Global Powerful Alliance in Strong Neutrosophic Graphs

Preprint · January 2022

DOI: 10.20944/preprints202201.0442.v1

 doi:10.20944/preprints202201.0442.v1

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Henry Garrett

Independent Researcher

DrHenryGarrett@gmail.com

Twitter's ID: @DrHenryGarrett | ©DrHenryGarrett.wordpress.com

Abstract

New setting is introduced to study the global powerful alliance. Global powerful alliance is about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define this notion. Also, neighborhood is defined based on strong edges. Strong edge gets a framework as neighborhood and after that, too close vertices have key role to define global powerful alliance based on strong edges. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs excluding empty, path, star, and wheel and containing complete, cycle and r-regular-strong are investigated in the terms of set, minimal set, number, and neutrosophic number. Neutrosophic number is used in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number. It's called "modified neutrosophic number". Summation of three values of vertex makes one number and applying it to a set makes neutrosophic number of set. This approach facilitates identifying minimal set and optimal set which forms minimal-global-powerful-alliance number and minimal-global-powerful-alliance-neutrosophic number. Two different types of sets namely global-powerful alliance and minimal-global-powerful alliance are defined. Global-powerful alliance identifies the sets in general vision but minimal-global-powerful alliance takes focus on the sets which deleting a vertex is impossible. Minimal-global-powerful-alliance number is about minimum cardinality amid the cardinalities of all minimal-global-powerful alliances in a given neutrosophic graph. New notions are applied in the settings both individual and family. Family of neutrosophic graphs has an open avenue, in the way that, the family only contains same classes of neutrosophic graphs. The results are about minimal-global-powerful alliance, minimal-global-powerful-alliance number and its corresponded sets, minimal-global-powerful-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-global-powerful alliances, minimal-t-powerful alliance, minimal-t-powerful-alliance number and its corresponded sets, minimal-t-powerful-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-t-powerful alliances. The connections amid t-powerful-alliances are obtained. The number of connected components has some relations with this new concept and it gets some results. Some classes of neutrosophic graphs behave differently when the parity of vertices are different and in this case, cycle, and complete illustrate these behaviors. Two applications concerning complete model as individual and family, under the titles of time table and scheduling conclude the results and they give more clarifications and closing remarks. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study deeply and with more results but it seems that analogous results are determined. Slight progress is obtained in the family of these models but there are open avenues to study family of other models as same models and different models. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on strong edges illustrate open way to get results. A set is global powerful alliance when two sets partitioning vertex set have uniform structure. All members of set have more amount of neighbors in the set than out of set and reversely for non-members of set with less members in the way that the set is simultaneously t-offensive and (t-2)-defensive. A set is global if t=0. It leads us to the notion of global powerful alliance. Different edges make different neighborhoods but it's used one style edge titled strong edge. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for these notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Modified Neutrosophic Number, Global Powerful Alliance, R-Regular-Strong

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [16], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2, 14, 15], graphs and new notions on them in **Refs.** [5–12], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [13], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "Global Powerful Alliance", "Modified Neutrosophic Number" and "Complete Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign global-powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number. Thus they're used to define new ideas which conclude to the structure global powerful alliance. The concept of having strong edge inspires me to study the behavior of strong edges in the way that, two types of numbers and set, e.g., global-powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number are the cases of study in the settings of individuals and in settings of families. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of global- powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number are introduced and are clarified as individuals. In section "Preliminaries", general sets have the key role in this way. General results are obtained and also, the results about the basic notions of global-powerful alliance are elicited. Two classes of neutrosophic graphs are studied in the terms of global-powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number in section "r-Regular-Strong-Neutrosophic Graph' as individuals. In section "r-Regular-Strong-Neutrosophic Graph", both numbers have applied into individuals. As a concluding result, there are three statements and remarks about r-regular-strong-neutrosophic graphs which are either cycle or complete. The clarifications are also presented in section "r-Regular-Strong-Neutrosophic Graph" for introduced results. In section "Applications in Time Table and Scheduling", two applications are posed for global-powerful alliance concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are complete as individual and uniform family. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1],\ \mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v \in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e\in E}\Sigma_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_n$ is $\bigwedge_{i=0,\cdots,n-1}\mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_n is

$$\mu^{\infty}(x, y) = \bigwedge_{P:x_0,x_1,\cdots,x_n}\ \bigwedge_{i=0,\cdots,n-1}\mu(x_ix_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

The notions of neighbor and neighborhood are about some vertices which have one edge with a fixed vertex. These notions present vertices which are close to a fixed vertex as possible. Based on strong edge, it's possible to define different neighborhood as follows.

Definition 1.5. (Strong Neighborhood).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

$$N_s(x) = \{y \in N(x) \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}.$$

New notion is defined between two types of neighborhoods for a fixed vertex. A minimal set and some numbers are introduced in this way. The next definition has main role in every results which are given in this essay.

Definition 1.6. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set S of vertices is called **t-offensive alliance** if

$$\forall a \in V \setminus S,\ |N_s(a) \cap S| - |N_s(a) \cap (V \setminus S)| > t;$$

(*ii*) a t-offensive alliance is called **global-offensive alliance** if $t = 0$;

(*iii*) a set S of vertices is called **t-defensive alliance** if

$$\forall a \in S,\ |N_s(a) \cap S| - |N_s(a) \cap (V \setminus S)| < t;$$

(*iv*) a t-defensive alliance is called **global-defensive alliance** if $t = 0$;

(*v*) a set S of vertices is called **t-powerful alliance** if it's both t-offensive alliance and (t-2)-defensive alliance;

(*vi*) a t-powerful alliance is called **global-powerful alliance** if $t = 0$;

(*vii*) $\forall S' \subseteq S$, S is global-powerful alliance but S' isn't global-powerful alliance. Then S is called **minimal-global-powerful alliance**;

(*viii*) **minimal-global-powerful-alliance number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-powerful alliance.}} |S|$$

and it's denoted by Γ;

(*ix*) **minimal-global-powerful-alliance-neutrosophic number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)$$

and it's denoted by Γ_s.

In the next result, the notions of t-defensive alliance and t-offensive alliance have been extended to present the classes of defensive alliance and offensive alliance which hold when one type of them holds for a given set of vertices.

Proposition 1.7. *Let* $NTG : (V, E, \sigma, \mu)$ *be a strong neutrosophic graph. Then following statements hold;*

(*i*) *if* $s \geq t$ *and a set* S *of vertices is t-defensive alliance, then* S *is s-defensive alliance;*

(*ii*) *if* $s \leq t$ *and a set* S *of vertices is t-offensive alliance, then* S *is s-offensive alliance.*

Proof. (*i*). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t \leq s;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < s.$$

Thus S is s-defensive alliance.

(*ii*). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t \geq s;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > s.$$

Thus S is s-offensive alliance. □

As a consequence of previous result, the relations amid a set which is both t-offensive alliance and t-defensive alliance lead us toward the notion of t-powerful alliance.

Proposition 1.8. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Then following statements hold;*

(i) *if $s \geq t + 2$ and a set S of vertices is t-defensive alliance, then S is s-powerful alliance;*

(ii) *if $s \leq t$ and a set S of vertices is t-offensive alliance, then S is t-powerful alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-defensive alliance. Then

$$\forall t \in S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t;$$
$$\forall t \in S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t \leq t + 2 \leq s;$$
$$\forall t \in S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < s.$$

Thus S is (t+2)-defensive alliance. By S is s-defensive alliance and S is (s+2)-offensive alliance, S is s-powerful alliance.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-offensive alliance. Then

$$\forall t \in V \setminus S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t;$$
$$\forall t \in V \setminus S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t \geq s > s - 2;$$
$$\forall t \in V \setminus S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > s - 2.$$

Thus S is (s-2)-defensive alliance. By S is (s-2)-defensive alliance and S is s-offensive alliance, S is s-powerful alliance. □

2 r-Regular-Strong-Neutrosophic Graph

r-regular is an attribute. This property facilitates the results when the condition is about the neighbors inside fixed set to determine 2-defensive alliance and 2-offensive alliance. Also, a condition about the neighbors outside of fixed set determines some results about r-defensive alliance and r-offensive alliance.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph. Then following statements hold;*

(i) *if $\forall a \in S, \ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1$, then $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(ii) *if $\forall a \in V \setminus S, \ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1$, then $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*

(iii) *if $\forall a \in S, \ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is r-defensive alliance;*

(iv) *if $\forall a \in V \setminus S, \ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is r-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1) < 2;$$
$$\forall t \in S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1) > 2;$$
$$\forall t \in V \setminus S, \ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r - 0 = r;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r.$$

Thus S is r-defensive alliance.

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r - 0 = r;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r.$$

Thus S is r-offensive alliance.

□

2-defensive alliance and 2-offensive alliance get some results about the neighbors inside fixed set. Also, r-defensive alliance and r-offensive alliance get some results about the neighbors outside of fixed set.

Proposition 2.2. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph. Then following statements hold;*

(i) $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1$ *if $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(ii) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1$ *if $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*

(iii) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if $NTG : (V, E, \sigma, \mu)$ is r-defensive alliance;*

(iv) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if $NTG : (V, E, \sigma, \mu)$ is r-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| = \lfloor \tfrac{r}{2} \rfloor + 1,\ |N_s(t) \cap (V \setminus S)| = \lfloor \tfrac{r}{2} \rfloor - 1.$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| = \lfloor \tfrac{r}{2} \rfloor + 1,\ |N_s(t) \cap (V \setminus S) = \lfloor \tfrac{r}{2} \rfloor - 1.$$

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and r-defensive alliance.

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r = r - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| = r,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and r-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r = r - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| = r,\ |N_s(t) \cap (V \setminus S)| = 0.$$

□

As a special case, complete neutrosophic graph gets specific result excerpt from r-regular neutrosophic graph. 2-defensive alliance and 2-offensive alliance get some results about the neighbors inside fixed set depending on order. Also, $(\mathcal{O}-1)$-defensive alliance and $(\mathcal{O}-1)$-offensive alliance get some results about the neighbors outside of fixed set depending on order.

Proposition 2.3. *Let $NTG:(V,E,\sigma,\mu)$ be a r-regular-strong-neutrosophic graph which is complete. Then following statements hold;*

(i) $\forall a\in S,\ |N_s(a)\cap S|<\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1$ *if $NTG:(V,E,\sigma,\mu)$ is 2-defensive alliance;*

(ii) $\forall a\in V\setminus S,\ |N_s(a)\cap S|>\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1$ *if $NTG:(V,E,\sigma,\mu)$ is 2-offensive alliance;*

(iii) $\forall a\in S,\ |N_s(a)\cap V\setminus S|=0$ *if $NTG:(V,E,\sigma,\mu)$ is $(\mathcal{O}-1)$-defensive alliance;*

(iv) $\forall a\in V\setminus S,\ |N_s(a)\cap V\setminus S|=0$ *if $NTG:(V,E,\sigma,\mu)$ is $(\mathcal{O}-1)$-offensive alliance.*

Proof. (i). Suppose $NTG:(V,E,\sigma,\mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance. Then

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2=\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in S,\ |N_s(t)\cap S|=\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor+1,\ |N_s(t)\cap(V\setminus S)|=\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor-1.$$

(ii). Suppose $NTG:(V,E,\sigma,\mu)$ is a r-regular-strong-neutrosophic graph and 2-offensive alliance. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2=\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|=\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor+1,\ |N_s(t)\cap(V\setminus S)=\lfloor\tfrac{\mathcal{O}-1}{2}\rfloor-1.$$

(iii). Suppose $NTG:(V,E,\sigma,\mu)$ is a r-regular-strong-neutrosophic graph and $(\mathcal{O}-1)$-defensive alliance.

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\mathcal{O}-1;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\mathcal{O}-1=\mathcal{O}-1-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\mathcal{O}-1-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|=\mathcal{O}-1,\ |N_s(t)\cap(V\setminus S)|=0.$$

(iv). Suppose $NTG:(V,E,\sigma,\mu)$ is a $(\mathcal{O}-1)$-regular-strong-neutrosophic graph and r-offensive alliance. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\mathcal{O}-1;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\mathcal{O}-1=\mathcal{O}-1-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\mathcal{O}-1-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|=\mathcal{O}-1,\ |N_s(t)\cap(V\setminus S)|=0.$$

□

As a special case of r-regular, complete is an attribute. This property facilitates the results when the condition is about the neighbors inside fixed set to determine 2-defensive alliance and 2-offensive alliance. Also, a condition about the neighbors outside of fixed set determines some results about $(\mathcal{O}-1)$-defensive alliance and $(\mathcal{O}-1)$-offensive alliance.

Proposition 2.4. *Let $NTG:(V,E,\sigma,\mu)$ be a r-regular-strong-neutrosophic graph which is complete. Then following statements hold;*

(*i*) *if* $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1,$ *then* $NTG : (V, E, \sigma, \mu)$ *is 2-defensive alliance;*

(*ii*) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1,$ *then* $NTG : (V, E, \sigma, \mu)$ *is 2-offensive alliance;*

(*iii*) *if* $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0,$ *then* $NTG : (V, E, \sigma, \mu)$ *is* $(\mathcal{O}-1)$*-defensive alliance;*

(*iv*) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0,$ *then* $NTG : (V, E, \sigma, \mu)$ *is* $(\mathcal{O}-1)$*-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \tfrac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \tfrac{\mathcal{O}-1}{2} \rfloor - 1) < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.
(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \tfrac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \tfrac{\mathcal{O}-1}{2} \rfloor - 1) > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.
(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0 = \mathcal{O} - 1;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1.$$

Thus S is $(\mathcal{O}-1)$-defensive alliance.
(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0 = \mathcal{O} - 1;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1.$$

Thus S is $(\mathcal{O}-1)$-offensive alliance.

□

In next example, the concept of r-defensive alliance and r-offensive alliance are applied into a r-regular-strong-neutrosophic graph which is complete and its order is five, it means $\mathcal{O} = 5$.

Example 2.5. Consider Figure (1). In this section, 1-powerful alliance is studied in the way that more clarifications are represented.

(*i*) Every 3-set of vertices, e.g.,

$$S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_3, s_5\}, S_3 = \{s_2, s_3, s_4\}, S_4 = \{s_3, s_4, s_5\}$$

is minimal-1-powerful alliance and it forms a minimal-1-powerful-alliance number but only $S_3 = \{s_3, s_4, s_5\}$ is optimal such that forms both minimal-1-powerful-alliance-neutrosophic number and minimal-1-powerful-alliance number;

(*ii*) $N = \{s_2, s_5\}$ isn't 1-powerful alliance. Since

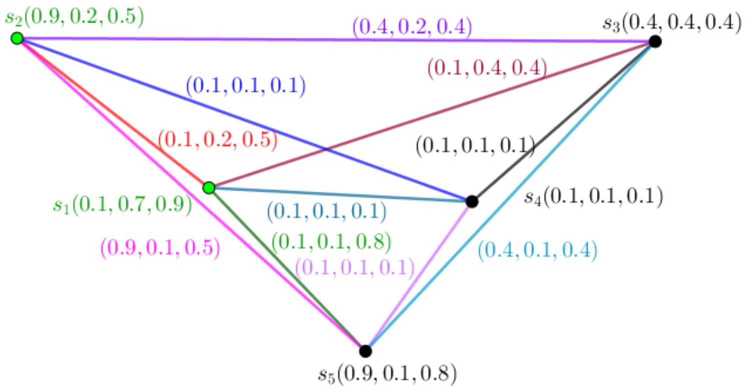

Figure 1. Black circles form a set which is 1-powerful alliance.

$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 2 - 2 = 0 < 1$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 2 - 2 = 0 \not> 1$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| \not> 1;$$

it implies $N = \{s_2, s_5\}$ isn't 1-offensive alliance. So $N = \{s_2, s_5\}$ isn't 1-powerful alliance. Also,

$$\exists s_2 \in N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 1 - 3 = -2 < 1$$
$$\exists s_2 \in N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 1 - 3 = -2 < 1$$
$$\exists s_2 \in N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| < 1;$$

it implies $N = \{s_2, s_5\}$ 1-defensive alliance but $N = \{s_2, s_5\}$ isn't 1-powerful alliance.

(iii) $\Gamma_s = 3.3$;

(iv) $\Gamma = 3$.

As a special case, cycle neutrosophic graph gets specific result excerpt from 2-regular neutrosophic graph. 2-defensive alliance and 2-offensive alliance get some results about the neighbors inside fixed set which their number is at most 2. Also, 2-defensive alliance and 2-offensive alliance get some results about the neighbors outside of fixed set which their number is at most 2.

Proposition 2.6. *Let* $NTG : (V, E, \sigma, \mu)$ *be a r-regular-strong-neutrosophic graph which is cycle. Then following statements hold;*

(i) $\forall a \in S,\ |N_s(a) \cap S| < 2$ *if* $NTG : (V, E, \sigma, \mu)$ *is 2-defensive alliance;*

(ii) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$ *if* $NTG : (V, E, \sigma, \mu)$ *is 2-offensive alliance;*

(iii) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $NTG : (V, E, \sigma, \mu)$ *is 2-defensive alliance;*

(iv) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $NTG : (V, E, \sigma, \mu)$ *is 2-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| < 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-offensive alliance. Then

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$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| > 2,\ |N_s(t) \cap (V \setminus S) = 0.$$

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance.

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| < 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a 2-regular-strong-neutrosophic graph and r-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| > 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

□

As a special case of r-regular, cycle is an attribute. This property facilitates the results when the condition is about the neighbors inside fixed set to determine 2-defensive alliance and 2-offensive alliance. Also, a condition about the neighbors outside of fixed set determines some results about 2-defensive alliance and 2-offensive alliance.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph which is cycle. Then following statements hold;*

- (i) *if $\forall a \in S,\ |N_s(a) \cap S| < 2$, then $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*
- (ii) *if $\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$, then $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*
- (iii) *if $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*
- (iv) *if $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0 = 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0 = 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0 = 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0 = 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

□

Example 2.8. Consider Figure (2). In this section, 3-powerful alliance is studied in the way that more clarifications are represented.

(i) Every 3-set of vertices, e.g.,

$$S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_3, s_5\}, S_3 = \{s_2, s_3, s_4\}, S_4 = \{s_3, s_4, s_5\}$$

is minimal-3-powerful alliance and it forms a minimal-3-powerful-alliance number but only $S_3 = \{s_3, s_4, s_5\}$ is optimal such that forms both minimal-3-powerful-alliance-neutrosophic number and minimal-3-powerful-alliance number; since

$$\exists s_3 \in S_4,\ |N_s(s_3) \cap S_4| - |N_s(s_3) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_3 \in S_4,\ |N_s(s_3) \cap S_4| - |N_s(s_3) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_3 \in S_4,\ |N_s(s_3) \cap S_4| - |N_s(s_3) \cap (V \setminus S_4)| < 3;$$

$$\exists s_5 \in S_4,\ |N_s(s_5) \cap S_4| - |N_s(s_5) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_5 \in S_4,\ |N_s(s_5) \cap N| - |N_s(s_5) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_5 \in S_4,\ |N_s(s_5) \cap S_4| - |N_s(s_5) \cap (V \setminus S_4)| < 3;$$

$$\exists s_4 \in S_4,\ |N_s(s_4) \cap S_4| - |N_s(s_4) \cap (V \setminus S_4)| = 2 - 0 = 2 < 3$$
$$\exists s_4 \in S_4,\ |N_s(s_4) \cap S_4| - |N_s(s_4) \cap (V \setminus S_4)| = 2 - 0 = 2 < 3$$
$$\exists s_4 \in S_4,\ |N_s(s_4) \cap S_4| - |N_s(s_4) \cap (V \setminus N)| < 3;$$

It implies S_4 is 3-defensive alliance. Also,

$$\exists s_1 \in V \setminus S_4,\ |N_s(s_1) \cap S_4| - |N_s(s_1) \cap (V \setminus S_4)| = 1 - 1 = 0 > -1$$
$$\exists s_1 \in V \setminus S_4,\ |N_s(s_1) \cap S_4| - |N_s(s_1) \cap (V \setminus S_4)| = 1 - 1 = 0 > -1$$
$$\exists s_1 \in V \setminus S_4,\ |N_s(s_1) \cap S_4| - |N_s(s_1) \cap (V \setminus S_4)| > -1;$$

$$\exists s_2 \in S_4,\ |N_s(s_2) \cap N| - |N_s(s_2) \cap (V \setminus N)| = 1 - 1 = 0 > -1$$
$$\exists s_2 \in N,\ |N_s(s_2) \cap N| - |N_s(s_2) \cap (V \setminus N)| = 1 - 1 = 0 > -1$$
$$\exists s_2 \in N,\ |N_s(s_2) \cap N| - |N_s(s_2) \cap (V \setminus N)| > -1;$$

It implies S_4 is (-1)-offensive alliance. S_4 isn't (-1)-powerful alliance.

(ii) Every 4-set of vertices, e.g.,

$$S_1 = \{s_1, s_2, s_3, s_4\}, S_2 = \{s_1, s_2, s_3, s_5\}, S_3 = \{s_2, s_3, s_4, s_5\}$$

is minimal-3-powerful alliance and it forms a minimal-3-powerful-alliance number but only $S = \{s_2, s_3, s_4, s_5\}$ is optimal such that forms both minimal-3-powerful-alliance-neutrosophic number and minimal-3-powerful-alliance number; since

$$\exists s_3 \in S,\ |N_s(s_3) \cap S| - |N_s(s_3) \cap (V \setminus S)| = 2 - 0 = 2 < 3$$
$$\exists s_3 \in S,\ |N_s(s_3) \cap S| - |N_s(s_3) \cap (V \setminus S)| = 2 - 0 = 0 < 3$$
$$\exists s_3 \in S,\ |N_s(s_3) \cap S| - |N_s(s_3) \cap (V \setminus S)| < 3;$$

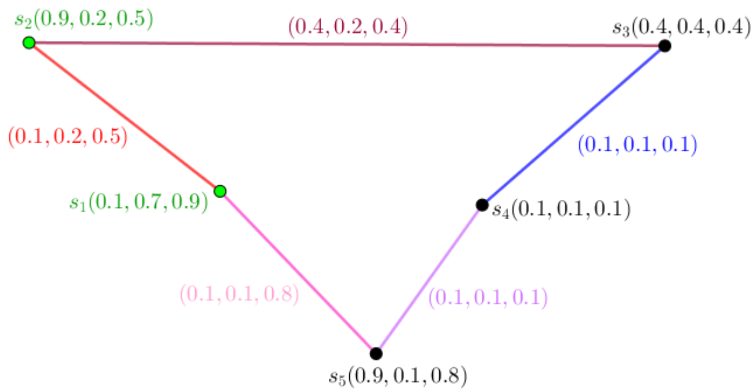

Figure 2. Black circles form a set which is 1-powerful alliance.

$$\exists s_4 \in S,\ |N_s(s_4) \cap S| - |N_s(s_4) \cap (V \setminus S)| = 2 - 0 = 2 < 3$$
$$\exists s_4 \in S,\ |N_s(s_4) \cap S| - |N_s(s_4) \cap (V \setminus S)| = 2 - 0 = 2 < 3$$
$$\exists s_4 \in S,\ |N_s(s_4) \cap S| - |N_s(s_4) \cap (V \setminus S)| < 3;$$

$$\exists s_5 \in S,\ |N_s(s_5) \cap S| - |N_s(s_5) \cap (V \setminus S)| = 1 - 1 = 0 < 3$$
$$\exists s_5 \in S,\ |N_s(s_5) \cap S| - |N_s(s_5) \cap (V \setminus S)| = 1 - 1 = 0 < 3$$
$$\exists s_5 \in S,\ |N_s(s_5) \cap S| - |N_s(s_5) \cap (V \setminus S)| < 3;$$

$$\exists s_2 \in S,\ |N_s(s_2) \cap S| - |N_s(s_2) \cap (V \setminus S)| = 1 - 1 = 0 < 3$$
$$\exists s_2 \in S,\ |N_s(s_2) \cap S| - |N_s(s_2) \cap (V \setminus S)| = 1 - 1 = 0 < 3$$
$$\exists s_2 \in S,\ |N_s(s_2) \cap S| - |N_s(s_2) \cap (V \setminus S)| < 3;$$

it implies S is 3-defensive alliance. Also,

$$\exists s_1 \in V \setminus S,\ |N_s(s_1) \cap S| - |N_s(s_1) \cap (V \setminus S)| = 2 - 0 = 2 > 1$$
$$\exists s_1 \in V \setminus S,\ |N_s(s_1) \cap S| - |N_s(s_1) \cap (V \setminus S)| = 2 - 0 = 2 > 1$$
$$\exists s_1 \in V \setminus S,\ |N_s(s_1) \cap S| - |N_s(s_1) \cap (V \setminus S)| > 1;$$

it implies S_4 is 1-offensive alliance. S_4 isn't 1-powerful alliance.

(iii) Γ_s isn't well-defined;

(iv) Γ isn't well-defined.

3 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importantance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NTG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

3.1 Case 1: Complete Model as Individual

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented as Figure (3). This model is strong. And the study proposes using specific set of objects which is called minimal-global-offensive alliance. There are also some analyses on other sets in the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (3).

(i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_6 = \{s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;

(iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;

(a) First counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N, \ |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N, \ |N_s(s_2) \cap N| = 1 \not> 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N, \ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

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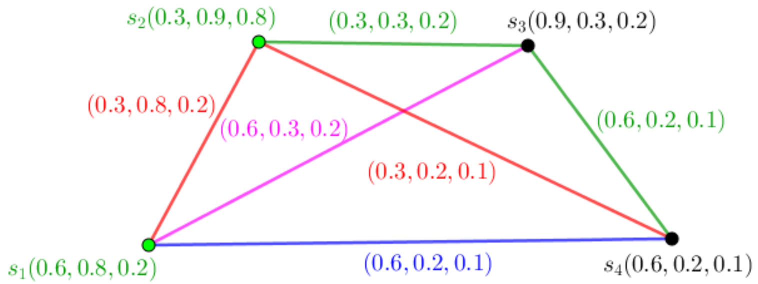

Figure 3. The set of black circles is minimal-global-offensive alliance.

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.".

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 2.3$ and corresponded set is $S_6 = \{s_3, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_6 = \{s_3, s_4\}$.

3.2 Case 2: Family of Complete Models

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented in the formation of family of models as Figure (3). These models are strong in family. And the study proposes using specific set of objects which is called minimal-global-offensive alliance for this family of models. There are also some analyses on other sets in the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this family of models and collection of situations to compare them with collection of situations to get more precise. Consider Figure (4).

(i) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

(ii) $S_3 = \{s_1, s_2, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for $\mathcal{G}$;

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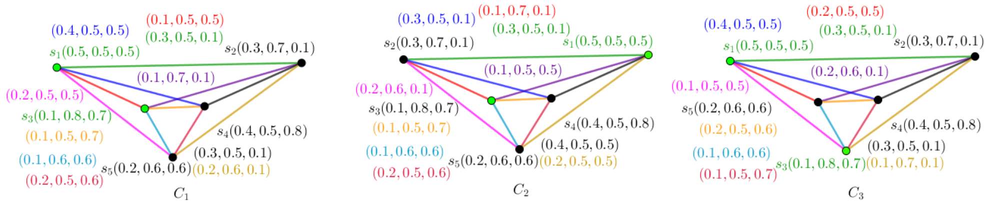

Figure 4. The set of black circles is minimal-global-offensive alliance.

(*iii*) $S_8 = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for $\mathcal{G}$;

(*iv*) $N = \{s_1, s_2\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for $\mathcal{G}$;

(*a*) First counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance."for $\mathcal{G}$;

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 2 = 2 = |N_s(s_3 \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 2 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(*b*) second counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$;

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 2 = 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 2 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(*c*) third counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$.

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 \not> 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(*v*) $\Gamma_s = 4$ and corresponded set is $S_3 = \{s_1, s_2, s_5\}$ for $\mathcal{G}$;

(*vi*) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances for $\mathcal{G}$.

4 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning alliance is defined in neutrosophic graphs. Neutrosophic number is also introduced. Thus,

Question 4.1. *Is it possible to use other types neighborhood arising from different types of edges to define new alliances?*

Question 4.2. *Are existed some connections amid different types of alliances in neutrosophic graphs?*

Question 4.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 4.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 4.5. *Which parameters are related to this parameter?*

Problem 4.6. *Which approaches do work to construct applications to create independent study?*

Problem 4.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

5 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses one definition concerning global powerful alliance to study neutrosophic graphs. New neutrosophic number is introduced which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a set

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Defining Global Powerful Alliances	1. General Results
2. Applying on Strong Neutrosophic Graphs	
3. Study on Complete Models	2. Study On Classes
4. Applying on Individuals	
5. Applying on Family	3. Same Models in Family

which is called global powerful alliance. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

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