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{S}(PTH) = \mathcal{O}(PTH) - 1.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)-1}$ be a path-neutrosophic graph. There are some neutrosophic paths but there's only one neutrosophic path related to strongest number of this path-neutrosophic graph. So the length of this path is strongest number. The length of this path is its order minus one related to neutrosophic path amid leaves. It means the length of this path is $\mathcal{O}(PTH) - 1$. Thus

$$\mathcal{S}(PTH) = \mathcal{O}(PTH) - 1.$$

□

Example 2.4. There are two sections for clarifications.

(a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it's strongest path and strongest number amid n_1 and n_5 is four. Also, $\mathcal{S}(PTH) = 4$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path but strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(PTH) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path but strongest number amid n_1 and n_4 is three. Also, $\mathcal{S}(PTH) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is four and $\mathcal{S}(PTH) = 4$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.4, 0.4, 0.1)$. Also, $\mathcal{S}_n(PTH) = (0.4, 0.4, 0.1)$;

(v) if n_4, n_5 is a neutrosophic path from n_4 to n_5, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_4 and n_5 is $(0.4, 0.4, 0.1)$. Also, $\mathcal{S}_n(PTH) = (0.4, 0.4, 0.1)$;

(vi) For every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.4, 0.4, 0.1)$ and $\mathcal{S}_n(PTH) = (0.4, 0.4, 0.1)$.

(b) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it's strongest path and strongest number amid n_1 and n_6 is five. Also, $\mathcal{S}(PTH) = 5$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path but strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(PTH) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path but strongest number amid n_1 and n_4 is three. Also, $\mathcal{S}(PTH) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is four and $\mathcal{S}(PTH) = 5$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_5 and n_6 is $(0.9, 0.1, 0.9)$. Also, $\mathcal{S}_n(PTH) = (0.9, 0.1, 0.9)$;

(v) if n_5, n_6 is a neutrosophic path from n_5 to n_6, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_5 and n_6 is $(0.9, 0.1, 0.9)$. Also, $\mathcal{S}_n(PTH) = (0.9, 0.1, 0.9)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.9, 0.1, 0.9)$ and $\mathcal{S}_n(PTH) = (0.9, 0.1, 0.9)$.

Proposition 2.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{S}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor.$$

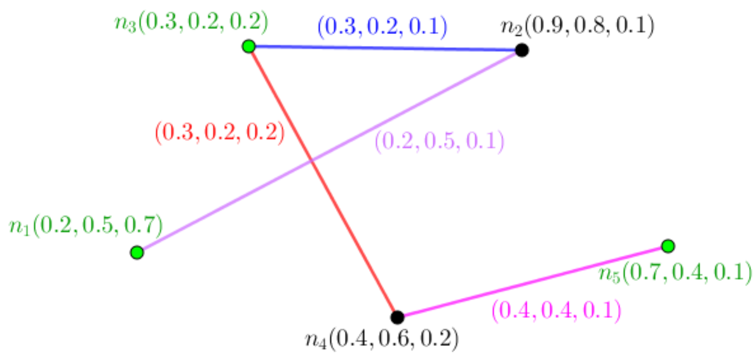

Figure 3. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

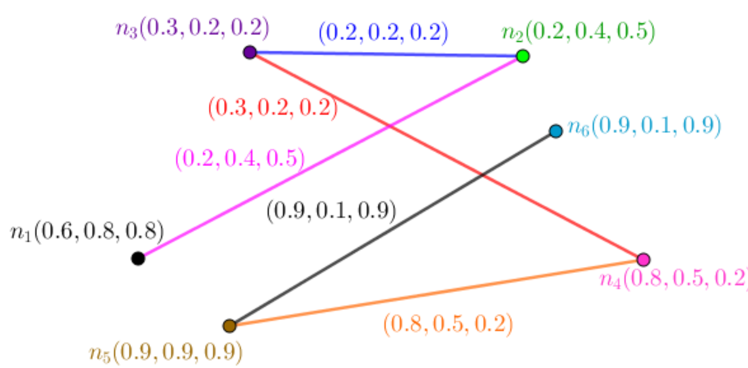

Figure 4. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a path is strongest number. For every given couple of vertices, there are two neutrosophic paths concerning two lengths s and $\mathcal{O}(CYC) - s$. If $s < \mathcal{O}(CYC) - s$, then s is intended length; otherwise, $\mathcal{O}(CYC) - s$ is intended length. Since minimum length amid two vertices are on demand. In next step, amid all lengths, the biggest number is strongest number. The biggest length of path is either order half or order half minus one. It means the length of this path is either $\frac{\mathcal{O}(CYC)}{2}$ or $\frac{\mathcal{O}(CYC)}{2} - 1$. Thus

$$\mathcal{S}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6. There are two sections for clarifications.

(a) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it isn't strongest path and strongest number amid n_1 and n_6 is one. Also, $\mathcal{S}(CYC) = 3$.

  (ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is two corresponded to n_1, n_2, n_3. Also, $\mathcal{S}(CYC) \neq 2$;

  (iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4 and n_1, n_6, n_5, n_4 Also, $\mathcal{S}(CYC) = 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is three and $\mathcal{S}(CYC) = 3$;

  (iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_1 and n_4 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

  (v) if n_2, n_3 is a neutrosophic path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

  (vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.1, 0.1, 0.2)$ and $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$.

(b) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it isn't strongest path and strongest number amid n_1 and n_5 is one. Also, $\mathcal{S}(CYC) = 2$;

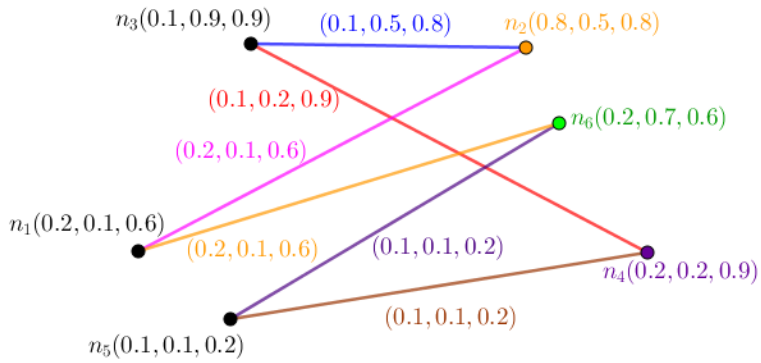

Figure 5. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

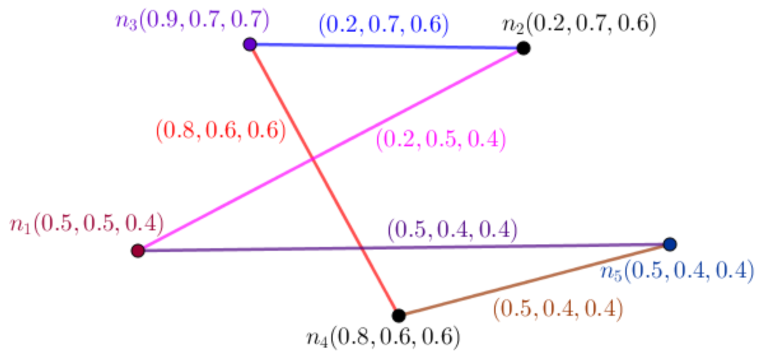

Figure 6. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it's strongest path and strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(CYC) = 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is two corresponded to n_1, n_5, n_4. Also, $\mathcal{S}(CYC) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is two and $\mathcal{S}(CYC) = 2$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path [strength is $(0.2, 0.5, 0.4)$] from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_4 is $(0.5, 0.4, 0.4)$ but neutrosophic strongest number amid n_1 and n_2 is $(0.2, 0.7, 0.6)$; neutrosophic strongest number amid n_2 and n_3 is $(0.2, 0.7, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(v) if n_3, n_4 is a neutrosophic path [strength is $(0.8, 0.6, 0.6)$] from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.2, 0.7, 0.6)$ and $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{S}(STR_{1,\sigma_2}) = 1.$$

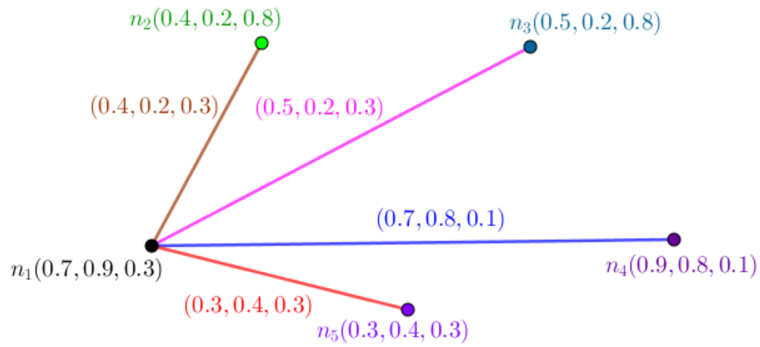

Figure 7. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. All paths have length 1. Thus

$$\mathcal{S}(STR_{1,\sigma_2}) = 1.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.8. There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a neutrosophic path from n_1 to n_2, then it's strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(STR_{1,\sigma_2}) = 1$;

(ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it's strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(STR_{1,\sigma_2}) = 1$;

(iii) if n_1, n_4 is a neutrosophic path from n_1 to n_4, then it's strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(STR_{1,\sigma_2}) = 1$. For every given couple of vertices x and y, only one strongest path is existed and strongest number is one. $\mathcal{S}(STR_{1,\sigma_2}) = 1$;

(iv) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.4, 0.2, 0.3)$. Also, $\mathcal{S}_n(STR_{1,\sigma_2}) = (0.4, 0.2, 0.3)$;

(v) if n_1, n_2 is a neutrosophic path from n_1 to n_4, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_1 and n_4 is $(0.4, 0.2, 0.3)$. Also, $\mathcal{S}_n(STR_{1,\sigma_2}) = (0.4, 0.2, 0.3)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.4, 0.2, 0.3)$ and $\mathcal{S}_n(STR_{1,\sigma_2}) = (0.4, 0.2, 0.3)$.

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Let $x_1, x_2, \cdots, x_s$ be a neutrosophic path. It's a sequence of consecutive vertices and there's no repetition of vertex in this sequence. The lengths are either one or two. So maximum number of all lengths is 2. It implies the length is 2. Thus

$$\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is 2. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2$;

(ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) \neq 1$;

(iii) if n_1, n_3, n_4, n_2 is a neutrosophic path from n_1 to n_2, then it isn't strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2$. For every given couple of vertices x and y, strongest path isn't existed where x and y have different part. But for every given couple of vertices x and y, strongest path is existed where x and y have same part and strongest number is two. $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2$;

(iv) if n_1, n_2, n_4, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_3 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = (0.3, 0.2, 0.3)$;

(v) if n_4, n_3 is a neutrosophic path from n_4 to n_3, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = (0.3, 0.2, 0.3)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.3, 0.2, 0.3)$ and $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = (0.3, 0.2, 0.3)$.

Proposition 2.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph. Then*

$$\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Let $x_1, x_2, \cdots, x_s$ be a neutrosophic path. It's a sequence of consecutive vertices and there's no repetition of vertex in this sequence. The lengths are either one or two. So maximum number of all lengths is 2. It implies the length is 2. Thus

$$\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.$$

□

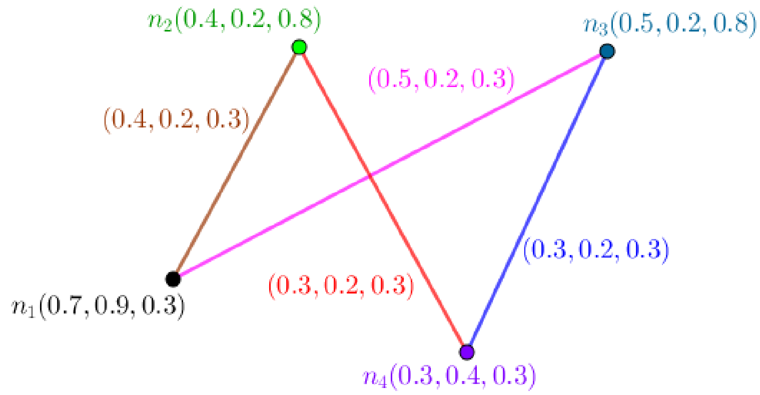

Figure 8. A Neutrosophic Graph in the Viewpoint of Eulerian(Hamiltonian) neutrosophic path.

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.12. There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is 2. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \neq 1$;

(iii) if n_1, n_3, n_4, n_2 is a neutrosophic path from n_1 to n_2, then it isn't strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$. For every given couple of vertices x and y, strongest path isn't existed where x and y have different part. But for every given couple of vertices x and y, strongest path is existed where x and y have same part and strongest number is two. $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) if n_1, n_2, n_4, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_3 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$;

(v) if n_4, n_3 is a neutrosophic path from n_4 to n_3, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.3, 0.2, 0.3)$ and $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$.

Proposition 2.13. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{S}(WHL_{1,\sigma_2}) = 2.$$

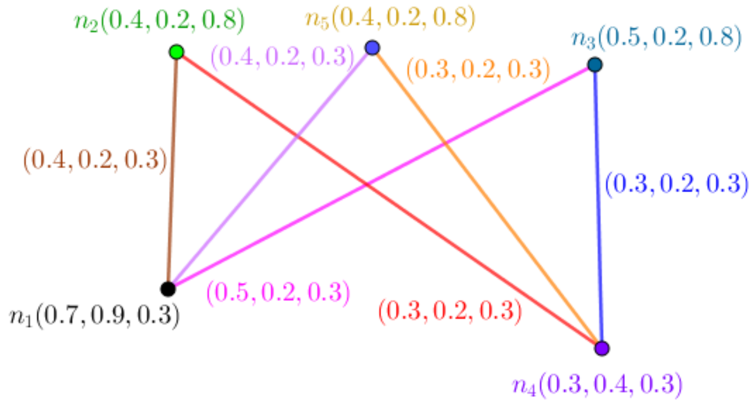

Figure 9. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. Minimum length of path is on demand but in other hands, the minimum length of path amid two given vertices is either one or two since every vertex which isn't a neighbor to another vertex, could choose the center to make a path with length two. In next step, maximum number is on demand. All numbers are either one or two which mean the maximum number is two. It's a sequence of consecutive vertices, there's no repetition of vertex in this sequence. It implies the length is 2. Thus

$$\mathcal{S}(WHL_{1,\sigma_2}) = 2.$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.14. There is one section for clarifications. In Figure (10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If s_1, s_2, s_3 is a neutrosophic path from s_1 to s_3, then it isn't strongest path and strongest number amid s_1 and s_3 is 1 corresponded to s_1, s_3. Also, $\mathcal{S}(WHL_{1,\sigma_2}) = 2$;

(*ii*) if s_2, s_3, s_4, s_5 is a neutrosophic path from s_2 to s_5, then it isn't strongest path and strongest number amid s_2 and s_5 is two corresponded to s_2, s_1, s_5. Also, $\mathcal{S}(WHL_{1,\sigma_2}) \neq 3$;

(*iii*) if s_2, s_1, s_4 is a neutrosophic path from s_2 to s_4, then it's strongest path and strongest number amid s_2 and s_4 is two. For every given couple of vertices x and y, strongest path is existed where x and y aren't neighbors and strongest number is two. $\mathcal{S}(WHL_{1,\sigma_2}) = 2$;

(*iv*) if s_1, s_2, s_3, s_4, s_5 is a neutrosophic path from s_1 to s_5, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid s_4 and s_5 is $(0.1, 0.1, 0.5)$. Since all paths from s_4 to s_5, have same strength $(0.1, 0.1, 0.5)$ implying maximum strength of all paths for these two vertices is $(0.1, 0.1, 0.5)$

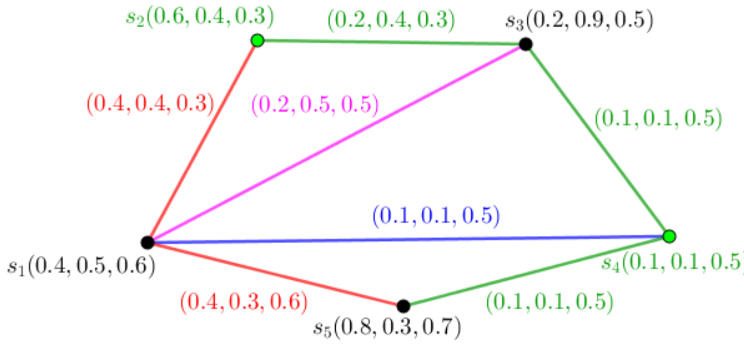

Figure 10. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

inducing minimum number amid strengths of intended paths from all given couple of vertices is $(0.1, 0.1, 0.5)$ since this corresponded edge is weakest. Edges form these numbers and the minimum amid all form neutrosophic strongest number. Also, $\mathcal{S}_n(WHL_{1,\sigma_2}) = (0.1, 0.1, 0.5)$;

(v) if s_4, s_5 is a neutrosophic path from s_4 to s_5, then it's a neutrosophic strongest path and neutrosophic strongest number amid s_4 and s_5 is $(0.1, 0.1, 0.5)$. Also, $\mathcal{S}_n(WHL_{1,\sigma_2}) = (0.1, 0.1, 0.5)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, there're two paths s_4, s_5, and s_3, s_4 neutrosophic strongest number is $(0.1, 0.1, 0.5)$ and $\mathcal{S}_n(WHL_{1,\sigma_2}) = (0.1, 0.1, 0.5)$.

3 Setting of Neutrosophic strongest Number

In this section, I provide some results in the setting of neutrosophic strongest number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, strong-path-neutrosophic graph, strong-cycle-neutrosophic graph, strong-star-neutrosophic graph, strong-bipartite-neutrosophic graph, strong-t-partite-neutrosophic graph, and strong-wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}_n(CMT_\sigma) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Minimum path is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's a path containing all vertices and there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. But the length of the path forms strongest number. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of path from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(CMT_\sigma) = \min_{v \in V} \sigma(v).$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.2. In Figure (2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(CMT_\sigma) = 1$;

(*ii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$;

(*iii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$. For every given couple of vertices x and y, strongest path is existed, strongest number is one and $\mathcal{S}(CMT_\sigma) = 1$;

(*iv*) if n_1, n_4, n_3, n_2 is a path from n_1 to n_2, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_1, n_4, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\ P_4 : n_1, n_2 &\Rightarrow (0.3, 0.8, 0.2) \\ \text{Maximum is } &(0.3, 0.8, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(*v*) if n_2, n_1, n_4, n_3 is a path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.6, 0.3, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_2, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1) \\ P_2 : n_2, n_4, n_3 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_2, n_1, n_3 &\Rightarrow (0.6, 0.3, 0.2) \\ P_4 : n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2) \\ \text{Maximum is } &(0.6, 0.3, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(*vi*) if n_3, n_2, n_1, n_4 is a path from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

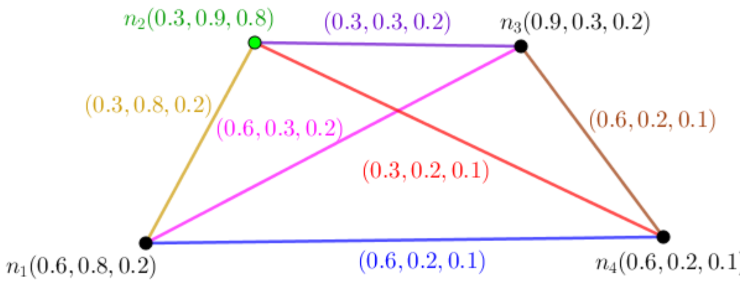

Figure 11. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

$$\begin{aligned} P_1 : n_3, n_2, n_1, n_4 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_3, n_1, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ P_3 : n_3, n_2, n_4 &\Rightarrow (0.3, 0.2, 0.1) \\ P_4 : n_3, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ \text{Maximum is } &(0.6, 0.2, 0.1) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-path-neutrosophic graph. Then*

$$\mathcal{S}_n(PTH) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a strong-path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)-1}$ be a path-neutrosophic graph. There are some neutrosophic paths but there's only one neutrosophic path related to strongest number of this path-neutrosophic graph. So the length of this path is strongest number. The length of this path is its order minus one related to neutrosophic path amid leaves. It means the length of this path is $\mathcal{O}(PTH) - 1$. There's only one path amid given couple of vertices. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(PTH) = \min_{v \in V} \sigma(v).$$

□

Example 3.4. There are two sections for clarifications.

(a) In Figure (12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it's strongest path and strongest number amid n_1 and n_5 is four. Also, $\mathcal{S}(PTH) = 4$;
- (ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path but strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(PTH) \neq 2$;

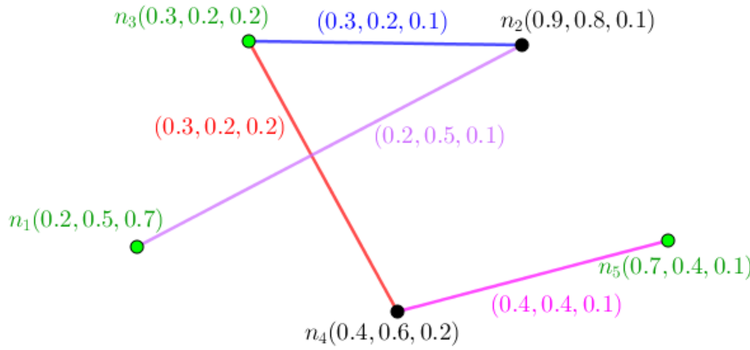

Figure 12. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path but strongest number amid n_1 and n_4 is three. Also, $\mathcal{S}(PTH) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is four and $\mathcal{S}(PTH) = 4$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.4, 0.4, 0.1)$. Also, $\mathcal{S}_n(PTH) = (0.4, 0.4, 0.1)$;

(v) if n_4, n_5 is a neutrosophic path from n_4 to n_5, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_4 and n_5 is $(0.4, 0.4, 0.1)$. Also, $\mathcal{S}_n(PTH) = (0.4, 0.4, 0.1)$;

(vi) For every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.4, 0.4, 0.1)$ and $\mathcal{S}_n(PTH) = (0.4, 0.4, 0.1)$.

(b) In Figure (13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it's strongest path and strongest number amid n_1 and n_6 is five. Also, $\mathcal{S}(PTH) = 5$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path but strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(PTH) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path but strongest number amid n_1 and n_4 is three. Also, $\mathcal{S}(PTH) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is four and $\mathcal{S}(PTH) = 5$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_5 and n_6 is $(0.9, 0.1, 0.9)$. Also, $\mathcal{S}_n(PTH) = (0.9, 0.1, 0.9)$;

(v) if n_5, n_6 is a neutrosophic path from n_5 to n_6, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_5 and n_6 is $(0.9, 0.1, 0.9)$. Also, $\mathcal{S}_n(PTH) = (0.9, 0.1, 0.9)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.9, 0.1, 0.9)$ and $\mathcal{S}_n(PTH) = (0.9, 0.1, 0.9)$.

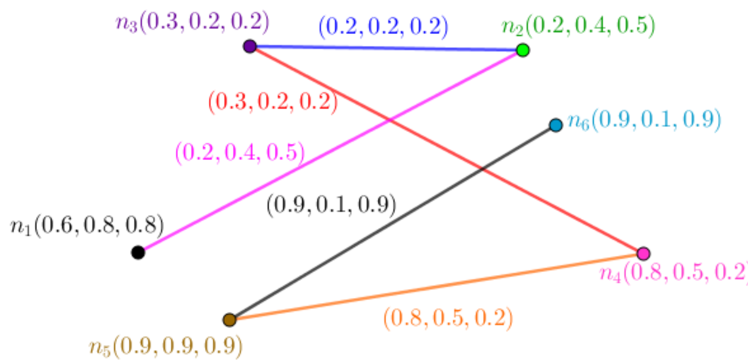

Figure 13. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

Proposition 3.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be a strong-cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{S}_n(CYC) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a path is strongest number. For every given couple of vertices, there are two neutrosophic paths concerning two lengths s and $\mathcal{O}(CYC) - s$. If $s < \mathcal{O}(CYC) - s$, then s is intended length; otherwise, $\mathcal{O}(CYC) - s$ is intended length. Since minimum length amid two vertices are on demand. In next step, amid all lengths, the biggest number is strongest number. The biggest length of path is either order half or order half minus one. It means the length of this path is either $\frac{\mathcal{O}(CYC)}{2}$ or $\frac{\mathcal{O}(CYC)}{2} - 1$. There are only two paths amid given couple of vertices. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(CYC) = \min_{v \in V} \sigma(v).$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There are two sections for clarifications.

(a) In Figure (14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it isn't strongest path and strongest number amid n_1 and n_6 is one. Also, $\mathcal{S}(CYC) = 3$.

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is two corresponded to n_1, n_2, n_3. Also, $\mathcal{S}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4 and n_1, n_6, n_5, n_4 Also, $\mathcal{S}(CYC) = 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is three and $\mathcal{S}(CYC) = 3$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_1 and n_4 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

(v) if n_2, n_3 is a neutrosophic path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.1, 0.1, 0.2)$ and $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$.

(b) In Figure (15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it isn't strongest path and strongest number amid n_1 and n_5 is one. Also, $\mathcal{S}(CYC) = 2$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it's strongest path and strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(CYC) = 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is two corresponded to n_1, n_5, n_4. Also, $\mathcal{S}(CYC) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is two and $\mathcal{S}(CYC) = 2$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path [strength is $(0.2, 0.5, 0.4)$] from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_4 is $(0.5, 0.4, 0.4)$ but neutrosophic strongest number amid n_1 and n_2 is $(0.2, 0.7, 0.6)$; neutrosophic strongest number amid n_2 and n_3 is $(0.2, 0.7, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(v) if n_3, n_4 is a neutrosophic path [strength is $(0.8, 0.6, 0.6)$] from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.2, 0.7, 0.6)$ and $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$.

Proposition 3.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{S}_n(STR_{1,\sigma_2}) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. All paths have length 1. There's only one path amid given couple of vertices. Consider $s \in S$ such that

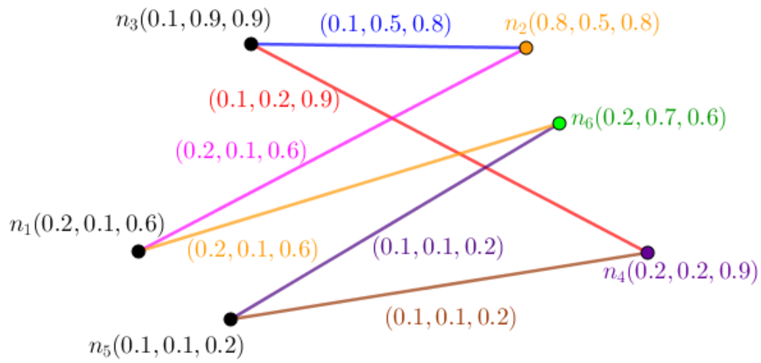

Figure 14. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

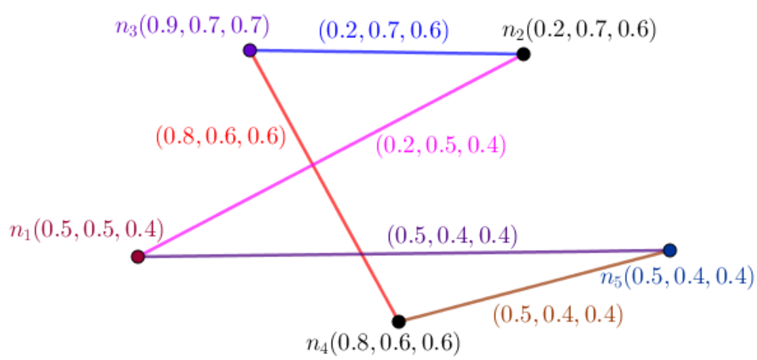

Figure 15. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

$\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(STR_{1,\sigma_2}) = \min_{v \in V} \sigma(v).$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If n_1, n_2 is a neutrosophic path from n_1 to n_2, then it's strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(STR_{1,\sigma_2}) = 1$;
- (ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it's strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(STR_{1,\sigma_2}) = 1$;
- (iii) if n_1, n_4 is a neutrosophic path from n_1 to n_4, then it's strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(STR_{1,\sigma_2}) = 1$. For every given couple of vertices x and y, only one strongest path is existed and strongest number is one. $\mathcal{S}(STR_{1,\sigma_2}) = 1$;
- (iv) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.4, 0.2, 0.3)$. Also, $\mathcal{S}_n(STR_{1,\sigma_2}) = (0.4, 0.2, 0.3)$;
- (v) if n_1, n_2 is a neutrosophic path from n_1 to n_4, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_1 and n_4 is $(0.4, 0.2, 0.3)$. Also, $\mathcal{S}_n(STR_{1,\sigma_2}) = (0.4, 0.2, 0.3)$;
- (vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.4, 0.2, 0.3)$ and $\mathcal{S}_n(STR_{1,\sigma_2}) = (0.4, 0.2, 0.3)$.

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-bipartite-neutrosophic graph. Then*

$$\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a strong-complete-bipartite-neutrosophic graph. Let $x_1, x_2, \cdots, x_s$ be a neutrosophic path. It's a sequence of consecutive vertices and there's no repetition of vertex in this sequence. The lengths are either one or two. So maximum number of all lengths is 2. It implies the length is 2. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given

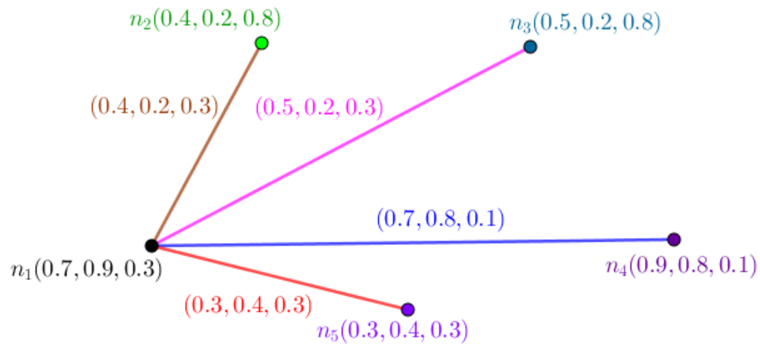

Figure 16. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = \min_{v \in V} \sigma(v).$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is 2. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2$;

(ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) \neq 1$;

(iii) if n_1, n_3, n_4, n_2 is a neutrosophic path from n_1 to n_2, then it isn't strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2$. For every given couple of vertices x and y, strongest path isn't existed where x and y have different part. But for every given couple of vertices x and y, strongest path is existed where x and y have same part and strongest number is two. $\mathcal{S}(CMC_{\sigma_1,\sigma_2}) = 2$;

(iv) if n_1, n_2, n_4, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_3 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = (0.3, 0.2, 0.3)$;

(v) if n_4, n_3 is a neutrosophic path from n_4 to n_3, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = (0.3, 0.2, 0.3)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.3, 0.2, 0.3)$ and $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2}) = (0.3, 0.2, 0.3)$.

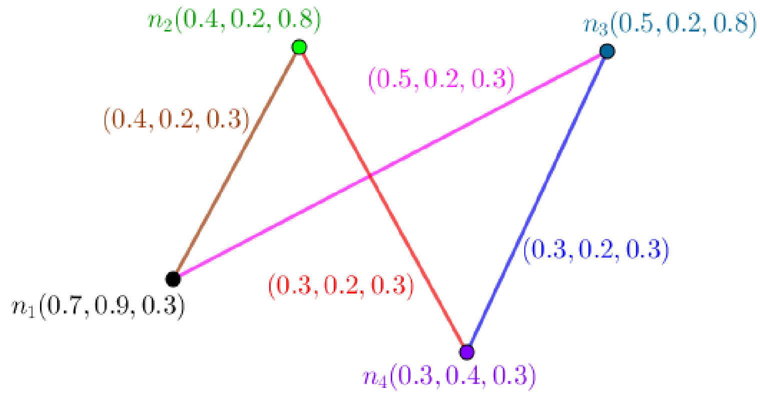

Figure 17. A Neutrosophic Graph in the Viewpoint of Eulerian(Hamiltonian) neutrosophic path.

Proposition 3.11. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-t-partite-neutrosophic graph. Then*

$$\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a strong-complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Let $x_1, x_2, \cdots, x_s$ be a neutrosophic path. It's a sequence of consecutive vertices and there's no repetition of vertex in this sequence. The lengths are either one or two. So maximum number of all lengths is 2. It implies the length is 2. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_{v \in V} \sigma(v).$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is 2. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \neq 1$;

(iii) if n_1, n_3, n_4, n_2 is a neutrosophic path from n_1 to n_2, then it isn't strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$. For

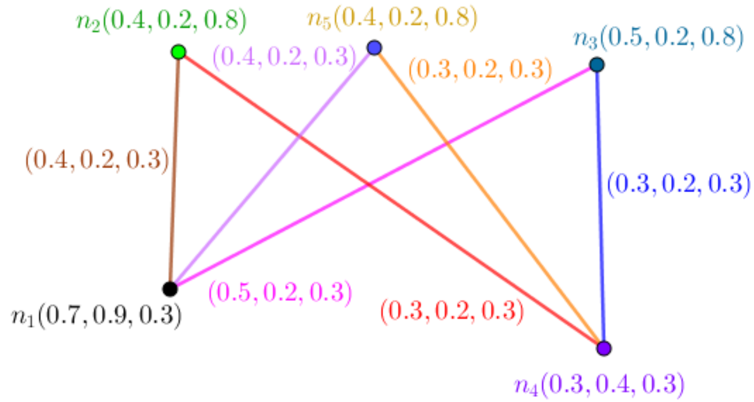

Figure 18. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

every given couple of vertices x and y, strongest path isn't existed where x and y have different part. But for every given couple of vertices x and y, strongest path is existed where x and y have same part and strongest number is two. $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) if n_1, n_2, n_4, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_3 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$;

(v) if n_4, n_3 is a neutrosophic path from n_4 to n_3, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.3, 0.2, 0.3)$ and $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$.

Proposition 3.13. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-wheel-neutrosophic graph. Then*

$$\mathcal{S}_n(WHL_{1,\sigma_2}) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a strong-wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. Minimum length of path is on demand but in other hands, the minimum length of path amid two given vertices is either one or two since every vertex which isn't a neighbor to another vertex, could choose the center to make a path with length two. In next step, maximum number is on demand. All numbers are either one or two which mean the maximum number is two. It's a sequence of consecutive vertices, there's no repetition of vertex in this sequence. It implies the length is 2. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(WHL_{1,\sigma_2}) = \min_{v \in V} \sigma(v).$$

□

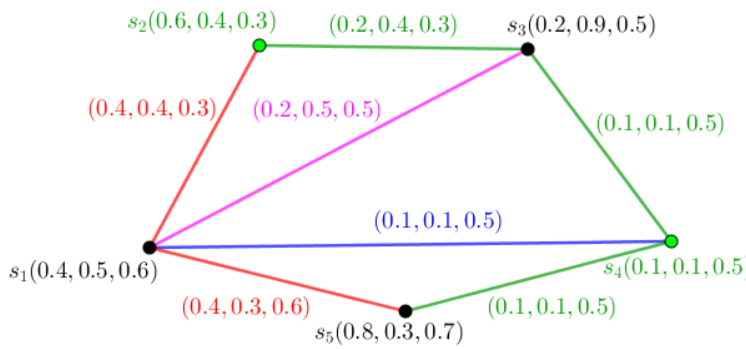

Figure 19. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number.

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.14. There is one section for clarifications. In Figure (19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2, s_3 is a neutrosophic path from s_1 to s_3, then it isn't strongest path and strongest number amid s_1 and s_3 is 1 corresponded to s_1, s_3. Also, $\mathcal{S}(WHL_{1,\sigma_2}) = 2$;

(ii) if s_2, s_3, s_4, s_5 is a neutrosophic path from s_2 to s_5, then it isn't strongest path and strongest number amid s_2 and s_5 is two corresponded to s_2, s_1, s_5. Also, $\mathcal{S}(WHL_{1,\sigma_2}) \neq 3$;

(iii) if s_2, s_1, s_4 is a neutrosophic path from s_2 to s_4, then it's strongest path and strongest number amid s_2 and s_4 is two. For every given couple of vertices x and y, strongest path is existed where x and y aren't neighbors and strongest number is two. $\mathcal{S}(WHL_{1,\sigma_2}) = 2$;

(iv) if s_1, s_2, s_3, s_4, s_5 is a neutrosophic path from s_1 to s_5, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid s_4 and s_5 is $(0.1, 0.1, 0.5)$. Since all paths from s_4 to s_5, have same strength $(0.1, 0.1, 0.5)$ implying maximum strength of all paths for these two vertices is $(0.1, 0.1, 0.5)$ inducing minimum number amid strengths of intended paths from all given couple of vertices is $(0.1, 0.1, 0.5)$ since this corresponded edge is weakest. Edges form these numbers and the minimum amid all form neutrosophic strongest number. Also, $\mathcal{S}_n(WHL_{1,\sigma_2}) = (0.1, 0.1, 0.5)$;

(v) if s_4, s_5 is a neutrosophic path from s_4 to s_5, then it's a neutrosophic strongest path and neutrosophic strongest number amid s_4 and s_5 is $(0.1, 0.1, 0.5)$. Also, $\mathcal{S}_n(WHL_{1,\sigma_2}) = (0.1, 0.1, 0.5)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, there're two paths s_4, s_5, and s_3, s_4 neutrosophic strongest number is $(0.1, 0.1, 0.5)$ and $\mathcal{S}_n(WHL_{1,\sigma_2}) = (0.1, 0.1, 0.5)$.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

4.1 Case 1: Complete-t-partite Model alongside its strongest Number and its Neutrosophic strongest Number

Step 4. (Solution) The neutrosophic graph alongside its strongest number and its neutrosophic strongest number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its strongest number and its neutrosophic strongest number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its strongest number and its neutrosophic strongest number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also,

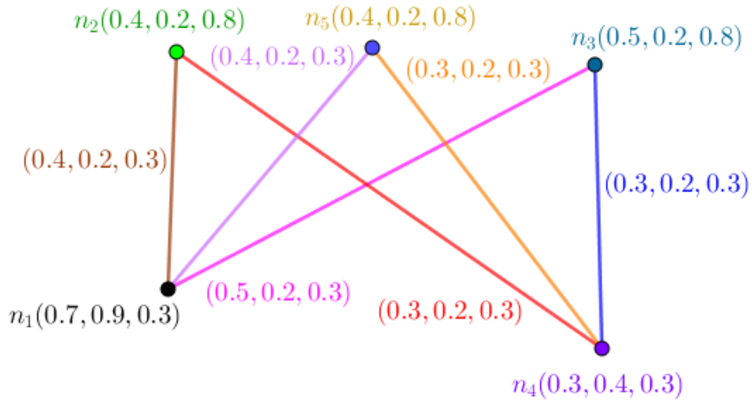

Figure 20. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number

in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (20). In Figure (20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is 2. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(ii) if n_1, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \neq 1$;

(iii) if n_1, n_3, n_4, n_2 is a neutrosophic path from n_1 to n_2, then it isn't strongest path and strongest number amid n_1 and n_2 is one. Also, $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$. For every given couple of vertices x and y, strongest path isn't existed where x and y have different part. But for every given couple of vertices x and y, strongest path is existed where x and y have same part and strongest number is two. $\mathcal{S}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) if n_1, n_2, n_4, n_3 is a neutrosophic path from n_1 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_3 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$;

(v) if n_4, n_3 is a neutrosophic path from n_4 to n_3, then it's a neutrosophic strongest path and neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.2, 0.3)$. Also, $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.3, 0.2, 0.3)$ and $\mathcal{S}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = (0.3, 0.2, 0.3)$.

4.2 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number

Step 4. (Solution) The neutrosophic graph alongside its strongest number and its neutrosophic strongest number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and

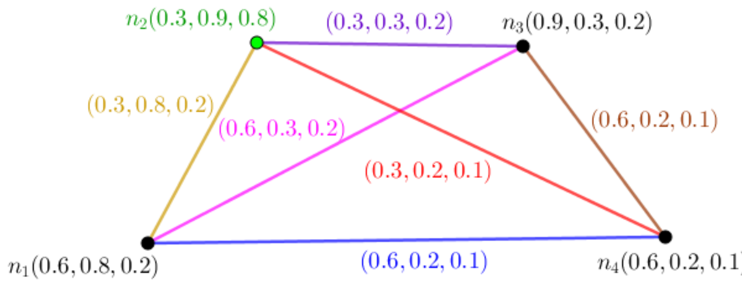

Figure 21. A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number

edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its strongest number and its neutrosophic strongest number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its strongest number and its neutrosophic strongest number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (21). There is one section for clarifications.

(i) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(CMT_\sigma) = 1$;

(ii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$;

(iii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$. For every given couple of vertices x and y, strongest path is existed, strongest number is one and $\mathcal{S}(CMT_\sigma) = 1$;

(iv) if n_1, n_4, n_3, n_2 is a path from n_1 to n_2, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned}
P_1 : n_1, n_4, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_2 : n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_3 : n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_4 : n_1, n_2 &\Rightarrow (0.3, 0.8, 0.2)\\
\text{Maximum is } &(0.3, 0.8, 0.2)
\end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(v) if n_2, n_1, n_4, n_3 is a path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.6, 0.3, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_2, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1) \\ P_2 : n_2, n_4, n_3 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_2, n_1, n_3 &\Rightarrow (0.6, 0.3, 0.2) \\ P_4 : n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2) \\ \text{Maximum is } &(0.6, 0.3, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(vi) if n_3, n_2, n_1, n_4 is a path from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_3, n_2, n_1, n_4 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_3, n_1, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ P_3 : n_3, n_2, n_4 &\Rightarrow (0.3, 0.2, 0.1) \\ P_4 : n_3, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ \text{Maximum is } &(0.6, 0.2, 0.1) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning its strongest number and its neutrosophic strongest number are defined in neutrosophic graphs. Thus,

Question 5.1. *Is it possible to use other types of its strongest number and its neutrosophic strongest number?*

Question 5.2. *Are existed some connections amid different types of its strongest number and its neutrosophic strongest number in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. The connections of consecutive vertices which aren't clarified by a neutrosophic path differ them from each other and put them in different categories to represent a number which is called strongest number and neutrosophic strongest number arising from length and strength of paths in neutrosophic graphs assigned to couple of vertices and to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

Table 2. A Brief Overview about Advantages and Limitations of this Study

Advantages	Limitations
1. strongest Number of Model	1. Connections amid Classes
2. Neutrosophic strongest Number of Model	
3. strongest Number amid two vertices	2. Study on Families
4. Neutrosophic strongest Number amid two vertices	
5. (Neutrosophic) strongest Paths	3. Same Models in Family

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