Algorithms for Computing M-Plithogenic Eigen Values and Vectors for Some Special Values of M

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Abstract

Plithogenic matrices are considered as advanced mathematical generalizations of classical square matrices. One of the most research problems that is related to them is finding the eigen-values and vectors. This paper aims to present an easy algorithm to find all eigen-values and eigen-vectors for 10 different symbolic square n-plithogenic matrices with plithogenic real entries, where n is between 20 and 29. All algorithms are explained through clear theorems and proofs.

Keywords: plithogenic matrix; plithogenic eigen-value; plithogenic eigen vector; neutrosophic matrix.

1. Introduction

The theory of neutrosophic logic and plithogenic sets was studied widely by many authors, especially those applications that are related to algebra and algebraic structures [1-2].

In [20-23], we find many great efforts done by mathematicians to improve and to understand the algebraic behaviors of these algebraic structures. These structures diverse in many categories such as commutative structures [16-18], and non-commutative structures [3-4, 12-15].

The study of plithogenic matrices and their substructures began in [7], with many algebraic concepts such as determinants and Invertibility of 2-plithogenic matrices. Recently, many authors keep generalizing results to higher orders of plithogenic matrices. For example, 3-plithogenic and 4-plithogenic matrices were studied in [8,26]. Also, very large orders were discussed in different works, see [10-11, 27-28].

This prompted us to close an interesting research gap by providing algorithms to compute eigen-values and vectors of plithogenic matrices of finite orders between 20 and 29, which is considered as a continuation of the efforts done previously to generalize results for extended classes of plithogenic matrices.

2. Main Discussion

Definition:

The square 20-plithogenic matrix is defined:

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$\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$; $(\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$ be a symbolic 20-plithogenic matrix, we have:

$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1} \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2} \aleph_i) - \det(\sum_{i=0}^{1} \aleph_i)]P_2 + [\det(\sum_{i=0}^{3} \aleph_i) - \det(\sum_{i=0}^{2} \aleph_i)]P_3 + [\det(\sum_{i=0}^{4} \aleph_i) - \det(\sum_{i=0}^{3} \aleph_i)]P_4 + \left[\det\left(\sum_{i=0}^{5} \aleph_i\right) - \det(\sum_{i=0}^{4} \aleph_i)\right]P_5 + \left[\det(\sum_{i=0}^{6} \aleph_i) - \det\left(\sum_{i=0}^{5} \aleph_i\right)\right]P_6 + [\det(\sum_{i=0}^{7} \aleph_i) - \det(\sum_{i=0}^{6} \aleph_i)]P_7 + [\det(\sum_{i=0}^{8} \aleph_i) - \det(\sum_{i=0}^{7} \aleph_i)]P_8 + [\det(\sum_{i=0}^{9} \aleph_i) - \det(\sum_{i=0}^{8} \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9} \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15} \aleph_i\right) - \det(\sum_{i=0}^{14} \aleph_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16} \aleph_i) - \det\left(\sum_{i=0}^{15} \aleph_i\right)\right]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{20} \delta_i P_i$ be a 20-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$; $(\aleph_i)_{n\times n}$ be a 20-plithogenic square real matrix, then δ is called 20-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 20-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{20} \delta_i P_i \in 20 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{20} \nabla_i P_i$ be a 20-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^{t} \delta_i$ is eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as eigen vector with $0 \le t \le 20$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:

$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t} \aleph_i \sum_{i=0}^{t} \nabla_i = \sum_{i=0}^{t} \delta_i \sum_{i=0}^{t} \nabla_i ; \ 1 \le t \le 20 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^{t} \delta_i$ is an eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as an eigen vector for all $1 \le t \le 20$.

Definition:

The square 21-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$; $(\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$ be a symbolic 20-plithogenic matrix, we have:

$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1} \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2} \aleph_i) - \det(\sum_{i=0}^{1} \aleph_i)]P_2 + [\det(\sum_{i=0}^{3} \aleph_i) - \det(\sum_{i=0}^{2} \aleph_i)]P_3 + [\det(\sum_{i=0}^{4} \aleph_i) - \det(\sum_{i=0}^{3} \aleph_i)]P_4 + \left[\det\left(\sum_{i=0}^{5} \aleph_i\right) - \det(\sum_{i=0}^{4} \aleph_i)\right]P_5 + \left[\det(\sum_{i=0}^{6} \aleph_i) - \det\left(\sum_{i=0}^{5} \aleph_i\right)\right]P_6 + [\det(\sum_{i=0}^{7} \aleph_i) - \det(\sum_{i=0}^{6} \aleph_i)]P_7 + [\det(\sum_{i=0}^{8} \aleph_i) - \det(\sum_{i=0}^{7} \aleph_i)]P_8 + [\det(\sum_{i=0}^{9} \aleph_i) - \det(\sum_{i=0}^{8} \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9} \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15} \aleph_i\right) - \det(\sum_{i=0}^{14} \aleph_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16} \aleph_i) - \det\left(\sum_{i=0}^{15} \aleph_i\right)\right]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{21} \delta_i P_i$ be a 21-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$; $(\aleph_i)_{n\times n}$ be a 21-plithogenic square real matrix, then δ is called 21-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 21-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{21} \delta_i P_i \in 21 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{21} \nabla_i P_i$ be a 21-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^{t} \delta_i$ is eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as eigen vector with $0 \le t \le 21$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:

$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t} \aleph_i \sum_{i=0}^{t} \nabla_i = \sum_{i=0}^{t} \delta_i \sum_{i=0}^{t} \nabla_i ; \ 1 \le t \le 2`1 \end{cases}$$

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Which is equivalent to:
$\sum_{i=0}^{t}\delta_i$ is an eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as an eigen vector for all $1 \leq t \leq 21$.

Definition:

The square 22-plithogenic matrix is defined:
$\aleph = \aleph_0 + \sum_{i=1}^{22}\aleph_i P_i\,; (\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{22}\aleph_i P_i$ be a symbolic 22-plithogenic matrix, we have:
$\det\aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1}\aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2}\aleph_i) - \det(\sum_{i=0}^{1}\aleph_i)]P_2 + [\det(\sum_{i=0}^{3}\aleph_i) - \det(\sum_{i=0}^{2}\aleph_i)]P_3 + [\det(\sum_{i=0}^{4}\aleph_i) - \det(\sum_{i=0}^{3}\aleph_i)]P_4 + [\det(\sum_{i=0}^{5}\aleph_i) - \det(\sum_{i=0}^{4}\aleph_i)]P_5 + [\det(\sum_{i=0}^{6}\aleph_i) - \det(\sum_{i=0}^{5}\aleph_i)]P_6 + [\det(\sum_{i=0}^{7}\aleph_i) - \det(\sum_{i=0}^{6}\aleph_i)]P_7 + [\det(\sum_{i=0}^{8}\aleph_i) - \det(\sum_{i=0}^{7}\aleph_i)]P_8 + [\det(\sum_{i=0}^{9}\aleph_i) - \det(\sum_{i=0}^{8}\aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9}\aleph_i)]P_{10} + [\det(\sum_{i=0}^{11}\aleph_i) - \det(\sum_{i=0}^{10}\aleph_i)]P_{11} + [\det(\sum_{i=0}^{12}\aleph_i) - \det(\sum_{i=0}^{11}\aleph_i)]P_{12} + [\det(\sum_{i=0}^{13}\aleph_i) - \det(\sum_{i=0}^{12}\aleph_i)]P_{13} + [\det(\sum_{i=0}^{14}\aleph_i) - \det(\sum_{i=0}^{13}\aleph_i)]P_{14} + [\det(\sum_{i=0}^{15}\aleph_i) - \det(\sum_{i=0}^{14}\aleph_i)]P_{15} + [\det(\sum_{i=0}^{16}\aleph_i) - \det(\sum_{i=0}^{15}\aleph_i)]P_{16} + [\det(\sum_{i=0}^{17}\aleph_i) - \det(\sum_{i=0}^{16}\aleph_i)]P_{17} + [\det(\sum_{i=0}^{18}\aleph_i) - \det(\sum_{i=0}^{17}\aleph_i)]P_{18} + [\det(\sum_{i=0}^{19}\aleph_i) - \det(\sum_{i=0}^{18}\aleph_i)]P_{19} + [\det(\sum_{i=0}^{20}\aleph_i) - \det(\sum_{i=0}^{19}\aleph_i)]P_{20} + [\det(\sum_{i=0}^{21}\aleph_i) - \det(\sum_{i=0}^{20}\aleph_i)]P_{21} + [\det(\sum_{i=0}^{22}\aleph_i) - \det(\sum_{i=0}^{21}\aleph_i)]P_{22}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{22}\delta_i P_i$ be a 22-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{22}\aleph_i P_i\,; (\aleph_i)_{n\times n}$ be a 22-plithogenic square real matrix, then δ is called 22-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.
∇ is called 22-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{22}\delta_i P_i \in 22 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{22}\nabla_i P_i$ be a 22-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{22}\aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:
$\sum_{i=0}^{t}\delta_i$ is eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as eigen vector with $0 \leq t \leq 22$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:
$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t}\aleph_i\sum_{i=0}^{t}\nabla_i = \sum_{i=0}^{t}\delta_i\sum_{i=0}^{t}\nabla_i \end{cases};\ 1 \leq t \leq 22$$

Which is equivalent to:
$\sum_{i=0}^{t}\delta_i$ is an eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as an eigen vector for all $1 \leq t \leq 22$.

Definition:

The square 23-plithogenic matrix is defined:
$\aleph = \aleph_0 + \sum_{i=1}^{23}\aleph_i P_i\,; (\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{23}\aleph_i P_i$ be a symbolic 23-plithogenic matrix, we have:
$\det\aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1}\aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2}\aleph_i) - \det(\sum_{i=0}^{1}\aleph_i)]P_2 + [\det(\sum_{i=0}^{3}\aleph_i) - \det(\sum_{i=0}^{2}\aleph_i)]P_3 + [\det(\sum_{i=0}^{4}\aleph_i) - \det(\sum_{i=0}^{3}\aleph_i)]P_4 + [\det(\sum_{i=0}^{5}\aleph_i) - \det(\sum_{i=0}^{4}\aleph_i)]P_5 + [\det(\sum_{i=0}^{6}\aleph_i) - \det(\sum_{i=0}^{5}\aleph_i)]P_6 + [\det(\sum_{i=0}^{7}\aleph_i) - \det(\sum_{i=0}^{6}\aleph_i)]P_7 + [\det(\sum_{i=0}^{8}\aleph_i) - \det(\sum_{i=0}^{7}\aleph_i)]P_8 + [\det(\sum_{i=0}^{9}\aleph_i) - \det(\sum_{i=0}^{8}\aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9}\aleph_i)]P_{10} + [\det(\sum_{i=0}^{11}\aleph_i) - \det(\sum_{i=0}^{10}\aleph_i)]P_{11} + [\det(\sum_{i=0}^{12}\aleph_i) - \det(\sum_{i=0}^{11}\aleph_i)]P_{12} + [\det(\sum_{i=0}^{13}\aleph_i) - \det(\sum_{i=0}^{12}\aleph_i)]P_{13} + [\det(\sum_{i=0}^{14}\aleph_i) - \det(\sum_{i=0}^{13}\aleph_i)]P_{14} + [\det(\sum_{i=0}^{15}\aleph_i) - \det(\sum_{i=0}^{14}\aleph_i)]P_{15} + [\det(\sum_{i=0}^{16}\aleph_i) - \det(\sum_{i=0}^{15}\aleph_i)]P_{16} + [\det(\sum_{i=0}^{17}\aleph_i) - \det(\sum_{i=0}^{16}\aleph_i)]P_{17} + [\det(\sum_{i=0}^{18}\aleph_i) - \det(\sum_{i=0}^{17}\aleph_i)]P_{18} + [\det(\sum_{i=0}^{19}\aleph_i) - \det(\sum_{i=0}^{18}\aleph_i)]P_{19} + [\det(\sum_{i=0}^{20}\aleph_i) - \det(\sum_{i=0}^{19}\aleph_i)]P_{20} + [\det(\sum_{i=0}^{21}\aleph_i) - \det(\sum_{i=0}^{20}\aleph_i)]P_{21} + [\det(\sum_{i=0}^{22}\aleph_i) - \det(\sum_{i=0}^{21}\aleph_i)]P_{22} + [\det(\sum_{i=0}^{23}\aleph_i) - \det(\sum_{i=0}^{22}\aleph_i)]P_{23}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{23}\delta_i P_i$ be a 23-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{23}\aleph_i P_i\,; (\aleph_i)_{n\times n}$ be a 23-plithogenic square real matrix, then δ is called 23-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.
∇ is called 23-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{23}\delta_i P_i \in 23 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{23}\nabla_i P_i$ be a 23-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{23}\aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:
$\sum_{i=0}^{t}\delta_i$ is eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as eigen vector with $0 \leq t \leq 23$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:
$\aleph\nabla = \delta\nabla$., thus:

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$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t}\aleph_i\sum_{i=0}^{t}\nabla_i = \sum_{i=0}^{t}\delta_i\sum_{i=0}^{t}\nabla_i \end{cases} ;\ 1 \le t \le 23$$

Which is equivalent to:

$\sum_{i=0}^{t}\delta_i$ is an eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as an eigen vector for all $1 \le t \le 23$.

Definition:

The square 24-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{24}\aleph_i P_i$; $(\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{24}\aleph_i P_i$ be a symbolic 24-plithogenic matrix, we have:

$\det\aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1}\aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2}\aleph_i) - \det(\sum_{i=0}^{1}\aleph_i)]P_2 + [\det(\sum_{i=0}^{3}\aleph_i) - \det(\sum_{i=0}^{2}\aleph_i)]P_3 + [\det(\sum_{i=0}^{4}\aleph_i) - \det(\sum_{i=0}^{3}\aleph_i)]P_4 + \left[\det\left(\sum_{i=0}^{5}\aleph_i\right) - \det(\sum_{i=0}^{4}\aleph_i)\right]P_5 + \left[\det(\sum_{i=0}^{6}\aleph_i) - \det\left(\sum_{i=0}^{5}\aleph_i\right)\right]P_6 + [\det(\sum_{i=0}^{7}\aleph_i) - \det(\sum_{i=0}^{6}\aleph_i)]P_7 + [\det(\sum_{i=0}^{8}\aleph_i) - \det(\sum_{i=0}^{7}\aleph_i)]P_8 + [\det(\sum_{i=0}^{9}\aleph_i) - \det(\sum_{i=0}^{8}\aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9}\aleph_i)]P_{10} + [\det(\sum_{i=0}^{11}\aleph_i) - \det(\sum_{i=0}^{10}\aleph_i)]P_{11} + [\det(\sum_{i=0}^{12}\aleph_i) - \det(\sum_{i=0}^{11}\aleph_i)]P_{12} + [\det(\sum_{i=0}^{13}\aleph_i) - \det(\sum_{i=0}^{12}\aleph_i)]P_{13} + [\det(\sum_{i=0}^{14}\aleph_i) - \det(\sum_{i=0}^{13}\aleph_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15}\aleph_i\right) - \det(\sum_{i=0}^{14}\aleph_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16}\aleph_i) - \det\left(\sum_{i=0}^{15}\aleph_i\right)\right]P_{16} + [\det(\sum_{i=0}^{17}\aleph_i) - \det(\sum_{i=0}^{16}\aleph_i)]P_{17} + [\det(\sum_{i=0}^{18}\aleph_i) - \det(\sum_{i=0}^{17}\aleph_i)]P_{18} + [\det(\sum_{i=0}^{19}\aleph_i) - \det(\sum_{i=0}^{18}\aleph_i)]P_{19} + [\det(\sum_{i=0}^{20}\aleph_i) - \det(\sum_{i=0}^{19}\aleph_i)]P_{20} + [\det(\sum_{i=0}^{21}\aleph_i) - \det(\sum_{i=0}^{20}\aleph_i)]P_{21} + [\det(\sum_{i=0}^{22}\aleph_i) - \det(\sum_{i=0}^{21}\aleph_i)]P_{22} + [\det(\sum_{i=0}^{23}\aleph_i) - \det(\sum_{i=0}^{22}\aleph_i)]P_{23} + [\det(\sum_{i=0}^{24}\aleph_i) - \det(\sum_{i=0}^{23}\aleph_i)]P_{24}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{24}\delta_i P_i$ be a 24-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{24}\aleph_i P_i$; $(\aleph_i)_{n\times n}$ be a 24-plithogenic square real matrix, then δ is called 24-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 24-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{24}\delta_i P_i \in 24 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{24}\nabla_i P_i$ be a 24-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{24}\aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^{t}\delta_i$ is eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as eigen vector with $0 \le t \le 24$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:

$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t}\aleph_i\sum_{i=0}^{t}\nabla_i = \sum_{i=0}^{t}\delta_i\sum_{i=0}^{t}\nabla_i \end{cases} ;\ 1 \le t \le 24$$

Which is equivalent to:

$\sum_{i=0}^{t}\delta_i$ is an eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as an eigen vector for all $1 \le t \le 24$.

Definition:

The square 25-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{25}\aleph_i P_i$; $(\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{25}\aleph_i P_i$ be a symbolic 25-plithogenic matrix, we have:

$\det\aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1}\aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2}\aleph_i) - \det(\sum_{i=0}^{1}\aleph_i)]P_2 + [\det(\sum_{i=0}^{3}\aleph_i) - \det(\sum_{i=0}^{2}\aleph_i)]P_3 + [\det(\sum_{i=0}^{4}\aleph_i) - \det(\sum_{i=0}^{3}\aleph_i)]P_4 + \left[\det\left(\sum_{i=0}^{5}\aleph_i\right) - \det(\sum_{i=0}^{4}\aleph_i)\right]P_5 + \left[\det(\sum_{i=0}^{6}\aleph_i) - \det\left(\sum_{i=0}^{5}\aleph_i\right)\right]P_6 + [\det(\sum_{i=0}^{7}\aleph_i) - \det(\sum_{i=0}^{6}\aleph_i)]P_7 + [\det(\sum_{i=0}^{8}\aleph_i) - \det(\sum_{i=0}^{7}\aleph_i)]P_8 + [\det(\sum_{i=0}^{9}\aleph_i) - \det(\sum_{i=0}^{8}\aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9}\aleph_i)]P_{10} + [\det(\sum_{i=0}^{11}\aleph_i) - \det(\sum_{i=0}^{10}\aleph_i)]P_{11} + [\det(\sum_{i=0}^{12}\aleph_i) - \det(\sum_{i=0}^{11}\aleph_i)]P_{12} + [\det(\sum_{i=0}^{13}\aleph_i) - \det(\sum_{i=0}^{12}\aleph_i)]P_{13} + [\det(\sum_{i=0}^{14}\aleph_i) - \det(\sum_{i=0}^{13}\aleph_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15}\aleph_i\right) - \det(\sum_{i=0}^{14}\aleph_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16}\aleph_i) - \det\left(\sum_{i=0}^{15}\aleph_i\right)\right]P_{16} + [\det(\sum_{i=0}^{17}\aleph_i) - \det(\sum_{i=0}^{16}\aleph_i)]P_{17} + [\det(\sum_{i=0}^{18}\aleph_i) - \det(\sum_{i=0}^{17}\aleph_i)]P_{18} + [\det(\sum_{i=0}^{19}\aleph_i) - \det(\sum_{i=0}^{18}\aleph_i)]P_{19} + [\det(\sum_{i=0}^{20}\aleph_i) - \det(\sum_{i=0}^{19}\aleph_i)]P_{20} + [\det(\sum_{i=0}^{21}\aleph_i) - \det(\sum_{i=0}^{20}\aleph_i)]P_{21} + [\det(\sum_{i=0}^{22}\aleph_i) - \det(\sum_{i=0}^{21}\aleph_i)]P_{22} + [\det(\sum_{i=0}^{23}\aleph_i) - \det(\sum_{i=0}^{22}\aleph_i)]P_{23} + [\det(\sum_{i=0}^{24}\aleph_i) - \det(\sum_{i=0}^{23}\aleph_i)]P_{24} + \left[\det\left(\sum_{i=0}^{25}\aleph_i\right) - \det(\sum_{i=0}^{24}\aleph_i)\right]P_{25}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{25}\delta_i P_i$ be a 25-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{25}\aleph_i P_i$; $(\aleph_i)_{n\times n}$ be a 25-plithogenic square real matrix, then δ is called 25-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 25-plithogenic eigenvector.

Theorem:

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Let $\delta = \delta_0 + \sum_{i=1}^{25} \delta_i P_i \in 25 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{25} \nabla_i P_i$ be a 25-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{25} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:
$\sum_{i=0}^{t} \delta_i$ is eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as eigen vector with $0 \le t \le 25$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:
$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t} \aleph_i \sum_{i=0}^{t} \nabla_i = \sum_{i=0}^{t} \delta_i \sum_{i=0}^{t} \nabla_i ; \ 1 \le t \le 25 \end{cases}$$

Which is equivalent to:
$\sum_{i=0}^{t} \delta_i$ is an eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as an eigen vector for all $1 \le t \le 25$.

Definition:

The square 26-plithogenic matrix is defined:
$\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i ; (\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i$ be a symbolic 26-plithogenic matrix, we have:
$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1} \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2} \aleph_i) - \det(\sum_{i=0}^{1} \aleph_i)]P_2 + [\det(\sum_{i=0}^{3} \aleph_i) - \det(\sum_{i=0}^{2} \aleph_i)]P_3 + [\det(\sum_{i=0}^{4} \aleph_i) - \det(\sum_{i=0}^{3} \aleph_i)]P_4 + \left[\det\left(\sum_{i=0}^{5} \aleph_i\right) - \det(\sum_{i=0}^{4} \aleph_i)\right]P_5 + \left[\det(\sum_{i=0}^{6} \aleph_i) - \det\left(\sum_{i=0}^{5} \aleph_i\right)\right]P_6 + [\det(\sum_{i=0}^{7} \aleph_i) - \det(\sum_{i=0}^{6} \aleph_i)]P_7 + [\det(\sum_{i=0}^{8} \aleph_i) - \det(\sum_{i=0}^{7} \aleph_i)]P_8 + [\det(\sum_{i=0}^{9} \aleph_i) - \det(\sum_{i=0}^{8} \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9} \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15} \aleph_i\right) - \det(\sum_{i=0}^{14} \aleph_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)\right]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22} + [\det(\sum_{i=0}^{23} \aleph_i) - \det(\sum_{i=0}^{22} \aleph_i)]P_{23} + [\det(\sum_{i=0}^{24} \aleph_i) - \det(\sum_{i=0}^{23} \aleph_i)]P_{24} + \left[\det\left(\sum_{i=0}^{25} \aleph_i\right) - \det(\sum_{i=0}^{24} \aleph_i)\right]P_{25} + \left[\det(\sum_{i=0}^{26} \aleph_i) - \det\left(\sum_{i=0}^{25} \aleph_i\right)\right]P_{26}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{26} \delta_i P_i$ be a 26-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i ; (\aleph_i)_{n\times n}$ be a 26-plithogenic square real matrix, then δ is called 26-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.
∇ is called 26-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{26} \delta_i P_i \in 26 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{26} \nabla_i P_i$ be a 26-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:
$\sum_{i=0}^{t} \delta_i$ is eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as eigen vector with $0 \le t \le 26$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:
$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t} \aleph_i \sum_{i=0}^{t} \nabla_i = \sum_{i=0}^{t} \delta_i \sum_{i=0}^{t} \nabla_i ; \ 1 \le t \le 26 \end{cases}$$

Which is equivalent to:
$\sum_{i=0}^{t} \delta_i$ is an eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as an eigen vector for all $1 \le t \le 26$.

Definition:

The square 27-plithogenic matrix is defined:
$\aleph = \aleph_0 + \sum_{i=1}^{27} \aleph_i P_i ; (\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{27} \aleph_i P_i$ be a symbolic 27-plithogenic matrix, we have:
$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1} \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2} \aleph_i) - \det(\sum_{i=0}^{1} \aleph_i)]P_2 + [\det(\sum_{i=0}^{3} \aleph_i) - \det(\sum_{i=0}^{2} \aleph_i)]P_3 + [\det(\sum_{i=0}^{4} \aleph_i) - \det(\sum_{i=0}^{3} \aleph_i)]P_4 + \left[\det\left(\sum_{i=0}^{5} \aleph_i\right) - \det(\sum_{i=0}^{4} \aleph_i)\right]P_5 + \left[\det(\sum_{i=0}^{6} \aleph_i) - \det\left(\sum_{i=0}^{5} \aleph_i\right)\right]P_6 + [\det(\sum_{i=0}^{7} \aleph_i) - \det(\sum_{i=0}^{6} \aleph_i)]P_7 + [\det(\sum_{i=0}^{8} \aleph_i) - \det(\sum_{i=0}^{7} \aleph_i)]P_8 + [\det(\sum_{i=0}^{9} \aleph_i) - \det(\sum_{i=0}^{8} \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9} \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15} \aleph_i\right) - \det(\sum_{i=0}^{14} \aleph_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)\right]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22} + [\det(\sum_{i=0}^{23} \aleph_i) - \det(\sum_{i=0}^{22} \aleph_i)]P_{23} + [\det(\sum_{i=0}^{24} \aleph_i) -$

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$\det(\sum_{i=0}^{23}\aleph_i)]P_{24} + [\det(\sum_{i=0}^{25}\aleph_i) - \det(\sum_{i=0}^{24}\aleph_i)]P_{25} + [\det(\sum_{i=0}^{26}\aleph_i) - \det(\sum_{i=0}^{25}\aleph_i)]P_{26} + [\det(\sum_{i=0}^{27}\aleph_i) - \det(\sum_{i=0}^{26}\aleph_i)]P_{27}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{27}\delta_i P_i$ be a 27-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{27}\aleph_i P_i\,;(\aleph_i)_{n\times n}$ be a 27-plithogenic square real matrix, then δ is called 27-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 27-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{27}\delta_i P_i \in 27 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{27}\nabla_i P_i$ be a 27-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{27}\aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^{t}\delta_i$ is eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as eigen vector with $0 \le t \le 27$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:

$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t}\aleph_i \sum_{i=0}^{t}\nabla_i = \sum_{i=0}^{t}\delta_i \sum_{i=0}^{t}\nabla_i \end{cases} ;\ 1 \le t \le 27$$

Which is equivalent to:

$\sum_{i=0}^{t}\delta_i$ is an eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as an eigen vector for all $1 \le t \le 27$.

Definition:

The square 28-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{28}\aleph_i P_i\,;(\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{28}\aleph_i P_i$ be a symbolic 28-plithogenic matrix, we have:

$\det\aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1}\aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2}\aleph_i) - \det(\sum_{i=0}^{1}\aleph_i)]P_2 + [\det(\sum_{i=0}^{3}\aleph_i) - \det(\sum_{i=0}^{2}\aleph_i)]P_3 + [\det(\sum_{i=0}^{4}\aleph_i) - \det(\sum_{i=0}^{3}\aleph_i)]P_4 + [\det(\sum_{i=0}^{5}\aleph_i) - \det(\sum_{i=0}^{4}\aleph_i)]P_5 + [\det(\sum_{i=0}^{6}\aleph_i) - \det(\sum_{i=0}^{5}\aleph_i)]P_6 + [\det(\sum_{i=0}^{7}\aleph_i) - \det(\sum_{i=0}^{6}\aleph_i)]P_7 + [\det(\sum_{i=0}^{8}\aleph_i) - \det(\sum_{i=0}^{7}\aleph_i)]P_8 + [\det(\sum_{i=0}^{9}\aleph_i) - \det(\sum_{i=0}^{8}\aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9}\aleph_i)]P_{10} + [\det(\sum_{i=0}^{11}\aleph_i) - \det(\sum_{i=0}^{10}\aleph_i)]P_{11} + [\det(\sum_{i=0}^{12}\aleph_i) - \det(\sum_{i=0}^{11}\aleph_i)]P_{12} + [\det(\sum_{i=0}^{13}\aleph_i) - \det(\sum_{i=0}^{12}\aleph_i)]P_{13} + [\det(\sum_{i=0}^{14}\aleph_i) - \det(\sum_{i=0}^{13}\aleph_i)]P_{14} + [\det(\sum_{i=0}^{15}\aleph_i) - \det(\sum_{i=0}^{14}\aleph_i)]P_{15} + [\det(\sum_{i=0}^{16}\aleph_i) - \det(\sum_{i=0}^{15}\aleph_i)]P_{16} + [\det(\sum_{i=0}^{17}\aleph_i) - \det(\sum_{i=0}^{16}\aleph_i)]P_{17} + [\det(\sum_{i=0}^{18}\aleph_i) - \det(\sum_{i=0}^{17}\aleph_i)]P_{18} + [\det(\sum_{i=0}^{19}\aleph_i) - \det(\sum_{i=0}^{18}\aleph_i)]P_{19} + [\det(\sum_{i=0}^{20}\aleph_i) - \det(\sum_{i=0}^{19}\aleph_i)]P_{20} + [\det(\sum_{i=0}^{21}\aleph_i) - \det(\sum_{i=0}^{20}\aleph_i)]P_{21} + [\det(\sum_{i=0}^{22}\aleph_i) - \det(\sum_{i=0}^{21}\aleph_i)]P_{22} + [\det(\sum_{i=0}^{23}\aleph_i) - \det(\sum_{i=0}^{22}\aleph_i)]P_{23} + [\det(\sum_{i=0}^{24}\aleph_i) - \det(\sum_{i=0}^{23}\aleph_i)]P_{24} + [\det(\sum_{i=0}^{25}\aleph_i) - \det(\sum_{i=0}^{24}\aleph_i)]P_{25} + [\det(\sum_{i=0}^{26}\aleph_i) - \det(\sum_{i=0}^{25}\aleph_i)]P_{26} + [\det(\sum_{i=0}^{27}\aleph_i) - \det(\sum_{i=0}^{26}\aleph_i)]P_{27} + [\det(\sum_{i=0}^{28}\aleph_i) - \det(\sum_{i=0}^{27}\aleph_i)]P_{28}$.

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{28}\delta_i P_i$ be a 28-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{28}\aleph_i P_i\,;(\aleph_i)_{n\times n}$ be a 28-plithogenic square real matrix, then δ is called 28-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 28-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{28}\delta_i P_i \in 28 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{28}\nabla_i P_i$ be a 28-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{28}\aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^{t}\delta_i$ is eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as eigen vector with $0 \le t \le 28$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:

$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t}\aleph_i \sum_{i=0}^{t}\nabla_i = \sum_{i=0}^{t}\delta_i \sum_{i=0}^{t}\nabla_i \end{cases} ;\ 1 \le t \le 28$$

Which is equivalent to:

$\sum_{i=0}^{t}\delta_i$ is an eigen value of $\sum_{i=0}^{t}\aleph_i$ with $\sum_{i=0}^{t}\nabla_i$ as an eigen vector for all $1 \le t \le 28$.

Definition:

The square 29-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{29}\aleph_i P_i\,;(\aleph_i)_{n\times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{29}\aleph_i P_i$ be a symbolic 29-plithogenic matrix, we have:

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$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^{1} \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^{2} \aleph_i) - \det(\sum_{i=0}^{1} \aleph_i)]P_2 + [\det(\sum_{i=0}^{3} \aleph_i) - \det(\sum_{i=0}^{2} \aleph_i)]P_3 + [\det(\sum_{i=0}^{4} \aleph_i) - \det(\sum_{i=0}^{3} \aleph_i)]P_4 + [\det(\sum_{i=0}^{5} \aleph_i) - \det(\sum_{i=0}^{4} \aleph_i)]P_5 + [\det(\sum_{i=0}^{6} \aleph_i) - \det(\sum_{i=0}^{5} \aleph_i)]P_6 + [\det(\sum_{i=0}^{7} \aleph_i) - \det(\sum_{i=0}^{6} \aleph_i)]P_7 + [\det(\sum_{i=0}^{8} \aleph_i) - \det(\sum_{i=0}^{7} \aleph_i)]P_8 + [\det(\sum_{i=0}^{9} \aleph_i) - \det(\sum_{i=0}^{8} \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^{9} \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22} + [\det(\sum_{i=0}^{23} \aleph_i) - \det(\sum_{i=0}^{22} \aleph_i)]P_{23} + [\det(\sum_{i=0}^{24} \aleph_i) - \det(\sum_{i=0}^{23} \aleph_i)]P_{24} + [\det(\sum_{i=0}^{25} \aleph_i) - \det(\sum_{i=0}^{24} \aleph_i)]P_{25} + [\det(\sum_{i=0}^{26} \aleph_i) - \det(\sum_{i=0}^{25} \aleph_i)]P_{26} + [\det(\sum_{i=0}^{27} \aleph_i) - \det(\sum_{i=0}^{26} \aleph_i)]P_{27} + [\det(\sum_{i=0}^{28} \aleph_i) - \det(\sum_{i=0}^{27} \aleph_i)]P_{28} + [\det(\sum_{i=0}^{29} \aleph_i) - \det(\sum_{i=0}^{28} \aleph_i)]P_{29}.$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{29} \delta_i P_i$ be a 29-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{29} \aleph_i P_i$; $(\aleph_i)_{n\times n}$ be a 29-plithogenic square real matrix, then δ is called 29-plithogenic eigenvalue if and only if $\aleph\nabla = \delta\nabla$.

∇ is called 29-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{29} \delta_i P_i \in 29 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{29} \nabla_i P_i$ be a 29-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{29} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^{t} \delta_i$ is eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as eigen vector with $0 \le t \le 29$.

Proof:

δ is an eigen value of $\aleph$ with ∇ as an eigen vector if:

$\aleph\nabla = \delta\nabla$., thus:

$$\begin{cases} \aleph_0\nabla_0 = \delta_0\nabla_0 \\ \sum_{i=0}^{t} \aleph_i \sum_{i=0}^{t} \nabla_i = \sum_{i=0}^{t} \delta_i \sum_{i=0}^{t} \nabla_i \end{cases} ; \ 1 \le t \le 29$$

Which is equivalent to:

$\sum_{i=0}^{t} \delta_i$ is an eigen value of $\sum_{i=0}^{t} \aleph_i$ with $\sum_{i=0}^{t} \nabla_i$ as an eigen vector for all $1 \le t \le 29$.

3. Conclusion

In this paper we presented an easy algorithm to find all eigen-values and eigen-vectors for 10 different symbolic square n-plithogenic matrices with plithogenic real entries, where n is between 20 and 29. All algorithms are explained through clear theorems and proofs. In the future, we aim to generalize our results for plithogenic matrices of orders higher than 30.

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