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A study in The Neutrosophic Square Complex Matrices

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Abstract: The objective of this paper is to study algebraic properties of neutrosophic matrices, where a necessary and sufficient condition for the invertibility of a square neutrosophic matrix is presented by defining the neutrosophic determinant. On the other hand, this work introduces the concept of neutrosophicPolynomial characterstic And Theorem neutrosophicKayley-Hamilton.

Keywords: Neutrosophiccomplex matrix, neutrosophicReal number, neutrosophic determinant, neutrosophic inverse

1. Introduction

Neutrosophy is a general form of logic founded by Smarandache to deal with indeterminacy in all fields of knowledge science. We find many applications in, decision making [5,6,12], optimization theory [18,45,47], topology [23], medical studies [33,36], energy studies [40], and number theory [1649,51,52],

Recently, there is an increasing interesting in algebraic applications of neutrosophy such as neutrosophic modules [1,2,3,4,8,9,10], spaces [11,13,14,15,20], rings [21,22,24,25,26,30,31], and their generalizations [32,35,39,41,42].

After the emergence of the neutrosophic logic at 1995 there were a lot of applications to handle the indeterminacy notion. It is common for anyone to say that an unknown data is indeterminate than saying it is not exist as well in mathematics. Because when that the unknown data is not exist to a common mind it means that this data is absent does not exist. However, indeterminacy is suitable, for we can say to any layman, "We cannot determine what you ask for", but we cannot say, "your inquiry is not exist". Therefore, when we are in a moderate position as we cannot perceive $\emptyset$ for un known data, so we felt it is appropriate under these circumstances to introduce the notion of indeterminacy I where $I^2 = I$. Using this indeterminacy, we construct some notion regarding neutrosophic matrices, which can be used in neutrosophic models. Researchers have already defined the concept of neutrosophic matrices and have used them in Neutrosophic Cognitive Maps model and in the Neutrosophic Relational Equations models, which are analogous to Fuzzy Cognitive Map and Fuzzy Relational Equations models respectively.

In [44], Malath Alaswad, proposed for the first time the notion of bi-matrices.and a neutrosophic complex numbers [43], integration by using a thick function [36] Also, a minimal study of their properties can be found in [50].

2. Preliminaries

Definition 2.1 [28]: Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such that $0 \cdot I = 0$ and $I^2 = I$ which results that
$I^n = I$ for all positive integers n.

Definition 2.2 [29]: Let $w_1 = a_1 + b_1 I$, $w_2 = a_2 + b_2 I$ Then we have:

$$\frac{w_1}{w_2} = \frac{a_1}{a_2} + \frac{a_1 b_2 - a_2 b_1}{a_2(a_2 + b_2)}$$

Definition 2.3 [10]: Let Kbe a field, the neutrosophic file generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 2.4 (Neutrosophic complex matrix) [16]. Let $M_{m\times n} = \{(a_{ij}): a_{ij} \in K(I)\}$,where $K(I)$is a neutrosophic complex field. We call to be the neutrosophic complex matrix.

Definition 2.4 : Let $M_{m\times n}$ is a neutrosophiccomplex matrix. We call to be the neutrosophic square complex matrix if $m = n$.

Now aneutrosophic$n$square complex matrix is defined by form $M = A + BI$where A and B are two$n$ squares complex matrices.

3. Main discussion

Definition 3.1:

Let $M = A + BI$ be a neutrosophic n squarecomplex matrix. The determinant of M is defined as

$detM = detA + I[det(A + B) - detA]$.

Definition 3.2:

Let $M = A + BI$ a neutrosophic square $n \times n$ matrix, where A , B are two squares $n \times n$complex matrices, then
M is invertible if and only if$A$ and $A + B$ are invertible matrices and

$M^{-1} = A^{-1} + I[(A + B)^{-1} - A^{-1}]$.

Theorem 3.3:

M is invertible matrix if and only if $detM \neq 0$.

Proof:

From **Definition 3.2**we find that M is invertible matrix if and only if $A + B, A$ are two invertible matrices, hence
$det[A + B] \neq 0, detA \neq 0$ which means

$detM = detA + I[\det(A + B) - detA] \neq 0$.

Example 3.4:

Consider the following neutrosophiccomplex matrix

$M = A + BI = \begin{pmatrix} i & -1 \\ 0 & 1-i \end{pmatrix} + I \begin{pmatrix} 0 & i \\ -i & -1+i \end{pmatrix}$.

(a)$detA = 1 + i, A + B = \begin{pmatrix} i & -1+i \\ -i & 0 \end{pmatrix}$, $\det(A + B) = -1 - i, detM = 1 + i + (-2 - 2i)I \neq 0$, hence M is invertible.

(b) We have:

$$A^{-1}=\begin{pmatrix}-i & \frac{1}{2}-\frac{i}{2}\\ 0 & \frac{1}{2}+\frac{i}{2}\end{pmatrix},(A+B)^{-1}=\begin{pmatrix}0 & i\\ -\frac{1}{2}-\frac{i}{2} & -\frac{1}{2}-\frac{i}{2}\end{pmatrix},$$

$$thus\ M^{-1}=(A^{-1})+I[(A+B)^{-1}-A^{-1}]=\begin{pmatrix}-i & \frac{1}{2}-\frac{i}{2}\\ 0 & \frac{1}{2}+\frac{i}{2}\end{pmatrix}+I\begin{pmatrix}i & -\frac{1}{2}+\frac{3i}{2}\\ -\frac{1}{2}-\frac{i}{2} & -1-i\end{pmatrix}.$$

(c) We can compute $MM^{-1}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=U_{2\times2}$.

Definition 3.5:

Let $M=A+BI$ be a neutrosophic n square complex matrix, were A and B are two n square complex matrices, then

$M^T=A^T+I[(A+B)^T-A^T]$.

Definition 3.6:

Let $M=A+BI$ be a neutrosophic n squarecomplex matrix, were A and B are two n square complexmatrices, then

$M^r=A^r+I[(A+B)^r-A^r]$.

Remark 3.7:

Let $M=A+BI$ and $N=C+DI$ be two neutrosophic n squarecomplexmatrices, then

(3.7.1) $det(M\cdot N)=detM\cdot detN$.

(3.7.2) $\det(M^{-1})=(detM)^{-1}$.

(3.7.3) $detM=detM^T$.

Remark: The result in the section (c) can be generalized easily to the following fact:

$detM=detA$ if and only if $detA=det(A+B)$.

Definition 3.8:

Let $M=A+BI$ be a neutrosophic n squarecomplexmatrix, where A and B are two n squarecomplexmatrices, And $Z=X+YI$. We definethe neutrosophic Polynomial charactersticby form:

$$\varphi(Z)=det[ZU_{n\times n}-M]=det[ZU_{n\times n}-(A+BI)]=det[(ZU_{n\times n}-A)+(-B)I]$$

$$\varphi(Z)=det(ZU_{n\times n}-A)+I\left[det\left(ZU_{n\times n}-(A+B)\right)-det(ZU_{n\times n}-A)\right]$$

$$\varphi(Z)=\alpha(Z)+I[\beta(Z)-\alpha(Z)].$$

Where:

$$\alpha(Z)=det(ZU_{n\times n}-A),\beta(Z)=det\left(ZU_{n\times n}-(A+B)\right)$$

Example 3.9:

Consider the following neutrosophiccomplex matrix

$M = A + BI = \begin{pmatrix} i & -1 \\ 0 & 1-i \end{pmatrix} + I\begin{pmatrix} 0 & i \\ -i & -1+i \end{pmatrix}, A + B = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$. Then.

$$\varphi(Z) = \alpha(Z) + I[\beta(Z) - \alpha(Z)]$$

$$\alpha(Z) = det(ZU_{2\times 2} - A) = \begin{vmatrix} Z - i & -1 \\ 0 & Z - (1-i) \end{vmatrix}$$

$$\alpha(Z) = Z^2 - (1-i)Z - iZ + 1 + i = Z^2 - Z + (1+i)$$

$$\beta(Z) = Z^2 - iZ - i - 1$$

Then.

$$\varphi(Z) = \alpha(Z) + I[\beta(Z) - \alpha(Z)] = Z^2 - Z + (1+i) + I[(1-i)Z - 2 - 2i]$$

Theorem 3.10:

A neutrosophicPolynomial characterstic of neutrosophicsquare complex matrix is equal a neutrosophicPolynomial charactersticof their transpose.

Proof:

Let $M = A + BI$ be a neutrosophic n squarecomplexmatrix, where A and B are two n squarecomplexmatrices.

Let$\varphi(Z) = \alpha(Z) + I[\beta(Z) - \alpha(Z)]$a neutrosophicPolynomial characterstic for $M$and $M^T$is transpose for M.

Let$\psi(Z)$a neutrosophicPolynomial characterstic forM^T. Then.

$$\varphi(Z) = det[ZU_{n\times n} - M]$$

$$\varphi(Z) = det(ZU_{n\times n} - A) + I\left[det\big(ZU_{n\times n} - (A+B)\big) - det(ZU_{n\times n} - A)\right]$$

Now we have.

$$\psi(Z) = det[ZU_{n\times n} - M]^T = det[(ZU_{n\times n} - A) + (-B)I]^T$$

$$\psi(Z) = \det\left[(ZU_{n\times n} - A)^T + I\left[\big(ZU_{n\times n} - (A+B)\big)^T - (ZU_{n\times n} - A)^T\right]\right]$$

$$\psi(Z) = \det(ZU_{n\times n} - A)^T + I\left[det\big(ZU_{n\times n} - (A+B)\big)^T - det(ZU_{n\times n} - A)^T\right]$$

By **Remark 3.7**we have.

$$[det(ZU_{n\times n} - A)]^T = det(ZU_{n\times n} - A)$$

$$det\big(ZU_{n\times n} - (A+B)\big)^T = det\big(ZU_{n\times n} - (A+B)\big)$$

Then.

$$\psi(Z) = det(ZU_{n\times n} - A) + I\left[det\big(ZU_{n\times n} - (A+B)\big) - det(ZU_{n\times n} - A)\right]$$

Then.

$$\varphi(Z) = \psi(Z)$$

Example 3.11:

Consider the neutrosophic matrix defined in Example 3.9, we have:

$\varphi(Z) = Z^2 - Z + (1 + i) + I[(1 - i)Z - 2 - 2i]$

Now.

$A^T = \begin{pmatrix} i & 0 \\ -1 & 1-i \end{pmatrix}, B^T = \begin{pmatrix} 0 & -i \\ i & -1+i \end{pmatrix}$ Then.

$\psi(Z) = \alpha^*(Z) + I[\beta^*(Z) - \alpha^*(Z)]$

$\alpha^*(Z) = det(ZU_{2\times2} - A^T) = \begin{vmatrix} Z-i & 0 \\ -1 & Z-(1-i) \end{vmatrix}$

$\alpha^*(Z) = Z^2 - Z + (1 + i)$

$\beta^*(Z) = det(ZU_{2\times2} - (A+B)^T) = \begin{vmatrix} Z & i \\ -i & Z-(-1+i) \end{vmatrix}$

$\beta^*(Z) = Z^2 - iZ - i - 1$

Then.

$\varphi(Z) = \psi(Z) = Z^2 - Z + (1 + i) + I[(1 - i)Z - 2 - 2i]$

Theorem 3.12:(A NeutrosophicKayely-Hamelton): Any neutrosophic squarecomplex matrix is root of this a neutrosophicPolynomial characterstic.

Example 3.13:

Consider the neutrosophic matrix defined in Example 3.9, we have:

$\varphi(Z) = Z^2 - Z + (1 + i) + I[(1 - i)Z - 2 - 2i]$

Now we find $\varphi(M)$.

$\varphi(M) = M^2 - M + (1+i)U_{2\times2} + (1-i)MI + (-2-2i)U_{2\times2}I$

$M^2 = A^2 + I[(A+B)^2 - A^2] = \begin{pmatrix} -1 & -1 \\ 0 & -2i \end{pmatrix} + I\begin{pmatrix} i+1 & -i \\ 1 & 1+3i \end{pmatrix}$

$$\varphi(M) = \begin{pmatrix} -1 & -1 \\ 0 & -2i \end{pmatrix} + I\begin{pmatrix} i+1 & -i \\ 1 & 1+3i \end{pmatrix} + \begin{pmatrix} -i & 1 \\ 0 & -1+i \end{pmatrix} + I\begin{pmatrix} 0 & -i \\ i & 1-i \end{pmatrix} + \begin{pmatrix} i+1 & 0 \\ 0 & i+1 \end{pmatrix} + I\begin{pmatrix} 1+i & 2i \\ -1-i & 0 \end{pmatrix} + I\begin{pmatrix} -2-2i & 0 \\ 0 & -2-2i \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + I\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\varphi(M) = 0$

Theorem 3.14:

A neutrosophicPolynomial characterstic of neutrosophicsquare complexmatrix is equal a neutrosophicPolynomial charactersticfor any neutrosophiccomplexmatrix similar it.

Proof:

Let $M = A + BI$ be a neutrosophic n squarecomplexmatrix, where A and B are two n squarecomplexmatrices and similara neutrosophic$n$ squarecomplexmatrix$N = C + DI$. Then $N = P^{-1}MP$ where $P = K + LI$.

Let$\varphi(Z) = \alpha(Z) + I[\beta(Z) - \alpha(Z)]$a neutrosophicPolynomial characterstic for M

Let$\psi(Z)$a neutrosophicPolynomial characterstic for$N$. Then.

$$\psi(Z) = det[ZU_{n\times n} - N] = det[ZU_{n\times n} - P^{-1}MP] = det[ZP^{-1}P - P^{-1}MP]$$

$$\psi(Z) = det[P^{-1}(ZU_{n\times n} - M)P]$$

Now we have by **Remark 3.7:**.

$$\psi(Z) = det(P^{-1})\, det(ZU_{n\times n} - M) det(P) = det(P^{-1})\, det(P)\, det(ZU_{n\times n} - M)$$

$$\psi(Z) = det(U_{n\times n})\, det(ZU_{n\times n} - M) = 1\, det(ZU_{n\times n} - M) = \varphi(Z)$$

4. Refined neutrosophic matrix

Definition 4.1: The structureof refined neutrosophic numbers is taken as $a + bI_1 + cI_2$ instead of (a, bI_1, cI_2).

Definition 4.2: ${I_1}^2 = I_1$, ${I_2}^2 = I_2$, $I_1.I_2 = I_2.I_1 = I_1$

Definition 4.3: (**Refined neutrosophiccomplex matrix).**

Let$A = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$be an $m \times n$matrix: if $a_{ij} = a + bI_1 + cI_2 \in C_2(I)$, then it is called an refined neutrosophic complex matrix, where $C_2(I)$ is an refined neutrosophic complex field.

Example 4.4: Let $A = \begin{pmatrix} 1 + i + (-1 - i)I_1 & (1 + i)I_1 - iI_2 \\ 3 - iI_1 & (2 - i)I_2 \end{pmatrix}$ as a 2×2 refined neutrosophiccomplex matrix.

Theorem 4.5: Let $M = A + BI_1 + CI_2$be a square $n \times n$refined neutrosophiccomplex matrix; then it is invertible if only of $A, A + C$ and $A + B + C$ are invertible. The inverse of M is

$$M^{-1} = A^{-1} + ((A + B + C)^{-1} - (A + C)^{-1})I_1 + ((A + C)^{-1} - A^{-1})I_2$$

Proof: The proof holds as a special case of invertible elememts in refined neutrosophic rings [30].

Definition4.6:

Let $M = A + BI_1 + CI_2$ be a refinedneutrosophic n square complexmatrix, where A, B and c are n square complexmatrices, then.

$$M^T = A^T + [(A + B + C)^T - (A + C)^T]I_1 + [(A + C)^T - A^T]I_2.$$

Definition4.7:

Let $M = A + BI_1 + CI_2$ be a refinedneutrosophic n square complexmatrix, where A, B and C are n square complexmatrices, then.

$$detM = det(A + BI_1 + CI_2) = detA + [det(A + B + C) - det(A + C)]I_1 + [det(A + C) - detA]I_2.$$

Remark 4.8:

(a).If $A$is an$m \times n$matrix, then it can be represented as an element of the refined neutrosophic ring of matrices such as the following: $M = A + BI_1 + CI_2$, where A, B and $C$are complex matrices with elements from ring $C$and from size$m \times n$.

For example, $M = \begin{pmatrix} (-1-i) + I_1 + 3iI_2 & 1 - (1-i)I_1 - I_2 \\ 3 + (1+i)I_2 & 1 + (2+i)I_1 \end{pmatrix} = \begin{pmatrix} -1-i & 1 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1+i \\ 0 & 2+i \end{pmatrix} I_1 + \begin{pmatrix} 3i & -1 \\ (1+i) & 0 \end{pmatrix} I_2.$

(b). Multiplication can be defined by using the same representation as a special case multiplication on refined neutrosophic rings as follows:

$$(A + BI_1 + CI_2)(X + YI_1 + ZI_2) = (AX) + (AY + BX + BY + BZ + CY)I_1 + (AZ + CZ + CX)I_2$$

Theorem 4.9:

Let $M = A + BI_1 + CI_2$ be a neutrosophic n squarecomplexmatrix, where A, B and C are two n squarecomplexmatrices, And $Z = X + YI_1 + TI_2$. We define the neutrosophic Polynomial characterstic by form:

$$\varphi(z) = det[ZU_{n\times n} - M] = det[ZU_{n\times n} - (A + BI_1 + CI_2)] = det[(ZU_{n\times n} - A) + (-B)I_1 + (-C)I_2]$$

$$\varphi(z) = det(ZU_{n\times n} - A) + \left[det\left(ZU_{n\times n} - (A + B + C)\right) - det\left(ZU_{n\times n} - (A + C)\right)\right]I_1 + \left[det\left(ZU_{n\times n} - (A + C)\right) - det(ZU_{n\times n} - A)\right]I_2$$

$$\varphi(z) = \alpha(Z) + [\beta(Z) - \gamma(Z)]I_1 + [\gamma(Z) - \alpha(Z)]I_2.$$

Where:

$$\alpha(Z) = det(ZU_{n\times n} - A)\ , \beta(Z) = det\left(ZU_{n\times n} - (A + B + C)\right), \gamma(Z) = det\left(ZU_{n\times n} - (A + C)\right)$$

Example4.10:

Consider the following neutrosophic matrix

$M = A + BI_1 + CI_2$. Where $A = \begin{pmatrix} 2+i & 1 \\ -i & -1+i \end{pmatrix}$, $B = \begin{pmatrix} 1 & -i \\ 0 & 2i \end{pmatrix}$, $C = \begin{pmatrix} 1-i & -1 \\ i & 0 \end{pmatrix}$

$A + B + C = \begin{pmatrix} 4 & -i \\ 0 & -1+3i \end{pmatrix}$, $A + C = \begin{pmatrix} 3 & 0 \\ 0 & -1+i \end{pmatrix}$.

Then.

$$\varphi(z) = \alpha(Z) + [\beta(Z) - \gamma(Z)]I_1 + [\gamma(Z) - \alpha(Z)]I_2$$

$$\alpha(Z) = det(ZU_{n\times n} - A) = \begin{vmatrix} Z - (2+i) & -1 \\ i & Z + (1-3i) \end{vmatrix}$$

$$\alpha(Z) = Z^2 - (1 + 2i)Z - 3 + 2i$$

$$\beta(Z) = det\big(ZU_{n\times n} - (A+B+C)\big) = \begin{vmatrix} Z-4 & i \\ 0 & Z+(1-3i) \end{vmatrix}$$

$$\beta(Z) = Z^2 - (1+4i)Z - (5-5i)$$

$$\gamma(Z) = det\big(ZU_{n\times n} - (A+C)\big) = \begin{vmatrix} Z-3 & 0 \\ 0 & Z+(1-i) \end{vmatrix}$$

$$\gamma(Z) = Z^2 - (2+i)Z + (-3+3i)$$

Then.

$$\varphi(Z) = \alpha(Z) + [\beta(Z) - \gamma(Z)]I_1 + [\gamma(Z) - \alpha(Z)]I_2$$

$$\varphi(Z) = Z^2 - (1+2i)Z - 3 + 2i + [(1-3i)Z + (-2+2i)]I_1 + [(-1+i)Z + i]I_2$$

Theorem 4.11: :(A refined NeutrosophicKayely-Hamelton): Any refined neutrosophic squarecomplex matrix is root of this a neutrosophicPolynomial characterstic.

Conclusion

In this article, we have determined necessary and sufficient conditions for the invertibility anddiagonalization of neutrosophic matrices. Also, we have found an easy algorithm to compute the inverse of a neutrosophic matrix and its Eigen values and vectors.

As a future research direction, we aim to find the representation of neutrosophic matrices by linear transformations in neutrosophic vector spaces.

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