Analyzing of Process Capability Indices based on Neutrosophic Sets

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Analyzing of Process Capability Indices based on Neutrosophic Sets

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Abstract

Process capability analysis (PCA) is an important statistical analysis approach for measuring and analyzing the ability of the process to meet specifications. This analysis has been applied by producing process capability indices (PCIs). C_p and C_{pk} are the most commonly used PCIs for this aim. Although they are completely effective statistics to analyze process' capability, the complexity of the production processes based on uncertainty arising from human thinking, incomplete or vague information makes it difficult to analyze the process capability with precise values. When there is uncertain, complex, incomplete and inaccurate information, the capability of the process is successfully analyzed by using the fuzzy sets. Neutrosophic sets (NSs), one of the new fuzzy set extensions, have a significant role in modeling uncertainty, since they contain the membership functions of truth, indeterminacy, and falsity definitions rather than an only membership function. This feature provides a strong advantage for modeling uncertainty. In this paper, PCA has been performed based on NSs to overcome uncertainties of the process. For this purpose, specification limits (SLs) have been reconsidered by using NSs and two of the well-known process capability indices (PCIs) named C_p and C_{pk} have been reformulated. Finally, the neutrosophic process capability indices (NPCIs) named C_p $\left(\ddot{\tilde{C}}_p\right)$ and C_{pk} $\left(\ddot{\tilde{C}}_{pk}\right)$ have been derived for three cases that are created by defining SLs. Additionally, the obtained NPCIs have also been applied and confirmed on real case problems from automotive industry. The obtained results show that the NPCIs support the quality engineers to easily define SLs and obtain more flexible and realistic evaluations for PCA.

Keywords: Process capability analysis, process capability index, neutrosophic sets, single valued neutrosophic numbers

1. Introduction

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Process capability analysis (PCA) is a critical methodology based on a statistical analysis of variability that occurs within the process to measure process' performance. PCA is used for purposes such as improving process activities by examining the activities of the process, measuring the variability of the process, ensuring variability reduction by evaluating variations with specification limits (SLs), choosing critical processes, meeting performance requirements for new equipment, determining which tolerances the process can fall within, ensuring continuous improvement in enhancing the quality of the product, preventing the production of defective parts, defining the boundaries between the samples taken to analyze the process and helping to determine the priorities (Kane 1986; Montgomery 2005) Each process has its own different conditions and requirements. Since these requirements and conditions cannot be calculated with a single index, various process capability indices (PCIs) have been developed (Wu et al. 2009). The process of producing a product includes many stages from raw material procurement to the formation of the final product. Quality inspectors find defective products in the samples they examine after the process is completed, causing high costs. In order to prevent this, the process should be checked regularly, the variations in the process should be noticed and the reasons causing the variation should be eliminated. The analysis of whether the process is under control is made with quality control charts (QCCs). The outliers, namely out of control points, in the process are clearly visible thanks to the QCCs. Correct interpretations cannot be made about the capability of processes that are not under control. Parameters are not fixed in processes that are not under the control (Montgomery 2009). In order to measure the process capability, the process must be under control. Analyzing PCIs after eliminating the causes that cause outliers in the process will enable us to obtain more accurate and realistic information about the capability of the process and give information about whether the process will meet the requirements requested in the future.

It is misleading to evaluate the process only with the mean and the standard deviation. The process should be evaluated by correlating the mean and the standard deviation of the process with specification limits (SLs) (Tannock 1995). The SLs of the process point what the customer expects or wants from the process. Specifications are a set of predefined rules for a product to occur (Gülbay and Kahraman 2008). For example, the hardness or strength of a bolt is determined by specifications. In other words, the specifications define the limits of the product or service. Determination of the acceptable quality limits of the process, namely the SLs, is vital for the analysis of the process. More flexible, more sensitive and accurate definition of parameters such as SLs, mean and variance will increase the effectiveness of PCIs. These parameters cannot be defined precisely in some situations where the measurement system is different, the sensitivity of the measuring instruments or the measurement errors due to the worker factor or human's hesitancy based on evaluations. Defining these parameters with the neutrosophic logic instead of exact values due to time, cost and sampling difficulty will ensure that the information content and sensitivity of the process is higher.

Neutrosophic logic is better to handle uncertainty than fuzzy logic. There are many similar problems involving uncertain conditions in which fuzzy logic does not do well, such as weather forecasting, stock price prediction and political choices (Zhang 2010). One of the fuzzy extension methods named NSs that are expressed with truth-membership, indeterminacy-membership and falsity-membership functions which are all independent from each other. Independency of truth-membership and indeterminacy-membership functions from each other and expressing the fact that an individual does not have full control of the issue with the indeterminacy-membership function has an important place in modeling uncertainty problems (Radwan et al. 2016). NSs provide an effective approach on PCA since they have more advantages compared to other fuzzy set extensions in terms of both ease of application and flexibility. Defining PCIs with more than one membership function instead of only one membership function will enable us to evaluate the process more broadly. The neutrosophic SLs provide more information, more sensitiveness and more precision in the analysis of the process.

The neutrosophic process capability indices (NPCIs) show all possible values of the process with functions of truth, indeterminacy and falsity values. This wide range of information makes it easy for process engineers to follow the process and interpret the results obtained. The fact that the developed NPCIs are more flexible and include more information than the traditional PCIs provides ease of application in real life problems. Additionally, they contain more information about the process capability allows realistic analyzes and ensures that the uncertainty that may arise due to the subjective thinking structure is eliminated when defining the SLs of the quality engineers.

In recent years, many studies have been conducted for the fuzzification of the PCA. Generally, traditional fuzzy sets called type-1 fuzzy sets have been used in these studies. Extensions of fuzzy sets such as hesitant, intuitionistic, pythagorean and type-2 fuzzy sets have become very popular as they overcome the criticism for traditional fuzzy sets (Kahraman et al. 2017). In the literature, some fuzzy extensions have been used on PCA. Aslam and Albassam (2019) calculated the mean and standard deviation parameters with the neutrosophic statistical method to use in the sampling plan. They also analyzed the index C_{pk}. However, it is realized that research with the extensions of fuzzy sets including hesitant, intuitionistic and type-2 fuzzy sets are very limited. These extensions have not received enough attention yet. Haktanır and Kahraman (2021) developed the indices C_p and C_{pk} by using penthagorean fuzzy sets and applied to a real application. Kahraman et al. (2017) utilized intuitionistic fuzzy SLs to obtain the indices C_p and C_{pk}. Parchami et al. (2017) calculated the indices C_p, C_{pk} and C_{pm} by taking interval type-2 fuzzy SLs. Senvar and Kahraman (2014) calculated the percentile-based the indices C_p and C_{pk} by using interval type-2 fuzzy sets for non-normal processes. Hesamian and Akbari (2019) analyzed the index C_{pm} that the SLs, mean, and target value were defined by using intuitionistic fuzzy sets (IFSs). Cao et al. (2016) also calculated the multivariate process capability index (MPCI) S_{pk}^T by using IFSs.

The quality engineer or quality inspector who defines the SLs may not always have full control of the subject and they can easily define parameters of the process. By the way, the process includes some uncertainties based on measurement systems, human's hesitancy and vagueness for evaluations etc. The traditional PCIs are not usable to analyze this type of process and the results obtained may be meaningless or lead to erroneous interpretations. In order to overcome this problem, the NSs can be used effectively. For example, the indeterminacy-membership function involved in defining NSs is very significant in expressing uncertainty. For these reasons, the SLs are defined by using NSs in this paper. The use of NSs in the PCA has led to a significant level of flexibility in defining SLs, and the results obtained to be more precise and contain more information. Based on the literature review, it is also observed that the PCA and SLs are not previously examined by using NSs. This study has been proposed to fill this gap in the literature.

In this paper, three cases based on NSs such as the single valued neutrosophic state of the SLs, the state of being flexible and more flexible NSs have been investigated. For this aim, the SLs are considered neutrosoficically, and the NPCIs such as $\tilde{\dddot{C}}_p$ and $\tilde{\dddot{C}}_{pk}$ have been derived for the first time. The flexibility of the SLs will facilitate the expression of uncertainty and will allow easier integration into the real case problems.

The rest of this paper has been organized as follows: The PCIs have briefly summarized in Section 2. The main characteristics of NSs are explained into Section 3. The PCIs based on NSs are derived into Section 4. Some real case examples from automotive industry to apply the proposed PCIs have been explained into Section 5. Finally, the results obtained and main advantages of the proposed NPCIs are discussed into Section 6. The general inferences, future research directions and some suggestions have been discussed into Section 7.

2. Process Capability Indices

Process' performance can be analyzed by using some statistical process capability indices (PCIs) that are summary statistics which measure the actual or the potential performance of the process characteristics relative to the target and SLs by considering process location and dispersion (Kaya and Kahraman 2010a). PCIs, which provide numerical measures on whether a process meets the customer expectations or not, have been popularly applied for evaluating process' performance. Several PCIs such as C_p, C_{pk}, C_{pm}, and C_{pmk} are used to estimate the capability of a process (Kotz and Johnson 2002;Kaya and Çolak 2020, Kaya and Kahraman 2011a).

The index C_p is defined as the ratio of specification width over the process spread. The specification width represents customer and/or product requirements. The process variations are represented by the specification width. If the process variation is very large, the C_p value is small and it represents a low process capability. It can be obtained by using the following formula (Montgomery 2005;Kaya and Kahraman 2011a):

$$C_p = \frac{\text{Allowable Process Spread}}{\text{Actual Process Spread}} = \frac{\text{USL} - \text{LSL}}{6\sigma} \tag{2.1}$$

where σ is the standard deviation of the process, USL and LSL represent the upper and lower SLs, respectively.

C_p also indicates how well the process fits between the upper and lower SLs. It never considers any process shift and simply measures the spread of the specifications relative to the six-sigma spread in the process. If the process average is not centered near the midpoint of SLs (m), the C_p index gives misleading results. We also know that the index C_p indicates how well the process fits within the two SLs. It never considers any process shift as presented in Figure 2.1 and it is calculated by using Eq. (2.1). The index C_p simply measures the spread of the specifications relative to the six-sigma spread in the process (Kotz and Johnson 2002; Montgomery 2005;Kaya and Kahraman 2011b).

Therefore Kane (1986) introduced the index C_{pk} which is used to provide an indication of the variability associated with a process. It shows how a process confirms to its specification. The index is usually used to relate the *natural tolerance* (3σ) to the SLs and describes how well the process fits within these limits, taking into account the location of the process mean. C_{pk} is calculated as follows(Montgomery 2005; Kaya and Kahraman 2011a):

$$C_{pk} = \min\left\{C_{pl}, C_{pu}\right\} = \frac{\min\left\{USL-\mu, \mu-LSL\right\}}{3\sigma} \tag{2.2}$$

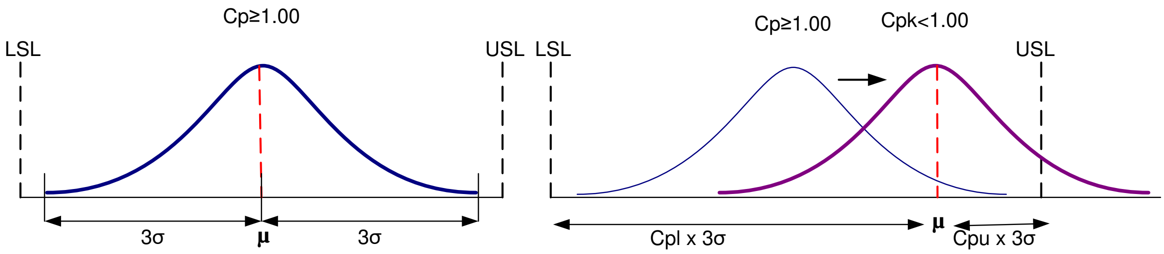

Figure 1.1. C_p and C_{pk} Indices (Kaya and Kahraman 2011b)

In this paper, these two types of PCIs have been re-considered with respect to NSs. The main design principles of these indices have been analyzed and mathematical formulations have been derived by using NSs.

3. Neutrosophic Sets

Zadeh (1965) first introduced the concept of the fuzzy set theory (FST). The fuzzy sets are used to solve uncertainty and ambiguity in the information and they only have degrees of membership. To increase the ability of fuzzy sets in modeling uncertainty, some new fuzzy extensions have recently been proposed. Atanassov (1999) developed IFSs theory by adding non-membership degree to fuzzy sets. The sum of degrees of these membership and non-membership functions cannot exceed 1 while it can exceed 1 in vague sets. Another type of the new fuzzy set extensions is pythagorean fuzzy sets (PFSs) that have membership and non-membership functions. The sum of degree of these membership functions (MFs) can be greater than 1, but sum of their squares cannot be greater than 1. Smarandache (1999) presented the neutrosophic logic as the generalized form of the fuzzy logic. The neutrosophic logic is a context that provides combining many existing logics. The main idea of neutrosophic logic is to describe each proposition in a three dimensions of neutrosophic space as a degree of truth, indeterminacy, and falsity (Çevik et al. 2018). For instance, the proposition “tomorrow it will be raining” is not a fixed-valued statement. If expressed in neutrosophic terms, it may be 70% true, 50% indeterminate and 20% false. The truth value also changes according to the observer. Also, the proposition “Tom is smart” can be (0.35, 0.67, 0.6) with respect to his friends, with respect to himself (0.8, 0.25, 0.1) or with respect to his wife (0.5, 0.2, 0.3) (Zhang 2010). This definition and expression flexibility of NSs makes a significant contribution to modeling uncertainty and creates a significant difference. In this paper these advantages of NSs have been used into PCIs by using a statistical perspective. The neutrosophic statistics refers to the methods used to analyze a set of data that the data or a portion of it has some indeterminacy. For example, the sample size may not be precisely known. In this case, it is taken as an interval n = [90, 100], instead of a crisp number n = 90 (or n = 100) as in classical statistics. Since neutrosophic statistic is a random variable, it has a neutrosophic probability distribution (Smarandache 2014). Smarandache also proposed a new conception of NSs, which independently include truth-membership, indeterminacy-membership and falsity-membership degrees to reflect humanitarian style of thinking or human's hesitancy. This theory is very significant to solve the real case problems since indeterminacy is quantified clearly and to provide more flexibility and more sensitivity by using three dimensions compared to the other extension of fuzzy sets. Neutrosophic theory is applied in various fields of life to solve problems concerned to indeterminacy, such as mathematical, engineering, geography, medicine, psychology (Alhasan, et al. 2021). In recent years, there are many studies in the literature using neutrosophic logic and neutrosophic statistics because of their advantages. Wang et al. (2010) developed single valued neutrosophic sets (SVNSs) that is a special form of NSs. Chakraborty et al. (2019) proposed the de-neutrosophication method for trapezoidal NNs and then applied it to the crash model problem in networking area and job-sequencing problem. Radwan et al. (2016) presented a new approach for expert system for learning management systems evaluation based on NNs. Abdel-Basset et al. (2018a) introduced a novel algorithm for the group decision making (GDM) problem based on NNs. Aslam (2019a) proposed an attribute sampling plan using neutrosophic statistical interval method and then compared the proposed sampling plan designed using the neutrosophic binomial distribution with the existing sampling plan designed using the classical binomial distribution. Chen et al. (2017b) developed a new concept of neutrosophic interval probability and neutrosophic interval statistical number and applied them on a real case. Aslam (2018) designed a new sampling plan using the neutrosophic statistics for the process loss function and used the factory data to illustrate the performance of the proposed sampling plan. Aslam and Al-Marshadi (2018) introduced the neutrosophic regression estimator for the acceptance sampling plan and discussed the efficiency and accuracy of the proposed sampling plan using neutrosophic statistics. Chen et al. (2017a) proposed a new neutrosophic statistics method for the joint roughness coefficient (JRC). The JRC is used to determine the shear strength in rock mechanics. Aslam (2019b) proposed neutrosophic analysis of variance (NANOVA) under the neutrosophic statistics and it can be applied to test more than one neutrosophic population means. Deli and Subas (2014) introduced single valued neutrosophic numbers (SVNNs) and discussed triangular single valued neutrosophic numbers

(TrSVNNs) and trapezoidal single valued neutrosophic numbers (TSVNNs) that are two special forms of SVNNs. The special forms of SVNNs are applied to multi-attribute decision making (MCDM) problem. Deli and Şubaş (2017) presented a methodology for solving multi attribute decision making (MADM) problems with single valued trapezoidal neutrosophic numbers (SVTNNs). Mohamed et al. (2017) suggested the integer programming problem under the neutrosophic environments. Abdel-Basset et al. (2018b) extended SWOT analysis with neutrosophic AHP to determine the best strategy in terms of organizational goals. Nabeeh et al. (2019) employed neutrosophic AHP to determine weights of criteria of personnel selection problem. Then, TOPSIS method was used to evaluate different alternatives. Yalçın and Kaya (2021) analyzed the indices C_{pm} and C_{pmk} by using single valued neutrosophic numbers. Gul et al. (2021) presented SVNS based TOPSIS under the concept of Fine-Kinney risk assessment. Wind turbine's risk assessment problem was handled with proposed methodology. Şahin and Yigider (2016) used neutrosophic TOPSIS to solve supplier selection problem. Otay and Kahraman (2018) employed interval neutrosophic TOPSIS based GDM methodology to evaluate different six sigma projects. In this paper, NSs have been integrated into PCIs to improve their flexibility and sensitiveness.

Some basic notions of NSs as follows:

Definition 3.1. (Deli and Subas 2014): Let X be a universe. A single valued neutrosophic set (SVNS) $\ddot{\tilde{A}}$ in X is characterized by a truth-membership $T_A(x)$, an indeterminacy-membership $I_A(x)$ and a falsity-membership $F_A(x)$ functions, respectively. As a SVNS $\ddot{\tilde{A}}$ is defined as follows:

$$\ddot{\tilde{A}} = \{< x, (T_A(x), I(x), F_A(x)) >: x \in X, T_A(x), I(x), F_A(x) \in [0,1]\} \tag{3.1}$$

There is no restriction on the sum of $T_A(x)$, $I(x)$ and $F_A(x)$. So that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 3.2. (Deli and Şubaş 2017;Ye 2017): Let $w_{\tilde{\tilde{a}}}, u_{\tilde{\tilde{a}}}, y_{\tilde{\tilde{a}}} \in [0,1]$ be any real numbers, $a_i, b_i, c_i, d_i \in \mathbb{R}$ and $a_i \leq b_i \leq c_i \leq d_i$ for $(i = 1,2,3)$. A single valued neutrosophic number (SVNN) $\tilde{\tilde{a}} = \left\langle \left((a_1, b_1, c_1, d_1), w_{\tilde{\tilde{a}}}\right), \left((a_2, b_2, c_2, d_2), u_{\tilde{\tilde{a}}}\right), ((a_3, b_3, c_3, d_3), y_{\tilde{\tilde{a}}}) \right\rangle$ is a special neutrosophic set in $\mathbb{R}$ with truth-membership function $\mu_{\tilde{\tilde{a}}}: \mathbb{R} \to [0, w_{\tilde{a}}]$, indeterminacy-membership function $v_{\tilde{\tilde{a}}}: \mathbb{R} \to [0, u_{\tilde{a}}]$ and falsity-membership function $\lambda_{\tilde{\tilde{a}}}: \mathbb{R} \to [0, y_{\tilde{a}}]$ as follows:

$$\mu_{\tilde{\tilde{a}}}(x)\begin{cases} f_\mu^l(x) & a_1 \le x < b_1 \\ w_{\tilde{\tilde{a}}} & b_1 \le x < c_1 \\ f_\mu^r(x) & c_1 \le x \le d_1 \\ 0 & Otherwise \end{cases} \tag{3.2}$$

$$v_{\tilde{\tilde{a}}}(x)\begin{cases} f_v^l(x) & a_2 \le x < b_2 \\ u_{\tilde{\tilde{a}}} & b_2 \le x < c_2 \\ f_v^r(x) & c_2 \le x \le d_2 \\ 1 & Otherwise \end{cases} \tag{3.3}$$

$$\lambda_{\tilde{\tilde{a}}}(x)\begin{cases} f_\lambda^l(x) & a_3 \le x < b_3 \\ y_{\tilde{\tilde{a}}} & b_3 \le x < c_3 \\ f_\lambda^r(x) & c_3 \le x \le d_3 \\ 0 & Otherwise \end{cases} \tag{3.4}$$

where the functions $f_\mu^l\colon [\, a_1, b_1] \to [0, w_{\tilde{\tilde{a}}}], f_v^r\colon [\, c_2, d_2] \to [u_{\tilde{\tilde{a}}}, 1], f_\lambda^r\colon [\, c_3, d_3] \to [y_{\tilde{\tilde{a}}}, 1]$ are increasing continuous functions. Then the functions $f_\mu^r\colon [\, c_1, d_1] \to [0, w_{\tilde{\tilde{a}}}], f_v^l\colon [\, a_2, b_2] \to [u_{\tilde{\tilde{a}}}, 1], f_\lambda^l\colon [\, a_3, b_3] \to [y_{\tilde{\tilde{a}}}, 1]$ are decreasing continuous functions.

Definition 3.3. (Deli and Şubaş 2017): A single valued trapezoidal neutrosophic number (SVTNN) $\tilde{\tilde{a}} = \langle (a, b, c, d); w_{\tilde{\tilde{a}}}, u_{\tilde{\tilde{a}}}, y_{\tilde{\tilde{a}}} \rangle$ is a special neutrosophic set in $\mathbb{R}$ with truth-membership function, indeterminacy-membership function and falsity-membership function as follows:

$$\mu_{\tilde{\tilde{a}}}(x)\begin{cases} (x-a)w_{\tilde{\tilde{a}}}/(b-a) & a \le x < b \\ w_{\tilde{\tilde{a}}} & b \le x \le c \\ (d-x)w_{\tilde{\tilde{a}}}/(d-c) & c < x \le d \\ 0 & Otherwise \end{cases} \tag{3.5}$$

$$v_{\tilde{\tilde{a}}}(x)\begin{cases} (b-x+u_{\tilde{\tilde{a}}}(x-a))/(b-a) & a \le x < b \\ u_{\tilde{\tilde{a}}} & b \le x \le c \\ (x-c+u_{\tilde{\tilde{a}}}(d-x))/(d-c) & c < x \le d \\ 1 & Otherwise \end{cases} \tag{3.6}$$

$$\lambda_{\tilde{\tilde{a}}}(x)\begin{cases} (b-x+y_{\tilde{\tilde{a}}}(x-a))/(b-a) & a \le x < b \\ y_{\tilde{\tilde{a}}} & b \le x \le c \\ (x-c+y_{\tilde{\tilde{a}}}(d-x))/(d-c) & c < x \le d \\ 0 & Otherwise \end{cases} \tag{3.7}$$

Definition 3.4. (Deli and Şubaş 2017): A single valued triangular neutrosophic number (SVTrNN) $\tilde{\tilde{a}} = \langle (a, b, c); w_{\tilde{\tilde{a}}}, u_{\tilde{\tilde{a}}}, y_{\tilde{\tilde{a}}} \rangle$ is a special neutrosophic set in $\mathbb{R}$ with truth-membership function, indeterminacy-membership function and falsity-membership function as follows:

$$\mu_{\tilde{\tilde{a}}}(x)\begin{cases}(x-a)w_{\tilde{\tilde{a}}}/(b-a) & a\le x<b\\ w_{\tilde{\tilde{a}}} & x=b\\ (c-x)w_{\tilde{\tilde{a}}}/(c-b) & b\le x\le c\\ 0 & Otherwise\end{cases}$$
(3.8)

$$v_{\tilde{\tilde{a}}}(x)\begin{cases}(b-x+u_{\tilde{\tilde{a}}}(x-a))/(b-a) & a\le x<b\\ u_{\tilde{\tilde{a}}} & x=b\\ (x-b+u_{\tilde{\tilde{a}}}(c-x))/(c-b) & b\le x\le c\\ 1 & Otherwise\end{cases} \quad (3.9)$$

$$\lambda_{\tilde{\tilde{a}}}(x)\begin{cases}(b-x+y_{\tilde{\tilde{a}}}(x-a))/(b-a) & a\le x<b\\ y_{\tilde{\tilde{a}}} & x=b\\ (x-b+y_{\tilde{\tilde{a}}}(d-x))/(d-c) & b\le x\le c\\ 0 & Otherwise\end{cases} \quad (3.10)$$

If $a\geq 0$ at least $c>0$ then STrVNN is called a positive SVTrNN, denoted by $\tilde{\tilde{a}}>0$. In the same way, if $c\leq 0$ at least $a<0$ then SVTrNN is called a negative SVTrNN, denoted by $\tilde{\tilde{a}}<0$.

Definition 3.5. (Deli and Subas 2014): Let $\tilde{\tilde{a}}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{\tilde{a}}},u_{\tilde{\tilde{a}}},y_{\tilde{\tilde{a}}}\rangle$ and $\tilde{\tilde{b}}=\langle(a_2,b_2,c_2,d_2);w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{b}}}\rangle$ be two SVTNNs and $\gamma\neq 0$ any real number. Then, the arithmetic operations of SVTNNs are defined as follows:

$$\tilde{\tilde{a}}\oplus\tilde{\tilde{b}}=\langle(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle \quad (3.11)$$

$$\tilde{\tilde{a}}\ominus\tilde{\tilde{b}}=\langle(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle \quad (3.12)$$

$$\tilde{\tilde{a}}\otimes\tilde{\tilde{b}}=\begin{cases}\langle(a_1a_2,b_1b_2,c_1c_2,d_1d_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle & d_1>0,d_2>0\\ \langle(a_1d_2,b_1c_2,c_1b_2,d_1a_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle & d_1<0,d_2>0\\ \langle(d_1d_2,c_1c_2,b_1b_2,a_1a_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle & d_1<0,d_2<0\end{cases} \quad (3.13)$$

$$\tilde{\tilde{a}}\oslash\tilde{\tilde{b}}=\begin{cases}\langle(a_1/d_2,b_1/c_2,c_1/b_2,d_1/a_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle & d_1>0,d_2>0\\ \langle(d_1/d_2,c_1/c_2,b_1/b_2,a_1/a_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle & d_1<0,d_2>0\\ \langle(d_1/a_2,c_1/b_2,b_1/c_2,a_1/d_2);w_{\tilde{\tilde{a}}}\wedge w_{\tilde{\tilde{b}}},u_{\tilde{\tilde{a}}}\vee u_{\tilde{\tilde{b}}},y_{\tilde{\tilde{a}}}\vee y_{\tilde{\tilde{b}}}\rangle & d_1<0,d_2<0\end{cases} \quad (3.14)$$

$$\gamma\tilde{\tilde{a}}=\begin{cases}\langle(\gamma a_1,\gamma b_1,\gamma c_1,\gamma d_1);w_{\tilde{\tilde{a}}},u_{\tilde{\tilde{a}}},y_{\tilde{\tilde{a}}}\rangle & \gamma>0\\ \langle(\gamma d_1,\gamma c_1,\gamma b_1,\gamma a_1);w_{\tilde{\tilde{a}}},u_{\tilde{\tilde{a}}},y_{\tilde{\tilde{a}}}\rangle & \gamma<0\end{cases} \quad (3.15)$$

$$\tilde{\tilde{a}}^{\gamma}\begin{cases}\langle(a_1^{\gamma},b_1^{\gamma},c_1^{\gamma},d_1^{\gamma});w_{\tilde{\tilde{a}}},u_{\tilde{\tilde{a}}},y_{\tilde{\tilde{a}}}\rangle & \gamma>0\\ \langle(d_1^{\gamma},c_1^{\gamma},b_1^{\gamma},a_1^{\gamma});w_{\tilde{\tilde{a}}},u_{\tilde{\tilde{a}}},y_{\tilde{\tilde{a}}}\rangle & \gamma<0\end{cases} \quad (3.16)$$

$$\tilde{\tilde{a}}^{-1}=\langle(1/d_1,1/c_1,1/b_1,1/a_1);w_{\tilde{\tilde{a}}},u_{\tilde{\tilde{a}}},y_{\tilde{\tilde{a}}}\rangle \quad (3.17)$$

Definition 3.6. (Broumi et al. 2016): Score and accuracy functions are used to compare between two SVNNs. Let $\tilde{\tilde{a}} = \langle (a, b, c); w_{\tilde{\tilde{a}}}, u_{\tilde{\tilde{a}}}, y_{\tilde{\tilde{a}}} \rangle$ be a SVTrNN. Score function denoted by $S(\tilde{\tilde{a}})$ and accuracy function denoted by $A(\tilde{\tilde{a}})$, are defined as follow:

$$S(\tilde{\tilde{a}}) = \frac{1}{12}[a + 2b + c]x(2 + \mu_{\tilde{\tilde{a}}} - v_{\tilde{\tilde{a}}} - \gamma_{\tilde{\tilde{a}}}) \tag{3.18}$$

$$A(\tilde{\tilde{a}}) = \frac{1}{12}[a + 2b + c]x(2 + \mu_{\tilde{\tilde{a}}} - v_{\tilde{\tilde{a}}} + \gamma_{\tilde{\tilde{a}}}) \tag{3.19}$$

Definition 3.7.(Ye 2017): Let $\tilde{\tilde{a}} = \langle (a, b, c, d); w_{\tilde{\tilde{a}}}, u_{\tilde{\tilde{a}}}, y_{\tilde{\tilde{a}}} \rangle$ be a SVTNN. $S(\tilde{\tilde{a}})$ and $A(\tilde{\tilde{a}})$ are defined as follow:

$$S(\tilde{\tilde{a}}) = \frac{1}{12}[a + b + c + d](2 + \mu_{\tilde{\tilde{a}}} - v_{\tilde{\tilde{a}}} - \gamma_{\tilde{\tilde{a}}}) \tag{3.20}$$

$$A(\tilde{\tilde{a}}) = \frac{1}{12}[a + b + c + d](2 + \mu_{\tilde{\tilde{a}}} - v_{\tilde{\tilde{a}}} + \gamma_{\tilde{\tilde{a}}}) \tag{3.21}$$

Definition 3.8. (Ye 2017): Let $\tilde{\tilde{a}}_1$ and $\tilde{\tilde{a}}_2$ be two SVTNNs. The ranking between $\tilde{\tilde{a}}_1$ and $\tilde{\tilde{a}}_2$ by score and accuracy functions are defined as follows:

- If $S(\tilde{\tilde{a}}_1) < S(\tilde{a}_2)$ then $\tilde{\tilde{a}}_1 < \tilde{\tilde{a}}_2$
- If $S(\tilde{\tilde{a}}_1) > S(\tilde{a}_2)$ then $\tilde{\tilde{a}}_1 > \tilde{\tilde{a}}_2$
- If $S(\tilde{\tilde{a}}_1) = S(\tilde{a}_2)$ then
    - If $A(\tilde{\tilde{a}}_1) < A(\tilde{a}_2)$ then $\tilde{\tilde{a}}_1 < \tilde{\tilde{a}}_2$
    - If $A(\tilde{\tilde{a}}_1) > A(\tilde{a}_2)$ then $\tilde{\tilde{a}}_1 > \tilde{\tilde{a}}_2$
    - If $A(\tilde{\tilde{a}}_1) = A(\tilde{a}_2)$ then $\tilde{\tilde{a}}_1 = \tilde{\tilde{a}}_2$

Definition 3.9. (Deli 2019): A single valued trapezoidal neutrosophic number (SVTNN) $\tilde{\tilde{a}} = \left\langle \left((a_1, b_1, c_1, d_1), w_{\tilde{\tilde{a}}}\right), \left((a_2, b_2, c_2, d_2), u_{\tilde{\tilde{a}}}\right), ((a_3, b_3, c_3, d_3), y_{\tilde{\tilde{a}}}) \right\rangle$ for $w_{\tilde{\tilde{a}}} = 1, u_{\tilde{\tilde{a}}} = 0,\ y_{\tilde{\tilde{a}}} = 0$ is defined by (Ye 2015) as follows:

$\tilde{\tilde{a}} = \langle (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3) \rangle$.

For SVTNN, there exist twelve numbers $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, a_3, b_3, c_3, d_3 \in$ R such that $a_3 \le a_2 \le a_1 \le b_3 \le b_2 \le b_1 \le c_1 \le c_2 \le c_3 \le d_1 \le d_2 \le d_3$ and the six functions $T_A^L(x), T_A^R(x), I_A^L(x), I_A^R(x), F_A^L(x), F_A^R(x): R \to [0,1]$ to represent the truth, indeterminacy and falsity membership degrees of $\tilde{\tilde{a}}$ (Biswas et al. 2016).

Definition 3.10. (Ye 2017): Let $\tilde{\ddot{a}} = \langle(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3)\rangle$ be a SVTNN. Score function denoted by $S(\tilde{\ddot{a}})$ and accuracy function denoted by $H(\tilde{\ddot{a}})$, are defined as:

$$S(\tilde{\ddot{a}}) = \frac{1}{3}\left(2 + \frac{a_1+b_1+c_1+d_1}{4} - \frac{a_2+b_2+c_2+d_2}{4} - \frac{a_3+b_3+c_3+d_3}{4}\right), S(\tilde{\ddot{a}}) \in [0,1] \quad (3.22)$$

$$H(\tilde{\ddot{a}}) = \left(\frac{a_1+b_1+c_1+d_1}{4} - \frac{a_3+b_3+c_3+d_3}{4}\right), H(\tilde{\ddot{a}}) \in [-1,1] \quad (3.23)$$

Definition 3.11. (Ye 2015): Let $\tilde{\ddot{a}} = \langle(a_1, b_1, c_1), (a_2, b_2, c_2), (a_3, b_3, c_3)\rangle$ be a SVTrNN. $S(\tilde{\ddot{a}})$ and $H(\tilde{\ddot{a}})$ are defined as follow:

$$S(\tilde{\ddot{a}}) = \frac{1}{3}\left(2 + \frac{a_1+2b_1+c_1}{4} - \frac{a_2+2b_2+c_2}{4} - \frac{a_3+2b_3+c_3}{4}\right), S(\tilde{\ddot{a}}) \in [0,1] \quad (3.24)$$

$$H(\tilde{\ddot{a}}) = \left(\frac{a_1+2b_1+c_1}{4} - \frac{a_3+2b_3+c_3}{4}\right), H(\tilde{\ddot{a}}) \in [-1,1] \quad (3.25)$$

Definition 3.12. (Deli 2019): Let $\tilde{\ddot{a}}_1 = \langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ and $\tilde{\ddot{a}}_2 = \langle(a_{12}, a_{22}, a_{32}, a_{42}), (b_{12}, b_{22}, b_{32}, b_{42}), (c_{12}, c_{22}, c_{32}, c_{42})\rangle$ be two SVTNNs. The ranking between $\tilde{\ddot{a}}_1$ and $\tilde{\ddot{a}}_2$ based on score and accuracy functions are defined as follows:

- If $S(\tilde{\ddot{a}}_1) > S(\tilde{\ddot{a}}_2)$ then $\tilde{\ddot{a}}_1 > \tilde{\ddot{a}}_2$
- If $S(\tilde{\ddot{a}}_1) = S(\tilde{\ddot{a}}_2)$ then
  - If $H(\tilde{\ddot{a}}_1) > H(\tilde{\ddot{a}}_2)$ then $\tilde{\ddot{a}}_1 > \tilde{\ddot{a}}_2$
  - If $H(\tilde{\ddot{a}}_1) = H(\tilde{\ddot{a}}_2)$ then $\tilde{\ddot{a}}_1 = \tilde{\ddot{a}}_2$

4. Process Capability Indices based on Neutrosophic Sets

In this study, the effects of NSs on PCA has been analyzed and different forms of SVNNs have been applied to PCIs. The use of NSs, which are more successful in modeling human thinking than other fuzzy set extensions, in defining the SLs, in cases where uncertainty arises due to human thinking structure, will bring flexibility, sensitivity and reality to the PCA. Thus, it is aimed to give a different perspective to the PCA. The application of the definitions of SNVNNs in different forms to PCA provides flexibility in evaluating the capability of the process. Previously, researchers would take the maximum value of the truth, indeterminacy, and falsity functions as 1. Thanks to the different definitions of NNs, these MFs are allowed to be defined as less than 1. The capability of the process has been analyzed by considering the SVNNs defined as $\tilde{\ddot{a}} = \langle(a, b, c, d); w_{\tilde{\ddot{a}}}, u_{\tilde{\ddot{a}}}, y_{\tilde{\ddot{a}}}\rangle$ and the asymmetric views of the functions of truth, indeterminacy and falsity defined as $\tilde{\ddot{a}} = \langle(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3)\rangle$. The meanings of both definitions are the similar. If $a_1 = a_2 = a_2 = a$; $b_1 = b_2 = b_3 = b$; $c_1 =$

$c_2 = c_3 = c; d_1 = d_2 = d_3 = d$ and the degree of MFs are taken $w_{\tilde{\ddot{a}}}, u_{\tilde{\ddot{a}}}, y_{\tilde{\ddot{a}}}$ instead of 1, the definition in Definition (3.3) comes (Chakraborty et al. 2019). In this study, in the most general sense, two of the well-known PCIs named C_p and C_{pk} have been re-formulated by using SVNNs to improve their flexibility and sensitiveness. Finally, the NPCIs are obtained based on mathematical backgrounds of them that defining by using NSs. As a result, the neutrosophic C_p ($\tilde{\ddot{C}}_p$) and neutrosophic C_{pk} ($\tilde{\ddot{C}}_{pk}$) indices have been obtained based on three cases as detailed below:

4.1. The Case That Specifications Limits are Single-Valued Neutrosophic Numbers

In this subsection, we consider the case that degrees of truth, falsity and indeterminacy are dependent. For this subsection, a real case example can be given as follows: Suppose a person applies for a visa to travel, but he/she doesn't know whether his/her application will be accepted or not (indeterminacy function). The authorized people can approve it (truth function) or they can reject it (falsity function). So all three functions are dependent in this way (Chakraborty et al. 2019). The indeterminacy membership function that involved in definition of NSs is very important in expressing uncertainty. For this reason, SLs are defined as SVNNs. We firstly consider that the SLs are defined by using SVTrNNs. Now assume that single valued triangular neutrosophic SLs are defined as follow:

$\widetilde{\ddot{LSL}} = \langle (lsl_1, lsl_2, lsl_3); w_{\widetilde{\ddot{LSL}}}, u_{\widetilde{\ddot{LSL}}}, y_{\widetilde{\ddot{LSL}}} \rangle$ and $\widetilde{\ddot{USL}} = \langle (usl_1, usl_2, usl_3); w_{\widetilde{\ddot{USL}}}, u_{\widetilde{\ddot{USL}}}, y_{\widetilde{\ddot{USL}}} \rangle$.

The MFs of neutrosophic SLs $(\widetilde{\ddot{SLs}})$ as SVTrNN are obtained as shown in Figure 4.1.

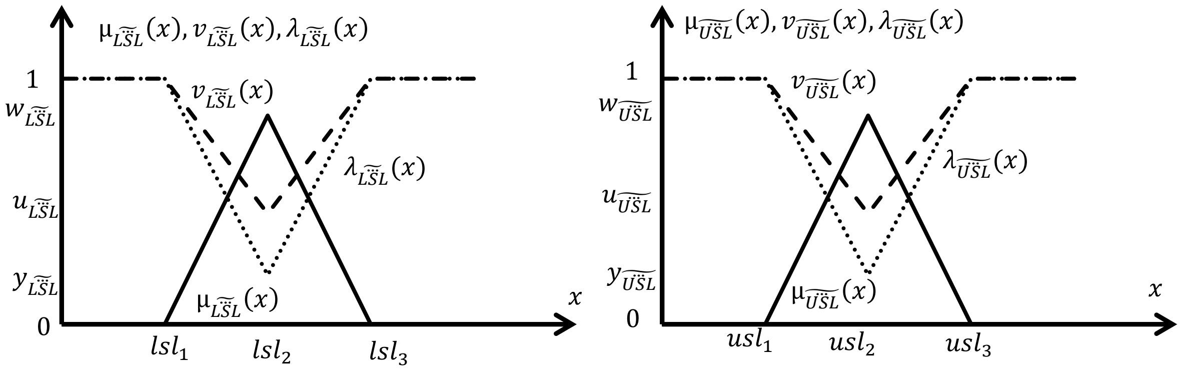

Figure 4.1. (a) The MFs of $\widetilde{\ddot{LSL}}$ derived by using SVTrNN. (b). The MFs of $\widetilde{\ddot{USL}}$ derived by using SVTrNN

Now assume that single valued trapezoidal neutrosophic SLs are defined as follow:

$\widetilde{\ddot{LSL}} = \langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{\widetilde{\ddot{LSL}}}, u_{\widetilde{\ddot{LSL}}}, y_{\widetilde{\ddot{LSL}}} \rangle$ and $\widetilde{\ddot{USL}} = \langle (usl_1, usl_2, usl_3, usl_4); w_{\widetilde{\ddot{USL}}}, u_{\widetilde{\ddot{USL}}}, y_{\widetilde{\ddot{USL}}} \rangle$.

The MFs of $\widetilde{\ddot{SLs}}$ as SVTNN are obtained as shown in Figure 4.2.

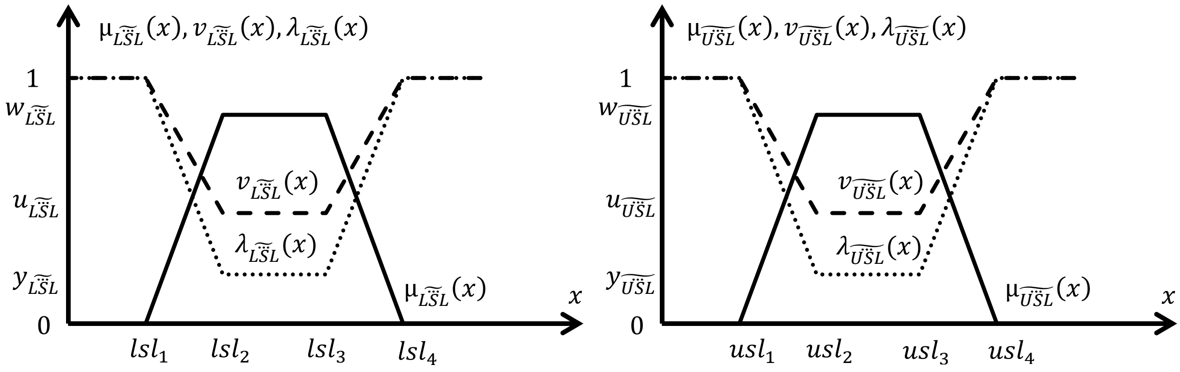

Figure 4.2. (a) The MFs of $\ddot{\widetilde{LSL}}$ derived by using SVTNN. (b) The MFs of $\ddot{\widetilde{USL}}$ derived by using SVTrNN.

4.1.1. The Index $\ddot{\tilde{C}}_p$

The general definition of the index $\ddot{\tilde{C}}_p$ is as shown in Eq. (4.1):

$$\ddot{\tilde{C}}_p = \frac{\ddot{\widetilde{USL}} \ominus \ddot{\widetilde{LSL}}}{6\sigma} \tag{4.1}$$

The SLs are also defined as SVTrNN. Then, the index $\ddot{\tilde{C}}_p$ is derived by using the addition, subtraction and multiplication operations in Definition (3.5) as follows:

$$\ddot{\tilde{C}}_p = \frac{\left\langle (usl_1, usl_2, usl_3); w_{\ddot{\widetilde{USL}}}, u_{\ddot{\widetilde{USL}}}, y_{\ddot{\widetilde{USL}}} \right\rangle \ominus \left\langle (lsl_1, lsl_2, lsl_3); w_{\ddot{\widetilde{LSL}}}, u_{\ddot{\widetilde{LSL}}}, y_{\ddot{\widetilde{LSL}}} \right\rangle}{6\sigma} \tag{4.2}$$

Eq. (4.2) can be written as Eq. (4.3):

$$\ddot{\tilde{C}}_p = \left\langle \left(\frac{usl_1 - lsl_3}{6\sigma}, \frac{usl_2 - lsl_2}{6\sigma}, \frac{usl_3 - lsl_1}{6\sigma} \right); w_{\ddot{\widetilde{USL}}} \wedge w_{\ddot{\widetilde{LSL}}}, u_{\ddot{\widetilde{USL}}} \vee u_{\ddot{\widetilde{LSL}}}, y_{\ddot{\widetilde{USL}}} \vee y_{\ddot{\widetilde{LSL}}} \right\rangle \tag{4.3}$$

Then, the MFs of the index $\ddot{\tilde{C}}_p$ is shown in Figure 4.3.

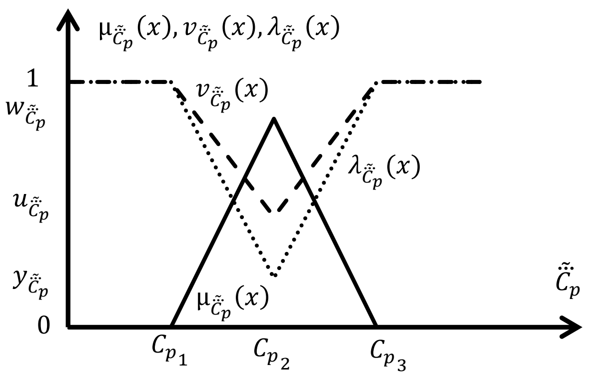

Figure 4.3. The MFs of the index $\ddot{\tilde{C}}_p$ derived by using SVTrNN

The SLs are defined as SVTNN and the index $\ddot{\tilde{C}}_p$ is obtained by using the addition, subtraction and multiplication operations in Definition (3.5) as follows:

$$\ddot{\tilde{C}}_p = \frac{\left\langle (usl_1, usl_2, usl_3, usl_4); w_{\ddot{\widetilde{USL}}}, u_{\ddot{\widetilde{USL}}}, y_{\ddot{\widetilde{USL}}} \right\rangle \ominus \left\langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{\ddot{\widetilde{LSL}}}, u_{\ddot{\widetilde{LSL}}}, y_{\ddot{\widetilde{LSL}}} \right\rangle}{6\sigma} \tag{4.4}$$

Eq. (4.4) can be written as Eq. (4.5).

$$\ddot{\tilde{C}}_p = \left\langle \left(\frac{usl_1 - lsl_4}{6\sigma}, \frac{usl_2 - lsl_3}{6\sigma}, \frac{usl_3 - lsl_2}{6\sigma}, \frac{usl_4 - lsl_1}{6\sigma} \right); w_{\widetilde{USL}} \wedge w_{\widetilde{LSL}}, u_{\widetilde{USL}} \vee u_{\widetilde{LSL}}, y_{\widetilde{USL}} \vee y_{\widetilde{LSL}} \right\rangle \quad (4.5)$$

Then, the MFs of the index $\ddot{\tilde{C}}_p$ is shown in Figure 4.4.

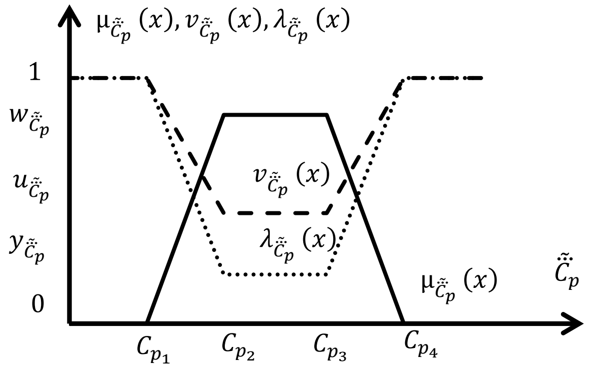

Figure 4.4. The MFs of the index $\ddot{\tilde{C}}_p$ derived by using SVTNN

4.1.2. The Index $\ddot{\tilde{C}}_{pk}$

The general definition of $\ddot{\tilde{C}}_{pk}$ index can be obtained as below:

$$\ddot{\tilde{C}}_{pk} = min \left\{ \ddot{\tilde{C}}_{pl}, \ddot{\tilde{C}}_{pu} \right\} \quad (4.6)$$

and

$$\ddot{\tilde{C}}_{pk} = min \left\{ \frac{\mu \ominus \widetilde{L\ddot{S}L}}{3\sigma}, \frac{\widetilde{U\ddot{S}L} \ominus \mu}{3\sigma} \right\} \quad (4.7)$$

Similarly, the index $\ddot{\tilde{C}}_{pk}$ is calculated by using arithmetic operations of SVNNs in Definition (3.5) as follows:

$$\ddot{\tilde{C}}_{pk} = min \left\{ \frac{\mu \ominus \left\langle (lsl_1, lsl_2, lsl_3); w_{\widetilde{LSL}}, u_{\widetilde{LSL}}, y_{\widetilde{LSL}} \right\rangle}{3\sigma}, \frac{\left\langle (usl_1, usl_2, usl_3); w_{\widetilde{USL}}, u_{\widetilde{USL}}, y_{\widetilde{USL}} \right\rangle \ominus \mu}{3\sigma} \right\} \quad (4.8)$$

then

$$\ddot{\tilde{C}}_{pk} = min \left\{ \left\langle \left(\frac{\mu - lsl_3}{3\sigma}, \frac{\mu - lsl_2}{3\sigma}, \frac{\mu - lsl_1}{3\sigma} \right); w_{\widetilde{LSL}}, u_{\widetilde{LSL}}, y_{\widetilde{LSL}} \right\rangle ; \left\langle \left(\frac{usl_1 - \mu}{3\sigma}, \frac{usl_2 - \mu}{3\sigma}, \frac{usl_3 - \mu}{3\sigma} \right); w_{\widetilde{USL}}, u_{\widetilde{USL}}, y_{\widetilde{USL}} \right\rangle \right\} \quad (4.9)$$

These one-sided capability indices are calculated by using Eq. (2.2). Additionally, the indices $\ddot{\tilde{C}}_{pl}$ and $\ddot{\tilde{C}}_{pu}$ named as one-sided capability indices can be also obtained by using SVTrNNs as follows:

$$\ddot{\tilde{C}}_{pl} = \left\langle \left(\frac{\mu - lsl_3}{3\sigma}, \frac{\mu - lsl_2}{3\sigma}, \frac{\mu - lsl_1}{3\sigma} \right); w_{\widetilde{LSL}}, u_{\widetilde{LSL}}, y_{\widetilde{LSL}} \right\rangle \quad (4.10)$$

and

$$\ddot{\tilde{C}}_{pu} = \left\langle \left(\frac{usl_1 - \mu}{3\sigma}, \frac{usl_2 - \mu}{3\sigma}, \frac{usl_3 - \mu}{3\sigma} \right); w_{\widetilde{USL}}, u_{\widetilde{USL}}, y_{\widetilde{USL}} \right\rangle \quad (4.11)$$

Then, the MFs of the index $\tilde{\ddot{C}}_{pk}$ is shown in Figure 4.5.

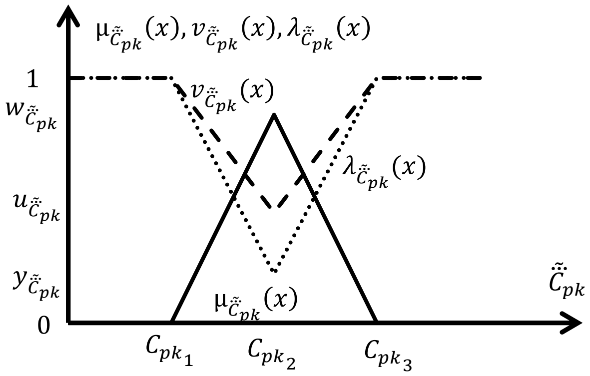

Figure 4.5. The MFs of the index $\tilde{\ddot{C}}_{pk}$ derived by using SVTrNN

The index $\tilde{\ddot{C}}_{pk}$ is derived by using SVTNNs by using similar operations as follows:

$$\tilde{\ddot{C}}_{pk} = min\left\{\frac{\mu\ominus\left\langle(lsl_1,lsl_2,lsl_3,lsl_4);w_{\widetilde{\ddot{LSL}}},u_{\widetilde{\ddot{LSL}}},y_{\widetilde{\ddot{LSL}}}\right\rangle}{3\sigma},\frac{\left\langle(usl_1,usl_2,usl_3,usl_4);w_{\widetilde{\ddot{USL}}},u_{\widetilde{\ddot{USL}}},y_{\widetilde{\ddot{USL}}}\right\rangle\ominus\mu}{3\sigma}\right\} \tag{4.12}$$

The Eq. (4.12) can be reconsider as follows:

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{c}\left\langle\left(\frac{\mu-lsl_4}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right);w_{\widetilde{\ddot{LSL}}},u_{\widetilde{\ddot{LSL}}},y_{\widetilde{\ddot{LSL}}}\right\rangle;\\ \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4-\mu}{3\sigma}\right);w_{\widetilde{\ddot{USL}}},u_{\widetilde{\ddot{USL}}},y_{\widetilde{\ddot{USL}}}\right\rangle\end{array}\right\} \tag{4.13}$$

Additionally, the indices $\tilde{\ddot{C}}_{pl}$ and $\tilde{\ddot{C}}_{pu}$ can be obtained by using SVTNNs as follows:

$$\tilde{\ddot{C}}_{pl} = \left\langle\left(\frac{\mu-lsl_4}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right);w_{\widetilde{\ddot{LSL}}},u_{\widetilde{\ddot{LSL}}},y_{\widetilde{\ddot{LSL}}}\right\rangle \tag{4.14}$$

and

$$\tilde{\ddot{C}}_{pu} = \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4-\mu}{3\sigma}\right);w_{\widetilde{\ddot{USL}}},u_{\widetilde{\ddot{USL}}},y_{\widetilde{\ddot{USL}}}\right\rangle \tag{4.15}$$

The minimum value for the C_{pk} index is calculated with the score and accuracy functions in Definitions (3.6) and (3.7). Based on SVTNNs, the MFs of the index $\tilde{\ddot{C}}_{pk}$ is shown in Figure 4.6.

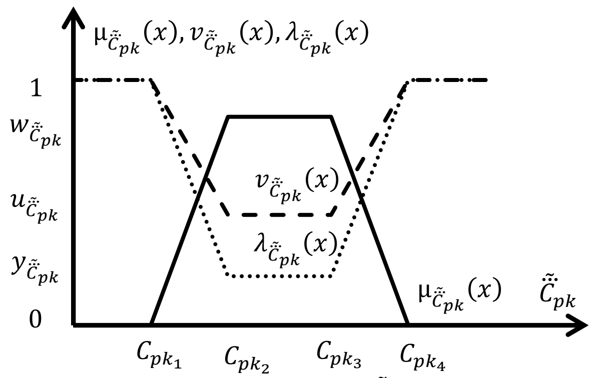

Figure 4.6. The MFs of the index $\tilde{\ddot{C}}_{pk}$ derived by using SVTNN

4.2. The Case That Specifications Limits are Flexible Structure Neutrosophic Numbers

In this subsection, we consider the case that degrees of truth, falsity and indeterminacy are independent. For this subsection, a real case example can be given as follows: Assume a person considers to surely vote for the political party X in an election (truth function). Another person considers to surely vote against the political party X (falsity function). Furthermore, another person hesitates to vote for the political party X (indeterminacy function) (Chakraborty et al. 2019). When the NSs are expressed as Definition (3.9) instead of Definition (3.4), the defined SLs can gain more flexibility. The flexibility of $\widetilde{\ddot{S}Ls}$ allow PCA to yield more realistic, sensitive and extended results. Now assume that the SLs are defined by using SVTrNN in flexible structure as follow:

$\widetilde{L\ddot{S}L} = \langle (lsl_1, lsl_2, lsl_3), (lsl'_1, lsl'_2, lsl'_3), (lsl''_1, lsl''_2, lsl''_3) \rangle$ and

$\widetilde{U\ddot{S}L} = \langle (usl_1, usl_2, usl_3), (usl'_1, usl'_2, usl'_3), (usl''_1, usl''_2, usl''_3) \rangle$.

The MFs of the $(\widetilde{\ddot{S}Ls})$ as SVTrNN are obtained in a flexible structure as shown in Figure 4.7.

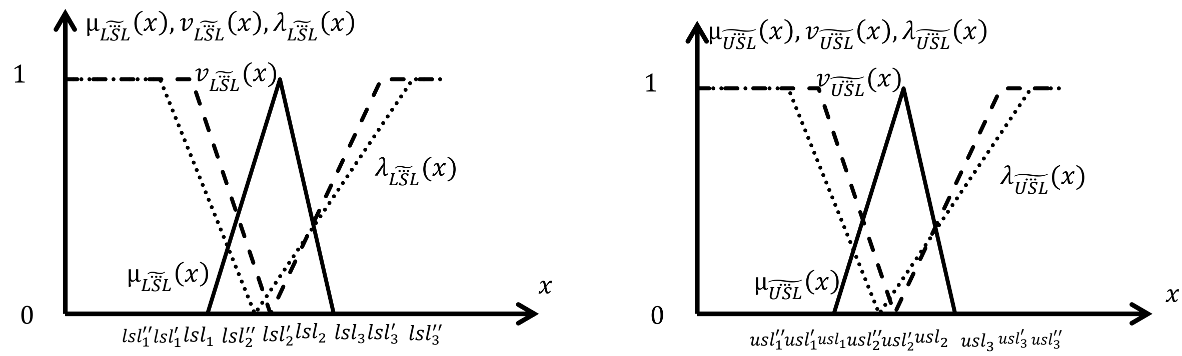

Figure 4.7. (a) The MFs of $\widetilde{L\ddot{S}L}$ derived by using SVTrNN (b) The MFs of $\widetilde{U\ddot{S}L}$

Now assume that the SLs are defined by using SVTNN in flexible structure as follow:

$\widetilde{L\ddot{S}L} = \langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl'_1, lsl'_2, lsl'_3, lsl'_4), (lsl''_1, lsl''_2, lsl''_3, lsl''_4) \rangle$ and

$\widetilde{U\ddot{S}L} = \langle (usl_1, usl_2, usl_3, usl_4), (usl'_1, usl'_2, usl'_3, usl'_4), (usl''_1, usl''_2, usl''_3, usl''_4) \rangle$.

The MFs of $\widetilde{\ddot{S}Ls}$ based on SVTNN in a flexible structure are shown in Figure 4.8.

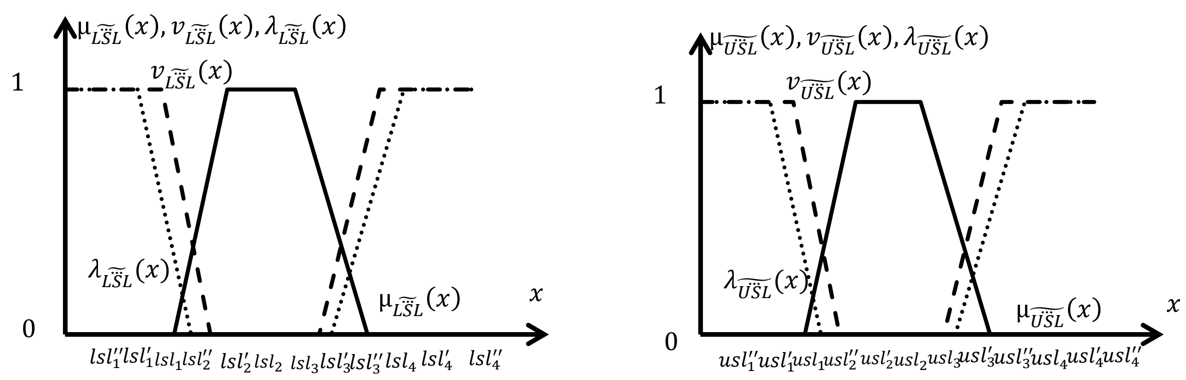

Figure 4.8. (a) The MFs of $\widetilde{L\ddot{S}L}$ derived by using SVTNN. (b) The MFs of $\widetilde{U\ddot{S}L}$ derived by using SVTNN

4.2.1. The Index $\tilde{\ddot{C}}_p$

The index $\tilde{\ddot{C}}_p$ based on SVTrNN in a flexible structure can be obtained by using Eq. (4.16) as follows:

$$\tilde{\ddot{C}}_p = \frac{\begin{array}{c}\langle (usl_1, usl_2, usl_3), (usl'_1, usl'_2, usl'_3), (usl''_1, usl''_2, usl''_3) \rangle \ominus \\ \langle (lsl_1, lsl_2, lsl_3), (lsl'_1, lsl'_2, lsl'_3), (lsl''_1, lsl''_2, lsl''_3) \rangle\end{array}}{6\sigma} \tag{4.16}$$

Then, it converts to Eq. (4.17) as shown in below:

$$\tilde{\ddot{C}}_p = \left\langle \left(\frac{usl_1 - lsl_3}{6\sigma}, \frac{usl_2 - lsl_2}{6\sigma}, \frac{usl_3 - lsl_1}{6\sigma}\right), \left(\frac{usl'_1 - lsl'_3}{6\sigma}, \frac{usl'_2 - lsl'_2}{6\sigma}, \frac{usl'_3 - lsl'_1}{6\sigma}\right), \left(\frac{usl''_1 - lsl''_3}{6\sigma}, \frac{usl''_2 - lsl''_2}{6\sigma}, \frac{usl''_3 - lsl''_1}{6\sigma}\right) \right\rangle \tag{4.17}$$

The asymmetric view of MFs for the index $\tilde{\ddot{C}}_p$ is shown in Figure 4.9.

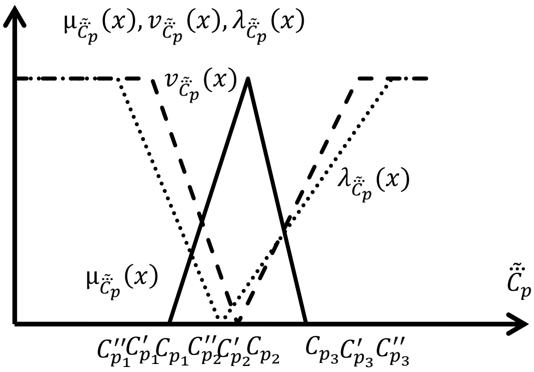

Figure 4.9. The MFs for the index $\tilde{\ddot{C}}_p$ derived by using SVTrNN

Similarly, the index $\tilde{\ddot{C}}_p$ based on SVTNN in a flexible structure like Eq. (4.2) and Eq. (4.3) can also be obtained by using following equations:

$$\tilde{\ddot{C}}_p = \frac{\begin{array}{c}\langle (usl_1, usl_2, usl_3, usl_4), (usl'_1, usl'_2, usl'_3, usl'_4), (usl''_1, usl''_2, usl''_3, usl''_4) \rangle \ominus \\ \langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl'_1, lsl'_2, lsl'_3, lsl'_4), (lsl''_1, lsl''_2, lsl''_3, lsl''_4) \rangle\end{array}}{6\sigma} \tag{4.18}$$

or

$$\tilde{\ddot{C}}_p = \left\langle \begin{array}{c} \left(\frac{usl_1 - lsl_4}{6\sigma}, \frac{usl_2 - lsl_3}{6\sigma}, \frac{usl_3 - lsl_2}{6\sigma}, \frac{usl_4 - lsl_1}{6\sigma}\right), \left(\frac{usl'_1 - lsl'_4}{6\sigma}, \frac{usl'_2 - lsl'_3}{6\sigma}, \frac{usl'_3 - lsl'_2}{6\sigma}, \frac{usl'_4 - lsl'_1}{6\sigma}\right), \\ \left(\frac{usl''_1 - lsl''_4}{6\sigma}, \frac{usl''_2 - lsl''_3}{6\sigma}, \frac{usl''_3 - lsl''_2}{6\sigma}, \frac{usl''_4 - lsl''_1}{6\sigma}\right) \end{array} \right\rangle \tag{4.19}$$

The asymmetric view of MFs for the index $\tilde{\ddot{C}}_p$ based on SVTNN in a flexible structure is shown in Figure 4.10.

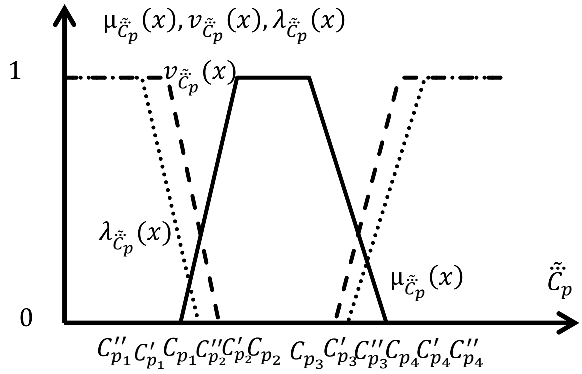

Figure 4.10. The MFs of the index $\tilde{\ddot{C}}_p$ derived by using SVTNN

4.2.2. The Index $\tilde{\ddot{C}}_{pk}$

The index $\tilde{\ddot{C}}_{pk}$ based on SVTrNN in a flexible structure can be obtained as detailed below:

$$\tilde{\ddot{C}}_{pk} = min\left\{ \begin{array}{c} \frac{\mu \ominus \langle (lsl_1, lsl_2, lsl_3), (lsl'_1, lsl'_2, lsl'_3), (lsl''_1, lsl''_2, lsl''_3) \rangle}{3\sigma}, \\ \frac{\langle (usl_1, usl_2, usl_3), (usl'_1, usl'_2, usl'_3), (usl''_1, usl''_2, usl''_3) \rangle \ominus \mu}{3\sigma} \end{array} \right\} \tag{4.20}$$

or

$$\tilde{\ddot{C}}_{pk} = min\left\{ \begin{array}{c} \left\langle \left(\frac{\mu - lsl_3}{3\sigma}, \frac{\mu - lsl_2}{3\sigma}, \frac{\mu - lsl_1}{3\sigma} \right), \left(\frac{\mu - lsl'_3}{3\sigma}, \frac{\mu - lsl'_2}{3\sigma}, \frac{\mu - lsl'_1}{3\sigma} \right), \left(\frac{\mu - lsl''_3}{3\sigma}, \frac{\mu - lsl''_2}{3\sigma}, \frac{\mu - lsl''_1}{3\sigma} \right) \right\rangle; \\ \left\langle \left(\frac{usl_1 - \mu}{3\sigma}, \frac{usl_2 - \mu}{3\sigma}, \frac{usl_3 - \mu}{3\sigma} \right), \left(\frac{usl'_1 - \mu}{3\sigma}, \frac{usl'_2 - \mu}{3\sigma}, \frac{usl'_3 - \mu}{3\sigma} \right), \left(\frac{usl''_1 - \mu}{3\sigma}, \frac{usl''_2 - \mu}{3\sigma}, \frac{usl''_3 - \mu}{3\sigma} \right) \right\rangle \end{array} \right\} \tag{4.21}$$

Then, the one-sided capability indices as using the arithmetic operators of the SVNNs in Eq. (3.5) are as follows:

$$\tilde{\ddot{C}}_{pl} = \left\langle \left(\frac{\mu - lsl_3}{3\sigma}, \frac{\mu - lsl_2}{3\sigma}, \frac{\mu - lsl_1}{3\sigma} \right), \left(\frac{\mu - lsl'_3}{3\sigma}, \frac{\mu - lsl'_2}{3\sigma}, \frac{\mu - lsl'_1}{3\sigma} \right), \left(\frac{\mu - lsl''_3}{3\sigma}, \frac{\mu - lsl''_2}{3\sigma}, \frac{\mu - lsl''_1}{3\sigma} \right) \right\rangle \tag{4.22}$$

and

$$\tilde{\ddot{C}}_{pu} = \left\langle \left(\frac{usl_1 - \mu}{3\sigma}, \frac{usl_2 - \mu}{3\sigma}, \frac{usl_3 - \mu}{3\sigma} \right), \left(\frac{usl'_1 - \mu}{3\sigma}, \frac{usl'_2 - \mu}{3\sigma}, \frac{usl'_3 - \mu}{3\sigma} \right), \left(\frac{usl''_1 - \mu}{3\sigma}, \frac{usl''_2 - \mu}{3\sigma}, \frac{usl''_3 - \mu}{3\sigma} \right) \right\rangle \tag{4.23}$$

The asymmetric view of MFs for the index $\tilde{\ddot{C}}_{pk}$ based on SVTrNN in a flexible structure is shown in Figure 4.11.

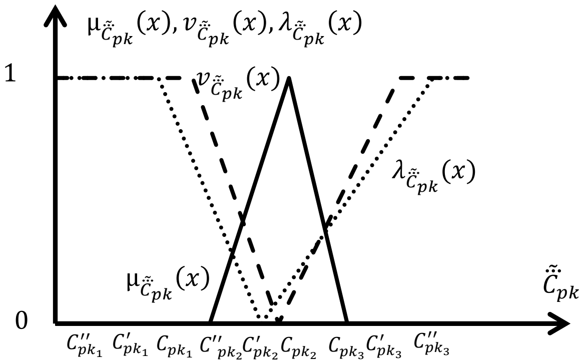

Figure 4.11. The MFs of the index $\tilde{\ddot{C}}_{pk}$ derived by using SVTrNN

Additionally, the index $\tilde{\ddot{C}}_{pk}$ based on SVTNN in a flexible structure like Eq. (4.12) can be obtained as follows:

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{c}\frac{\mu\ominus\langle(lsl_1,lsl_2,lsl_3,lsl_4),(lsl'_1,lsl'_2,lsl'_3,lsl'_4),(lsl''_1,lsl''_2,lsl''_3,lsl''_4)\rangle}{3\sigma},\\ \frac{\langle(usl_1,usl_2,usl_3,usl_4),(usl'_1,usl'_2,usl'_3,usl'_4),(usl''_1,usl''_2,usl''_3,usl''_4)\rangle\ominus\mu}{3\sigma}\end{array}\right\} \tag{4.24}$$

or

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{c}\left\langle\left(\frac{\mu-lsl_4}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right),\left(\frac{\mu-lsl'_4}{3\sigma},\frac{\mu-lsl'_3}{3\sigma},\frac{\mu-lsl'_2}{3\sigma},\frac{\mu-lsl'_1}{3\sigma}\right),\left(\frac{\mu-lsl''_4}{3\sigma},\frac{\mu-lsl''_3}{3\sigma},\frac{\mu-lsl''_2}{3\sigma},\frac{\mu-lsl''_1}{3\sigma}\right)\right\rangle;\\ \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4-\mu}{3\sigma}\right),\left(\frac{usl'_1-\mu}{3\sigma},\frac{usl'_2-\mu}{3\sigma},\frac{usl'_3-\mu}{3\sigma},\frac{usl'_4-\mu}{3\sigma}\right),\left(\frac{usl''_1-\mu}{3\sigma},\frac{usl''_2-\mu}{3\sigma},\frac{usl''_3-\mu}{3\sigma},\frac{usl''_4-\mu}{3\sigma}\right)\right\rangle\end{array}\right\} \tag{4.25}$$

Thus, the indices $\tilde{\ddot{C}}_{pl}$ and $\tilde{\ddot{C}}_{pu}$ are obtained by using SVTNN as follows:

$$\tilde{\ddot{C}}_{pl} = \left\langle\left(\frac{\mu-lsl_4}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right),\left(\frac{\mu-lsl'_4}{3\sigma},\frac{\mu-lsl'_3}{3\sigma},\frac{\mu-lsl'_2}{3\sigma},\frac{\mu-lsl'_1}{3\sigma}\right),\left(\frac{\mu-lsl''_4}{3\sigma},\frac{\mu-lsl''_3}{3\sigma},\frac{\mu-lsl''_2}{3\sigma},\frac{\mu-lsl''_1}{3\sigma}\right)\right\rangle \tag{4.32}$$

$$\tilde{\ddot{C}}_{pu} = \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4-\mu}{3\sigma}\right),\left(\frac{usl'_1-\mu}{3\sigma},\frac{usl'_2-\mu}{3\sigma},\frac{usl'_3-\mu}{3\sigma},\frac{usl'_4-\mu}{3\sigma}\right),\left(\frac{usl''_1-\mu}{3\sigma},\frac{usl''_2-\mu}{3\sigma},\frac{usl''_3-\mu}{3\sigma},\frac{usl''_4-\mu}{3\sigma}\right)\right\rangle \tag{4.26}$$

The minimum value of the C_{pk} index is calculated with the score and accuracy functions in Definitions (3.10) and (3.11). Consequently, the asymmetric view of MFs for the index $\tilde{\ddot{C}}_{pk}$ based on SVTNN is obtained as shown in Figure 4.16.

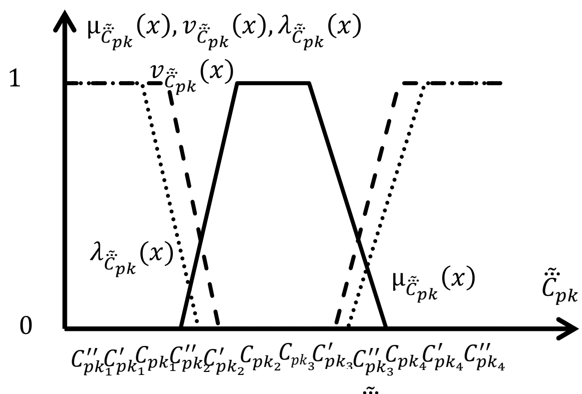

Figure 4.12. The MFs of the index $\tilde{\ddot{C}}_{pk}$ derived by using SVTNN

4.3. The Case That Specifications Limits are More Flexible Structure of Neutrosophic numbers

In this subsection, a new case related with a more flexible structure that is obtained by using the SLs. Instead of expressed as Definition (3.4), the expanding border values of truth, indeterminacy and falsity MFs provide more flexibility to SLs. The flexibility of the SLs makes them easier to apply for real life problems.

Now assume that the SLs are defined in a more flexible structure of SVTrNN as

$\widetilde{\ddot{LSL}} = \langle (lsl_1, lsl_2, lsl_3), (lsl'_1, lsl_2, lsl'_3), (lsl''_1, lsl_2, lsl''_3) \rangle$ and

$\widetilde{\ddot{USL}} = \langle (usl_1, usl_2, usl_3), (usl'_1, usl_2, usl'_3), (usl''_1, usl_2, usl''_3) \rangle.$

The asymmetric view of MFs for the $\widetilde{\ddot{SLs}}$ as SVTrNN are obtained in a more flexible structure as shown in Figure 4.13.

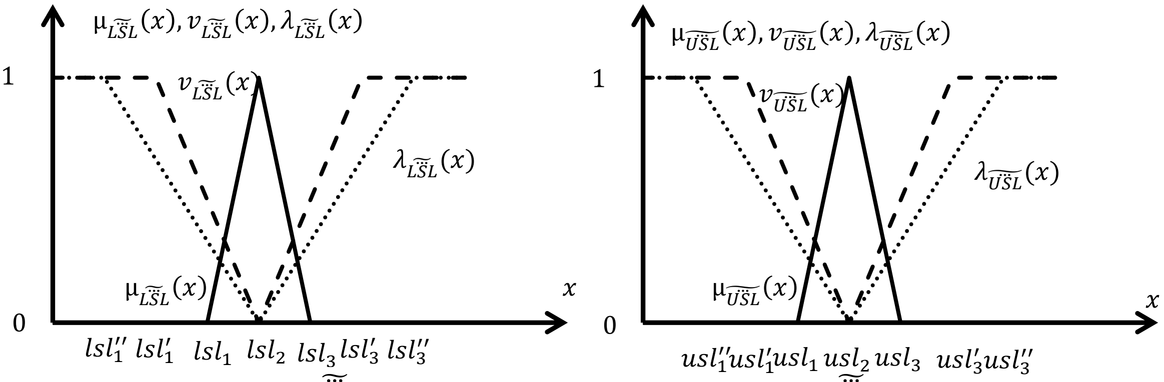

Figure 4.13. (a) The MFs of $\widetilde{\ddot{LSL}}$ for more flexible structure (b) The MFs of $\widetilde{\ddot{USL}}$ for more flexible structure

SLs are defined in a more flexible structure of SVTNN as

$\widetilde{\ddot{LSL}} = \langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl'_1, lsl_2, lsl_3, lsl'_4), (lsl''_1, lsl_2, lsl_3, lsl''_4) \rangle$ and

$\widetilde{\ddot{USL}} = \langle (usl_1, usl_2, usl_3, usl_4), (usl'_1, usl_2, usl_3, usl'_4), (usl''_1, usl_2, usl_3, usl''_4) \rangle.$

The asymmetric view of MFs for $\widetilde{\ddot{SLs}}$ as SVTNN are obtained in a more flexible structure as shown in Figures 4.14.

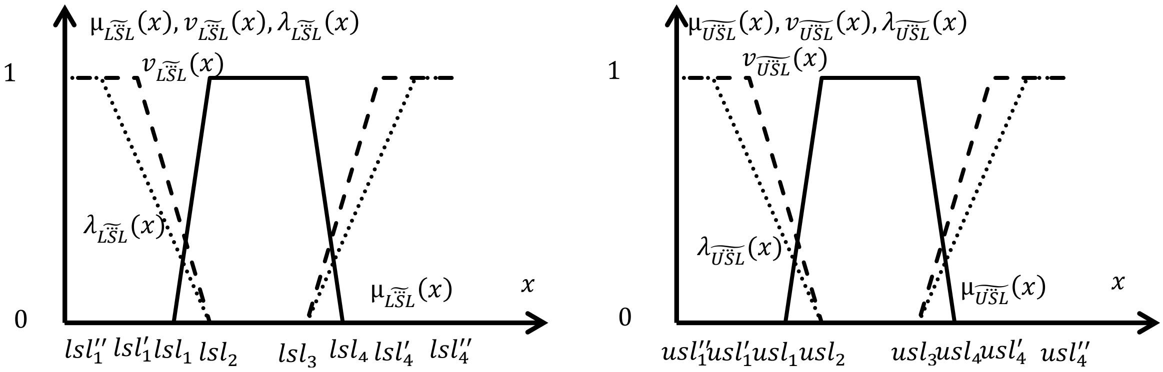

Figure 4.14. (a) The MFs of $\widetilde{\ddot{LSL}}$ for more flexible structure (b) The MFs of $\widetilde{\ddot{USL}}$ for more flexible structure

4.3.1. The Index $\tilde{\ddot{C}}_p$

The index $\tilde{\ddot{C}}_p$ for a more flexible structure that is obtained by using the arithmetic operators of the SVNNs in Eq. (3.5) that the SLs which expressed by extending the limit values of the MFs can be obtained as Eq. (4.27). For this is used the arithmetic operators of the SVNNs.

$$\tilde{\ddot{C}}_p = \frac{\langle (usl_1,usl_2,usl_3),(usl_1',usl_2,usl_3'),(usl_1'',usl_2,usl_3'')\rangle \ominus \langle (lsl_1,lsl_2,lsl_3),(lsl_1',lsl_2,lsl_3'),(lsl_1'',lsl_2,lsl_3'')\rangle}{6\sigma} \tag{4.27}$$

$$\tilde{\ddot{C}}_p = \left\langle \left(\frac{usl_1-lsl_3}{6\sigma},\frac{usl_2-lsl_2}{6\sigma},\frac{usl_3-lsl_1}{6\sigma}\right),\left(\frac{usl_1'-lsl_3'}{6\sigma},\frac{usl_2-lsl_2}{6\sigma},\frac{usl_3'-lsl_1'}{6\sigma}\right),\left(\frac{usl_1''-lsl_3''}{6\sigma},\frac{usl_2-lsl_2}{6\sigma},\frac{usl_3''-lsl_1''}{6\sigma}\right)\right\rangle \tag{4.28}$$

The asymmetric view of MFs for the index $\tilde{\ddot{C}}_p$ based on SVTrNN in a more flexible structure is shown in Figure 4.15.

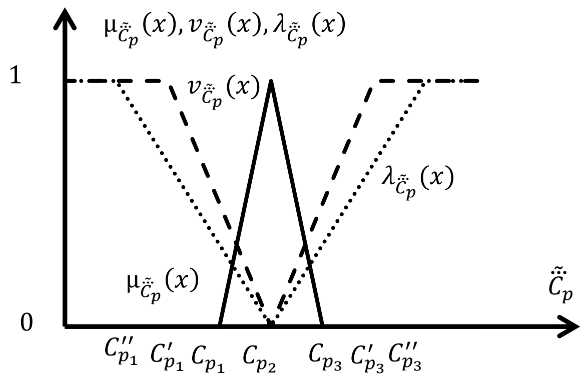

Figure 4.15. The MFs of the index $\tilde{\ddot{C}}_p$ based on SVTrNN for more flexible structure

Based on SVTNN, the index $\tilde{\ddot{C}}_p$ can be derivate as follows:

$$\tilde{\ddot{C}}_p = \frac{\begin{array}{c}\langle (usl_1,usl_2,usl_3,usl_4),(usl_1',usl_2,usl_3,usl_4'),(usl_1'',usl_2,usl_3,usl_4'')\rangle \ominus \\ \langle (lsl_1,lsl_2,lsl_3,lsl_4),(lsl_1',lsl_2,lsl_3,lsl_4'),(lsl_1'',lsl_2,lsl_3,lsl_4'')\rangle\end{array}}{6\sigma} \tag{4.29}$$

So, the Eq. (4.30) can be written in a more flexible structure as shown in Eq. (4.30).

$$\tilde{\ddot{C}}_p = \left\langle \begin{array}{c}\left(\frac{usl_1-lsl_4}{6\sigma},\frac{usl_2-lsl_3}{6\sigma},\frac{usl_3-lsl_2}{6\sigma},\frac{usl_4-lsl_1}{6\sigma}\right),\left(\frac{usl_1'-lsl_4'}{6\sigma},\frac{usl_2-lsl_3}{6\sigma},\frac{usl_3-lsl_2}{6\sigma},\frac{usl_4'-lsl_1'}{6\sigma}\right),\\ \left(\frac{usl_1''-lsl_4''}{6\sigma},\frac{usl_2-lsl_3}{6\sigma},\frac{usl_3-lsl_2}{6\sigma},\frac{usl_4''-lsl_1''}{6\sigma}\right)\end{array}\right\rangle \tag{4.30}$$

Then, the asymmetric view of MFs for the index $\tilde{\ddot{C}}_p$ based on SVTNN in a more flexible structure can be seen as shown in Figure 4.16.

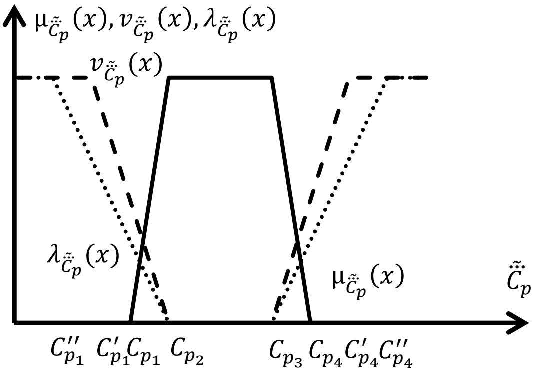

Figure 4.16. The MFs of the index $\ddot{\tilde{C}}_p$ based on SVTNN for more flexible structure

4.3.2. The Index $\ddot{\tilde{C}}_{pk}$

Similarly, the index $\ddot{\tilde{C}}_{pk}$ based on SVTrNN for a more flexible structure can be obtained as detailed in below:

$$\ddot{\tilde{C}}_{pk} = min\left\{\begin{array}{c}\frac{\mu\ominus\langle(lsl_1,lsl_2,lsl_3),(lsl'_1,lsl_2,lsl'_3),(lsl''_1,lsl_2,lsl''_3)\rangle}{3\sigma}, \\ \frac{\langle(usl_1,usl_2,usl_3),(usl'_1,usl_2,usl'_3),(usl''_1,usl_2,usl''_3)\rangle\ominus\mu}{3\sigma}\end{array}\right\} \tag{4.31}$$

and

$$\ddot{\tilde{C}}_{pk} = min\left\{\begin{array}{c}\left\langle\left(\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right),\left(\frac{\mu-lsl'_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl'_1}{3\sigma}\right),\left(\frac{\mu-lsl''_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl''_1}{3\sigma}\right)\right\rangle, \\ \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma}\right),\left(\frac{usl'_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl'_3-\mu}{3\sigma}\right),\left(\frac{usl''_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl''_3-\mu}{3\sigma}\right)\right\rangle\end{array}\right\} \tag{4.32}$$

Then, the one-sided capability indices based on SVTrNN are defined as follows:

$$\ddot{\tilde{C}}_{pl} = \left\langle\left(\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right),\left(\frac{\mu-lsl'_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl'_1}{3\sigma}\right),\left(\frac{\mu-lsl''_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl''_1}{3\sigma}\right)\right\rangle \tag{4.33}$$

and

$$\ddot{\tilde{C}}_{pu} = \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma}\right),\left(\frac{usl'_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl'_3-\mu}{3\sigma}\right),\left(\frac{usl''_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl''_3-\mu}{3\sigma}\right)\right\rangle \tag{4.34}$$

The asymmetric view of MFs for the index $\ddot{\tilde{C}}_{pk}$ based on SVTrNN for more flexible structure is drawn in Figure 4.17.

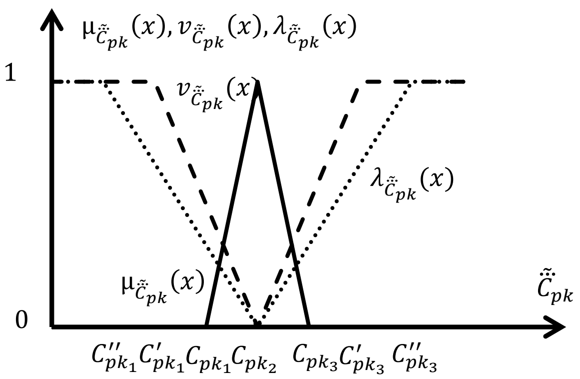

Figure 4.17. The MFs of the index $\ddot{\tilde{C}}_{pk}$ based on SVTrNN for more flexible structure

Similarly, the index $\dddot{\tilde{C}}_{pk}$ based on SVTNN for a more flexible structure is derivate as follows:

$$\dddot{\tilde{C}}_{pk} = min\left\{\begin{array}{c}\frac{\mu\ominus\langle(lsl_1,lsl_2,lsl_3,lsl_4),(lsl_1',lsl_2,lsl_3,lsl_4'),(lsl_1'',lsl_2,lsl_3,lsl_4'')\rangle}{3\sigma},\\ \frac{(usl_1,usl_2,usl_3,usl_4;usl_1',usl_2,usl_3,usl_4';usl_1'',usl_2,usl_3,usl_4'')\ominus\mu}{3\sigma}\end{array}\right\} \tag{4.35}$$

$$\dddot{\tilde{C}}_{pk} = min\left\{\begin{array}{c}\left\langle\left(\frac{\mu-lsl_4}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right),\left(\frac{\mu-lsl_4'}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1'}{3\sigma}\right),\left(\frac{\mu-lsl_4''}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1''}{3\sigma}\right)\right\rangle,\\ \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4-\mu}{3\sigma}\right),\left(\frac{usl_1'-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4'-\mu}{3\sigma}\right),\left(\frac{usl_1''-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4''-\mu}{3\sigma}\right)\right\rangle\end{array}\right\} \tag{4.36}$$

Then, the indices $\dddot{\tilde{C}}_{pl}$ and $\dddot{\tilde{C}}_{pu}$ based on SVTNN are defined as follows:

$$\dddot{\tilde{C}}_{pl} = \left\langle\left(\frac{\mu-lsl_4}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1}{3\sigma}\right),\left(\frac{\mu-lsl_4'}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1'}{3\sigma}\right),\left(\frac{\mu-lsl_4''}{3\sigma},\frac{\mu-lsl_3}{3\sigma},\frac{\mu-lsl_2}{3\sigma},\frac{\mu-lsl_1''}{3\sigma}\right)\right\rangle \tag{4.37}$$

and

$$\dddot{\tilde{C}}_{pu} = \left\langle\left(\frac{usl_1-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4-\mu}{3\sigma}\right),\left(\frac{usl_1'-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4'-\mu}{3\sigma}\right),\left(\frac{usl_1''-\mu}{3\sigma},\frac{usl_2-\mu}{3\sigma},\frac{usl_3-\mu}{3\sigma},\frac{usl_4''-\mu}{3\sigma}\right)\right\rangle \tag{4.38}$$

The minimum value of the C_{pk} index is calculated with the score and accuracy functions in Definitions (3.10) and (3.11). Consequently, the asymmetric view of MFs for the index $\dddot{\tilde{C}}_{pk}$ based on SVTNN for more flexible structure is obtained as shown in Figure 4.18.

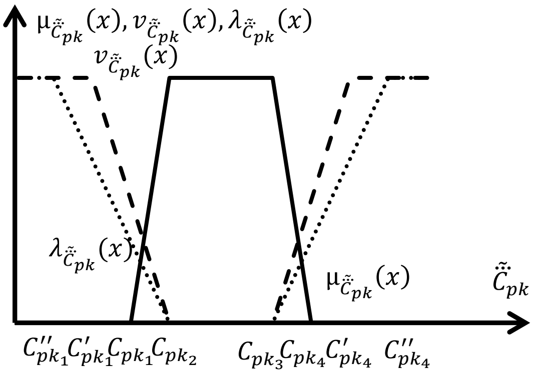

Figure 4.18. The MFs of the index $\dddot{\tilde{C}}_{pk}$ based on SVTNN for more flexible structure

5. Real Case Applications for the Proposed PCIs

In this section, some real case applications have been analyzed for the proposed methodologies. For this purpose, a company, which started its investments to produce pistons in Konya in 1998, successfully continues its automobile spare parts marketing activities has been analyzed. The main products of the company are piston, liner and piston ring. (Kaya and Kahraman 2010b; Engin et al. 2008; Kaya and Engin 2007; Kaya 2009a; Kaya 2009b). In this study, piston diameter measurements have been handled as an application. The piston is an apparatus connected to the crank system in motor vehicles. It is possible to extend the life of the engine by minimizing the wear of the piston. Although it may seem like a simple apparatus, the piston is one of the most important parts in a motor vehicle, the measurement of which require engineering knowledge. The piston dimensions are shown in Fig. 5.1.

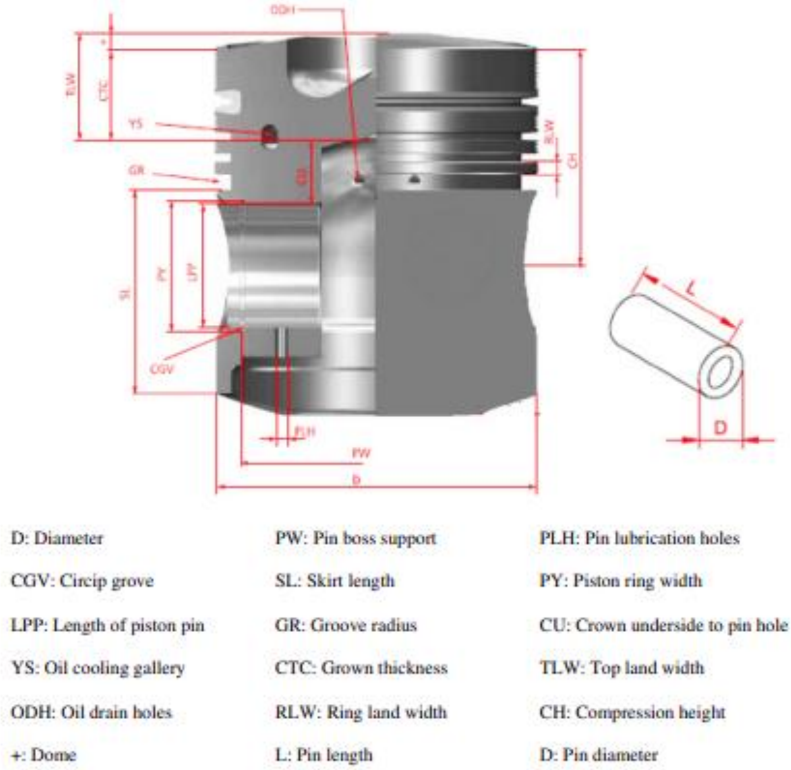

Figure 5.1. The representation of piston dimensions (Kaya and Kahraman 2011a)

SLs are a set of predetermined rules in order to produce a product that meets the expectations of the customer. To improve the flexibility and sensitiveness, the SLs have been defined by using NSs. Defining the piston diameter, which requires sensitive measurement, with SVNNs will increase the sensitivity, flexibility and information content of the results of the PCA. For the application stage, three real case problems based on piston production stages have been analyzed by using the proposed NPCIs. NPCIs is a new tool and one of the most important indicators in measuring the quality. Instead of analyzing SLs with exact numbers or crisp definitions due to time, cost and sampling difficulty, the quality inspector's definition of SLs with NSs saves time and cost. The produced pistons have been examined and the SLs have been defined as SVNNs by quality inspectors to improve the sensitivity and flexibility. The mean and variance values of piston diameters are determined as 130.17931 and 0.000096341, respectively by using historical data of measurements for diameters. For three cases detailed above, the indices $\ddot{\tilde{C}}_p$ and $\ddot{\tilde{C}}_{pk}$ have been obtained as explained below:

5.1. A Real Case Application for The Case That Specifications Limits are Single-Valued Neutrosophic Numbers

A real case application for piston diameter has been provided in this subsection to illustrate the performance of the proposed approach for the NPCIs. The SLs based SVTrNN and SVTNN for these dimensions can be defined by quality engineers as follows:

$\widetilde{\widetilde{LSL}} = \langle(130.1490, 130.1500, 130.1510); 0.95, 0.05, 0.03\rangle$ and

$\widetilde{\widetilde{USL}} = \langle(130.2090, 130.2100, 130.2110); 0.95, 0.05, 0.03\rangle$;

$\widetilde{\widetilde{LSL}} = \langle(130.1490, 130.1500, 130.1510, 130.1520); 0.950, 0.055, 0.025\rangle$ and

$\widetilde{\widetilde{USL}} = \langle(130.2090, 130.2100, 130.2110, 130.2120); 0.950, 0.055, 0.025\rangle$,

respectively.

The indices $\tilde{\tilde{C}}_p$ and $\tilde{\tilde{C}}_{pk}$ based SVTrNN are calculated as follows:

$$\tilde{\tilde{C}}_p = \left\langle \begin{matrix} \left(\dfrac{130.2090 - 130.1510}{6x0.00981}, \dfrac{130.2100 - 130.1500}{6x0.00981}, \dfrac{130.2110 - 130.1490}{6x0.00981}\right); \\ 0.95 \wedge 0.95, 0.05 \vee 0.05, 0.03 \vee 0.03 \end{matrix} \right\rangle$$

$$\tilde{\tilde{C}}_p = \langle(0.98, 1.02, 1.05); 0.95, 0.05, 0.03\rangle$$

$$\tilde{\tilde{C}}_{pk} = min \left\{ \begin{matrix} \left\langle\left(\dfrac{130.1793 - 130.1510}{3x0.00981}, \dfrac{130.1793 - 130.1500}{3x0.00981}, \dfrac{130.1793 - 130.1490}{3x0.00981}\right); 0.95, 0.05, 0.03\right\rangle; \\ \left\langle\left(\dfrac{130.2090 - 130.1793}{3x0.00981}, \dfrac{130.2100 - 130.1793}{3x0.00981}, \dfrac{130.2110 - 130.1793}{3x0.00981}\right); 0.95, 0.05, 0.03\right\rangle \end{matrix} \right\}$$

$$\tilde{\tilde{C}}_{pk} = min\{\langle(0.96, 1.02, 1.05); 0.95, 0.05, 0.03\rangle; \langle(1.01, 1.04, 1.08); 0.95, 0.05, 0.03\rangle\}$$

$$\tilde{\tilde{C}}_{pk} = \langle((0.96, 1.02, 1.05)); 0.95, 0.05, 0.03\rangle.$$

The score values of $\tilde{\tilde{C}}_{pl}$ and $\tilde{\tilde{C}}_{pu}$ indices are calculated by using Eq. (3.18). Score values are determined as 0.95 and 1.00, respectively. Since $\tilde{\tilde{C}}_{pl}$ index is smaller than $\tilde{\tilde{C}}_{pu}$ index, it was chosen as $\tilde{\tilde{C}}_{pk}$ index. The process capability of this process within the based on crisp SLs is calculated approximately 1.02. However, the quality inspectors may not be able to define these SLs precisely. Consequently, the indeterminacy and falsity degrees of the process is expressed as 0.05 and 0.03.

Then, the indices $\tilde{\tilde{C}}_p$ and $\tilde{\tilde{C}}_{pk}$ based on SVTNN indices are calculated as follows:

$$\tilde{\tilde{C}}_p = \left\langle \begin{matrix} \left(\dfrac{130.2090 - 130.1520}{6x0.00981}, \dfrac{130.2100 - 130.1510}{6x0.00981}, \dfrac{130.2110 - 130.1500}{6x0.00981}, \dfrac{130.2120 - 130.1490}{6x0.00981}\right); \\ 0.950 \wedge 0.950, 0.055 \vee 0.055, 0.025 \vee 0.025 \end{matrix} \right\rangle$$

$\ddot{\tilde{C}}_p = \langle (0.97,1.00,1.04,1.07);0.950,0.055,0.025\rangle$

$$\ddot{\tilde{C}}_{pk} = min\left\{ \begin{array}{l} \left\langle \left(\frac{130.1793-130.1520}{3x0.00981}, \frac{130.1793-130.1510}{3x0.00981}, \frac{130.1793-130.1500}{3x0.00981}, \frac{130.1793-130.1490}{3x0.00981} \right); 0.950,0.055,0.025 \right\rangle; \\ \left\langle \left(\frac{130.2090-130.1793}{3x0.00981}, \frac{130.2100-130.1793}{3x0.00981}, \frac{130.2110-130.1793}{3x0.00981}, \frac{130.2120-130.1793}{3x0.00981} \right); 0.950,0.055,0.025 \right\rangle \end{array} \right\}$$

$$\ddot{\tilde{C}}_{pk} = min\{\langle (0.93,0.96,1.00,1.03);0.950,0.055,0.025\rangle;\langle (1.01,1.04,1.08,1.11);0.950,0.055,0.025\rangle\}$$

$$\ddot{\tilde{C}}_{pk} = \langle (0.93,0.96,1.00,1.03);0.950,0.055,0.025\rangle$$

The score values of $\ddot{\tilde{C}}_{pl}$ and $\ddot{\tilde{C}}_{pu}$ indices are calculated by using Eq. (3.20). Score values are found as 0.94 and 1.01, respectively. Since $\ddot{\tilde{C}}_{pl}$ index is smaller than $\ddot{\tilde{C}}_{pu}$ index, it was chosen as $\ddot{\tilde{C}}_{pk}$ index.

5.2. A Real Case Application for The Case That Specifications Limits are Flexible Structure of Neutrosophic Numbers

A real case application for SLs that define in a flexible environment is also analyzed based on similar piston diameter measurements in this subsection. The SLs for a flexible structure based on SVTrNN and SVTNN, respectively can be defined as follows:

$$\widetilde{\widetilde{LSL}} = \left\langle \begin{array}{c} (130.1490,130.1500,130.1510),(130.1480,130.1496,130.1520), \\ (130.1470,130.14930,130.1530) \end{array} \right\rangle,$$

$$\widetilde{\widetilde{USL}} = \left\langle \begin{array}{c} (130.2090,130.2100,130.2110),(130.2080,130.2096,130.2120), \\ (130.2070,130.2093,130.2130) \end{array} \right\rangle.$$

and

$$\widetilde{\widetilde{LSL}} = \left\langle \begin{array}{c} (130.1490,130.1500,130.1510,130.1520),(130.1480,130.1496,130.1513,130.1530), \\ (130.1470,130.1493,130.1516,130.1540) \end{array} \right\rangle$$

$$\widetilde{\widetilde{USL}} = \left\langle \begin{array}{c} (130.2090,130.2100,130.2110,130.2120),(130.2080,130.2096,130.2113,130.2130), \\ (130.2070,130.2093,130.2116,130.2140) \end{array} \right\rangle.$$

The indices $\ddot{\tilde{C}}_p$ and $\ddot{\tilde{C}}_{pk}$ based on SVTrNN for flexible structure are obtained as follows:

$$\ddot{\tilde{C}}_p = \left\langle \begin{array}{l} \left(\frac{130.2090-130.1510}{6x0.00981}, \frac{130.2100-130.1500}{6x0.00981}, \frac{130.2110-130.1490}{6x0.00981} \right), \\ \left(\frac{130.2080-130.1520}{6x0.00981}, \frac{130.2096-130.1496}{6x0.00981}, \frac{130.2120-130.1480}{6x0.00981} \right), \\ \left(\frac{130.2070-130.1530}{6x0.00981}, \frac{130.2093-130.14930}{6x0.00981}, \frac{130.2130-130.1470}{6x0.00981} \right) \end{array} \right\rangle$$

$\ddot{\tilde{C}}_p = \langle (0.98,1.02,1.05),(0.95,1.02,1.09),(0.92,1.02,1.12)\rangle.$

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{l}\left\langle\begin{array}{l}\left(\frac{130.1793-130.1510}{3x0.00981},\frac{130.1793-130.1500}{3x0.00981},\frac{130.1793-130.1490}{3x0.00981}\right),\\ \left(\frac{130.1793-130.1520}{3x0.00981},\frac{130.1793-130.1496}{3x0.00981},\frac{130.1793-130.1480}{3x0.00981}\right),\\ \left(\frac{130.1793-130.1530}{3x0.00981},\frac{130.1793-130.14930}{3x0.00981},\frac{130.1793-130.1470}{3x0.00981}\right)\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\frac{130.2090-130.1793}{3x0.00981},\frac{130.2100-130.1793}{3x0.00981},\frac{130.2110-130.1793}{3x0.00981}\right),\\ \left(\frac{130.2080-130.1793}{3x0.00981},\frac{130.2096-130.1793}{3x0.00981},\frac{130.2120-130.1793}{3x0.00981}\right),\\ \left(\frac{130.2070-130.1793}{3x0.00981},\frac{130.2093-130.1793}{3x0.00981},\frac{130.2130-130.1793}{3x0.00981}\right)\end{array}\right\rangle\end{array}\right\}$$

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{l}\langle(0.96,1.00,1.03),(0.93,1.01,1.06),(0.89,1.02,1.10)\rangle,\\ \langle(1.01,1.04,1.08),(0.97,1.03,1.11),(0.94,1.02,1.14)\rangle\end{array}\right\}$$

$$\tilde{\ddot{C}}_{pk} = \langle(0.96,1.00,1.03),(0.93,1.01,1.06),(0.89,1.02,1.10)\rangle.$$

The process capability of piston diameter within the defined SLs is approximately 1.00. The neutrosophic process capability analysis shows that, considering the indeterminacy-membership function, the capability of the process cannot be less than 0.93 and more than 1.06, and considering the falsity-membership function, it cannot be less than 0.89 and more than 1.10.

The indices $\tilde{\ddot{C}}_p$ and $\tilde{\ddot{C}}_{pk}$ based on SVTNN for flexible structure are also obtained as follows:

$$\tilde{\ddot{C}}_p = \left\langle\begin{array}{l}\left(\frac{130.2090-130.1520}{6x0.00981},\frac{130.2100-130.1510}{6x0.00981},\frac{130.2110-130.1500}{6x0.00981},\frac{130.2120-130.1490}{6x0.00981}\right),\\ \left(\frac{130.2080-130.1530}{6x0.00981},\frac{130.2096-,130.1513}{6x0.00981},\frac{130.2113-130.1496}{6x0.00981},\frac{130.2130-130.1480}{6x0.00981}\right),\\ \left(\frac{130.2070-130.1540}{6x0.00981},\frac{130.2093-,130.1516}{6x0.00981},\frac{130.2116-130.1493}{6x0.00981},\frac{130.2140-130.1470}{6x0.00981}\right)\end{array}\right\rangle$$

$$\tilde{\ddot{C}}_p = \langle(0.97,1.00,1.04,1.07),(0.93,0.99,1.05,1.10),(0.90,0.98,1.06,1.14)\rangle.$$

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{l}\left\langle\begin{array}{l}\left(\frac{130.1793-130.1520}{3x0.00981},\frac{130.1793-130.1510}{3x0.00981},\frac{130.1793-130.1500}{3x0.00981},\frac{130.1793-130.1490}{3x0.00981}\right),\\ \left(\frac{130.1793-130.1530}{3x0.00981},\frac{130.1793-130.1513}{3x0.00981},\frac{130.1793-130.1496}{3x0.00981},\frac{130.1793-130.1480}{3x0.00981}\right),\\ \left(\frac{130.1793-130.1540}{3x0.00981},\frac{130.1793-130.1516}{3x0.00981},\frac{130.1793-130.1493}{3x0.00981},\frac{130.1793-130.1470}{3x0.00981}\right)\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\frac{130.2090-130.1793}{3x0.00981},\frac{130.2100-130.1793}{3x0.00981},\frac{130.2110-130.1793}{3x0.00981},\frac{130.2120-130.1793}{3x0.00981}\right),\\ \left(\frac{130.2080-130.1793}{3x0.00981},\frac{130.2096-130.1793}{3x0.00981},\frac{130.2113-130.1793}{3x0.00981},\frac{130.2130-130.1793}{3x0.00981}\right),\\ \left(\frac{130.2070-130.1793}{3x0.00981},\frac{130.2093-130.1793}{3x0.00981},\frac{130.2116-130.1793}{3x0.00981},\frac{130.2140-130.1793}{3x0.00981}\right)\end{array}\right\rangle\end{array}\right\}$$

$$\tilde{\ddot{C}}_{pk} = min\left\{\begin{array}{l}\langle(0.93,0.96,1.00,1.03),(0.89,0.95,1.01,1.06),(0.86,0.94,1.02,1.10)\rangle\\ \langle(1.01,1.04,1.08,1.11),(0.97,1.03,1.09,1.14),(0.94,1.02,1.10,1.18)\rangle\end{array}\right\}$$

$$\tilde{\ddot{C}}_{pk} = \langle(1.01,1.04,1.08,1.11),(0.97,1.03,1.09,1.14),(0.94,1.02,1.10,1.18)\rangle.$$

The score values of $\tilde{\ddot{C}}_{pl}$ and $\tilde{\ddot{C}}_{pu}$ indices are calculated by using Eq. (3.24). Score values are found as 0.340 and 0.314, respectively.

5.3. A Real Case Application for The Case That Specifications Limits are More Flexible Structure of Neutrosophic Numbers

A real case application is also analyzed based on similar piston diameter measurements for a more flexible structure in this subsection. For this aim, the quality engineers can define the SLs in a more flexible and precisely way. The SLs based on SVTrNN and SVTNN, respectively can be defined as follows:

$$\widetilde{\ddot{LSL}} = \left\langle \begin{matrix} (130.1490,130.1500,130.1510), (130.1470,130.1500,130.1530), \\ (130.1450,130.1500,130.1550) \end{matrix} \right\rangle,$$

$$\widetilde{\ddot{USL}} = \left\langle \begin{matrix} (130.2090,130.2100,130.2110), (130.2070,130.2100,130.2130), \\ (130.2050,130.2100,130.2150) \end{matrix} \right\rangle$$

and

$$\widetilde{\ddot{LSL}} = \left\langle \begin{matrix} (130.1490,130.1500,130.1510,130.1520), (130.1470,130.1500,130.1510,130.1540), \\ (130.1450,130.1500,130.1510,130.1560) \end{matrix} \right\rangle$$

$$\widetilde{\ddot{USL}} = \left\langle \begin{matrix} (130.2090,130.2100,130.2110,130.2120), (130.2070,130.2100,130.2110,130.2140), \\ (130.2050,130.2100,130.2110,130.2160) \end{matrix} \right\rangle,$$

respectively.

The indices $\tilde{\ddot{C}}_p$ and $\tilde{\ddot{C}}_{pk}$ based on SVTrNN for a more flexible structure can be obtained as follows:

$$\tilde{\ddot{C}}_p = \left\langle \begin{matrix} \left(\frac{130.2090-130.1510}{6x0.00981}, \frac{130.2100-130.1500}{6x0.00981}, \frac{130.2110-130.1490}{6x0.00981} \right), \\ \left(\frac{130.2070-130.1530}{6x0.00981}, \frac{130.2100-130.1500}{6x0.00981}, \frac{130.2130-130.1470}{6x0.00981} \right), \\ \left(\frac{130.2050-130.1550}{6x0.00981}, \frac{130.2100-130.1500}{6x0.00981}, \frac{130.2150-130.1450}{6x0.00981} \right) \end{matrix} \right\rangle$$

$$\tilde{\ddot{C}}_p = \langle (0.98,1.02,1.05); (0.92,1.02,1.12); (0.85,1.02,1.19) \rangle.$$

$$\tilde{\ddot{C}}_{pk} = min \left\{ \begin{matrix} \left\langle \begin{matrix} \left(\frac{130.1793-130.1510}{3x0.00981}, \frac{130.1793-130.1500}{3x0.00981}, \frac{130.1793-130.1490}{3x0.00981} \right), \\ \left(\frac{130.1793-130.1530}{3x0.00981}, \frac{130.1793-130.1500}{3x0.00981}, \frac{130.1793-130.1470}{3x0.00981} \right), \\ \left(\frac{130.1793-130.1550}{3x0.00981}, \frac{130.1793-130.1500}{3x0.00981}, \frac{130.1793-130.1450}{3x0.00981} \right) \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} \left(\frac{130.2090-130.1793}{3x0.00981}, \frac{130.2100-130.1793}{3x0.00981}, \frac{130.2110-130.1793}{3x0.00981} \right), \\ \left(\frac{130.2070-130.1793}{3x0.00981}, \frac{130.2100-130.1793}{3x0.00981}, \frac{130.2130-130.1793}{3x0.00981} \right), \\ \left(\frac{130.2050-130.1793}{3x0.00981}, \frac{130.2100-130.1793}{3x0.00981}, \frac{130.2150-130.1793}{3x0.00981} \right) \end{matrix} \right\rangle \end{matrix} \right\}$$

$$\tilde{\ddot{C}}_{pk} = min \left\{ \begin{matrix} \langle (0.96,1.00,1.03), (0.89,1.00,1.10), (0.83,1.00,1.17) \rangle, \\ \langle (1.01,1.04,1.08), (0.94,1.04,1.14), (0.87,1.04,1.21) \rangle \end{matrix} \right\}$$

$\tilde{\ddot{C}}_{pk} = \langle(1.01,1.04,1.08),(0.94,1.04,1.14),(0.87,1.04,1.21)\rangle.$

The score values of $\tilde{\ddot{C}}_{pl}$ and $\tilde{\ddot{C}}_{pu}$ indices are calculated by using Eq. (3.22). Since the score values of $\tilde{\ddot{C}}_{pl}$ and $\tilde{\ddot{C}}_{pu}$ indices are determined as 0.335 and 0.319, respectively. The index $\tilde{\ddot{C}}_{pk}$ is determined based on the index $\tilde{\ddot{C}}_{pu}$. The process capability of this process based on the defined SLs is approximately 1.04. Differently from this process, the neutrosophic process capability shows that, considering the indeterminacy-membership function, the capability of the process cannot be less than 0.94 and more than 1.14, and considering the falsity-membership function, it cannot be less than 0.87 and more than 1.21.

Similarly, the indices $\tilde{\ddot{C}}_{p}$ and $\tilde{\ddot{C}}_{pk}$ based on STVNN for more flexible structure can be calculated as follows:

$$\tilde{\ddot{C}}_{p} = \left\langle \begin{array}{l} \left(\frac{130.2090-130.1520}{6x0.00981},\frac{130.2100-130.1510}{6x0.00981},\frac{130.2110-130.1500}{6x0.00981},\frac{130.2120-130.1490}{6x0.00981}\right), \\ \left(\frac{130.2070-130.1540}{6x0.00981},\frac{130.2100-130.1510}{6x0.00981},\frac{130.2110-130.1500}{6x0.00981},\frac{130.2140-130.1470}{6x0.00981}\right), \\ \left(\frac{130.2050-130.1560}{6x0.00981},\frac{130.2100-130.1510}{6x0.00981},\frac{130.2110-130.1500}{6x0.00981},\frac{130.2160-130.1450}{6x0.00981}\right) \end{array} \right\rangle$$

$\tilde{\ddot{C}}_{p} = \langle(0.97,1.00,1.04,1.07),(0.90,1.00,1.04,1.14),(0.83,1.00,1.04,1.21)\rangle.$

$$\tilde{\ddot{C}}_{pk} = min \left\{ \begin{array}{l} \left\langle \begin{array}{l} \left(\frac{130.1793-130.1520}{3x0.00981},\frac{130.1793-130.1510}{3x0.00981},\frac{130.1793-130.1500}{3x0.00981},\frac{130.1793-130.1490}{3x0.00981}\right), \\ \left(\frac{130.1793-130.1540}{3x0.00981},\frac{130.1793-130.1510}{3x0.00981},\frac{130.1793-130.1500}{3x0.00981},\frac{130.1793-130.1470}{3x0.00981}\right), \\ \left(\frac{130.1793-130.1560}{3x0.00981},\frac{130.1793-130.1510}{3x0.00981},\frac{130.1793-130.1500}{3x0.00981},\frac{130.1793-130.1450}{3x0.00981}\right) \end{array} \right\rangle, \\ \left\langle \begin{array}{l} \left(\frac{130.2090-130.1793}{3x0.00981},\frac{130.2100-130.1793}{3x0.00981},\frac{130.2110-130.1793}{3x0.00981},\frac{130.2120-130.1793}{3x0.00981}\right), \\ \left(\frac{130.2070-130.1793}{3x0.00981},\frac{130.2100-130.1793}{3x0.00981},\frac{130.2110-130.1793}{3x0.00981},\frac{130.2140-130.1793}{3x0.00981}\right), \\ \left(\frac{130.2050-130.1793}{3x0.00981},\frac{130.2100-130.1793}{3x0.00981},\frac{130.2110-130.1793}{3x0.00981},\frac{130.2160-130.1793}{3x0.00981}\right) \end{array} \right\rangle \end{array} \right\}$$

$$\tilde{\ddot{C}}_{pk} = min \left\{ \begin{array}{l} \langle(0.93,0.96,1.00,1.03),(0.86,0.96,1.00,1.10),(0.79,0.96,1.00,1.17)\rangle \\ \langle(1.01,1.04,1.08,1.11),(0.94,1.04,1.08,1.18),(0.87,1.04,1.08,1.25)\rangle \end{array} \right\}$$

$\tilde{\ddot{C}}_{pk} = \langle(1.01,1.04,1.08,1.11),(0.94,1.04,1.08,1.18),(0.87,1.04,1.08,1.25)\rangle.$

Similar comments are also valid for the more flexible structure based on SVTNN.

6. The Results and Discussion

In this paper, PCA has been analyzed based on NSs. When the SLs of the piston's diameter are compared with the case where it is defined with NSs and the case where it is defined with type-1 fuzzy sets, it is obtained that some differences are valid. Since the membership degrees of type-1 fuzzy sets are expressed as crisp values and NPCIs include the functions of truth, indeterminacy and falsity, NSs provide more information, more flexibility and sensitivity in evaluating the capability of the process compared to type-1 fuzzy sets. While the neutrosophic C_p index, shows that the degree of indeterminacy is 0.05 and the degree of falsity is 0.03. The results obtained allow for a more detailed interpretation and provide a wider evaluation perspective about the process. If PCIs are analyzed with type-2 fuzzy sets, they will contain more information than type-1 fuzzy sets, but will provide both less information and less flexibility about process capability analysis compared to NPCIs. It is obtained that NSs are more advantageous than type-2 fuzzy sets in terms of ease of operation for PCA. If the PCIs are defined with IFSs, the SLs defined by NSs will provide more information as they contain more MFs with respect to IFSs. The more flexible C_p index contains more detailed information than IFSs as it clearly expresses both the indeterminacy limits and the falsity limits of the process. It is clearly seen that NSs are advantageous in real life applications due to the flexibility and expressing of uncertainty compared to the fuzzy set extensions in the PCA.

7. Conclusions

Sometimes, a quality engineer may not have enough information about each subject. For this reason, the NSs have an important role in modeling uncertainty problems. Increasing the usage areas of NSs can be beneficial in the solution of uncertain models. Additionally, it is clear that the NSs have more advantageous such as flexibility and easily adaptation on uncertainty than other the fuzzy set extensions. PCA using NSs provides wider knowledge about capability of process than the analysis that used traditional fuzzy sets. Many studies have been carried out in the literature on the examination of PCA with the help of traditional fuzzy sets. However, when studies on PCA are examined, it is noticeable that studies with the extensions of fuzzy sets such as hesitant fuzzy sets, intuitionistic fuzzy sets and type-2 fuzzy sets are very limited. In this study, PCA has been performed by examining the status of SLs to be neutrosophic with the help of NSs. We also try to integrate advantages of NSs on PCA. For this aim, traditional PCIs have been analyzed and restructured based on NSs. Then, two of the well-known PCIs named C_p and C_{pk} have been reformulated by using SVNNs. Finally, the NPCIs are obtained. As a result, the neutrosophic C_p ($\ddot{\tilde{C}}_p$) and neutrosophic C_{pk} ($\ddot{\tilde{C}}_{pk}$) indices have been obtained based on three cases. Additionally, a real case application is added and this real case application shows that the indices $\ddot{\tilde{C}}_p$ and $\ddot{\tilde{C}}_{pk}$ are more effective than the PCIs using traditional fuzzy sets under the uncertainty environment. Sensitivity analysis is the analysis of the change in the process by amending the parameter values of the process with different numbers. As the values of the SLs, mean and variance parameters in the PCI change, the sensitivity of the process analysis is also ensured. Since NSs contain the three MFs, they provide flexibility in analyzing the capability of the process, such as interval-valued sets, and help us gain more information about process capability, such as in sensitivity analysis, thanks to this flexibility. In the future studies, further development of fuzzy set extensions can be achieved by using different PCIs and obtained results can be compared.

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