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Independent Set in Neutrosophic Graphs

Henry Garrett

Independent Researcher

DrHenryGarrett@gmail.com

Twitter's ID: @DrHenryGarrett | ©DrHenryGarrett.wordpress.com

Abstract

New setting is introduced to study neutrosophic independent number and independent neutrosophic-number arising neighborhood of different vertices. Neighbor is a key term to have these notions. Having no edge amid vertices in a set is a key type of approach to have these notions namely neutrosophic independent number and independent neutrosophic-number. Two numbers are obtained but now both settings leads to approach is on demand which is finding biggest set which doesn't have some vertices which are neighbors. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then independent number $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously; independent neutrosophic-number $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-t-partite-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Neutrosophic Independent Number," and "Setting of Independent Neutrosophic-Number," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form neutrosophic independent number and independent neutrosophic-number arising neighborhoods of vertices. In path-neutrosophic graphs, either odd indexes or even indexes, forms maximal set but with slightly differences, in cycle-neutrosophic graphs, either odd indexes or even indexes, forms maximal set. Other classes have same approaches. In complete-neutrosophic graphs, a set of vertices containing one vertex leads us to neutrosophic independent number and independent neutrosophic-number. In star-neutrosophic graphs, a set of vertices excluding only center, makes maximal set. In complete-bipartite-neutrosophic graphs, a set of vertices excluding (four) vertices from one part as possible makes intended set but with slightly differences, in complete-t-partite-neutrosophic graphs, a set of vertices excluding vertices from all parts but one part, makes intended set. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. Using basic set not to extend this set to set of all vertices has key role to have these notions in the form of neutrosophic independent number and independent neutrosophic-number arising neighborhood of vertices. The cardinality of a set has eligibility to neutrosophic independent number but the neutrosophic cardinality of a set has eligibility to call independent neutrosophic-number. Some results get more frameworks and perspective about these definitions. The way in that, two vertices don't have connection amid each other, opens the way to do some approaches. A vertex could affect on other vertex but there's no usage of edges. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Neutrosophic Independent Number, Independent Neutrosophic-Number, Maximal Set

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [16], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2, 14, 15], graphs and new notions on them in **Refs.** [5–12], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [13], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "Neutrosophic Independent Number", "Independent Neutrosophic-Number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. One connection amid two vertices have key roles to assign neutrosophic independent number, independent neutrosophic-number arising neighborhood of vertices. Thus they're used to define new ideas which conclude to the structure neutrosophic independent number, independent neutrosophic-number arising neighborhood of vertices. The concept of not having edge and extra condition inspire us to study the behavior of vertices in the way that, some types of numbers, neutrosophic independent number, independent neutrosophic-number arising neighborhood of vertices are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of neutrosophic independent number, independent neutrosophic-number are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sets of vertices have the key role in this way. General results are obtained and also, the results about the basic notions of neutrosophic independent number, independent neutrosophic-number are elicited. Some classes of neutrosophic graphs are studied in

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the terms of neutrosophic independent number, in section "Setting of Neutrosophic Independent Number," as individuals. In section "Setting of Independent Neutrosophic-Number," independent neutrosophic-number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-t-partite-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Neutrosophic Independent Number," and "Setting of Independent Neutrosophic-Number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-t-neutrosophic graphs and complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

$(i):$ a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

$(ii):$ **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

$(iii):$ **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

$(iv):$ a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

$(v):$ it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

$(vi):$ t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

$(vii):$ complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii):$ a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

$(ix):$ it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

$(x):$ it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Definition 1.5. (Independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously;

(ii) **independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.6. In Figure (1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

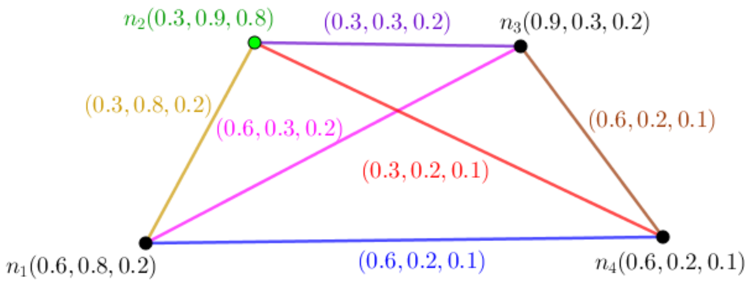

Figure 1. A Neutrosophic Graph in the Viewpoint of its Independent Number and its Independent Neutrosophic-Number.

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_2\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to both independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 1 is independent number and its corresponded sets are $\{n_1\}, \{n_2\}$, $\{n_3\}$, and $\{n_4\}$;

(vi) 0.9 is independent neutrosophic-number and its corresponded set is $\{n_4\}$.

2 Setting of Neutrosophic Independent Number

In this section, I provide some results in the setting of neutrosophic neutrosophic independent number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, and star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph are both of cases of study and classes which the results are about them.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. It induces if $S = \{n\}$ is a set of vertices, then there's no vertex in S but n. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n\}$ is corresponded to independent number. Thus

$$\mathcal{I}(NTG) = 1.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.2. In Figure (2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*ii*) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_2\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iii*) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to both independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iv*) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*v*) 1 is independent number and its corresponded sets are $\{n_1\}$, $\{n_2\}$, $\{n_3\}$, and $\{n_4\}$;

(*vi*) 0.9 is independent neutrosophic-number and its corresponded set is $\{n_4\}$.

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil.$$

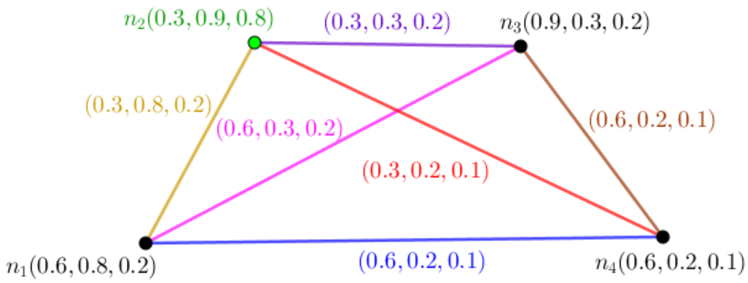

Figure 2. A Neutrosophic Graph in the Viewpoint of its Independent Number.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lceil \frac{\mathcal{O}(NTG)}{2} \rceil$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| > \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$. It implies that $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$ is corresponded to independent number. Thus

$$\mathcal{I}(NTG) = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil.$$

□

Example 2.4. There are two sections for clarifications.

(a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;
- (ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;
- (iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(v) 3 is independent number and its corresponded set is $\{n_1, n_3, n_5\}$;

(vi) 3.3 is independent neutrosophic-number and its corresponded set is $\{n_1, n_3, n_5\}$.

(b) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|\neq\lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(ii) if $S = \{n_2, n_4, n_6\}$ is a set of vertices, then there's no vertex in S but n_2, n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_2, n_4, n_6\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(v) 3 is independent number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(vi) 4.5 is independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

Proposition 2.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor$. Then there are x and y in S

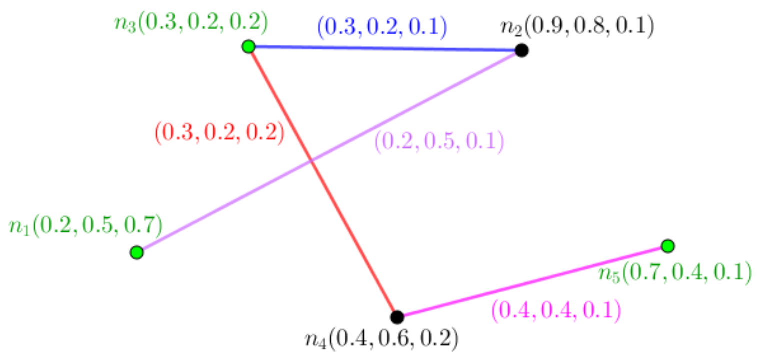

Figure 3. A Neutrosophic Graph in the Viewpoint of its Independent Number.

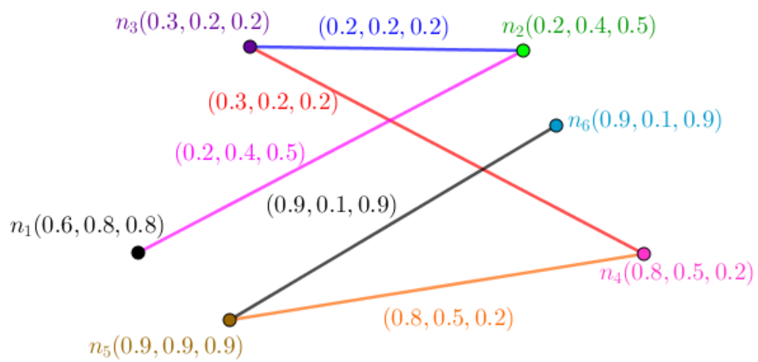

Figure 4. A Neutrosophic Graph in the Viewpoint of its Independent Number.

such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$. It implies that $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ is corresponded to independent number. Thus

$$\mathcal{I}(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6. There are two sections for clarifications.

(a) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(ii) if $S = \{n_2, n_4, n_6\}$ is a set of vertices, then there's no vertex in S but n_2, n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_2, n_4, n_6\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 3 is independent number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(vi) 3.2 is independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(b) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S and it implies that $S = \{n_2, n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(ii) if $S = \{n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It implies that $S = \{n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 or n_5, n_1 of S, it's possible to have endpoints of an edge either n_3n_4 or

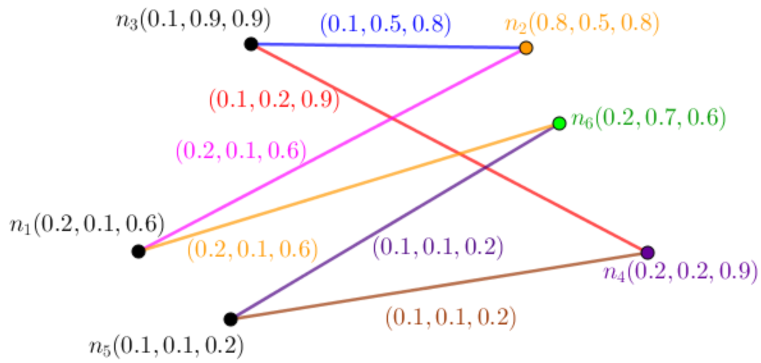

Figure 5. A Neutrosophic Graph in the Viewpoint of its Independent Number.

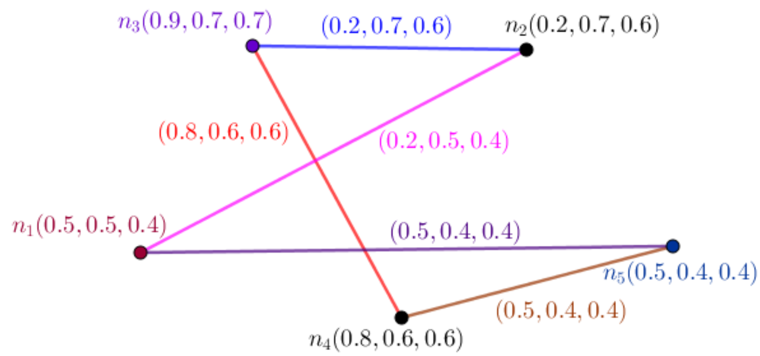

Figure 6. A Neutrosophic Graph in the Viewpoint of its Independent Number.

n_4n_5 or n_5n_1. There are three edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge n_1n_5. There's one edge n_1n_5 to have exclusive endpoints n_1 and n_5 from S hence it implies that $S = \{n_1, n_3, n_5\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 2 is independent number and its corresponded sets are $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, and $\{n_3, n_5\}$;

(vi) 2.8 is independent neutrosophic-number and its corresponded set is $\{n_2, n_5\}$.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{I}(NTG) = \mathcal{O}(NTG) - 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. Hence all vertices excluding center are only members of S is a set which its cardinality is independent number $\mathcal{I}(NTG)$. In other words, if $|S| > \mathcal{O}(NTG) - 1$, then center belongs to S. It implies that there are $\mathcal{O}(NTG) - 1$

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edges in the way that, the endpoints of these edges only belong to S. It induces S isn't corresponded set to independent number $\mathcal{I}(NTG)$. In other words, if $|S| > \mathcal{O}(NTG) - 1$, then $|S| = |V|$. It induces NTG is empty neutrosophic-graph. So it isn't a star neutrosophic-graph. By another way, if $|S| > \mathcal{O}(NTG) - 1$, then $S = V$. It means there's no edge. Thus

$$\mathcal{I}(NTG) = \mathcal{O}(NTG) - 1.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.8. There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| < \mathcal{O}(NTG)-1}$;

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_3, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| < \mathcal{O}(NTG)-1}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)-1}$ but by using the members either n_1, n_3 or n_1, n_4 or n_1, n_5 of S, it's possible to have endpoints of an edge either n_1n_3 or n_1n_4 or n_1n_5. There are three edges to have exclusive endpoints from S and it doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S| = \mathcal{O}(NTG)-1}$;

(iv) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_2, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)-1}$ and by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S. thus it implies that $S = \{n_2, n_3, n_4, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S| = \mathcal{O}(NTG)-1}$;

(v) 4 is independent number and its corresponded set is $\{n_2, n_3, n_4, n_5\}$;

(vi) 5.9 is independent neutrosophic-number and its corresponded set is $\{n_2, n_3, n_4, n_5\}$.

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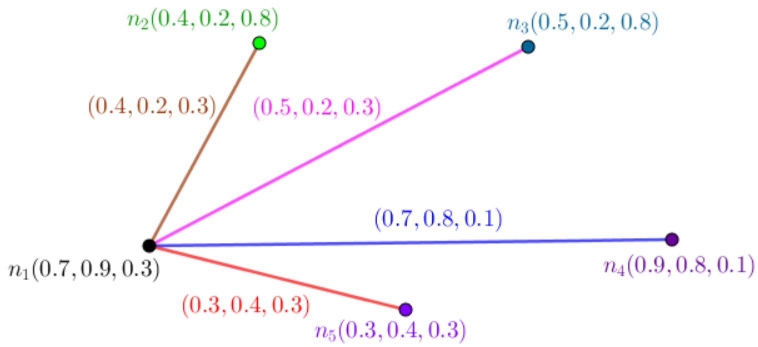

Figure 7. A Neutrosophic Graph in the Viewpoint of its Independent Number.

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*
$$\mathcal{I}(NTG) = \max\{|V_1|, |V_2|\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence all vertices excluding vertices from different part are only members of S is a set which its cardinality is independent number $\mathcal{I}(NTG)$. There are two parts. Thus
$$\mathcal{I}(NTG) = \max\{|V_1|, |V_2|\}.$$
□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There's one edge to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(*ii*) if $S = \{n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There are two edges to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_3, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\max\{|V_1|,|V_2|\}}$;

(*iii*) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_1\}$ is corresponded to

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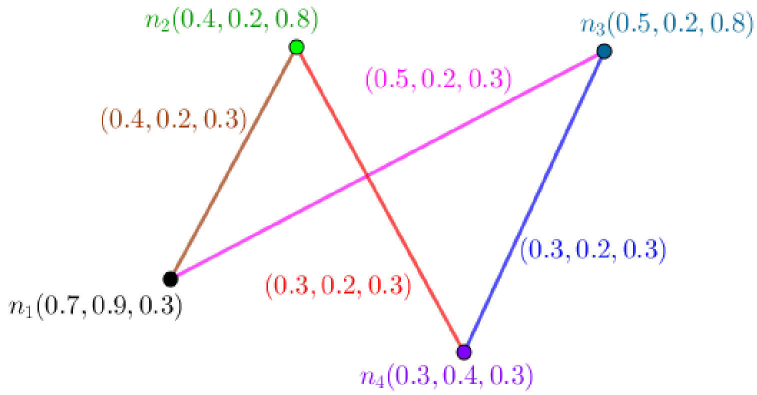

Figure 8. A Neutrosophic Graph in the Viewpoint of its Independent Number.

either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|<\max\{|V_1|,|V_2|\}}$;

(iv) if $S = \{n_1, n_4\}$ is a set of vertices, then there's no vertex in S but n_1 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S thus it implies that $S = \{n_1, n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(v) 2 is independent number and its corresponded sets are $\{n_1, n_4\}$ and $\{n_2, n_3\}$;

(vi) 2.9 is independent neutrosophic-number and its corresponded sets are $\{n_1, n_4\}$ and $\{n_2, n_3\}$.

Proposition 2.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph such that $t \neq 2$. Then*

$$\mathcal{I}(NTG) = \max\{|V_1|, |V_2|, \cdots, |V_t|\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another parts. Hence all vertices excluding vertices from different parts are only members of S is a set which its cardinality is independent number $\mathcal{I}(NTG)$. There are t parts. Thus

$$\mathcal{I}(NTG) = \max\{|V_1|, |V_2|, \cdots, |V_t|\}.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.12. There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

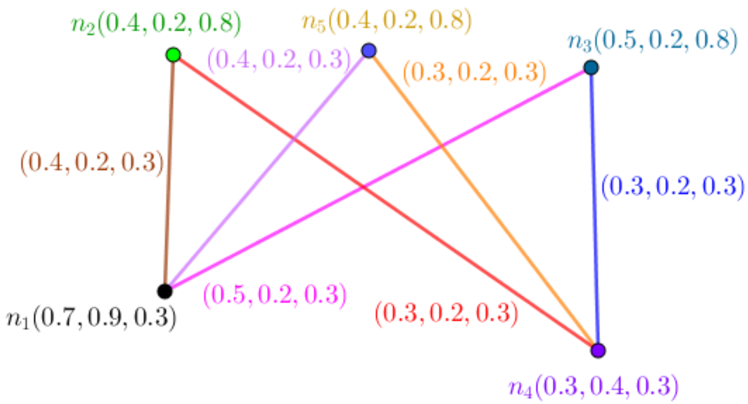

Figure 9. A Neutrosophic Graph in the Viewpoint of its Independent Number.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There's one edge to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There are two edges to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_3, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\max\{|V_1|,|V_2|\}}$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_1\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|<\max\{|V_1|,|V_2|\}}$;

(iv) if $S = \{n_2, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S thus it implies that $S = \{n_2, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(v) 3 is independent number and its corresponded set is $\{n_2, n_3, n_5\}$;

(vi) 4.3 is independent neutrosophic-number and its corresponded set is $\{n_2, n_3, n_5\}$.

3 Setting of Independent Neutrosophic-Number

In this section, I provide some results in the setting of independent neutrosophic-number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, and star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph are both of cases of study and classes which the results are about them.

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. It induces if $S = \{n\}$ is a set of vertices, then there's no vertex in S but n. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n\}$ is corresponded to independent number. Thus

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.2. In Figure (10), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_2\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to both independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

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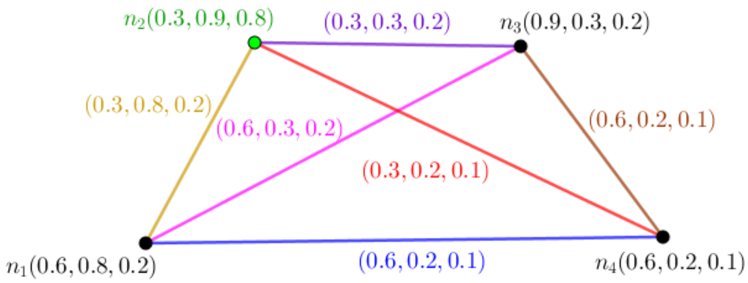

Figure 10. A Neutrosophic Graph in the Viewpoint of its Independent Number.

(*iv*) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*v*) 1 is independent number and its corresponded sets are $\{n_1\}, \{n_2\}$, $\{n_3\}$, and $\{n_4\}$;

(*vi*) 0.9 is independent neutrosophic-number and its corresponded set is $\{n_4\}$.

Proposition 3.3. *Let* $NTG : (V, E, \sigma, \mu)$ *be a path-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3}\sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_i x_{i+1} \in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lceil \frac{\mathcal{O}(NTG)}{2} \rceil$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| > \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$. It implies that $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$ is corresponded to independent number. Thus

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3}\sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_i x_{i+1} \in E}.$$

□

Example 3.4. There are two sections for clarifications.

(*a*) In Figure (11), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

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(i) If $S=\{n_2,n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S=\{n_2,n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|\neq\lceil\frac{\mathcal{O}(NTG)}{2}\rceil}$;

(ii) if $S=\{n_1,n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S=\{n_1,n_3\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|\neq\lceil\frac{\mathcal{O}(NTG)}{2}\rceil}$;

(iii) if $S=\{n_1,n_3,n_4,n_5\}$ is a set of vertices, then there's no vertex in S but n_1,n_3,n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3,n_4 or n_4,n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S=\{n_1,n_3,n_4,n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|>\lceil\frac{\mathcal{O}(NTG)}{2}\rceil}$;

(iv) if $S=\{n_1,n_3,n_5\}$ is a set of vertices, then there's no vertex in S but n_1,n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S=\{n_1,n_3,n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|=\lceil\frac{\mathcal{O}(NTG)}{2}\rceil}$;

(v) 3 is independent number and its corresponded set is $\{n_1,n_3,n_5\}$;

(vi) 3.3 is independent neutrosophic-number and its corresponded set is $\{n_1,n_3,n_5\}$.

(b) In Figure (12), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S=\{n_2,n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S=\{n_2,n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|\neq\lceil\frac{\mathcal{O}(NTG)}{2}\rceil}$;

(ii) if $S=\{n_2,n_4,n_6\}$ is a set of vertices, then there's no vertex in S but n_2,n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S=\{n_2,n_4,n_6\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|=\lceil\frac{\mathcal{O}(NTG)}{2}\rceil}$;

(iii) if $S=\{n_1,n_3,n_4,n_5\}$ is a set of vertices, then there's no vertex in S but n_1,n_3,n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3,n_4 or n_4,n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply

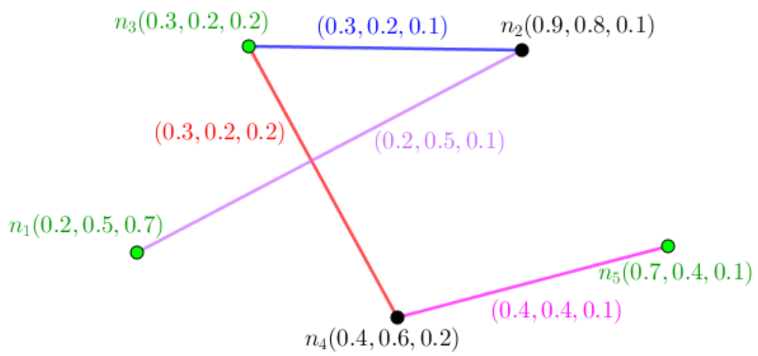

Figure 11. A Neutrosophic Graph in the Viewpoint of its Independent Number.

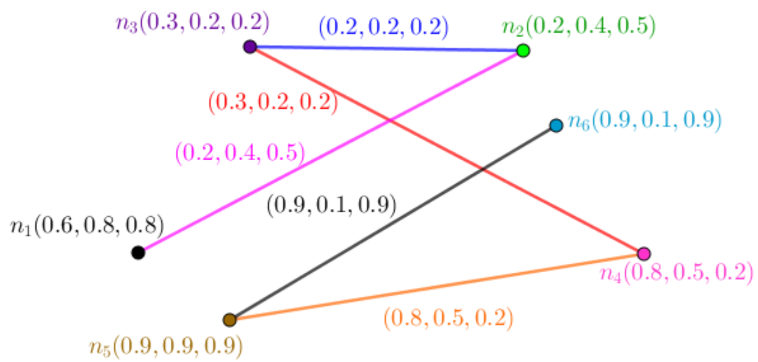

Figure 12. A Neutrosophic Graph in the Viewpoint of its Independent Number.

that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lceil \frac{\mathcal{O}(NTG)}{2} \rceil}$;

(v) 3 is independent number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(vi) 4.5 is independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

Proposition 3.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} (\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3} \sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_i x_{i+1} \in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor$. Then there are x and y in S

such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S=\{n_i\}_{|S|>\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S=\{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S=\{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$. It implies that $S=\{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$ is corresponded to independent number. Thus

$$\mathcal{I}_n(NTG)=\max\{\sum_{i=1}^{3}(\sigma_i(x_1)+\sigma_i(x_3)+\cdots+\sigma_i(x_t)),$$

$$\sum_{i=1}^{3}\sigma_i(x_2)+\sigma_i(x_4)+\cdots+\sigma_i(x'_t))\}_{x_ix_{i+1}\in E}.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There are two sections for clarifications.

(a) In Figure (13), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S=\{n_2,n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S=\{n_2,n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|\neq\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$;

(ii) if $S=\{n_2,n_4,n_6\}$ is a set of vertices, then there's no vertex in S but n_2,n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S=\{n_2,n_4,n_6\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$;

(iii) if $S=\{n_1,n_3,n_4,n_5\}$ is a set of vertices, then there's no vertex in S but n_1,n_3,n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3,n_4 or n_4,n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S=\{n_1,n_3,n_4,n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S=\{n_i\}_{|S|>\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$;

(iv) if $S=\{n_1,n_3,n_5\}$ is a set of vertices, then there's no vertex in S but n_1,n_3 and n_5. In other side, for having an edge, there's a need to have two vertices.

So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 3 is independent number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(vi) 3.2 is independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(b) In Figure (14), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S and it implies that $S = \{n_2, n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(ii) if $S = \{n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It implies that $S = \{n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 or n_5, n_1 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5 or n_5n_1. There are three edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge n_1n_5. There's one edge n_1n_5 to have exclusive endpoints n_1 and n_5 from S hence it implies that $S = \{n_1, n_3, n_5\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 2 is independent number and its corresponded sets are $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, and $\{n_3, n_5\}$;

(vi) 2.8 is independent neutrosophic-number and its corresponded set is $\{n_2, n_5\}$.

Proposition 3.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{I}_n(NTG) = \mathcal{O}_n(NTG) - \sigma(c) = \sum_{i=1}^{3} \sum_{x_j \neq c} \sigma_i(x_j).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center

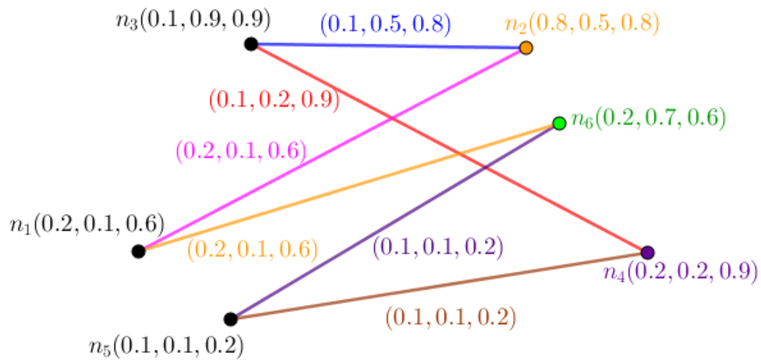

Figure 13. A Neutrosophic Graph in the Viewpoint of its Independent Number.

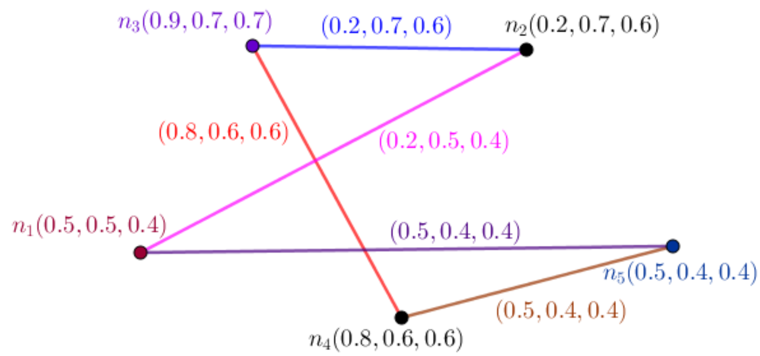

Figure 14. A Neutrosophic Graph in the Viewpoint of its Independent Number.

is only neighbor for all vertices. Hence all vertices excluding center are only members of S is a set which its cardinality is independent number $\mathcal{I}(NTG)$. In other words, if $|S| > \mathcal{O}(NTG) - 1$, then center belongs to S. It implies that there are $\mathcal{O}(NTG) - 1$ edges in the way that, the endpoints of these edges only belong to S. It induces S isn't corresponded set to independent number $\mathcal{I}(NTG)$. In other words, if $|S| > \mathcal{O}(NTG) - 1$, then $|S| = |V|$. It induces NTG is empty neutrosophic-graph. So it isn't a star neutrosophic-graph. By another way, if $|S| > \mathcal{O}(NTG) - 1$, then $S = V$. It means there's no edge. Thus

$$\mathcal{I}_n(NTG) = \mathcal{O}_n(NTG) - \sigma(c) = \sum_{i=1}^{3} \sum_{x_j \neq c} \sigma_i(x_j).$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (15), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the

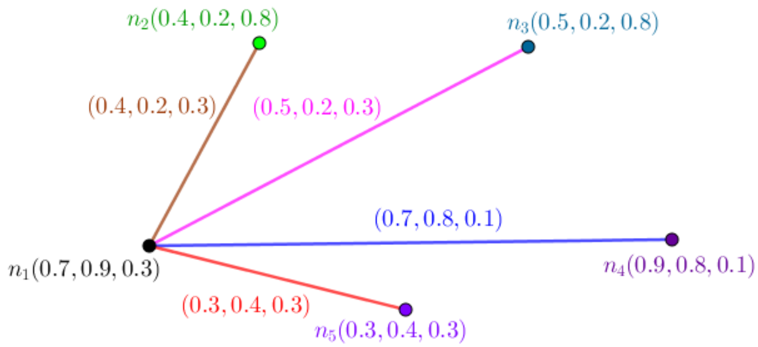

Figure 15. A Neutrosophic Graph in the Viewpoint of its Independent Number.

members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|<\mathcal{O}(NTG)-1}$;

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_3, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|<\mathcal{O}(NTG)-1}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. $S = \{n_i\}_{|S|=\mathcal{O}(NTG)-1}$ but by using the members either n_1, n_3 or n_1, n_4 or n_1, n_5 of S, it's possible to have endpoints of an edge either n_1n_3 or n_1n_4 or n_1n_5. There are three edges to have exclusive endpoints from S and it doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\mathcal{O}(NTG)-1}$;

(iv) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_2, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. $S = \{n_i\}_{|S|=\mathcal{O}(NTG)-1}$ and by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S. thus it implies that $S = \{n_2, n_3, n_4, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\mathcal{O}(NTG)-1}$;

(v) 4 is independent number and its corresponded set is $\{n_2, n_3, n_4, n_5\}$;

(vi) 5.9 is independent neutrosophic-number and its corresponded set is $\{n_2, n_3, n_4, n_5\}$.

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{(\sum_{i=1}^{3} \sum_{x_j \in V_1} \sigma_i(x_j)), (\sum_{i=1}^{3} \sum_{x_j \in V_2} \sigma_i(x_j))\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence all vertices excluding vertices from different part are only members of S is a set which its cardinality is independent number $\mathcal{I}(NTG)$. There are two parts. Thus

$$\mathcal{I}_n(NTG) = \max\{(\sum_{i=1}^{3}\sum_{x_j \in V_1} \sigma_i(x_j)), (\sum_{i=1}^{3}\sum_{x_j \in V_2} \sigma_i(x_j))\}.$$

$\square$

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (16), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There's one edge to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;
- (ii) if $S = \{n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There are two edges to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_3, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\max\{|V_1|,|V_2|\}}$;
- (iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_1\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|<\max\{|V_1|,|V_2|\}}$;
- (iv) if $S = \{n_1, n_4\}$ is a set of vertices, then there's no vertex in S but n_1 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S thus it implies that $S = \{n_1, n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;
- (v) 2 is independent number and its corresponded sets are $\{n_1, n_4\}$ and $\{n_2, n_3\}$;
- (vi) 2.9 is independent neutrosophic-number and its corresponded sets are $\{n_1, n_4\}$ and $\{n_2, n_3\}$.

Proposition 3.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph such that $t \neq 2$. Then*

$$\mathcal{I}_n(NTG) = \max\{(\sum_{i=1}^{3}\sum_{x_j \in V_1} \sigma_i(x_j)), (\sum_{i=1}^{3}\sum_{x_j \in V_2} \sigma_i(x_j)), \cdots,$$

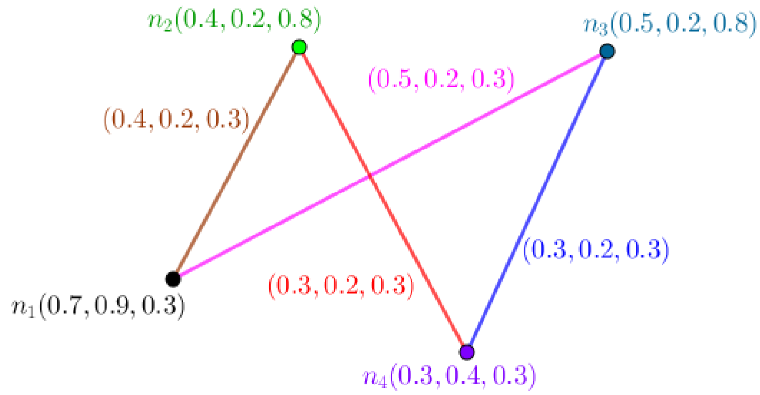

Figure 16. A Neutrosophic Graph in the Viewpoint of its Independent Number.

$$(\sum_{i=1}^{3}\sum_{x_j\in V_t}\sigma_i(x_j))\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another parts. Hence all vertices excluding vertices from different parts are only members of S is a set which its cardinality is independent number $\mathcal{I}(NTG)$. There are t parts. Thus

$$\mathcal{I}_n(NTG) = \max\{(\sum_{i=1}^{3}\sum_{x_j\in V_1}\sigma_i(x_j)), (\sum_{i=1}^{3}\sum_{x_j\in V_2}\sigma_i(x_j)), \cdots,$$

$$(\sum_{i=1}^{3}\sum_{x_j\in V_t}\sigma_i(x_j))\}.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (17), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There's one edge to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There are two edges to have exclusive endpoints from S thus it doesn't imply that

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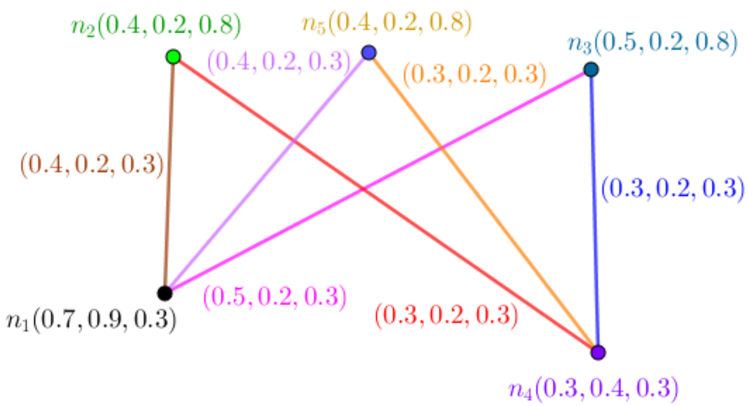

Figure 17. A Neutrosophic Graph in the Viewpoint of its Independent Number.

$S = \{n_2, n_3, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \max\{|V_1|,|V_2|\}}$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_1\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| < \max\{|V_1|,|V_2|\}}$;

(iv) if $S = \{n_2, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S thus it implies that $S = \{n_2, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \max\{|V_1|,|V_2|\}}$;

(v) 3 is independent number and its corresponded set is $\{n_2, n_3, n_5\}$;

(vi) 4.3 is independent neutrosophic-number and its corresponded set is $\{n_2, n_3, n_5\}$.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

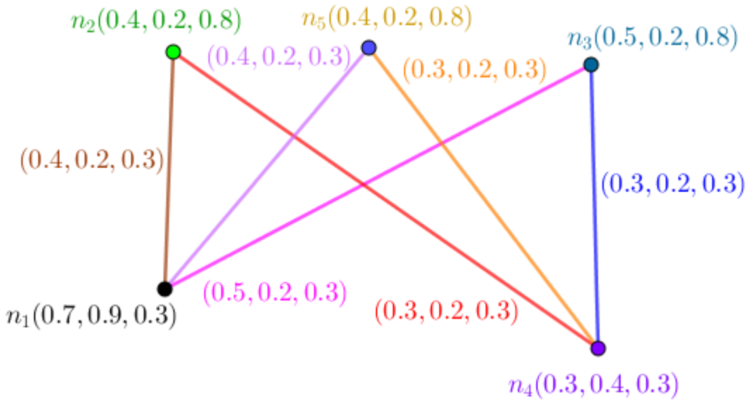

Figure 18. A Neutrosophic Graph in the Viewpoint of its Independent Number and its Independent Neutrosophic-Number.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

4.1 Case 1: Complete-t-partite Model alongside its Independent Number and its Independent Neutrosophic-Number

Step 4. (Solution) The neutrosophic graph alongside its independent number and its independent neutrosophic-number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of independent number and its independent neutrosophic-number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (18). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called independent number and independent neutrosophic-number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (18). In Figure (18), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There's one edge to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. But $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints n_2 and n_4 of an edge n_2n_4. There are two edges to have exclusive endpoints from S thus it doesn't imply that $S = \{n_2, n_3, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\max\{|V_1|,|V_2|\}}$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S but it doesn't imply that $S = \{n_1\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|<\max\{|V_1|,|V_2|\}}$;

(iv) if $S = \{n_2, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_2, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There is no edge to have exclusive endpoints from S thus it implies that $S = \{n_2, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\max\{|V_1|,|V_2|\}}$;

(v) 3 is independent number and its corresponded set is $\{n_2, n_3, n_5\}$;

(vi) 4.3 is independent neutrosophic-number and its corresponded set is $\{n_2, n_3, n_5\}$.

4.2 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its Independent Number and its Independent Neutrosophic-Number.

Step 4. (Solution) The neutrosophic graph alongside its independent number and its independent neutrosophic-number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is

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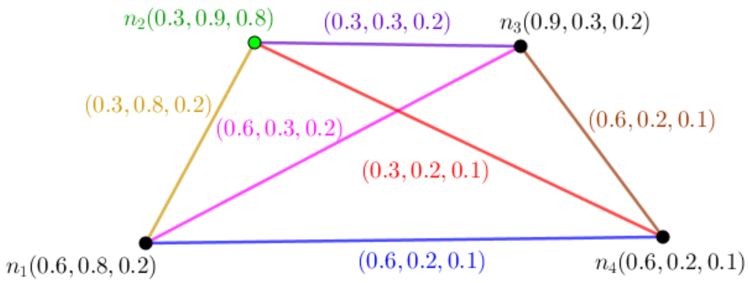

Figure 19. A Neutrosophic Graph in the Viewpoint of its Independent Number and its Independent Neutrosophic-Number.

another application of independent number and independent neutrosophic-number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (19). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called independent number and independent neutrosophic-number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (19). There is one section for clarifications.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_2\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to both independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 1 is independent number and its corresponded sets are $\{n_1\}, \{n_2\}$, $\{n_3\}$, and $\{n_4\}$;

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(vi) 0.9 is independent neutrosophic-number and its corresponded set is $\{n_4\}$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning independent number and independent neutrosophic-number are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 5.1. *Is it possible to use other types of independent number and independent neutrosophic-number?*

Question 5.2. *Are existed some connections amid different types of independent number and independent neutrosophic-number in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning independent number and independent neutrosophic-number arising neighborhoods of vertices to study neutrosophic graphs. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid number and edges are done by using neutrosophic tool. The connections of vertices which aren't clarified by one edge differ them from each other and put them in different categories to represent a number which is called independent

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Neutrosophic Independent Number	1. Wheel-Neutrosophic Graphs
2. Independent Neutrosophic-Number	
3. Neutrosophic Number	2. Study on Families
4. Study on Classes of Neutrosophic Graphs	
5. Using Neighborhood of Vertices	3. Same Models in Family

number and independent neutrosophic-number. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

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