A study of cyclic refined neutrosophic complex numbers

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Abstract

This papers dedicated to define for t first time the concept of complex cyclic refined neutrosophic numbers as a direct application of refined neutrosophic sets. Also, it presents some of their elementary properties such as, conjugates, absolute values, invertibility, and algebraic operations. The importance of the dfinitions in this article lies in the use of them by dfining the polar form of cyclic refined neutrosophic complex numbers.

Keywords:refined neutrosophic complex number,cyclic refined neutrosophic complex number, refined neutrosophic real number, invertible number.

1.Introduction

Neutrosophy is a new branch of philosophy founded by Smarandache [6,36], to study the indeterminacy in the real world problems and science. It has a master effect in many areas such as topology [7,27,29], equations [3,30], decision making [8], abstract algebra [25,26,39,41], and number theory [35].

Neutrosophic algebra began with the definitions of neutrosophic groups [9,17], and rings [13]. The neutrosophic rings and their generalizations such as refined neutrosophic rings [19], and n-refined neutrosophic rings [11,12], were very useful in the study of neutrosophic algebraic structures.

Neutrosophicalgebraic structures were defined as new generalizations of classical ones based on neutrosophic rings and fields, where we find many concepts from linear algebra were generalized into neutrosophic systems such as neutrosophic matrices and spaces over neutrosophic fields [1,42], refined neutrosophic spaces and matrices over refined neutrosophic fields [24], n-refined neutrosophic spaces over n-refined neutrosophic fields [21,32], linear modules and ideals [4,5,20,22].

Neutrosophic complex numbers were firstly studied in [43]. Recently, many of their properties were discussed in [44], especially their invertibility, absolute values, and complex functions.

Through this paper, we define cyclic refined neutrosophic complex numbers for the first time. On the other hand, we study many related properties of these numbers such as the invertibles, conjugates, and absolute values.

2. Neutrosophic complex number

Definition 2.1.Neutrosophic Real Number:

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n.

For example:

$w = 1 + 2I, w = 3 = 3 + 0I.$

Definition 2.2. NeutrosophicComplex Number:

Suppose that z is a neutrosophic complex number, then it takes the following standard form: $z = a + bI + i(\mathrm{c} + \mathrm{d}I)$ where a, b, c, d are real coefficients, and I is the indeterminacy element, where $i^2 = -1$ i.e. $i = \sqrt{-1}$.

We recall $a + bI$ the real part, then it takes the following standard form $Re(z) = a + bI$.

We recall $c + dI$ the imagine part, then it takes the following standard form $Im(z) = c + dI$.

For example:

$$z = 4 + I + i(2 + 2I)$$

Note: we can say that any real number can be considered a neutrosophic number.

For example: $z = 3 = 3 + 0.I + i(0 + 0.I)$

Definition 2.3. Division of neutrosophic real numbers:

Suppose that w_1, w_2 are two neutrosophic numbers,where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

3. cyclic refinedneutrosophic complex numbers.

Definition 3.1.

We define a cyclic refined neutrosophiccomplex numbers by the following form:

$z = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$, where $a_o, a_1, a_2, b_o, b_1, b_2$are real coeffients. For example:

$z = (1 - I_1 + 2I_2) + i(3 + 2I_1 - I_2)$.

We recall $a_o + a_1 I_1 + a_2 I_2$ the real part, then it takes the following standard form $Re(z) = a_o + a_1 I_1 + a_2 I_2$.

We recall $b_o + b_1 I_1 + b_2 I_2$ the image part, then it takes the following standard form $Im(z) = b_o + b_1 I_1 + b_2 I_2$.

Remark 3.1. A cyclic refined neutrosophic complex number can be defined as follows:

$\boldsymbol{z = a + bI_1 + cI_2}$where $\boldsymbol{a, b, c}$ are complex numbers. For example:

$\boldsymbol{z = (1 - i) + (2 + i)I_1 + (3 - 2i)I_2}$.

Remark 3.2.

$I_i \times I_j = I_{(i+j \ mod \ 2)}$.

Mod 2 means the remainder is $\{1,2\}$

Definition 3.2.The conjugate of cyclic refined neutrosophiccomplex number:

Let$z = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$a cyclic refined neutrosophic complex number.We denote the conjugate of a cyclic refindneutrosophic complex number by $\bar{z}$ and define it by the following form:

$$\bar{z} = (a_o + a_1 I_1 + a_2 I_2) - i(b_o + b_1 I_1 + b_2 I_2)$$

For example:

$z = (-1 + I_1 + 2I_2) + i(1 + 2I_1 - I_2)$, Then $\bar{z} = (-1 + I_1 + 2I_2) - i(1 + 2I_1 - I_2)$.

Definition 3.3. The absolute value of a cyclic refined neutrosophic complex number:

Suppose that $z = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$ is a cyclic refined neutrosophic complex number. The absolute value of a z can be defined by the following form:

$$|z| = \sqrt{(a_o + a_1 I_1 + a_2 I_2)^2 + (b_o + b_1 I_1 + b_2 I_2)^2}$$

Remark3.3.

$(\mathbf{1}).\overline{(\bar{\boldsymbol{z}})} = \boldsymbol{z}.$

Proof: Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$, then $\bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$.

Now.

$\overline{(\bar{\boldsymbol{z}})} = \overline{\left((\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) - \boldsymbol{\imath}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})\right)} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2}) = \boldsymbol{z}$

$(\mathbf{2}).\boldsymbol{z} + \bar{\boldsymbol{z}} = \mathbf{2}\boldsymbol{Re}(\boldsymbol{z})$

Proof:Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$, then $\bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$.

Now.

$\boldsymbol{z} + \bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2}) + (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$

$\boldsymbol{z} + \bar{\boldsymbol{z}} = \mathbf{2}[(\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2})] = \mathbf{2}\boldsymbol{Re}(\boldsymbol{z})$

$(3).z - \bar{z} = 2Im(z)$

Proof:

Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$, then$\bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$.

Now.

$z - \bar{z} = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2) - [(a_o + a_1 I_1 + a_2 I_2) - i(b_o + b_1 I_1 + b_2 I_2)]$

$z - \bar{z} = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2) - (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$

$z - \bar{z} = 2i(b_o + b_1 I_1 + b_2 I_2) = 2Im(z)$

$(4).\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

Proof:

Let $\boldsymbol{z_1} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2})$, $\boldsymbol{z_2} = (\boldsymbol{c_o} + \boldsymbol{c_1 I_1} + \boldsymbol{c_2 I_2}) + \boldsymbol{i}(\boldsymbol{d_o} + \boldsymbol{d_1 I_1} + \boldsymbol{d_2 I_2})$.

Now.

$$z_1 + z_2 = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2] + i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2]$$

Then.

$$\overline{z_1 + z_2} = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2] - i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2]$$

$$\overline{z_1 + z_2} = [(a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)] + [(c_o + c_1I_1 + c_2I_2) - i(d_o + d_1I_1 + d_2I_2)] = \overline{z_1} + \overline{z_2}$$

Definition 3.4. The multiplication of two cyclic refined neutrosophic real numbers:

Let $w_1 = a_o + a_1I_1 + a_2I_2, w_2 = b_o + b_1I_1 + b_2I_2$ are two cyclic refined neutrosophic real number, we define a multiplication$w_1 . w_2$ as follows:

$$\begin{aligned} w_1 \times w_2 &= (a_o + a_1I_1 + a_2I_2) \times (b_o + b_1I_1 + b_2I_2) \\ &= (a_o \times b_o) + (a_o \times b_1)I_1 + (a_o \times b_2)I_2 + (a_1 \times b_o)I_1 + (a_1 \times b_1)I_1 \times I_1 + (a_1 \times b_2)I_1 \times I_2 \\ &+ (a_2 \times b_o)I_2 + (a_2 \times b_1)I_2 \times I_1 + (a_2 \times b_2)I_2 \times I_2 \end{aligned}$$

By remark 3.2, we have.

$$I_1 \times I_1 = I_2, I_1 \times I_2 = I_1, I_2 \times I_1 = I_1, I_2 \times I_2 = I_2.$$

Then.

$$\begin{aligned} w_1 \times w_2 &= (a_o \times b_o) + [(a_o \times b_1) + (a_1 \times b_o) + (a_1 \times b_2) + (a_2 \times b_1)]I_1 \\ &+ [(a_1 \times b_1) + (a_o \times b_2) + (a_2 \times b_0) + (a_2 \times b_2)]I_2 \end{aligned}$$

Remark 3.4. The definition of product can be useful in refined neutrosophic rings. See reference [19].

Definition 3.5. The multiplication of two cyclic refined neutrosophic complex numbers:

Let $\boldsymbol{z_1 = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2), z_2 = (c_o + c_1I_1 + c_2I_2) + i(d_o + d_1I_1 + d_2I_2)}$.
A product $z_1 \times z_2$ is defined by form:

$$z_1 \times z_2 = [(a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)] \times [(c_o + c_1I_1 + c_2I_2) + i(d_o + d_1I_1 + d_2I_2)]$$

$$\begin{aligned} z_1 \times z_2 &= (a_o + a_1I_1 + a_2I_2) \times (c_o + c_1I_1 + c_2I_2) - (b_o + b_1I_1 + b_2I_2) \times (d_o + d_1I_1 + d_2I_2) \\ &+ i[(a_o + a_1I_1 + a_2I_2) \times (d_o + d_1I_1 + d_2I_2) + (c_o + c_1I_1 + c_2I_2) \times (b_o + b_1I_1 + b_2I_2)] \end{aligned}$$

By using Definition 3.4, we get the product $z_1 \times z_2$.

Remark 3.5.

(1). $\overline{z_1 \times z_2} = \overline{z_1} \times \overline{z_2}$

(1). $z \times \bar{z} = |z|^2$.

Where $a_0 + a_1 + a_2, a_0 + a_2, a_0$ are not zero elements.

Definition 3.6. The invertible a cyclic refined neutrosophic complex number.

Let $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$, then the invertible of z defined as follows:

$$z^{-1} = \frac{1}{z} = \frac{1}{(a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)} = \frac{\bar{z}}{z \times \bar{z}} = \frac{\bar{z}}{|z|^2}$$

Example3.1. Let $z = (1 + I_1 - I_2) + i(2 + 2I_1 - I_2), \bar{z} = (1 + I_1 - I_2) - i(2 + 2I_1 - I_2)$.
then.

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{(1 + I_1 - I_2)^2 + (2 + 2I_1 - I_2)^2} =$$

Now we have.

$$(1 + I_1 - I_2)^2 = (1 + I_1)^2 - 2(1 + I_1) \times I_2 + (I_2 \times I_2) = 1 + 2I_1 + (I_1 \times I_1) - 2I_2 - 2I_1 \times I_2 + I_2 \times I_2$$

$$(1 + I_1 - I_2)^2 = 1 + 2I_1 + I_2 - 2I_2 - 2I_1 + I_2 = 1$$

$$(2 + 2I_1 - I_2)^2 = (2 + 2I_1)^2 - 2(2 + 2I_1) \times I_2 + (I_2 \times I_2) = 4 + 8I_1 + 4(I_1 \times I_1) - 4I_2 - 4I_1 \times I_2 + I_2 \times I_2$$

$$(2 + 2I_1 - I_2)^2 = 4 + 8I_1 + 4I_2 - 4I_2 - 4I_1 + I_2 = 4 + 4I_1 + I_2$$

Then.

$$z^{-1} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{1 + 4 + 4I_1 + I_2} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{5 + 4I_1 + I_2}$$

$$z^{-1} = [(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)] \times (5 + 4I_1 + I_2)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2) \times (5 + 4I_1 + I_2)^{-1} - i(2 + 2I_1 - I_2) \times (5 + 4I_1 + I_2)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) - i(2 + 2I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right)$$

By using the Definition 3.4 we get:

$$(1 + I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) = \frac{1}{5} + \frac{1}{6}I_1 - \frac{4}{15}I_2$$

$$(2 + 2I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) = \frac{2}{5} + \frac{4}{15}I_1 - \frac{11}{30}I_2$$

Hence,

$$z^{-1} = \left(\frac{1}{5} + \frac{1}{6}I_1 - \frac{4}{15}I_2\right) - i\left(\frac{2}{5} + \frac{4}{15}I_1 - \frac{11}{30}I_2\right).$$

The condition of invertibility can be found in [41].

Remark 3.6.

A cyclic refined neutrosophic number $(a_o + a_1I_1 + a_2I_2)$ can be written as $a_o + a_1I_1 + a_2I_2$.

4. Conclusion

In this paper, we have defined for the first time the concept of cyclic refined neutrosophic complex numbers. Also, we have discussed some of their elementary properties such as the conjugate, the multiplication, absolute values and other related topics.

As a future research direction, we aim to study the natural generalization of those numbers by n-cyclic refined neutrosophic complex numbers.

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