A note on neutrosophic soft set with rough set

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Abstract: Rough set is a very powerful invention to the whole world for dealing with uncertain, incomplete and imprecise problems. Also soft set theory and neutrosophic set theory both are advance mathematical tools to handle these uncertain, incomplete, inconsistent information in a better way. The purpose of this article is to expand the scope of rough set, soft set and neutrosophic set theory. We have introduced the concept of neutrosophic soft set with roughness without using full soft set. Some definition, properties and examples have been established on neutrosophic soft rough set. Moreover, dispensable and equalities are written on roughness with neutrosophic soft set.

Keywords: Rough set, Soft set, Neutrosophic set and Neutrosophic soft lower approximation and Neutrosophic upper approximation operators, Neutrosophic soft rough set.

1 Introduction

The rough set theory was introduced by Z. Pawlak[13] in 1982, which is enlightened to the researcher of Artificial Intelligence, Mathematics and Computer Science. Here an inexact set "S" is categorized by two exact sets that is the lower approximation and upper approximation of the set "S" through an equivalence relation.

Rough set is based on the knowledge about ones ability to discern the objects, data, phenomenon etc. In 1983, W. Zakowski[17] defines the rough set using the covering instead of equivalence relation (or partition) where the lower approximation of the set "S' is the interior of "S" and upper approximation of "S" is the closure of "S". Later T.Y. Lin([9], 1988) defines rough set through neighborhood operators(a new covering). Many researcher found different approximation operators based on the covering and 1-neighborhood operators.

In 1999, F. Smarandache[16] introduced the concept of neutrospohic set(NS). Neutrosophic set is described by three functions: a membership function, indeterminacy function and a non-membership function that are independently related, where the membership, indeterminacy and non-membership functional values belong to $]^{-}0, 1^{+}[$.

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In the same year, that is in 1999, D. Molodtsov[12] proposed the soft set theory for dealing with uncertainties. The soft set is defined through a parameter set A and a mapping F is defined from A to power set of the universal set. Soft set theory has a potential for application in several direction. Maji, Biswas, Roy[7] in 2002 narrates the operations AND, OR, Compliment and other set theoretic operations which attracts the researchers to study more on soft set theory. P.K. Maji [8] in 2010 produced the concept "neutrosophic soft set". Feng feng et al[4] in 2009 published a paper on soft rough set and rough soft set. K. Bhutani and S. Aggarwal([2],2017) defined neutrosophic soft rough sets and also in 2018 A. Al-Quran, N. Hassan and E. Marei[1] defined the same.

In this note, we define roughness on neutrosophic soft set in a new manner which is different from the above two and we believe it is the best approach to handle the uncertainty, vague and imprecise data.

2 Rough set

Here we present the definition and some properties of rough set introduced by Z. Pawlak. Let U be the universal set and M be an equivalence relation(a knowledge) on U called an indiscernibility relation. U/M be the family of all equivalence classes of M known as categories or concepts of M and for $x \in U$, $[x]_M$ is an equivalence class of x. The relational system $K = (U, M)$ is called a approximation space. The M-lower and M-upper approximations of a set $X \subseteq U$ under the indiscernibilty relation M are defined as

$$\begin{aligned} \underline{M}X &= \{x \in U : [x]_M \subseteq X\} \ \ and \\ \overline{M}X &= \{x \in U : [x]_M \cap X \neq \phi\}, \ respectively. \end{aligned}$$

Definition 2.1. [13] Let U be universal set and M be an equivalence relation(a knowledge) on U. For the set $X \subseteq U$, X is rough with respect to knowledge M if and only if $\underline{M}X \neq \overline{M}X$. Otherwise X is called definable (or exact) set with respect to M. Also the M-positive, M-negative and M-boundary region of X are defined as $POS_M(X) = \underline{M}X$, $NEG_M(X) = U - \overline{M}X$ and $BN_M(X) = \overline{M}X - \underline{M}X$, respectively.

Example 2.2.
Let $U = \{h_1, h_2, \cdots, h_8\}$ be the universe of discourse, there are eight pebbles of different colors. Let M be the knowledge(an equivalence relation) on U, so we get a partition of U as

$$U/M = \{\{h_1, h_5\}, \{h_2, h_4, h_8\}, \{h_3\}, \{h_6, h_7\}\},$$

that is $\{h_1, h_5\}$ are the pebbles of blue color, $\{h_2, h_4, h_8\}$ are of red color, $\{h_3\}$ is of green color and $\{h_6, h_7\}$ are of yellow color.

Let $X = \{h_4, h_6, h_7\} \subset U$. The lower and upper approximation of X is

$$\begin{aligned} \underline{M}X &= \{h_6, h_7\} \\ \overline{M}X &= \{h_2, h_4, h_6, h_7, h_8\} \end{aligned}$$

Hence, $\overline{M}X \neq \underline{M}X$. So, the set X is rough with respect to knowledge M.

Proposition 2.3. *[14] Suppose that (U, M) is an approximation space and $X, Y \subseteq U$. Then*

$$\begin{aligned}
&(1) \quad \underline{M}X \subseteq X \subseteq \overline{M}X \\
&(2) \quad \underline{M}\phi = \overline{M}\phi = \phi,\ \underline{M}U = \overline{M}U = U \\
&(3) \quad \overline{M}(X \cup Y) = \overline{M}X \cup \overline{M}Y \\
&(4) \quad \underline{M}(X \cap Y) = \underline{M}X \cap \underline{M}Y \\
&(5) \quad X \subseteq Y \Rightarrow \underline{M}X \subseteq \underline{M}Y \\
&(6) \quad X \subseteq Y \Rightarrow \overline{M}X \subseteq \overline{M}Y \\
&(7) \quad \underline{M}(X \cup Y) \supseteq \underline{M}X \cup \underline{M}Y \\
&(8) \quad \overline{M}(X \cap Y) \subseteq \overline{M}X \cap \overline{M}Y \\
&(9) \quad \underline{M}(-X) = -\overline{M}X \\
&(10) \quad \overline{M}(-X) = -\underline{M}X
\end{aligned}$$

3 Soft Set

The concept of soft set was introduced by Molodtsov[12] in 1999. Here we discuss about the soft set theory with some properties.

Definition 3.1. ([7, 10, 12]) Let U be an initial universe, E be the set of parameters related to U. Let $P(U)$ denotes the power set of U, $A \subseteq E$ and F be a mapping given by $F : A \to P(U)$, then the pair (F, A) is called soft set over U. In other words the soft set is characterized by a parameter set and a mapping on parameters. For every $e \in A$, $F(e)$ is said to be e-approximate elements of U and soft set can be viewed as a parameterized family of subsets of U.

A soft set (F, A) is called full soft set if $\cup_{e \in A} F(e) = U$.

Definition 3.2. ([4, 7]) For $A, B \subseteq E$, two soft sets (F, A) and (G, B) over a common universe U, we find the set theoretic operations on soft set as

(1) **Soft Subset**: The soft set (F, A) is soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if $A \subseteq B$ and $F(x) \subseteq G(x)$ for all $x \in A$. At that time (G, B) is said to be a soft super set of (F, A).

(2) **Soft Equality**:

Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal, denoted by $(F, A) = (G, B)$, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) that is $(F, A) \subseteq (G, B)$ and $(G, B) \supseteq (F, A)$.

(3) **Soft Union**:

The union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), \text{if } e \in A - B \\ G(e), \text{if } e \in B - A \\ F(e) \cup G(e), \text{if } e \in A \cap B \end{cases}$$

We denote $(H, C) = (F, A) \cup (G, B)$.

(4) **Soft Intersection**: The intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. We denote $(F, A) \cap (G, B) = (H, C)$.

(5) **NOT set of a set of parameters**:

Let $E = \{e_1, e_2, \cdots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E$ and is defined by $\rceil E = \{\rceil e_1, \rceil e_2, \cdots, \rceil e_n\}$, where $\rceil e_i = not\ e_i$, for all i, $1 \leq i \leq n$.

(6) **Soft Complement**:

The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to P(U)$ is a mapping given by $F^c(e) = U - F(\rceil e)$, for all $e \in \rceil A$. Clearly, complement of $(F^c, \rceil A)$ is (F, A), that is $((F, A)^c)^c = (F, A)$. But in general the complement of a soft set (F, A) that is, $(F^c, \rceil A)$ is not a soft set, since $\rceil e_i \neq E$.

Example 3.3.
Let $U = \{x_1, x_2, \cdots, x_7\}$ be the set of houses under consideration, $E = \{a_1, a_2, a_3, a_4, a_5\}$ be set of parameters on U that is a_1 stands for expensive, a_2 stands for beautiful, a_3 stands for wooden, a_4 stands for cheap and a_5 stands for green surrounding. Let a mapping $F : E \to P(U)$ be given by that is an expert is giving his views as $F(a_1) = \{x_5, x_6\}$, $F(a_2) = \phi$, $F(a_3) = \{x_4\}$, $F(a_4) = \{x_3, x_7\}$, $F(a_5) = \{x_1, x_6\}$ and $G : E \to P(U)$ be a mapping(that is another expert giving his views) given by $G(a_1) = \{x_4, x_5, x_6\}$, $G(a_2) = \phi$, $G(a_3) = \{x_2, x_4\}$, $G(a_4) = \{x_6, x_7\}$, $G(a_5) = \{x_1, x_6\}$. Let $A = \{a_1, a_4\} \subseteq E$, $B = \{a_1, a_4, a_5\} \subseteq E$ then the soft set

$$\begin{aligned}(F,A) &= \{(a_1,G(a_1)),(a_4,G(a_4))\}\\ &= \{(a_1,\{x_5,x_6\}),(a_4,\{x_3,x_7\})\}\\ (G,B) &= \{(a_1,G(a_1)),(a_4,G(a_4)),(a_5,G(a_5))\}\\ &= \{(a_1,\{x_4,x_5,x_6\}),(a_4,\{x_3,x_7\}),(a_5,\{x_1,x_6\})\}\end{aligned}$$

Hence, $(F,A)\subseteq(G,B)$. Since $A\subseteq B$ and $F(a)\subseteq G(a)$, for all $a\in A$.

Soft Complement $(G,B)^c = (G^c,\rceil B)$={(not expensive houses, $\{x_1,x_2,x_3,x_7\}$), (not cheap houses, $\{x_1,x_2,x_3,x_4,x_5\}$), (not in the green surrounding, $\{x_2,x_3,x_4,x_5,x_7\}$)}
$=\{(\rceil a_1,\{x_1,x_2,x_3,x_7\}),(\rceil a_4,\{x_1,x_2,x_3,x_4,x_5\}),(\rceil a_5,\{x_2,x_3,x_4,x_5,x_7\})\}$

Example 3.4.
Let $U=\{x_1,x_2,\cdots,x_8\}$ be the initial universe, $E=\{a_1,a_2,\cdots,a_5\}$ be set of parameters with respect to U. Let $F:E\to P(U)$ be a mapping given by $F(a_1)=\{x_2,x_5,x_6,x_7\}$, $F(a_2)=\{x_1,x_2,x_6\}$, $F(a_3)=\{x_6,x_7\}$, $F(a_4)=\{x_2,x_5,x_7\}$, $F(a_5)=\{x_1,x_7\}$, and $G:E\to P(U)$ be a mapping given by $G(a_1)=\{x_2,x_8\}$, $G(a_2)=\{x_2,x_6,x_8\}$, $G(a_3)=\{x_2,x_5,x_7,x_8\}$, $G(a_4)=\{x_1,x_6,x_8\}$, $G(a_5)=\{x_2,x_3,x_7\}$. Let $A=\{a_1,a_2,a_5\}\subseteq E$, $B=\{a_2,a_4,a_5\}\subseteq E$ then the soft set are

$$\begin{aligned}(F,A) &= \{(a_1,F(a_1)),(a_2,F(a_2)),(a_5,F(a_5))\}\\ &= \{(a_1,\{x_2,x_5,x_6,x_7\}),(a_2,\{x_1,x_2,x_6\}),(a_5,\{x_1,x_7\})\}\\ (G,B) &= \{(a_2,G(a_2)),(a_4,G(a_4)),(a_5,G(a_5))\}\\ &= \{(a_2,\{x_2,x_6,x_8\}),(e_4,\{x_1,x_6,x_8\}),(a_5,\{x_2,x_3,x_7\})\}\end{aligned}$$

The union of two soft set (F,A) and (G,B) is denoted by $(F,A)\cup(G,B)$, is defined by the soft set (H,C), where $C=A\cup B$ and for all $e\in C$, that is

$$\begin{aligned}(F,A)\cup(G,B) &= (H,C)=(H,\{a_1,a_2,a_4,a_5\})\\ &= \{(a_1,H(a_1)),(a_2,H(a_2)),(a_4,H(a_4)),(a_5,H(a_5))\}\\ &= \{(a_1,\{x_2,x_5,x_6,x_7\}),(a_2,\{x_1,x_2,x_6,x_8\}),(a_4,\{x_1,x_7,x_8\}),\\ &\quad (a_5,\{x_1,x_2,x_3,x_7\})\}\end{aligned}$$

The intersection of two soft set (F,A) and (G,B) is denoted by $(F,A)\cap(G,B)$, is defined by the soft set (H,C), where $C=A\cap B$ and for all $e\in C$, that is

$$\begin{aligned}(F,A)\cap(G,B) &= (H,C)=(H,\{a_2,a_5\})\\ &= \{(a_2,H(a_2)),(a_5,H(a_5))\}=\{(a_2,\{x_2,x_6\}),(a_5,\{x_7\})\}\end{aligned}$$

4 Neutrosophic Set

It is some how a generalization of fuzzy set and intuitionistic fuzzy set theories known as neutrosophic set theory, introduced by F. Smarandache. In this section we present the definition and some operations on neutrosophic set.

Definition 4.1. [3, 6] The neutrosophic set A is a set of objects which defines on the universe of discourse U as

$$A = \{\langle\frac{x}{\mu_A(x), \nu_A(x), \omega_A(x)}\rangle, x \in U\},$$

where the function $\mu, \nu, \omega : U \to]^{-}0, 1^{+}[$ and $^{-}0 \leq \mu_A(x) + \nu_A(x) + \omega_A(x) \leq 3^{+}$ for all $x \in U$. Here, $\mu_A(x)$, $\nu_A(x)$, and ω_A(x) are named as the degree of membership(or Truthness), the degree of indeterminacy, and the degree of non-membership(or Falsehood) of the element $x \in U$ to the set A. For two neutrosophic sets A and B,

$$\begin{aligned} A &= \{\langle\frac{x}{\mu_A(x), \nu_A(x), \omega_A(x)}\rangle : x \in U\}, \textit{ and} \\ B &= \{\langle\frac{x}{\mu_B(x), \nu_B(x), \omega_B(x)}\rangle : x \in U\}, \end{aligned}$$

the relations on neutrosophic set are given as follows:

(1) **Subset**:

The neutrosophic set A is a subset of neutrosophic set B if and only if $\mu_A(x) \leq \mu_B(x),\ \nu_A(x) \geq \nu_B(x),\ \omega_A(x) \geq \omega_B(x)$.

(2) **Equality**:

The neutrosophic set A is equal to neutrosophic set B if and only if $\mu_A(x) = \mu_B(x),\ \nu_A(x) = \nu_B(x),\ \omega_A(x) = \omega_B(x)$.

(3) **Intersection**:

The intersection of these two neutrosophic sets A and B is given by

$$A \cap B = \{\langle\frac{x}{min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)), max(\omega_A(x), \omega_B(x))}\rangle | x \in U\},$$

(4) **Union**:

The union of these two neutrosophic sets A and B is given by

$$A \cup B = \{\langle\frac{x}{max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)), min(\omega_A(x), \omega_B(x))}\rangle | x \in U\},$$

(5) **Complement**:

The complement of neutrosophic sets A is denoted by A^c and defined by

$$A^c = \{\langle\frac{x}{\omega_A(x), 1-\nu_A(x), \mu_A(x)}\rangle | x \in U\},$$

(6) $0_n = (0, \alpha, 1)$ and $1_n = (1, 0, 0)$ for $0 \leq \alpha \leq 1$, are called null neutrosophic condition and unit neutrosophic condition respectively.

Example 4.2.
Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of quality of features that is, x_1 is for sharpness, x_2 is for sound, x_3 is for color, x_4 is for internet facilities and x_5 is for video. Here A is LG TV and B is Sony TV.

$$\begin{aligned} Let\ A &= \{\langle\frac{x_1}{0.9, 0.5, 0.4}\rangle, \langle\frac{x_2}{0.7, 0.5, 0.3}\rangle, \langle\frac{x_3}{0.8, 0.2, 0.7}\rangle, \langle\frac{x_4}{0.6, 0.7, 0.1}\rangle, \langle\frac{x_5}{0.0, 0.8, 1.0}\rangle\} \\ &= \{\langle\frac{x_1}{0.9, 0.5, 0.4}\rangle, \langle\frac{x_2}{0.7, 0.5, 0.3}\rangle, \langle\frac{x_3}{0.8, 0.2, 0.7}\rangle, \langle\frac{x_4}{0.6, 0.7, 0.1}\rangle\} \end{aligned}$$

Here, for the sharpness x_1 in LG TV, the degree of quality of goodness is 0.9, the degree of quanlity of indeterminacy is 0.5 and the degree of worstness is 0.4 and so on. The video quality x_5 is not considered in LG TV, because the degree of membership is 0 and degree of non-membership is 1.

$$Let\ B = \{\langle\frac{x_1}{0.9, 0.5, 0.3}\rangle, \langle\frac{x_2}{0.7, 0.5, 0.2}\rangle, \langle\frac{x_3}{0.9, 0.1, 0.5}\rangle, \langle\frac{x_4}{0.7, 0.5, 0.1}\rangle, \langle\frac{x_5}{0.6, 0.4, 0.8}\rangle\}$$

Here, for the sharpness x_1 in Sony TV, the degree of quality of goodness is 0.9, the degree of quanlity of indeterminacy is 0.5 and the degree of worstness is 0.3 and so on. Hence. $A \subseteq B$.

Example 4.3.
Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be different treatments that is, x_1 stands for physical therapy, x_2 stands for radiology, x_3 stands for immunotherapy, x_4 stands for phototherapy and x_5 stands for chemotherapy. Here A is Delhi city and B is Mumbai.

$$\begin{aligned} Let\ A &= \{\langle\frac{x_1}{0.7, 0.2, 0.5}\rangle, \langle\frac{x_2}{0.8, 0.4, 0.3}\rangle, \langle\frac{x_3}{0.5, 0.5, 0.5}\rangle, \langle\frac{x_4}{0.5, 0.7, 0.2}\rangle, \langle\frac{x_5}{0.8, 0.5, 0.3}\rangle\} \\ and\ B &= \{\langle\frac{x_1}{0.9, 0.2, 0.5}\rangle, \langle\frac{x_2}{0.7, 0.4, 0.3}\rangle, \langle\frac{x_3}{0.8, 0.7, 0.1}\rangle, \langle\frac{x_4}{0.5, 0.7, 0.6}\rangle, \langle\frac{x_5}{0.6, 0.4, 0.2}\rangle\} \end{aligned}$$

be neutrosophic sets. Then union, intersection and compliment of two neutrosophic sets A and B is

$$\begin{aligned} A \cup B &= \{\langle\frac{x_1}{0.9, 0.2, 0.5}\rangle, \langle\frac{x_2}{0.8, 0.4, 0.3}\rangle, \langle\frac{x_3}{0.8, 0.5, 0.1}\rangle, \langle\frac{x_4}{0.5, 0.7, 0.2}\rangle, \langle\frac{x_5}{0.8, 0.4, 0.2}\rangle\} \\ A \cap B &= \{\langle\frac{x_1}{0.7, 0.2, 0.5}\rangle, \langle\frac{x_2}{0.7, 0.4, 0.3}\rangle, \langle\frac{x_3}{0.5, 0.7, 0.5}\rangle, \langle\frac{x_4}{0.5, 0.7, 0.6}\rangle, \langle\frac{x_5}{0.6, 0.5, 0.3}\rangle\} \end{aligned}$$

For the city Delhi and Mumbai, we get the degree of quality of goodness in physical therapy is atleast 0.7, degree of indeterminacy is atmost 0.2 and degree of quality of worstness is atmost 0.5.

$$A^c = \{\langle\frac{x_1}{0.5,0.8,0.7}\rangle,\ \langle\frac{x_2}{0.3,0.6,0.8}\rangle,\ \langle\frac{x_3}{0.5,0.5,0.5}\rangle,\ \langle\frac{x_4}{0.2,0.3,0.5}\rangle,\ \langle\frac{x_5}{0.3,0.5,0.8}\rangle\}$$

5 Neutrosophic soft set(NSS)

Maji[8] introduced the concept of neutrosophic soft set in 2013, also see more details K. Bhutani and S. Aggarwal[2], D.Mohanty and N. Kalia[11].

Definition 5.1. [8] Let U be an initial universe set and E be the set of parameters on U, $A \subseteq E$. Let NS(U) be the set of all neutrosophic sets of U, then the collection (F_N, A) is called neutrosophic soft set over U, where F_N is a mapping given by $F_N : A \to NS(U)$.

Example 5.2.
Let U be the set of dresses under consideration and E be the set of parameters. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ and $A = \{a_1, a_2, a_3\} \subseteq E$,where $E = \{a_1, a_2, a_3, a_4, a_5\}$ that is E=$\{$beautiful, shinning, costly, modern, regular$\}$. Suppose that

$$\begin{aligned}
F_N(a_1) &= \{\langle\frac{x_1}{0.6,0.5,0.4}\rangle,\langle\frac{x_2}{0.4,0.7,0.3}\rangle,\langle\frac{x_3}{0.8,0.5,0.4}\rangle,\langle\frac{x_4}{0.9,0.6,0.2}\rangle,\langle\frac{x_5}{0.0,0.7,1.0}\rangle\}\\
F_N(a_2) &= \{\langle\frac{x_1}{0.9,0.6,0.3}\rangle,\langle\frac{x_2}{0.7,0.4,0.2}\rangle,\langle\frac{x_3}{0.6,0.2,0.3}\rangle,\langle\frac{x_4}{0.7,0.3,0.2}\rangle,\langle\frac{x_5}{0.8,0.2,0.3}\rangle\}\\
F_N(a_3) &= \{\langle\frac{x_1}{0.7,0.4,0.6}\rangle,\langle\frac{x_2}{0.9,0.3,0.5}\rangle,\langle\frac{x_3}{0.7,0.8,0.5}\rangle,\langle\frac{x_4}{0.0,0.5,1.0}\rangle,\langle\frac{x_5}{0.6,0.1,0.8}\rangle\}
\end{aligned}$$

Then the neutrosophic soft set (F_N, A), where $F_N : A \to NS(U)$ is defined as

$$\begin{aligned}
NS_A(U) &= (F_N, A) = \{(a_1, F_N(a_1)), (a_2, F_N(a_2)), (a_3, F_N(a_3))\}\\
&= \{(a_1, \{\langle\frac{x_1}{0.6,0.5,0.4}\rangle,\langle\frac{x_2}{0.4,0.7,0.3}\rangle,\langle\frac{x_3}{0.8,0.5,0.4}\rangle,\langle\frac{x_4}{0.9,0.6,0.2}\rangle\}),\\
&\quad (a_2, \{\langle\frac{x_1}{0.9,0.6,0.3}\rangle,\langle\frac{x_2}{0.7,0.4,0.2}\rangle,\langle\frac{x_3}{0.6,0.2,0.3}\rangle,\langle\frac{x_4}{0.7,0.3,0.2}\rangle,\langle\frac{x_5}{0.8,0.2,0.3}\rangle\}),\\
&\quad (a_3, \{\langle\frac{x_1}{0.7,0.4,0.6}\rangle,\langle\frac{x_2}{0.9,0.3,0.5}\rangle,\langle\frac{x_3}{0.7,0.8,0.5}\rangle,\langle\frac{x_5}{0.6,0.1,0.8}\rangle\})\}
\end{aligned}$$

Now we define neutrosophic soft subset, equal, union and intersection on U.

Definition 5.3. [8] Let (F_N, A) and (G_N, B) be two neutrosophic soft sets over the common universe U. (F_N, A) is said to be neutrosophic soft subset of (G_N, B) if $A \subset B$, and $T_{F_N(a)}(x) \leq T_{G_N(a)}(x),\ I_{F_N(a)}(x) \geq I_{G_N(a)}(x),\ F_{F_N(a)}(x) \geq F_{G_N(a)}(x), \forall a \in A,\ x \in U$. We denote it by $(F_N, A) \subseteq (G_N, B)$. At that time we say (G_N, B) be the neutrosophic soft super set of (F_N, A).

Definition 5.4. [8] Two NSSs (F_N, A) and (G_N, B) over the common universe U are said to be equal if (F_N, A) is neutrosophic soft subset of (G_N, B) and (G_N, B) is neutrosophic soft subset of (F_N, A). We denote it by $(F_N, A) = (G_N, B)$.

Definition 5.5. [8] Let (H_N, A) and (G_N, B) be two neutrosophic soft sets over the common universe U. Then the union of (H_N, A) and (G_N, B) is denoted by $(H_N, A) \cup (G_N, B)$ and is defined by $(H_N, A) \cup (G_N, B) = (K_N, C)$, where $C = A \cup B$ and for $m \in U$ the truth-membership, indeterminacy-membership and falsity-membership of (K_N, C) are follows, respectively

$$T_{K_N(a)}(m) = \begin{cases} T_{H_N(a)}(m), \text{ if } a \in A - B \\ T_{G_N(a)}(m), \text{ if } a \in B - A \\ max(T_{H_N(a)}(m), T_{G_N(a)}(m)), \text{ if } a \in A \cap B \end{cases}$$

$$I_{K_N(a)}(m) = \begin{cases} I_{H_N(a)}(m), \text{ if } a \in A - B \\ I_{G_N(a)}(m), \text{ if } a \in B - A \\ \dfrac{I_{H_N(a)}(m) + I_{G_N(a)}(m)}{2}, \text{ if } a \in A \cap B \end{cases}$$

$$F_{K_N(a)}(m) = \begin{cases} F_{H_N(a)}(m), \text{ if } a \in A - B \\ F_{G_N(a)}(m), \text{ if } a \in B - A \\ min(F_{H_N(a)}(m), F_{G_N(a)}(m)), \text{ if } a \in A \cap B \end{cases}$$

Definition 5.6. [8] Let (H_N, A) and (G_N, B) be two neutrosophic soft sets over the common universe U. Then the intersection of (H_N, A) and (G_N, B) is denoted by $(H_N, A) \cap (G_N, B)$ and is defined by $(H_N, A) \cap (G_N, B) = (K_N, C)$, where $C = A \cap B$ and for $m \in U$ the truth-membership, indeterminacy-membership and falsity-membership of (K_N, C) are follows, respectively

$$\begin{aligned} T_{K_N(a)}(m) &= min(T_{H_N(a)}(m), T_{G_N(a)}(m)), \\ I_{K_N(a)}(m) &= max(I_{H_N(a)}(m), I_{G_N(a)}(m)) \\ F_{K_N(a)}(m) &= max(F_{H_N(a)}(m), F_{G_N(a)}(m)), \ \forall \ a \in C. \end{aligned}$$

For example see P.K. Maji[8]

6 Neutrosophic soft set with roughness

In this section, NS_A-lower and NS_A-upper approximations are introduced and their properties are deduced and illustrated by examples. We can find the notation $S_A(X)$, for

$X \subseteq U$ and E is a set of parameters, $A \subseteq E$,

$$S_A(X) = \{u \in U : \exists e \in A[u \in F(e) \subseteq X]\}.$$

Clearly, $S_A(X) \subseteq X$, $S_A(\phi) = \phi$ and $S_A(U) \subseteq U$.

We note here that, neutrosophic soft rough set is defined without using full soft set.

Definition 6.1. Let U be a nonempty universe. Let E be set of parameters, $A \subseteq E$ and NS(U) be the set of all neutrosophic sets of U. The collection $S = (F_N, A)$ be the neutrosophic soft set(NSS) over U, where F_N be a mapping given by $F_N : A \to NS(U)$. Then $P = (U, F_N, A)$ is called neutrosophic soft approximation space. Now for $X \subseteq U$, we define NS_A-lower and NS_A-upper approximation as

$$\begin{aligned} \underline{apr}_{NS_A}(X) &= \big\{\big(e, \{\langle \frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)} \rangle\}\big) \in NS_A(U) : u \in S_A(X)\big\} \\ \textit{and } \overline{apr}_{NS_A}(X) &= NS_A(U) - \underline{apr}_{NS_A}(X^c), \end{aligned}$$

where $S_A(X) = \{u \in U : \exists\ e \in A\ [u \in F(e) \subset X]\}$.
If $\underline{apr}_{NS_A}(X) \neq \overline{apr}_{NS_A}(X)$, then X is neutrosophic soft rough set. Otherwise X is called neutrosophic soft definable set.

Example 6.2.
Let $U = \big\{x_1, x_2, \cdots, x_{10}\big\}$ be ten private new buildings are be sold and $E = \big\{f_1, f_2, f_3, f_4, f_5\big\}$ be a set of parameters such that f_1 be the parameter that the building looks like police quarter, the parameter f_2 be the buildings looks like peons quarters, f_3 be the doctor's quarter, f_4 be officer's quarter and f_5 be teacher's quarter. Let $F : E \to P(U)$ be a mapping given by $F(f_1) = \big\{x_1, x_4, x_5\big\}$, $F(f_2) = \phi$, $F(f_3) = \big\{x_4, x_5\big\}$, $F(f_4) = \big\{x_1, x_2, x_3\big\}$, $F(f_5) = \big\{x_3, x_6\big\}$, and $F(f_6) = \big\{x_3, x_4, x_7\big\}$.
Now we define a mapping $F_N : E \to NS(U)$ by

$$\begin{aligned} F_N(f_1) &= F_N(\text{building looks like police quarter}) \\ &= \big\{\langle \frac{x_1}{0.7, 0.5, 0.3} \rangle, \langle \frac{x_2}{0.0, 0.5, 1.0} \rangle, \langle \frac{x_3}{0.0, 0.9, 1.0} \rangle, \langle \frac{x_4}{0.8, 0.3, 0.5} \rangle, \langle \frac{x_5}{0.9, 0.7, 0.2} \rangle, \\ &\quad \langle \frac{x_6}{0.0, 0.8, 1.0} \rangle, \langle \frac{x_7}{0.0, 0.2, 1.0} \rangle, \langle \frac{x_8}{0.0, 0.7, 1.0} \rangle, \langle \frac{x_9}{0.0, 0.6, 1.0} \rangle, \langle \frac{x_{10}}{0.0, 0.5, 1.0} \rangle\big\} \\ &= \big\{\langle \frac{x_1}{0.7, 0.5, 0.3} \rangle, \langle \frac{x_4}{0.8, 0.3, 0.5} \rangle, \langle \frac{x_5}{0.9, 0.7, 0.2} \rangle\big\} \end{aligned}$$

Here, building $x_2, x_3, x_6, x_7, x_8, x_9$ and x_{10} are not considered because the degree of membership is 0 and degree of non-membership is 1.

$$\begin{aligned} F_N(f_2) &= F_N(\text{building looks like peons quarter}) = \phi \\ F_N(f_3) &= F_N(\text{building looks like doctor's quarter}) = \big\{\langle \frac{x_4}{0.9, 0.5, 0.6} \rangle, \langle \frac{x_5}{0.8, 0.6, 0.3} \rangle\big\} \end{aligned}$$

$$F_N(f_4) = F_N(\text{building looks like officer's quarter}) = \{\langle\frac{x_1}{0.8,0.3,0.2}\rangle,\langle\frac{x_2}{0.9,0.4,0.7}\rangle, \langle\frac{x_3}{0.6,0.7,0.3}\rangle\}$$

$$F_N(f_5) = F_N(\text{building looks like teacher's quarter}) = \{\langle\frac{x_3}{0.7,0.4,0.1}\rangle,\langle\frac{x_6}{0.5,0.3,0.6}\rangle\}$$

Now neutrosophic soft set over U is

$$\begin{aligned}(F_N,E) &= \{(f_1,F_N(f_1)),(f_2,F_N(f_2)),(f_3,F_N(f_3)),(f_4,F_N(f_4)),(f_5,F_N(f_5))\}\\ &= \{(f_1,\{\langle\frac{x_1,}{0.7,0.5,0.3}\rangle,\langle\frac{x_4}{0.8,0.3,0.5}\rangle,\langle\frac{x_5}{0.9,0.7,0.2}\rangle\}),(f_2,\phi),(f_3,\{\langle\frac{x_4}{0.9,0.5,0.6}\rangle,\\ &\quad \langle\frac{x_5}{0.8,0.6,0.3}\rangle\}),(f_4,\{\langle\frac{x_1}{0.8,0.3,0.2}\rangle,\langle\frac{x_2}{0.9,0.4,0.7}\rangle,\langle\frac{x_3}{0.6,0.7,0.3}\rangle\}),\\ &\quad (f_5,\{\langle\frac{x_3}{0.7,0.4,0.1}\rangle,\langle\frac{x_6}{0.5,0.3,0.6}\rangle\})\}\end{aligned}$$

Let $X=\{x_4,x_5,x_8\}$, $X^c=\{x_1,x_2,x_3,x_6,x_7,x_9,x_{10}\}\subset U$ and $A=\{f_1,f_3,f_5\}\subset E$. Here, $S_A(X)=\{x_4,x_5\}\subset X\subset U$. Then the neutrosophic soft set (F_N,A) is

$$\begin{aligned}(F_N,A) &= \{(f_1,F_N(f_1)),(f_3,F_N(f_3)),(f_5,F_N(f_5))\}\\ &= \{(f_1,\{\langle\frac{x_1,}{0.7,0.5,0.3}\rangle,\langle\frac{x_4}{0.8,0.3,0.5}\rangle,\langle\frac{x_5}{0.9,0.7,0.2}\rangle\}),(f_3,\{\langle\frac{x_4}{0.9,0.5,0.6}\rangle,\\ &\quad \langle\frac{x_5}{0.8,0.6,0.3}\rangle\}),(f_5,\{\langle\frac{x_3}{0.7,0.4,0.1}\rangle,\langle\frac{x_6}{0.5,0.3,0.6}\rangle\})\} = NS_A(U)\end{aligned}$$

Now, NS_A-lower approximation of X is

$$\begin{aligned}\underline{apr}_{NS_A}(X) &= \{(f_3,\{\langle\frac{x_4}{0.9,0.5,0.6}\rangle,\langle\frac{x_5}{0.8,0.6,0.3}\rangle\})\}\\ \underline{apr}_{NS_A}(X^c) &= \{(f_5,\{\langle\frac{x_3}{0.7,0.4,0.1}\rangle,\langle\frac{x_6}{0.5,0.3,0.6}\rangle\})\}\end{aligned}$$

The NS_A-upper approximation of X is

$$\begin{aligned}\overline{apr}_{NS_A}(X) &= \{(f_1,\{\langle\frac{x_1,}{0.7,0.5,0.3}\rangle,\langle\frac{x_4}{0.8,0.3,0.5}\rangle,\langle\frac{x_5}{0.9,0.7,0.2}\rangle\}),(f_3,\{\langle\frac{x_4}{0.9,0.5,0.6}\rangle,\\ &\quad \langle\frac{x_5}{0.8,0.6,0.3}\rangle\})\}\end{aligned}$$

Thus, X is rough with respect to knowledge NS_A, since $\underline{apr}_{NS_A}(X)\neq\overline{apr}_{NS_A}(X)$.

Theorem 6.3. *Let $S=(F_N,A)$ be neutrosophic soft set over U, $P=(U,F_N,A)$ be a neutrosophic soft approximation space and $X,Y\subseteq U$, then we have*
1. $\underline{apr}_{NS_A}(\phi)=\phi$, $\overline{apr}_{NS_A}(\phi)=\phi$,
2. $\underline{apr}_{NS_A}(U)=NS_A(U)=\overline{apr}_{NS_A}(U)$,
3. $X\subseteq Y\Rightarrow\underline{apr}_{NS_A}(X)\subseteq\underline{apr}_{NS_A}(Y)$,
4. $X\subseteq Y\Rightarrow\overline{apr}_{NS_A}(X)\subseteq\overline{apr}_{NS_A}(Y)$,

5. $\underline{apr}_{NS_A}(X \cap Y) \subseteq \underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y)$,
6. $\underline{apr}_{NS_A}(X \cup Y) \supseteq \underline{apr}_{NS_A}(X) \cup \underline{apr}_{NS_A}(Y)$,
7. $\overline{apr}_{NS_A}(X \cap Y) \subseteq \overline{apr}_{NS_A}(X) \cap \overline{apr}_{NS_A}(Y)$.

Proof. From definition of NS_A-lower and NS_A upper approximation, (1) and (2) are straightforward. So we prove the remaining.
Assume that $X \subseteq Y$.
Let $u \in \underline{apr}_{NS_A}(X)$, by definition there exists one $a \in A$ such that $u \in F(a) \subseteq X$ and $(a, \{\langle\frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)}\rangle\}) \in NS_A(U)$.
So that $u \in F(a) \subseteq X \subseteq Y$ and $(a, \{\langle\frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)}\rangle\}) \in NS_A(U)$.
This implies $u \in \underline{apr}_{NS_A}(Y)$.
Hence, $\underline{apr}_{NS_A}(X) \subseteq \underline{apr}_{NS_A}(Y)$. This proves (3).
Remaining properties comes directly.

□

Definition 6.4. Let $S = (F_N, A)$ be a neutrosophic soft set over U. For any $e_1, e_2 \in A$, there exists $e_3 \in A$ such that $F(e_3) = F(e_1) \cap F(e_2) \neq \phi$ and $(e_1, \{\langle\frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)}\rangle\}) \in NS_A(U)$, $(e_2, \{\langle\frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)}\rangle\}) \in NS_A(U)$, $\mu_A(u), \nu_A(u), \omega_A(u) \in [0, 1]$ for $u \in U$, then S is called intersection complete neutrosophic soft set.

Proposition 6.5. *Let $S = (F_N, A)$ be an intersection complete neutrosophic soft set over U and $P = (U, F_N, A)$ be a neutrosophic soft approximation space. Then we have*

$$\underline{apr}_{NS_A}(X \cap Y) = \underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y).$$

Proof. We have only to show $\underline{apr}_{NS_A}(X \cap Y) \supseteq \underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y)$, since,

$$\underline{apr}_{NS_A}(X \cap Y) \subseteq \underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y) \text{ is proved in theorem 6.3.}$$

Let $u \in \underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y)$, then there exists $e_1, e_2 \in A$ such that $u \in F(e_1) \subseteq X$, $(e_1, \{\langle\frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)}\rangle\}) \in NS_A(U)$, and $u \in F(e_2) \subseteq Y$ $(e_2, \{\langle\frac{u}{\mu_A(u), \nu_A(u), \omega_A(u)}\rangle\}) \in NS_A(U)$. By definition of intersection complete soft set , there exists $e_3 \in A$ such that $u \in F(e_3) = F(e_1) \cap F(e_2) \subseteq X \cap Y$.
Hence, $u \in \underline{apr}_{NS_A}(X \cap Y)$.
Therefore,

$$\underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y) \subseteq \underline{apr}_{NS_A}(X \cap Y)$$

□

Example 6.6.

Let $U = \{h_1, h_2, \cdots, h_{10}\}$ be universe of discourse and $E = \{e_1, e_2, \cdots, e_7\}$ be a set of parameters. Let $F : E \to P(U)$ be a mapping given by $F(e_1) = \{h_1, h_5\}$, $F(e_2) = \phi$, $F(e_3) = \{h_3\}$, $F(e_4) = \{h_1, h_{10}\}$, $F(e_5) = \{h_5\}$, $F(e_6) = \{h_6\}$ and $F(e_7) = \{h_9, h_{10}\}$.

Let $X = \{h_1, h_3, h_5, h_7, h_8, h_9\}$, $Y = \{h_1, h_2, h_3, h_{10}\}, then\ X^c = \{h_2, h_4, h_6, h_{10}\},, Y^c = \{h_4, h_5, h_6, h_7, h_8, h_9\} \subseteq U\ and\ let A = \{e_1, e_3, e_4, e_5, e_6\} \subseteq E$.

Thus, $S_A(X) = \{h_1, h_3, h_5\} \subset X$ and $S_A(Y) = \{h_1, h_3, h_5, h_{10}\} \subset Y$. Then neutrosophic soft set over U be

$$\begin{aligned}
(F_N, E) &= \{(e_1, \{\langle\frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle\frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_2, \phi), (e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\}),\\
&\quad (e_4, \{\langle\frac{h_1}{0.7, 0.2, 0.1}\rangle, \langle\frac{h_{10}}{1.0, 0.1, 0.5}\rangle\}), (e_5, \{\langle\frac{h_5}{0.8, 0.5, 0.5}\rangle\}),\\
&\quad (e_6, \{\langle\frac{h_6}{0.7, 0.1, 0.3}\rangle\}), (e_7, \{\langle\frac{h_9}{0.9, 0.1, 0.0}\rangle, \langle\frac{h_{10}}{1.0, 0.3, 0.5}\rangle\})\}\\
(F_N, A) &= \{(e_1, \{\langle\frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle\frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\}),\\
&\quad (e_4, \{\langle\frac{h_1}{0.7, 0.2, 0.1}\rangle, \langle\frac{h_{10}}{1.0, 0.1, 0.5}\rangle\}), (e_5, \{\langle\frac{h_5}{0.8, 0.5, 0.5}\rangle\}),\\
&\quad (e_6, \{\langle\frac{h_6}{0.7, 0.1, 0.3}\rangle\})\}\\
\underline{apr}_{NS_A}(X) &= \{(e_1, \{\langle\frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle\frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\})\}\\
\underline{apr}_{NS_A}(Y) &= \{(e_1, \{\langle\frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle\frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\}),\\
&\quad (e_4, \{\langle\frac{h_1}{0.7, 0.2, 0.1}\rangle, \langle\frac{h_{10}}{1.0, 0.1, 0.5}\rangle\})\}
\end{aligned}$$

Now, $X \cap Y = \{h_1, h_3\}$, $(X \cap Y)^c = \{h_2, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$.

$$\underline{apr}_{NS_A}(X \cap Y) = \{(e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\})\}$$

Hence, $\underline{apr}_{NS_A}(X \cap Y) \subseteq \underline{apr}_{NS_A}(X) \cap \underline{apr}_{NS_A}(Y) = \{(e_1, \{\langle\frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle\frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\})\}$.

Next, $X \cup Y = \{h_1, h_2, h_3, h_5, h_7, h_8, h_9, h_{10}\}$ and

$$\begin{aligned}
\underline{apr}_{NS_A}(X \cup Y) &= \{(e_1, \{\langle\frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle\frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle\frac{h_3}{0.9, 0.2, 0.4}\rangle\}),\\
&\quad (e_4, \{\langle\frac{h_1}{0.7, 0.2, 0.1}\rangle, \langle\frac{h_{10}}{1.0, 0.1, 0.5}\rangle\}), (e_5, \{\langle\frac{h_5}{0.8, 0.5, 0.5}\rangle\})\}
\end{aligned}$$

Therefore, $\underline{apr}_{NS_A}(X) \cup \underline{apr}_{NS_A}(Y) \subseteq \underline{apr}_{NS_A}(X \cup Y)$.

$$\begin{aligned}
\text{Now, } \underline{apr}_{NS_A}((X\cap Y)^c) &= \{(e_5, \{\langle \frac{h_5}{0.8, 0.5, 0.5}\rangle\}), (e_6, \{\langle \frac{h_6}{0.7, 0.1, 0.3}\rangle\})\} \\
\overline{apr}_{NS_A}(X \cap Y) &= \{(e_1, \{\langle \frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle \frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle \frac{h_3}{0.9, 0.2, 0.4}\rangle\}), \\
&\quad (e_4, \{\langle \frac{h_1}{0.7, 0.2, 0.1}\rangle, \langle \frac{h_{10}}{1.0, 0.1, 0.5}\rangle\})\} \\
\underline{apr}_{NS_A}(X^c) &= \{(e_6, \{\langle \frac{h_6}{0.7, 0.1, 0.3}\rangle\})\} = \underline{apr}_{NS_A}(Y^c). \text{ So that} \\
\overline{apr}_{NS_A}(X) = \overline{apr}_{NS_A}(Y) &= \{(e_1, \{\langle \frac{h_1}{0.2, 0.4, 0.5}\rangle, \langle \frac{h_5}{0.9, 0.7, 0.1}\rangle\}), (e_3, \{\langle \frac{h_3}{0.9, 0.2, 0.4}\rangle\}), \\
&\quad (e_4, \{\langle \frac{h_1}{0.7, 0.2, 0.1}\rangle, \langle \frac{h_{10}}{1.0, 0.1, 0.5}\rangle\}), (e_5, \{\langle \frac{h_5}{0.8, 0.5, 0.5}\rangle\})\}
\end{aligned}$$

Therefore, $\overline{apr}_{NS_A}(X \cap Y) \subseteq \overline{apr}_{NS_A}(X) \cap \overline{apr}_{NS_A}(Y)$.

7 Equality on neutrosophic soft rough set

Definition 7.1. Let (F_N, A) be an NSS on a universe U, $A \subseteq E$. For all $X, Y \subseteq U$, we define binary relation

1. Sets X and Y are bottom NS_A equal $(X \preccurlyeq_{NS_A} Y)$ if and only if $\underline{apr}_{NS_A}(X) = \underline{apr}_{NS_A}(Y)$
2. Sets X and Y are top NS_A equal $(X \curlyeqprec_{NS_A} Y)$ if and only if $\overline{apr}_{NS_A}(X) = \overline{apr}_{NS_A}(Y)$
3. Sets X and Y are NS_A equal $(X \approx_{NS_A} Y)$ if and only if $\underline{apr}_{NS_A}(X) = \underline{apr}_{NS_A}(Y)$ and $\overline{apr}_{NS_A}(X) = \overline{apr}_{NS_A}(Y)$

Proposition 7.2. *Let (F_N, A) be NSS over universe U, $A \subseteq E$. Then*

1. *If $X \subseteq Y$ and $Y \curlyeqprec_{NS_A} \phi$, then $X \curlyeqprec_{NS_A} \phi$,*
2. *If $X \subseteq Y$ and $X \curlyeqprec_{NS_A} U$, then $Y \curlyeqprec_{NS_A} U$,*
3. *If $X \subseteq Y$ and $Y \preccurlyeq_{NS_A} \phi$, then $X \preccurlyeq_{NS_A} \phi$,*
4. *If $X \subseteq Y$ and $Y \preccurlyeq_{NS_A} U$, then $X \preccurlyeq_{NS_A} U$.*

Proof. (1) Given $X \subseteq Y$ and $Y \preccurlyeq_{NS_A} \phi$, so that $\overline{apr}_{NS_A}(X) \subseteq \overline{apr}_{NS_A}(Y)$ and $\overline{apr}_{NS_A}(Y) = \phi$.

$$\text{Hence,} \quad \overline{apr}_{NS_A}(X) \;=\; \phi = \overline{apr}_{NS_A}(\phi)$$

(2) Given, $X \preccurlyeq_{NS_A} U$ and $X \subseteq Y$, then $\overline{apr}_{NS_A}(X) = \overline{apr}_{NS_A}(U)$ and $\overline{apr}_{NS_A}(X) \subseteq \overline{apr}_{NS_A}(Y)$. But we know that

$$\overline{apr}_{NS_A}(Y) \subseteq \overline{apr}_{NS_A}(U), \text{ hence}$$

$$\overline{apr}_{NS_A}(Y) = \overline{apr}_{NS_A}(U)$$

Remaining properties comes directly.
We note here that $X \preccurlyeq_{NS_A} Y$ if and only if $X \cap Y \preccurlyeq_{NS_A} X$ and $X \cap Y \preccurlyeq_{NS_A} Y$ is not true in general. □

8 Dispensable

In this section, we shall discuss about dispensable and indispensable of NSS. Let (L_N, A), (M_N, B) and (P_N, C) be NSS on U, where $A,\ B,\ C \subseteq S$ and $L_N : A \to NS(U)$, $M_N : B \to NS(U)$, $P_N : C \to NS(U)$ be the mappings. Let $(F_N, S) = \big\{(L_N, A), (M_N, B), (P_N, C)\big\}$. We define approximate NSS, which is denoted by APP and defined by

$$\begin{aligned} APP((F_N, S)) &= APP\big\{(L_N, A), (M_N, B), (P_N, C)\big\} \\ &= \big\{(l_\alpha, m_\beta, p_\delta), \cap\{L_N(l_\alpha), M_N(m_\beta), P_N(n_\delta)\}|\ 1 \leq \alpha, \beta, \delta \leq n\big\} \\ &= \{(e_i, F_N(e_i))|e_i \in S\}, \end{aligned}$$

where $l_\alpha \in A$, $m_\beta \in B$, $p_\delta \in C$ and $e_i \in S \subseteq A \times B \times C$, $F_N(e_i) = \cap\{L_N(l_\alpha), M_N(m_\beta), P_N(n_\delta)\}$. Also we write the difference in approximate NSS as $APP((F_N, S) - (L_N, A)) = APP((M_N, B), (P_N, C))$

Definition 8.1. Two approximate Neutrosophic soft sets APP(I_N, X) and APP(J_N, Y) are said to be equal that is, APP(I_N, X)=APP(J_N, Y) if for every $x_i \in X$ there exists one $y_j \in Y$ such that $I_N(x_i) = J_N(y_j)$ for some $1 \leq i, j \leq n$ and for every $y_j \in Y$ there exists one $x_i \in X$ such that $J_N(y_j) = I_N(x_i)$ for some $1 \leq i, j \leq n$, where $X, Y \subseteq E$ and $I_N : X \to NS(U)$, $J_N : Y \to NS(U)$.

Definition 8.2. The NSS (L_N, A) is dispensable in $\{(L_N, A), (M_N, B), (P_N, C)\}$ if $APP((F_N, S)) = APP((F_N, S) - (L_N, A))$. And if APP$((F_N, S)) \neq$ APP$((F_N, S) - (L_N, A))$, then (L_N, A) is indispensable in (F_N, S).

Definition 8.3. The NSS (F_N, S) is independent if each $(L_N, A) \subseteq (F_N, S)$ is indispensable in (F_N, S). Otherwise NSS (F_N, S) is dependent.

Example 8.4.
Let $U = \{x_1, x_2, \cdots, x_6\}$ be six most affected states in India due to Corona virus infection. Here x_1 is a group of persons from the state Maharashtra whose corona positive is detected, x_2 is a group of persons from the state Kerala whose corona positive is detected, x_3 is from Tamil Nadu, x_4 is from Delhi, x_5 is from Uttar Pradesh and x_6 is from Karnnataka. Let $E = \{e_1, e_2, \cdots, e_{11}\}$ be the set of parameters with respect to corona virus infection in the human body such that e_1 is aches, e_2 is difficult in breathing, e_3 is tiredness, e_4 is chill. e_5 is fever and cough, e_6 is sore throat, e_7 is loss of smell, e_8 is loss of taste, e_9 is headache, e_{10} is diaarhea and e_{11} is severe vomiting. Let $A = \{e_1, e_2, e_5\}$, $B = \{e_6, e_7, e_8\}$, $C = \{e_9, e_{10}, e_{11}\} \subseteq E$. Assume that $L_N(e_1) = \{x_3, x_5, x_6\}$ that is, aches symptoms is marked in human body due to corona infection in states Tamil Nadu, Uttar Pradesh and Karnataka, $L_N(e_2) = \{x_2, x_4\}$ that is, difficulty breathing is marked in human body due to corona infection in states Kerala and Delhi and $L_N(e_5) = \{x_1, x_3, x_4\}$ that is, fever and cough is marked in human body due to corona infection in states Maharashtra, Tamil Nadu and Delhi. $M_N(e_6) = \{x_1, x_5, x_6\}$, $M_N(e_7) = \{x_2, x_3, x_4\}$ and $M_N(e_8) = \{x_2, x_6\}$. Similary, $P_N(e_9) = \{x_2, x_5, x_6\}$, $P_N(e_{10}) = \{x_1, x_5\}$ and $P_N(e_{11}) = \{x_3, x_4\}$.

$$\begin{aligned}
\text{Now, } (L_N, A) &= \big\{(e_1, L_N(e_1)), (e_2, L_N(e_2)), (e_5, L_N(e_5))\big\} \\
&= \big\{(e_1, \{\langle\frac{x_3}{1.0, 0.9, 0.8}\rangle, \langle\frac{x_5}{0.7, 0.6, 0.9}\rangle, \langle\frac{x_6}{0.8, 0.3, 0.7}\rangle\}), (e_2, \{\langle\frac{x_2}{0.7, 0.6, 0.9}\rangle, \\
&\quad \langle\frac{x_4}{0.8, 0.6, 0.9}\rangle\}), (e_5, \{\langle\frac{x_1}{0.6, 0.3, 0.2}\rangle, \langle\frac{x_3}{0.7, 0.8, 0.6}\rangle, \langle\frac{x_4}{0.3, 0.5, 0.8}\rangle\})\big\} \\
(M_N, B) &= \big\{(e_6, M_N(e_6)), (e_7, M_N(e_7)), (e_8, M_N(e_8))\big\} \\
&= \big\{(e_6, \{\langle\frac{x_1}{0.7, 0.4, 0.1}\rangle, \langle\frac{x_5}{0.8, 0.5, 0.7}\rangle, \langle\frac{x_6}{0.8, 0.3, 0.7}\rangle\}), (e_7, \{\langle\frac{x_1}{0.7, 0.4, 0.1}\rangle, \\
&\quad \langle\frac{x_3}{0.9, 0.4, 0.6}\rangle, \langle\frac{x_4}{0.6, 0.4, 0.3}\rangle\}), (e_8, \{\langle\frac{x_2}{0.9, 0.4, 0.6}\rangle, \langle\frac{x_6}{1.0, 0.5, 0.3}\rangle\})\big\} \\
(P_N, C) &= \big\{(e_9, P_N(e_9)), (e_{10}, P_N(e_{10})), (e_{11}, P_N(e_{11}))\big\} \\
&= \big\{(e_9, \{\langle\frac{x_2}{1.0, 0.6, 0.4}\rangle, \langle\frac{x_5}{0.9, 0.2, 0.7}\rangle, \langle\frac{x_6}{0.7, 0.4, 0.3}\rangle\}), (e_{10}, \{\langle\frac{x_1}{0.9, 0.3, 0.1}\rangle, \\
&\quad \langle\frac{x_5}{0.7, 0.4, 0.1}\rangle\}), (e_{11}, \{\langle\frac{x_3}{0.8, 0.4, 0.3}\rangle, \langle\frac{x_4}{0.7, 0.2, 0.1}\rangle\})\big\}
\end{aligned}$$

$$\begin{aligned}
APP((L_N, A), (M_N, B), (P_N, C)) &= APP((F_N, S))(say) \\
&= \big\{((e_1, e_6, e_9), \{\langle\frac{x_5}{0.7, 0.6, 0.9}\rangle, \langle\frac{x_6}{0.8, 0.4, 0.7}\rangle\}), ((e_1, e_6, e_{10}), \\
&\quad \{\langle\frac{x_5}{0.7, 0.6, 0.9}\rangle\}), ((e_1, e_7, e_{11}), \{\langle\frac{x_3}{0.8, 0.9, 0.8}\rangle\}), \\
&\quad ((e_1, e_8, e_9), \{\langle\frac{x_6}{0.7, 0.5, 0.7}\rangle\}), ((e_2, e_7, e_{11}), \{\langle\frac{x_4}{0.6, 0.6, 0.9}\rangle\}),
\end{aligned}$$

$$((e_5,e_6,e_{10}),\{\langle\frac{x_1}{0.6,0.4,0.2}\rangle\}),((e_5,e_7,e_{11}),\{\langle\frac{x_3}{0.7,0.8,0.6}\rangle,\langle\frac{x_4}{0.3,0.5,0.8}\rangle\})\}$$
$$= \big\{(s_1,F_N(s_1)),(s_2,F_N(s_2)),(s_3,F_N(s_3)),(s_4,F_N(s_4)),(s_5,F_N(s_5)),$$
$$(s_6,F_N(s_6)),(s_7,F_N(s_7))\big\};$$

where $s_1 = (e_1,e_6,e_9)$, $F_N(s_1) = \{\langle\frac{x_5}{0.8,0.5,0.6}\rangle,\langle\frac{x_6}{0.6,0.4,0.3}\rangle\}$ and so on. As $L_N(e_1)\cap M_N(e_6)\cap P_N(e_{11}) = \phi$ the element $((e_1,e_6,e_{11}),\phi)$ in $APP(F_N,S)$ is not written.

$$\begin{aligned}
\text{Now, } APP((F_N,S)-(L_N,A)) &= APP((M_N,B),(P_N,C)) = APP(Z_N,T),\ \text{suppose}\\
&= \big\{((e_6,e_9),\{\langle\frac{x_5}{0.8,0.5,0.7}\rangle,\langle\frac{x_6}{0.7,0.4,0.7}\rangle\}),((e_6,e_{10}),\\
&\quad \{\langle\frac{x_1}{0.7,0.4,0.1}\rangle,\langle\frac{x_5}{0.7,0.5,0.7}\rangle\}),((e_7,e_{10}),\{\langle\frac{x_1}{0.7,0.4,0.1}\rangle\}),\\
&\quad ((e_7,e_{11}),\{\langle\frac{x_3}{0.8,0.4,0.6}\rangle,\langle\frac{x_4}{0.6,0.4,0.3}\rangle\}),((e_8,e_9),\\
&\quad \{\langle\frac{x_2}{0.9,0.6,0.6}\rangle,\langle\frac{x_6}{0.7,0.5,0.3}\})\}\\
&= \big\{(t_1,Z_N(t_1)),(t_2,Z_N(t_2)),(t_3,Z_N(t_3)),(t_4,Z_N(t_4)),\\
&\quad (t_5,Z_N(t_5))\big\}\\
&\neq APP((F_N,S))
\end{aligned}$$

Since, though $F_N(s_1) = \{\langle\frac{x_5}{0.7,0.6,0.9}\rangle\langle\frac{x_6}{0.8,0.4,0.7}\rangle\} = Z_N(t_1)$, but $F_N(s_2) = \{\langle\frac{x_5}{0.7,0.6,0.9}\rangle\} \neq Z_N(t_i)$, $1 \leq i \leq 5$, $F_N(s_3) = \{\langle\frac{x_3}{0.8,0.9,0.8}\rangle\} \neq Z_N(t_i)$, $F_N(s_4) = \{\langle\frac{x_6}{0.7,0.5,0.7}\rangle\} \neq Z_N(t_i)$, $F_N(s_5) = \{\langle\frac{x_4}{0.6,0.4,0.9}\rangle\} \neq Z_N(t_i)$, $F_N(s_6) = \{\langle\frac{x_1}{0.6,0.4,0.2}\rangle\} \neq Z_N(t_i)$, $Z_N(t_2) = \{\langle\frac{x_1}{0.7,0.4,0.1}\rangle,\langle\frac{x_5}{0.7,0.5,0.7}\rangle\} \neq F_N(s_j)$ and $Z(t_5) = \{\langle\frac{x_2}{0.9,0.6,0.6}\rangle,\langle\frac{x_6}{0.7,0.5,0.3}\rangle\} \neq F_N(s_j)$, where $1 \leq j \leq 7$.
Hence, (L_N,A) is indispensable in (F_N,S).

$$\begin{aligned}
\text{Now, } APP((F_N,S)-(M_N,B)) &= APP((L_N,A),(P_N,C)) = APP(Q_N,R),\ \text{say}\\
&= \big\{((e_1,e_9),\{\langle\frac{x_5}{0.7,0.6,0.9}\rangle,\langle\frac{x_6}{0.7,0.6,0.9}\rangle\}),((e_1,e_{10}),\\
&\quad \{\langle\frac{x_5}{0.7,0.6,0.9}\rangle),((e_1,e_{11}),\{\langle\frac{x_3}{0.8,0.9,0.8}\rangle\}),((e_2,e_9),\\
&\quad \{\langle\frac{x_2}{0.7,0.6,0.9}\rangle\}),((e_2,e_{11}),\{\langle\frac{x_4}{0.7,0.6,0.9}\rangle),((e_5,e_{10}),\\
&\quad \{\langle\frac{x_1}{0.6,0.3,0.2}\rangle),((e_5,e_{11}),\{\langle\frac{x_3}{0.7,0.8,0.6}\rangle,\langle\frac{x_4}{0.3,0.5,0.8}\rangle)\}\\
&= \big\{(r_1,Q_N(r_1)),(r_2,Q_N(r_2)),(r_3,Q_N(r_3)),(r_4,Q_N(r_4)),\\
&\quad (r_5,Q_N(r_5))\big\}\\
&\neq APP((F_N,S))
\end{aligned}$$

Hence, (M_N, B) is indispensable in (F_N, S).

$$
\begin{aligned}
APP((F_N, S) - (P_N, C)) &= APP((L_N, A), (M_N, B)) = APP(V_N, D) \\
&= \Big\{((e_1, e_6), \{\langle \frac{x_5}{0.7, 0.6, 0.9}\rangle, \langle \frac{x_6}{0.6, 0.3, 0.7}\rangle\}), ((e_1, e_7), \{\langle \frac{x_3}{0.9, 0.9, 0.8}\rangle), \\
&\quad ((e_1, e_8), \{\langle \frac{x_6}{0.8, 0.5, 0.7}\rangle\}), ((e_2, e_7), \{\langle \frac{x_4}{0.6, 0.6, 0.9}\}), ((e_2, e_8), \\
&\quad \{\langle \frac{x_2}{0.7, 0.6, 0.9}\rangle), ((e_5, e_6), \{\langle \frac{x_1}{0.6, 0.4, 0.2}\rangle), ((e_5, e_7), \{\langle \frac{x_1}{0.6, 0.4, 0.2}\rangle, \\
&\quad \langle \frac{x_3}{0.7, 0.8, 0.6}\rangle, \langle \frac{x_4}{0.3, 0.5, 0.8}\rangle)\Big\} \\
&= \Big\{(d_1, V_N(d_1)), (d_2, V_N(d_2)), (d_3, V_N(d_3)), (d_4, V_N(d_4)), (d_5, V_N(d_5))\Big\} \\
&\neq APP((F_N, S))
\end{aligned}
$$

Hence, (P_N, C) is indispensable in (F_N, S). Therefore, (F_N, S) is independent.

9 Conclusion

In this note it is defined the notion of neutrosophic soft rough set in new manner which is a combination of three theories that is rough set theory, soft set theory and neutrosophic set theory. We have studied some of their basic properties like union, intersection and complement. Some authors have defined neutrosophic soft rough set using full soft set which is not convenient to handle indeterminant and incomplete data. In this article neutrosophic soft rough set is established without using full soft set and also equality and dispensability on neutrosophic soft rough set are illustrated with examples.

10 Compliance with Ethical Standards

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Ethical Approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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☐ This is one original research work.

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