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Extending Sets Type-Results in Neutrosophic Graphs

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Abstract

New setting is introduced to study zero forcing number, zero forcing neutrosophic-number, 1-zero forcing number, 1-zero forcing neutrosophic-number, failed 1-zero forcing number, and failed 1-zero forcing neutrosophic-number. Some classes of neutrosophic graphs are investigated.

Keywords: Zero Forcing Number, Set, Vertex
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Chromatic number and neutrosophic chromatic number in **Ref.** [2], closing numbers and super-closing numbers as (dual)resolving and (dual)coloring alongside (dual)dominating in (neutrosophic)n-SuperHyperGraph in **Ref.** [3], co-degree and degree of classes of neutrosophic hypergraphs in **Ref.** [4], different types of neutrosophic chromatic number in **Ref.** [5], dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [6], e-matching number and e-matching polynomials in neutrosophic graphs in **Ref.** [7], independent set in neutrosophic graphs in **Ref.** [8], some polynomials related to numbers in classes of (strong) neutrosophic graphs in **Ref.** [9], three types of neutrosophic alliances based on connectedness and (strong) edges in **Ref.** [10], neutrosophic chromatic number based on connectedness in **Ref.** [11], are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [1].

2 Preliminaries

Definition 2.1. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 2.2. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_{σ} if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH_n where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **cycle** and denoted by CYC_n where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1,σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1,σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1,σ_2};

Definition 2.3. (Zero Forcing Number).

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Zero forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Zero forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

3 Setting of Neutrosophic Zero Forcing Number

Proposition 3.1.

$$\mathcal{Z}(CMT_{\sigma}) = \mathcal{O}(CMT_{\sigma}) - 1.$$

Proposition 3.2.

$$\mathcal{Z}(PTH_n) = 1.$$

Proposition 3.3.

$$\mathcal{Z}(CYC_n) = 2.$$

Proposition 3.4.

$$\mathcal{Z}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 2.$$

Proposition 3.5.

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2}) - 2.$$

Proposition 3.6.

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 1.$$

4 Setting of Zero Forcing Neutrosophic-Number

Proposition 4.1.

$$\mathcal{Z}_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \max\{\Sigma_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

Proposition 4.2.

$$\mathcal{Z}_n(PTH_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \ is \ a \ leaf}.$$

Proposition 4.3.

$$\mathcal{Z}_n(CYC_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{xy\in E.}.$$

Proposition 4.4.

$$\mathcal{Z}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\Sigma_{i=1}^{3}\sigma_i(c) + \Sigma_{i=1}^{3}\sigma_i(x)\}_{x\in V}.$$

Proposition 4.5.

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(x')\}_{x,x'\in V}.$$

Proposition 4.6.

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x\in V}.$$

5 Setting of Neutrosophic 1-Zero-Forcing Number

Definition 5.1. (1-Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-zero-forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **1-zero-forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

Definition 5.2. (Failed 1-Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Failed 1-zero-forcing number** $\mathcal{Z}'(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, Black can change any vertex from white to black. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **Failed 1-zero-forcing neutrosophic-number** $\mathcal{Z}'_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, Black can change any vertex from white to black. The last condition is as follows. For one time, black can change any vertex from white to black.

Proposition 5.3.

$$\mathcal{Z}(CMT_{\sigma}) = \mathcal{O}(CMT_{\sigma}) - 2.$$

Proposition 5.4.

$$\mathcal{Z}(PTH_n) = 1.$$

Proposition 5.5.

$$\mathcal{Z}(CYC_n) = 1.$$

Proposition 5.6.

$$\mathcal{Z}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 3.$$

Proposition 5.7.

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2}) - 3.$$

Proposition 5.8.

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2.$$

6 Setting of 1-Zero-Forcing Neutrosophic-Number

Proposition 6.1.

$$\mathcal{Z}_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{x,y\in V}.$$

Proposition 6.2.

$$\mathcal{Z}_n(PTH_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \ is \ a \ vertex}.$$

Proposition 6.3.

$$\mathcal{Z}_n(CYC_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \ is \ a \ vertex}.$$

Proposition 6.4.

$$\mathcal{Z}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\Sigma_{i=1}^{3}\sigma_i(c) + \Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{x,y\in V}.$$

Proposition 6.5.

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(x') + \Sigma_{i=1}^{3}\sigma_i(x'')\}_{x,x',x''\in V}.$$

Proposition 6.6.

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(x')\}_{x,x'\in V}.$$

7 Setting of Neutrosophic Failed 1-Zero-Forcing Number

Proposition 7.1.

$$\mathcal{Z}'(CMT_{\sigma}) = \mathcal{O}(CMT_{\sigma}) - 3.$$

Proposition 7.2.

$$\mathcal{Z}'(PTH_n) = 0.$$

Proposition 7.3.

$$\mathcal{Z}'(CYC_n) = 0.$$

Proposition 7.4.

$$\mathcal{Z}'(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 4.$$

Proposition 7.5.

$$\mathcal{Z}'(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2}) - 4.$$

Proposition 7.6.

$$\mathcal{Z}'(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 3.$$

8 Setting of Failed 1-Zero-Forcing Neutrosophic-Number

Proposition 8.1.

$$\mathcal{Z}'_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y) + \Sigma_{i=1}^{3}\sigma_i(z)\}_{x,y,z\in V}.$$

Proposition 8.2.

$$\mathcal{Z}'_n(PTH_n) = 0.$$

Proposition 8.3.

$$\mathcal{Z}'_n(CYC_n) = 0$$

Proposition 8.4.

$$\mathcal{Z}'_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \min\{\Sigma_{i=1}^{3}\sigma_i(c) + \Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y) + \Sigma_{i=1}^{3}\sigma_i(z)\}_{x,y,z\in V}.$$

Proposition 8.5.

$$\mathcal{Z}'_n(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2}) - \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(x') + \Sigma_{i=1}^{3}\sigma_i(x'') + \Sigma_{i=1}^{3}\sigma_i(x''')\}_{x,x',x'',x'''\in V}.$$

Proposition 8.6.

$$\mathcal{Z}'_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(x') + \Sigma_{i=1}^{3}\sigma_i(x'')\}_{x,x'\in V}.$$

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