Matching Number in Neutrosophic Graphs

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Abstract

New setting is introduced to study matching number and matching neutrosophic-number arising from edges. Being endpoints for two edges, simultaneously is key type of approach to have these notions namely neutrosophic matching number and matching neutrosophic-number. Two numbers are obtained but now both settings leads to approach is on demand which is finding biggest set which have any edges which have no common edge inside set. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then matching number $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common; matching neutrosophic-number $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-t-partite-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Neutrosophic Matching Number," and "Setting of Matching Neutrosophic-Number," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of an edge are used and they've same share to construct this number to compare with other edges. Summation of three values of edge makes one number and applying it to a comparison. This approach facilitates identifying edges which form matching number and matching neutrosophic-number arising from being specific edge. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. Using basic set to extend this set to set of all edges has key role to have these notions in the form of matching number and matching neutrosophic-number arising from edges. The cardinality of a set has eligibility to neutrosophic matching number but the neutrosophic cardinality of a set has eligibility to call matching neutrosophic-number. Some results get more frameworks and perspective about these definitions. The way in that, two edges have no connection with each other, opens the way to do some approaches. An edge could affect on other edge but there's no usage of vertices. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Neutrosophic Matching Number, Matching Neutrosophic-Number, Maximal Set of Edges

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [16], neutrosophic set in **Ref.** [3], related definitions of other sets in **Refs.** [3, 13, 15], graphs and new notions on them in **Refs.** [1, 4, 8–11, 14, 17], neutrosophic graphs in **Ref.** [5], studies on neutrosophic graphs in **Ref.** [2], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [12], related definitions of other graphs based on neutrosophic graphs in **Ref.** [6], are proposed. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [7].

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "Neutrosophic Matching Number", "Matching Neutrosophic-Number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Lack of connection amid two edges have key roles to assign neutrosophic matching number and matching neutrosophic-number arising from edges. Thus they're used to define new ideas which conclude to the structure neutrosophic matching number and matching neutrosophic-number arising from edges. The concept of having common endpoints of an edge inspires us to study the behavior of vertices in the way that, some types of numbers, neutrosophic matching number, matching neutrosophic-number arising from arising from edges are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of neutrosophic matching number and matching neutrosophic-number are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sets of edges have the key role in this way. General results are obtained and also, the results about the basic notions of neutrosophic matching number, matching neutrosophic-number are elicited. Some classes of neutrosophic graphs are studied in the terms of neutrosophic matching number, in section "Setting of Neutrosophic Matching Number," as individuals. In section "Setting of Matching Neutrosophic-Number," matching neutrosophic-number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-t-partite-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Neutrosophic Matching Number," and "Setting of Matching Neutrosophic-Number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-t-neutrosophic graphs and complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Definition 1.5. (Matching Number).

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common;

(ii) **matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.6. In Figure (1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

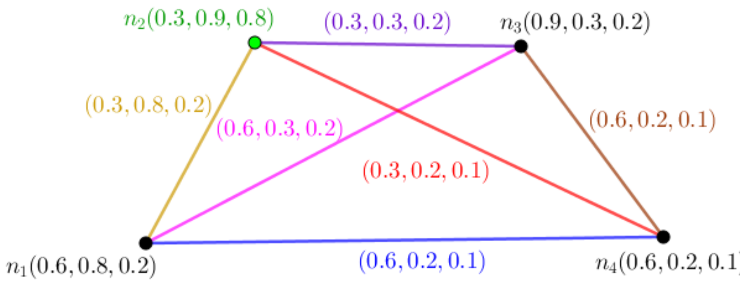

Figure 1. A Neutrosophic Graph in the Viewpoint of its Matching Number and its Matching Neutrosophic-Number.

(*iv*) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$;

(*vi*) 2.2 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$.

2 Setting of Neutrosophic Matching Number

In this section, I provide some results in the setting of neutrosophic matching number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, and star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

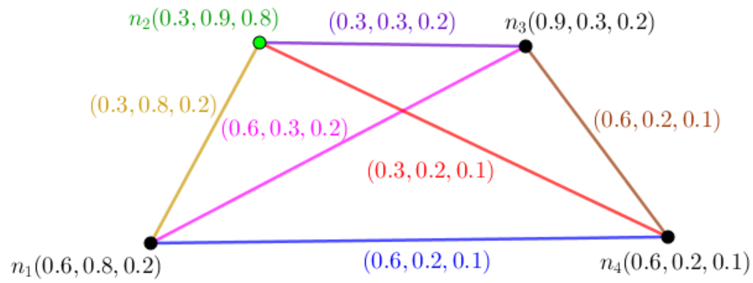

Figure 2. A Neutrosophic Graph in the Viewpoint of its matching Number.

Example 2.2. In Figure (2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$;

(vi) 2.2 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Every leaf with its neighbor identify an edge. Let $x_0, x_1, \cdots, x_t$ be a path-neutrosophic graph. $S = \{x_0x_1, x_2x_3, \cdots, x_{t-1}x_t\}_{t=\lfloor \frac{n}{2} \rfloor}$ has maximum cardinality of set of edges which have no common edge. So

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

□

Example 2.4. There are two sections for clarifications.

(a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are no edge in S. Cardinality of S implies that $S = \{n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 1.5, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(vi) 1.5 is matching neutrosophic-number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(b) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from

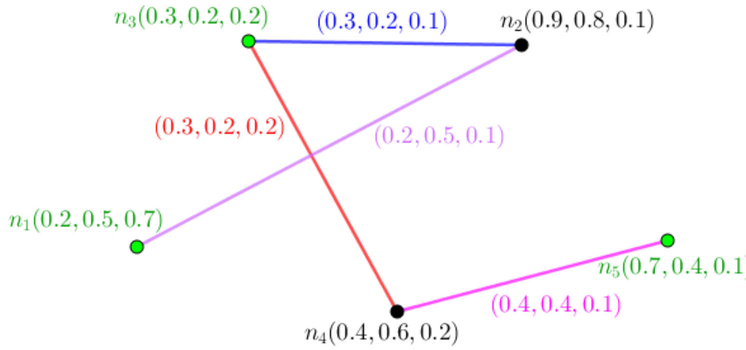

Figure 3. A Neutrosophic Graph in the Viewpoint of its Matching Number.

S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are no edge in S. Cardinality of S implies that $S = \{n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there's no edge in S but n_1n_2, n_3n_4 and n_5n_6. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 3.7, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 3 is matching number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$;

(vi) 3.7 is matching neutrosophic-number and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$.

Proposition 2.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_ix_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

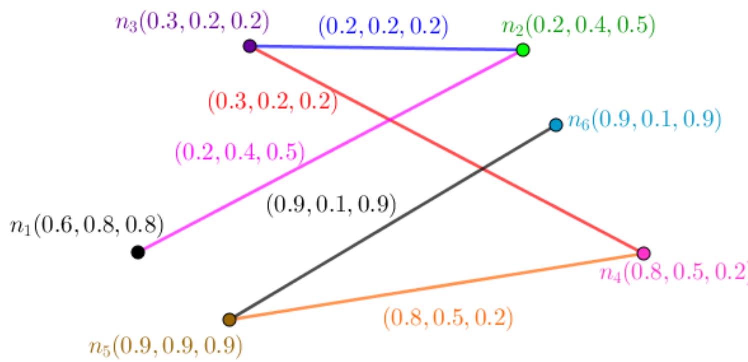

Figure 4. A Neutrosophic Graph in the Viewpoint of its Matching Number.

In S, there aren't two edges which have common endpoints. S is matching set and it has maximum cardinality amid such these sets which are matching set which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6. There are two sections for clarifications.

(a) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there are three edges in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges in S. Cardinality and structure of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.5, of S implies $S=\{n_1n_2,n_3n_4,n_5n_6\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S=\{n_2n_3,n_4n_5,n_6n_1\}$ is a set of edges, then there are three edges in S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S=\{n_2n_3,n_4n_5,n_6n_1\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.7, of S implies $S=\{n_1n_2,n_3n_4\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 3 is matching number and its corresponded sets are $\{n_1n_2,n_3n_4,n_5n_6\}$, and $\{n_2n_3,n_4n_5,n_6n_1\}$;

(vi) 2.5 is matching neutrosophic-number and its corresponded set is $\{n_1n_2,n_3n_4,n_5n_6\}$.

(b) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S=\{n_1n_3,n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S=\{n_1n_3,n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S=\{n_2n_3,n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S=\{n_2n_3,n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S=\{n_2n_3,n_4n_5\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S=\{n_2n_3,n_4n_5\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.8, of S implies $S=\{n_2n_3,n_4n_5\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S=\{n_1n_2,n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S=\{n_1n_2,n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 3.1, of S implies $S=\{n_1n_2,n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2,n_3n_4\}$, and $\{n_2n_3,n_4n_5\}$;

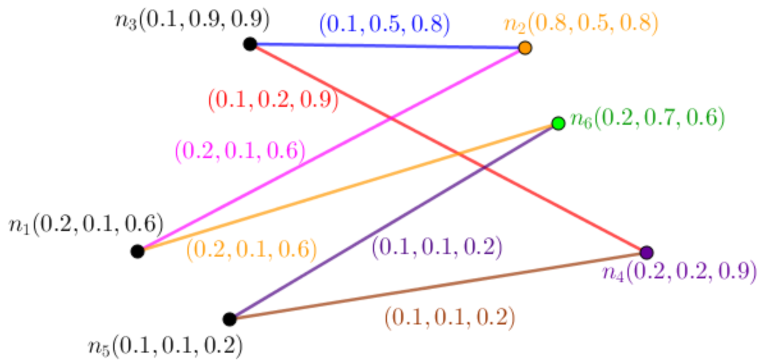

Figure 5. A Neutrosophic Graph in the Viewpoint of its Matching Number.

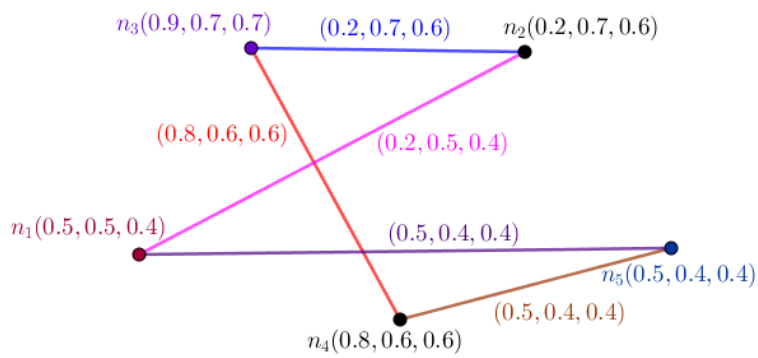

Figure 6. A Neutrosophic Graph in the Viewpoint of its Matching Number.

(vi) 2.8 is matching neutrosophic-number and its corresponded set is $\{n_2n_3, n_4n_5\}$.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{M}(NTG) = 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. Hence one edge including center and one other vertex is only member of S is a set which its cardinality is matching number $\mathcal{M}(NTG)$. In other words, if $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. Thus there is no triangle but there are two edges. One edge has two endpoints which one of them is center. Every edge is corresponded to matching number $\mathcal{M}(NTG)$. So

$$\mathcal{M}(NTG) = 1.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

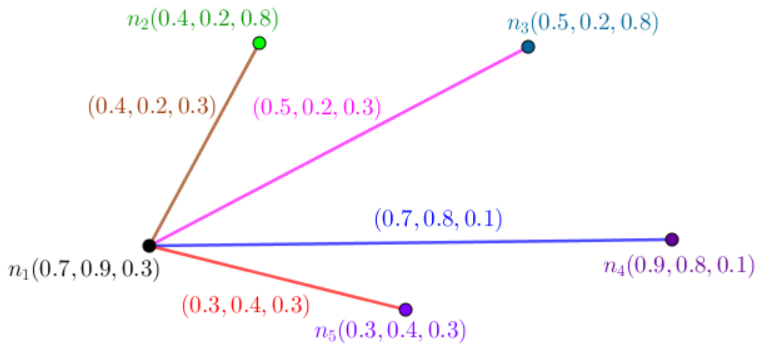

Figure 7. A Neutrosophic Graph in the Viewpoint of its Matching Number.

Example 2.8. There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies that $S = \{n_1n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_5\}$ is a set of edges, then there's no edge in S but n_1n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_5\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_5\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of S implies that $S = \{n_1n_5\}$ is corresponded matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 1 is matching number and its corresponded sets are $\{n_1n_2\}$, $\{n_1n_3\}$, $\{n_1n_4\}$, and $\{n_1n_5\}$;

(vi) 1.6 is matching neutrosophic-number and its corresponded set is $\{n_1n_4\}$.

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \min\{|V_1|, |V_2|\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence from any part, $\min\{|V_1|, |V_2|\}$ vertices are chosen to be only members of S is a set which its cardinality is matching number $\mathcal{M}(NTG)$. There are two parts but same numbers from them are chosen. Thus

$$\mathcal{M}(NTG) = \min\{|V_1|, |V_2|\}.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $S = \{n_1n_3, n_4n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_4n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3, n_4n_2\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $S = \{n_1n_3, n_1n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is possible. So by using the members of S, it's possible to have at least one endpoint for two edges. There are two edges from S. Structure of S implies that $S = \{n_1n_3, n_1n_2\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $S = \{n_1n_2, n_4n_3\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_4n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) 2 is matching number and its corresponded sets are $\{n_1n_2, n_4n_3\}$ and $\{n_1n_3, n_4n_2\}$;

(*vi*) 1.8 is matching neutrosophic-number and its corresponded set is $\{n_1n_3, n_4n_2\}$.

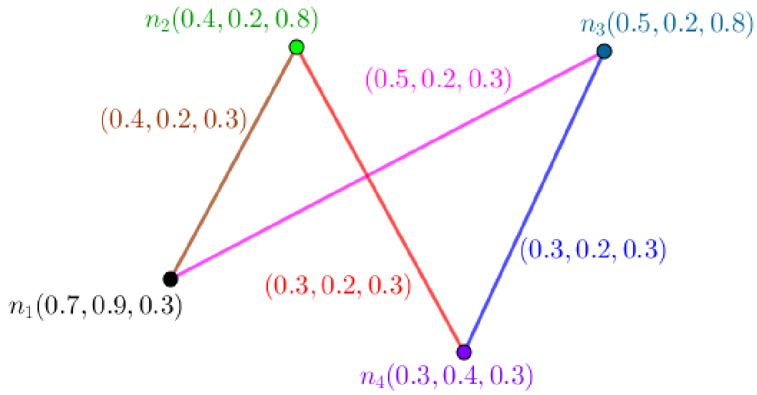

Figure 8. A Neutrosophic Graph in the Viewpoint of its Matching Number.

Proposition 2.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph such that $t \neq 2$. Then*

$$\mathcal{M}(NTG) = \min |V_i|_{i=1}^{t}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence from any part, $\min |V_i|_{i=1}^{t}$ vertices are chosen to be only members of S is a set which its cardinality is matching number $\mathcal{M}(NTG)$. There are t parts but same numbers from them are chosen. Thus

$$\mathcal{M}(NTG) = \min |V_i|_{i=1}^{t}.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.12. There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $S = \{n_1n_3, n_4n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_4n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3, n_4n_2\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

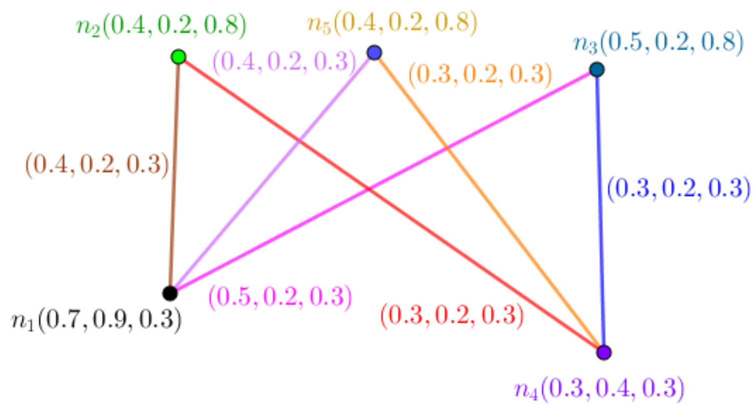

Figure 9. A Neutrosophic Graph in the Viewpoint of its Matching Number.

(iii) if $S = \{n_1n_3, n_1n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is possible. So by using the members of S, it's possible to have at least one endpoint for two edges. There are two edges from S. Structure of S implies that $S = \{n_1n_3, n_1n_2\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_4n_3\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_4n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_4n_3\}$ and $\{n_1n_3, n_4n_2\}$;

(vi) 1.8 is matching neutrosophic-number and its corresponded set is $\{n_1n_3, n_4n_2\}$.

3 Setting of Matching Neutrosophic-Number

In this section, I provide some results in the setting of matching neutrosophic-number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, and star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_1x_2) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{j=\lfloor \frac{n}{2} \rfloor}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_1x_2) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{j=\lfloor \frac{n}{2} \rfloor}.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.2. In Figure (10), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$;

(vi) 2.2 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

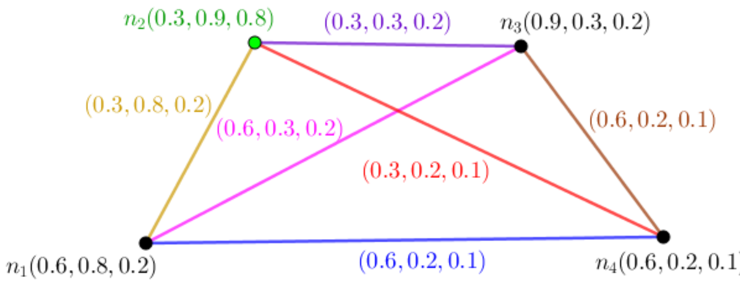

Figure 10. A Neutrosophic Graph in the Viewpoint of its matching Number.

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{|S|=\lfloor \frac{n}{2} \rfloor}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Every leaf with its neighbor identify an edge. Let $x_0, x_1, \cdots, x_t$ be a path-neutrosophic graph. $S = \{x_0x_1, x_2x_3, \cdots, x_{t-1}x_t\}_{|S|=\lfloor \frac{n}{2} \rfloor}$ has maximum cardinality of set of edges which have no common edge. So

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{|S|=\lfloor \frac{n}{2} \rfloor}.$$

□

Example 3.4. There are two sections for clarifications.

(a) In Figure (11), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are no edge in S. Cardinality of S implies that $S = \{n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 1.5, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(vi) 1.5 is matching neutrosophic-number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(b) In Figure (12), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are no edge in S. Cardinality of S implies that $S = \{n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there's no edge in S but n_1n_2, n_3n_4 and n_5n_6. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 3.7, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 3 is matching number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$;

(vi) 3.7 is matching neutrosophic-number and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$.

Proposition 3.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{|S|=\lfloor \frac{n}{2} \rfloor}.$$

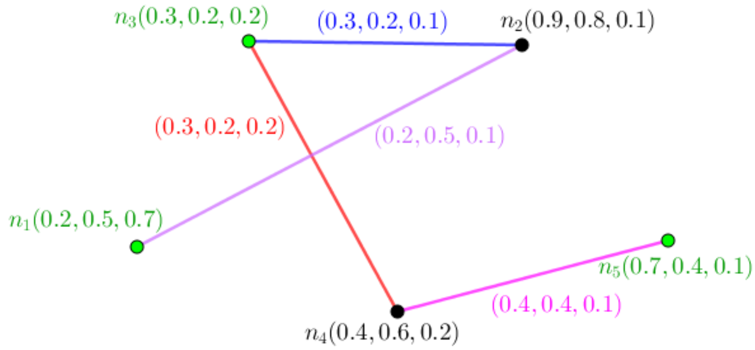

Figure 11. A Neutrosophic Graph in the Viewpoint of its Matching Number.

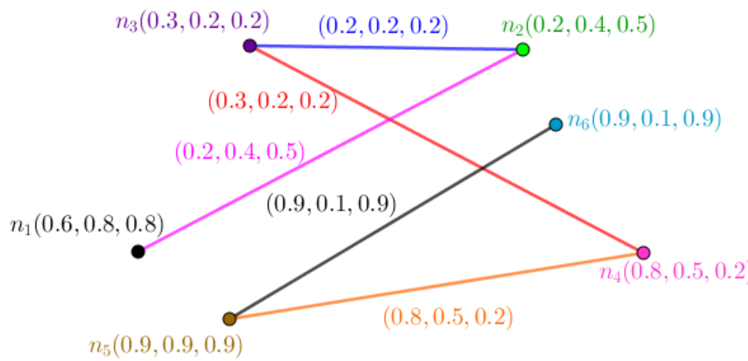

Figure 12. A Neutrosophic Graph in the Viewpoint of its Matching Number.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1 x_2, x_3 x_4, \cdots, x_i x_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is matching set and it has maximum cardinality amid such these sets which are matching set which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0 x_1) + \sum_{i=1}^{3} \mu_i(x_2 x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1} x_j)\}_{|S| = \lfloor \frac{n}{2} \rfloor}.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There are two sections for clarifications.

(a) In Figure (13), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there are three edges in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges in S. Cardinality and structure of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.5, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges in S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.7, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 3 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$, and $\{n_2n_3, n_4n_5, n_6n_1\}$;

(vi) 2.5 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$.

(b) In Figure (14), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

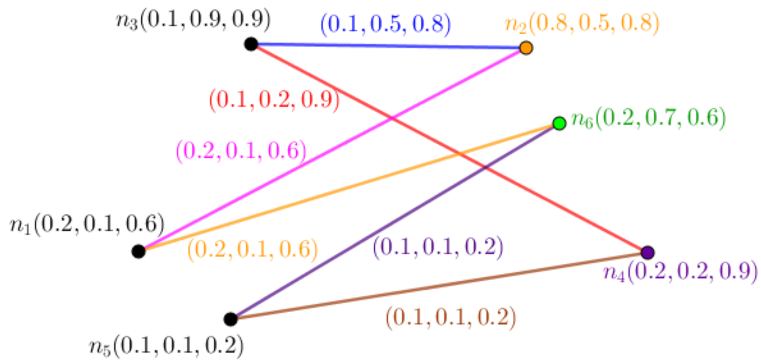

Figure 13. A Neutrosophic Graph in the Viewpoint of its Matching Number.

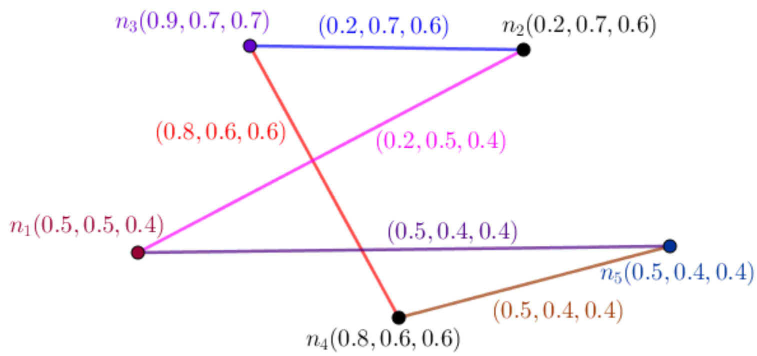

Figure 14. A Neutrosophic Graph in the Viewpoint of its Matching Number.

(iii) if $S = \{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.8, of S implies $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 3.1, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(vi) 2.8 is matching neutrosophic-number and its corresponded set is $\{n_2n_3, n_4n_5\}$.

Proposition 3.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c.*

Then

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{x_{j-1}x_j \in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. Hence one edge including center and one other vertex is only member of S is a set which its cardinality is matching number $\mathcal{M}(NTG)$. In other words, if $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. Thus there is no triangle but there are two edges. One edge has two endpoints which one of them is center. Every edge is corresponded to matching number $\mathcal{M}(NTG)$. So

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{x_{j-1}x_j \in E}.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (15), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies that $S = \{n_1n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_5\}$ is a set of edges, then there's no edge in S but n_1n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_5\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_5\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have

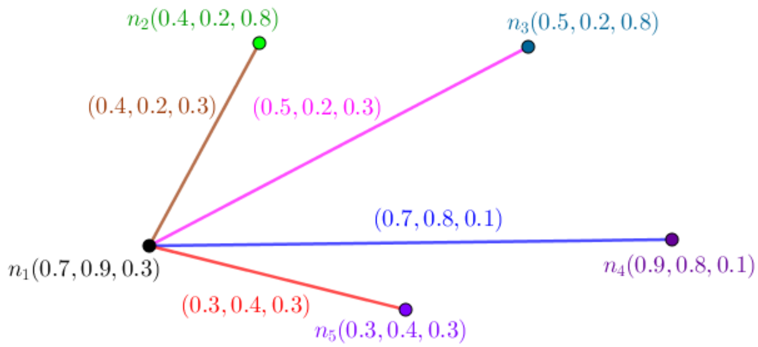

Figure 15. A Neutrosophic Graph in the Viewpoint of its Matching Number.

at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of S implies that $S = \{n_1n_5\}$ is corresponded matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 1 is matching number and its corresponded sets are $\{n_1n_2\}$, $\{n_1n_3\}$, $\{n_1n_4\}$, and $\{n_1n_5\}$;

(vi) 1.6 is matching neutrosophic-number and its corresponded set is $\{n_1n_4\}$.

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1) + \sum_{i=1}^{3}\mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{|S|=\min\{|V_1|,|V_2|\}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence from any part, $\min\{|V_1|, |V_2|\}$ vertices are chosen to be only members of S is a set which its cardinality is matching number $\mathcal{M}(NTG)$. There are two parts but same numbers from them are chosen. Thus

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1) + \sum_{i=1}^{3}\mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{|S|=\min\{|V_1|,|V_2|\}}.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (16), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

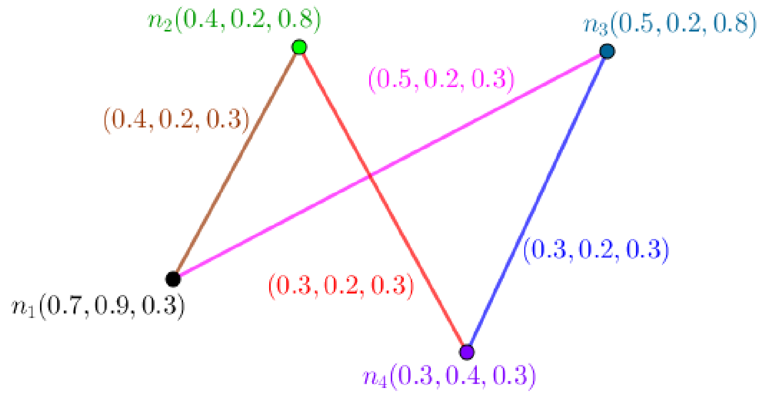

Figure 16. A Neutrosophic Graph in the Viewpoint of its Matching Number.

(*ii*) if $S = \{n_1n_3, n_4n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_4n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3, n_4n_2\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $S = \{n_1n_3, n_1n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is possible. So by using the members of S, it's possible to have at least one endpoint for two edges. There are two edges from S. Structure of S implies that $S = \{n_1n_3, n_1n_2\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $S = \{n_1n_2, n_4n_3\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_4n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) 2 is matching number and its corresponded sets are $\{n_1n_2, n_4n_3\}$ and $\{n_1n_3, n_4n_2\}$;

(*vi*) 1.8 is matching neutrosophic-number and its corresponded set is $\{n_1n_3, n_4n_2\}$.

Proposition 3.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph such that $t \neq 2$. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{|S|=\min |V_i|_{i=1}^{t}\}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence from any part, min $|V_i|_{i=1}^{t}$

vertices are chosen to be only members of S is a set which its cardinality is matching number $\mathcal{M}(NTG)$. There are t parts but same numbers from them are chosen. Thus

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1) + \sum_{i=1}^{3}\mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{|S|=\min|V_i|_{i=1}^{t}}\}.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (17), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- *(i)* If $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;
- *(ii)* if $S = \{n_1n_3, n_4n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_4n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3, n_4n_2\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;
- *(iii)* if $S = \{n_1n_3, n_1n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is possible. So by using the members of S, it's possible to have at least one endpoint for two edges. There are two edges from S. Structure of S implies that $S = \{n_1n_3, n_1n_2\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;
- *(iv)* if $S = \{n_1n_2, n_4n_3\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_4n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;
- *(v)* 2 is matching number and its corresponded sets are $\{n_1n_2, n_4n_3\}$ and $\{n_1n_3, n_4n_2\}$;
- *(vi)* 1.8 is matching neutrosophic-number and its corresponded set is $\{n_1n_3, n_4n_2\}$.

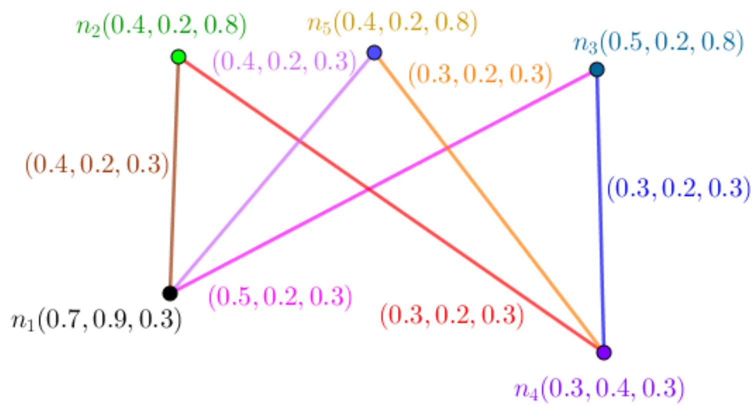

Figure 17. A Neutrosophic Graph in the Viewpoint of its Matching Number.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

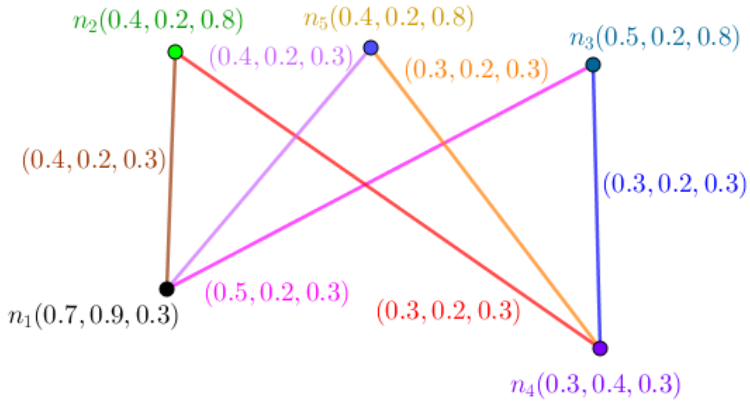

Figure 18. A Neutrosophic Graph in the Viewpoint of its Matching Number and its Matching Neutrosophic-Number

4.1 Case 1: Complete-t-partite Model alongside its Matching Number and its Matching Neutrosophic-Number

Step 4. (Solution) The neutrosophic graph alongside its matching number and its matching neutrosophic-number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of matching number and its matching neutrosophic-number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (18). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called matching number and matching neutrosophic-number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (18). In Figure (18), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3\}$ is a set of edges, then there's no edge in S but n_1n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_1n_3, n_4n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_4n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_3, n_4n_2\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality of

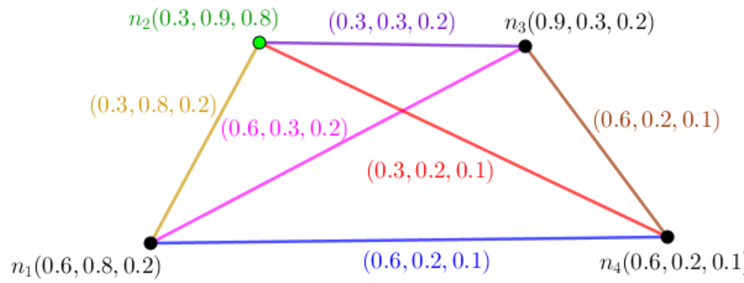

Figure 19. A Neutrosophic Graph in the Viewpoint of its Matching number and its Matching Neutrosophic-Number

S implies that $S = \{n_1n_3\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_3, n_1n_2\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_1n_2. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is possible. So by using the members of S, it's possible to have at least one endpoint for two edges. There are two edges from S. Structure of S implies that $S = \{n_1n_3, n_1n_2\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_4n_3\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_4n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality of S implies that $S = \{n_1n_2, n_4n_3\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_4n_3\}$ and $\{n_1n_3, n_4n_2\}$;

(vi) 1.8 is matching neutrosophic-number and its corresponded set is $\{n_1n_3, n_4n_2\}$.

4.2 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its Matching number and its Matching Neutrosophic-Number

Step 4. (Solution) The neutrosophic graph alongside its matching number and its matching neutrosophic-number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of matching number and matching neutrosophic-number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (19). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called matching number and matching neutrosophic-number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (19). There is one section for clarifications.

(*i*) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$;

(*vi*) 2.2 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning matching number and matching neutrosophic-number are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 5.1. *Is it possible to use other types of matching number and matching neutrosophic-number?*

Question 5.2. *Are existed some connections amid different types of matching number and matching neutrosophic-number in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning matching number and matching neutrosophic-number arising from edges to study neutrosophic graphs. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid number and edges are done by using neutrosophic tool. The connections of vertices which aren't clarified by one edge differ them from each other and put them in different categories to represent a number which is called matching number and matching

Table 2. A Brief Overview about Advantages and Limitations of this Study

Advantages	Limitations
1. Neutrosophic Matching Number	1. Wheel-Neutrosophic Graphs
2. Matching Neutrosophic-Number	
3. Neutrosophic Number	2. Study on Families
4. Study on Classes of Neutrosophic Graphs	
5. Using Edges	3. Same Models in Family

neutrosophic-number. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

References

1. N.H. Abughazalah, and M. Khan, "*Selection of Optimum Nonlinear Confusion Component of Information Confidentiality Mechanism Using Grey Theory Based Decision-Making Technique.*" Preprints 2021, 2021120310 (doi: 10.20944/preprints202112.0310.v1).
2. M. Akram, and G. Shahzadi, "*Operations on Single-Valued Neutrosophic Graphs*", Journal of uncertain systems 11 (1) (2017) 1-26.
3. K. Atanassov, "*Intuitionistic fuzzy sets*", Fuzzy Sets Syst. 20 (1986) 87-96.
4. S. Boumova, P. Boyvalenkov, and M. Stoyanova, "*Bounds for the Minimum Distance and Covering Radius of Orthogonal Arrays via Their Distance Distributions.*" Preprints 2021, 2021120293 (doi: 10.20944/preprints202112.0293.v1).
5. S. Broumi, M. Talea, A. Bakali and F. Smarandache, "*Single-valued neutrosophic graphs*", Journal of New Theory 10 (2016) 86-101.
6. N. Shah, and A. Hussain, "*Neutrosophic soft graphs*", Neutrosophic Set and Systems 11 (2016) 31-44.
7. Henry Garrett, (2022). "*Beyond Neutrosophic Graphs*", Ohio: E-publishing: Educational Publisher 1091 West 1st Ave Grandview Heights, Ohio 43212 United States. ISBN: 978-1-59973-725-6 (http://fs.unm.edu/BeyondNeutrosophicGraphs.pdf).
8. Henry Garrett, "*Co-degree and Degree of classes of Neutrosophic Hypergraphs*", Preprints 2022, 2022010027 (doi: 10.20944/preprints202201.0027.v1).
9. D. Jean, and S. Fault-Tolerant Seo, "*Detection Systems on the King's Grid.*" Preprints 2022, 2022010249 (doi: 10.20944/preprints202201.0249.v1).
10. H. Oliveira, "*Synthesis of Strategic Games With Multiple Pre-set Nash Equilibria - An Artificial Inference Approach Using Fuzzy ASA.*" Preprints 2019, 2019080128 (doi: 10.20944/preprints201908.0128.v2).
11. R.E. Perez, F.N. Gonzalez, M.C. Molina, and J.M. Moreno "*Methodological Analysis of Damage Estimation in Hydraulic Infrastructures.*" Preprints 2022, 2022010376 (doi: 10.20944/preprints202201.0376.v1).
12. A. Shannon and K.T. Atanassov, "*A first step to a theory of the intuitionistic fuzzy graphs*", Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61.
13. F. Smarandache, "*A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic, Rehoboth:* " American Research Press (1998).
14. F. Tang, X. Zhu, J. Hu, J. Tie, J. Zhou, and Y. Fu, "*Generative Adversarial Unsupervised Image Restoration in Hybrid Degradation Scenes.*" Preprints 2022, 2022020159 (doi: 10.20944/preprints202202.0159.v1).
15. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "*Single-valued neutrosophic sets*", Multispace and Multistructure 4 (2010) 410-413.
16. L. A. Zadeh, "*Fuzzy sets*", Information and Control 8 (1965) 338-354.
17. S. Zuev, Z. Hussain, and P. Kabalyants,"*Nanoparticle Based Water Treatment Model in Squeezing Channel under Magnetic Field.*" Preprints 2021, 2021120295 (doi: 10.20944/preprints202112.0295.v1).