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An Algebraic Approach to n-Plithogenic Square Matrices For $18 \leq n \leq 19$

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Abstract

This paper is dedicated to study the algebraic properties and substructures that are related to the symbolic 18 and 19 plithogenic square matrices, where we present many different theorems and examples to clarify the computation of symbolic 18 and 19 plithogenic determinants, and finding 18-plithogenic and 19-plithogenic eigenvalues and eigenvectors. On the other hand, we provide algorithms for computing the inverses of symbolic 18-plithogenic and 19-plithogenic matrices with necessary and sufficient conditions for the orthogonality of those matrices.

Keywords: symbolic 18-plithogenic matrix; symbolic 19-plithogenic matrix; eigenvalue; eigenvector; orthogonal matrix

1. Introduction

The study of plithogenic sets began as a generalization of both fuzzy and neutrosophic sets in [2], where these sets were used in the construction of extended algebraic constructions of classical algebraic structures, where we see a direct use of plithogenic sets in the expansion of matrices [8,19], vector spaces and modules [3-5], and even integers [6, 17-18].

The study of matrices is fundamental in mathematics in general because of its wide applications, where many researchers around the world have studied them in detail, in terms of finding methods, values and associated eigen-spaces, and also applied in solving equations and ring theory [1, 12-16, 19-25].

Plithogenic matrices of different ranks from the second to the seventeenth rank [7,9, 10-11, 27-28] were studied, where the researchers determined effective algorithms for calculating the determinants of these matrices and their special values, in addition to their orthogonal conditions.

This is what prompted us to follow up the previous efforts and generalize the results to include the plithogenic matrices of the eighteenth and nineteenth ranks, where we were able to find relationships that allow calculating the scales of these matrices and their natural forces, in addition to calculating the special vectors and their special values.

The necessary and sufficient conditions have also been defined for these matrices to be orthogonal.

2. Main Discussion

Definition:

The square 18-plithogenic matrix is defined:

$$\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i; (\tau_i)_{n \times n} \text{ is square matrix of real entries.}$$

Example.

Consider the symbolic 18-plithogenic matrix:

$$\Delta = \begin{pmatrix} -6 & -9 \\ 1 & 56 \end{pmatrix} + \begin{pmatrix} 1 & 12 \\ 65 & 31 \end{pmatrix} P_1 + \begin{pmatrix} 123 & -1 \\ 12 & 31 \end{pmatrix} P_2 + \begin{pmatrix} -96 & -8 \\ -19 & -16 \end{pmatrix} P_3 + \begin{pmatrix} 38 & 65 \\ 116 & 31 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \\ \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 42 & -41 \\ 2 & -8 \end{pmatrix} P_{12} + \\ \begin{pmatrix} 41 & -1 \\ 21 & -9 \end{pmatrix} P_{13} + \begin{pmatrix} 4 & -11 \\ 21 & -8 \end{pmatrix} P_{14} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{15} + \begin{pmatrix} 41 & -31 \\ 32 & -8 \end{pmatrix} P_{16} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{17} + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{18}.$$

Definition.

Let $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a symbolic 18-plithogenic matrix of size $n \times n$, hence:

$$\det \tau = \det(\tau_0) + [\det(\sum_{i=0}^1 \tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^2 \tau_i) - \det(\sum_{i=0}^1 \tau_i)]P_2 + [\det(\sum_{i=0}^3 \tau_i) - \det(\sum_{i=0}^2 \tau_i)]P_3 + [\det(\sum_{i=0}^4 \tau_i) - \det(\sum_{i=0}^3 \tau_i)]P_4 + [\det(\sum_{i=0}^5 \tau_i) - \det(\sum_{i=0}^4 \tau_i)]P_5 + [\det(\sum_{i=0}^6 \tau_i) - \det(\sum_{i=0}^5 \tau_i)]P_6 + [\det(\sum_{i=0}^7 \tau_i) - \det(\sum_{i=0}^6 \tau_i)]P_7 + [\det(\sum_{i=0}^8 \tau_i) - \det(\sum_{i=0}^7 \tau_i)]P_8 + [\det(\sum_{i=0}^9 \tau_i) - \det(\sum_{i=0}^8 \tau_i)]P_9 + [\det(\sum_{i=0}^{10} \tau_i) - \det(\sum_{i=0}^9 \tau_i)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i) - \det(\sum_{i=0}^{10} \tau_i)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i) - \det(\sum_{i=0}^{11} \tau_i)]P_{12} + [\det(\sum_{i=0}^{13} \tau_i) - \det(\sum_{i=0}^{12} \tau_i)]P_{13} + [\det(\sum_{i=0}^{14} \tau_i) - \det(\sum_{i=0}^{13} \tau_i)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i) - \det(\sum_{i=0}^{14} \tau_i)]P_{15} + [\det(\sum_{i=0}^{16} \tau_i) - \det(\sum_{i=0}^{15} \tau_i)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i) - \det(\sum_{i=0}^{16} \tau_i)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i) - \det(\sum_{i=0}^{17} \tau_i)]P_{18}.$$

Theorem

Let $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a symbolic 18-plithogenic matrix of size $n \times n$, hence:

1. τ is invertible if and only if $\det \tau$ is an invertible symbolic 18-plithogenic real number.
2. $\tau^{-1} = \tau_0^{-1} + [(\sum_{i=0}^1 \tau_i)^{-1} - \tau_0^{-1}]P_1 + [(\sum_{i=0}^2 \tau_i)^{-1} - (\sum_{i=0}^1 \tau_i)^{-1}]P_2 + [(\sum_{i=0}^3 \tau_i)^{-1} - (\sum_{i=0}^2 \tau_i)^{-1}]P_3 + [(\sum_{i=0}^4 \tau_i)^{-1} - (\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [(\sum_{i=0}^5 \tau_i)^{-1} - (\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [(\sum_{i=0}^6 \tau_i)^{-1} - (\sum_{i=0}^5 \tau_i)^{-1}]P_6 + [(\sum_{i=0}^7 \tau_i)^{-1} - (\sum_{i=0}^6 \tau_i)^{-1}]P_7 + [(\sum_{i=0}^8 \tau_i)^{-1} - (\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [(\sum_{i=0}^9 \tau_i)^{-1} - (\sum_{i=0}^8 \tau_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^9 \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + [(\sum_{i=0}^{15} \tau_i)^{-1} - (\sum_{i=0}^{14} \tau_i)^{-1}]P_{15} + [(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}]P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - (\sum_{i=0}^{17} \tau_i)^{-1}]P_{18}.$

Proof

- 1). Let $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$, then τ is invertible if and only if there exists $I = I_0 + \sum_{i=1}^{18} I_i P_i$ such that:
 $\tau \times I = U_{n \times n}$, hence:

$$\left\{ \begin{array}{l}
\tau_0 I_0 = U_{n \times n} \\
\sum_{i=0}^1 \tau_i \sum_{i=0}^1 I_i - \tau_0 I_0 = O_{n \times n} \\
\sum_{i=0}^2 \tau_i \sum_{i=0}^2 I_i - \sum_{i=0}^1 \tau_i \sum_{i=0}^1 I_i = O_{n \times n} \\
\sum_{i=0}^3 \tau_i \sum_{i=0}^3 I_i - \sum_{i=0}^2 \tau_i \sum_{i=0}^2 I_i = O_{n \times n} \\
\sum_{i=0}^4 \tau_i \sum_{i=0}^4 I_i - \sum_{i=0}^3 \tau_i \sum_{i=0}^3 I_i = O_{n \times n} \\
\sum_{i=0}^5 \tau_i \sum_{i=0}^5 I_i - \sum_{i=0}^4 \tau_i \sum_{i=0}^4 I_i = O_{n \times n} \\
\sum_{i=0}^6 \tau_i \sum_{i=0}^6 I_i - \sum_{i=0}^5 \tau_i \sum_{i=0}^5 I_i = O_{n \times n} \\
\sum_{i=0}^7 \tau_i \sum_{i=0}^7 I_i - \sum_{i=0}^6 \tau_i \sum_{i=0}^6 I_i = O_{n \times n} \\
\sum_{i=0}^8 \tau_i \sum_{i=0}^8 I_i - \sum_{i=0}^7 \tau_i \sum_{i=0}^7 I_i = O_{n \times n} \\
\sum_{i=0}^9 \tau_i \sum_{i=0}^9 I_i - \sum_{i=0}^8 \tau_i \sum_{i=0}^8 I_i = O_{n \times n} \\
\sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} I_i - \sum_{i=0}^9 \tau_i \sum_{i=0}^9 I_i = O_{n \times n} \\
\sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} I_i - \sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} I_i = O_{n \times n} \\
\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} I_i - \sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} I_i = O_{n \times n} \\
\sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} I_i - \sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} I_i = O_{n \times n} \\
\sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} I_i - \sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} I_i = O_{n \times n} \\
\sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} I_i - \sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} I_i = O_{n \times n} \\
\sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} I_i - \sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} I_i = O_{n \times n} \\
\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} I_i - \sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} I_i = O_{n \times n} \\
\sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} I_i - \sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} I_i = O_{n \times n}
\end{array} \right.$$

This implies that:

$$\begin{cases} \tau_0 I_0 = U_{n \times n} \\ \sum_{i=0}^r \tau_i \sum_{i=0}^r I_i = U_{n \times n} ; 1 \leq r \leq 18 \end{cases}$$

Hence $\det(\sum_{i=0}^r \tau_i) \neq 0$ for all $1 \leq r \leq 18$, so that $\det(\tau)$ is invertible in $18 - SP_R$.

2). $\sum_{i=0}^r I_i = (\sum_{i=0}^r \tau_i)^{-1}$ for $1 \leq r \leq 18$, therefor

$$\begin{aligned} \tau^{-1} &= \tau_0^{-1} + [(\sum_{i=0}^1 \tau_i)^{-1} - \tau_0^{-1}]P_1 + [(\sum_{i=0}^2 \tau_i)^{-1} - (\sum_{i=0}^1 \tau_i)^{-1}]P_2 + [(\sum_{i=0}^3 \tau_i)^{-1} - (\sum_{i=0}^2 \tau_i)^{-1}]P_3 + \\ &+ [(\sum_{i=0}^4 \tau_i)^{-1} - (\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [(\sum_{i=0}^5 \tau_i)^{-1} - (\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [(\sum_{i=0}^6 \tau_i)^{-1} - (\sum_{i=0}^5 \tau_i)^{-1}]P_6 + \\ &+ [(\sum_{i=0}^7 \tau_i)^{-1} - (\sum_{i=0}^6 \tau_i)^{-1}]P_7 + [(\sum_{i=0}^8 \tau_i)^{-1} - (\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [(\sum_{i=0}^9 \tau_i)^{-1} - (\sum_{i=0}^8 \tau_i)^{-1}]P_9 + \\ &+ [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^9 \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + \\ &+ [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + [(\sum_{i=0}^{15} \tau_i)^{-1} - (\sum_{i=0}^{14} \tau_i)^{-1}]P_{15} + \\ &+ [(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}]P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - (\sum_{i=0}^{17} \tau_i)^{-1}]P_{18}. \end{aligned}$$

Definition.

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{18} \varepsilon_i P_i$ be a symbolic 18-plithogenic real number and $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a symbolic 18-plithogenic square real matrix, then ε is called symbolic 18-plithogenic eigen value if and only if $\tau X = \varepsilon X$.

X is called symbolic 18-plithogenic eigenvector.

Theorem:

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{18} \varepsilon_i P_i \in 18 - SP_R$, $X = X_0 + \sum_{i=1}^{18} X_i P_i$ be a symbolic 18-plithogenic vector, then ε is eigen value of $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ with X as the corresponding eigen vector if and only if:

$\sum_{i=0}^j \varepsilon_i$ is eigen value of $\sum_{i=0}^j \tau_i$ with $\sum_{i=0}^j X_i$ as eigen vector with $0 \leq j \leq 18$.

Proof:

It is clear that r is an eigen value of τ with X as an eigen vector if and only if:

$\tau \cdot X = \varepsilon \cdot X$, which is equivalent to:

$$\begin{cases} \tau_0 X_0 = \varepsilon_0 X_0 \\ \sum_{i=0}^j \tau_i \sum_{i=0}^j X_i = \sum_{i=0}^j \varepsilon_i \sum_{i=0}^j X_i ; 1 \leq j \leq 18 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^j \varepsilon_i$ is an eigen value of $\sum_{i=0}^j \tau_i$ with $\sum_{i=0}^j X_i$ as an eigen vector for all $1 \leq j \leq 18$.

Theorem:

$$\begin{aligned} \tau^n &= \tau_0^n + P_1 \left[\left(\sum_{i=0}^1 \tau_i \right)^n - \tau_0^n \right] + \left[\left(\sum_{i=0}^2 \tau_i \right)^n - \left(\sum_{i=0}^1 \tau_i \right)^n \right] P_2 + \left[\left(\sum_{i=0}^3 \tau_i \right)^n - \left(\sum_{i=0}^2 \tau_i \right)^n \right] P_3 \\ &+ \left[\left(\sum_{i=0}^4 \tau_i \right)^n - \left(\sum_{i=0}^3 \tau_i \right)^n \right] P_4 + \left[\left(\sum_{i=0}^5 \tau_i \right)^n - \left(\sum_{i=0}^4 \tau_i \right)^n \right] P_5 + \left[\left(\sum_{i=0}^6 \tau_i \right)^n - \left(\sum_{i=0}^5 \tau_i \right)^n \right] P_6 \\ &+ \left[\left(\sum_{i=0}^7 \tau_i \right)^n - \left(\sum_{i=0}^6 \tau_i \right)^n \right] P_7 + \left[\left(\sum_{i=0}^8 \tau_i \right)^n - \left(\sum_{i=0}^7 \tau_i \right)^n \right] P_8 + \left[\left(\sum_{i=0}^9 \tau_i \right)^n - \left(\sum_{i=0}^8 \tau_i \right)^n \right] P_9 \\ &+ \left[\left(\sum_{i=0}^{10} \tau_i \right)^n - \left(\sum_{i=0}^9 \tau_i \right)^n \right] P_{10} + \left[\left(\sum_{i=0}^{11} \tau_i \right)^n - \left(\sum_{i=0}^{10} \tau_i \right)^n \right] P_{11} + \left[\left(\sum_{i=0}^{12} \tau_i \right)^n - \left(\sum_{i=0}^{11} \tau_i \right)^n \right] P_{12} \\ &+ \left[\left(\sum_{i=0}^{13} \tau_i \right)^n - \left(\sum_{i=0}^{12} \tau_i \right)^n \right] P_{13} + \left[\left(\sum_{i=0}^{14} \tau_i \right)^n - \left(\sum_{i=0}^{13} \tau_i \right)^n \right] P_{14} + \left[\left(\sum_{i=0}^{15} \tau_i \right)^n - \left(\sum_{i=0}^{14} \tau_i \right)^n \right] P_{15} \\ &+ \left[\left(\sum_{i=0}^{16} \tau_i \right)^n - \left(\sum_{i=0}^{15} \tau_i \right)^n \right] P_{16} + \left[\left(\sum_{i=0}^{17} \tau_i \right)^n - \left(\sum_{i=0}^{16} \tau_i \right)^n \right] P_{17} + \left[\left(\sum_{i=0}^{18} \tau_i \right)^n - \left(\sum_{i=0}^{17} \tau_i \right)^n \right] P_{18} \end{aligned}$$

Theorem:

Let $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a square 18-plithogenic invertible real matrix, then:

1). $\det(\tau^{-1}) = (\det \tau)^{-1}$

2). $\det \tau^t = \det \tau$

3). $\det(\tau.Y) = \det \tau \cdot \det Y; Y = Y_0 + \sum_{i=1}^{18} Y_i P_i$.

Proof:

$$\begin{aligned} 1). \det \tau^{-1} &= \det(\tau_0^{-1}) + P_1[\det(\sum_{i=0}^1 \tau_i)^{-1} - \det(\tau_0^{-1})] + [\det(\sum_{i=0}^2 \tau_i)^{-1} - \det(\sum_{i=0}^1 \tau_i)^{-1}]P_2 + \\ &[\det(\sum_{i=0}^3 \tau_i)^{-1} - \det(\sum_{i=0}^2 \tau_i)^{-1}]P_3 + [\det(\sum_{i=0}^4 \tau_i)^{-1} - \det(\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [\det(\sum_{i=0}^5 \tau_i)^{-1} - \\ &\det(\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [\det(\sum_{i=0}^6 \tau_i)^{-1} - \det(\sum_{i=0}^5 \tau_i)^{-1}]P_6 + [\det(\sum_{i=0}^7 \tau_i)^{-1} - \det(\sum_{i=0}^6 \tau_i)^{-1}]P_7 + \\ &[\det(\sum_{i=0}^8 \tau_i)^{-1} - \det(\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [\det(\sum_{i=0}^9 \tau_i)^{-1} - \det(\sum_{i=0}^8 \tau_i)^{-1}]P_9 + [\det(\sum_{i=0}^{10} \tau_i)^{-1} - \\ &\det(\sum_{i=0}^9 \tau_i)^{-1}]P_{10} + [\det(\sum_{i=0}^{11} \tau_i)^{-1} - \det(\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [\det(\sum_{i=0}^{12} \tau_i)^{-1} - \det(\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + \\ &[\det(\sum_{i=0}^{13} \tau_i)^{-1} - \det(\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [\det(\sum_{i=0}^{14} \tau_i)^{-1} - \det(\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + [\det(\sum_{i=0}^{15} \tau_i)^{-1} - \\ &\det(\sum_{i=0}^{14} \tau_i)^{-1}]P_{15} + [\det(\sum_{i=0}^{16} \tau_i)^{-1} - \det(\sum_{i=0}^{15} \tau_i)^{-1}]P_{16} + [\det(\sum_{i=0}^{17} \tau_i)^{-1} - \det(\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + \\ &[\det(\sum_{i=0}^{18} \tau_i)^{-1} - \det(\sum_{i=0}^{17} \tau_i)^{-1}]P_{18} = (\det \tau)^{-1}. \end{aligned}$$

$$2). \tau^t = \tau_0^t + \tau_1^t P_1 + \tau_2^t P_2 + \tau_3^t P_3 + \tau_4^t P_4 + \tau_5^t P_5 + \tau_6^t P_6 + \tau_7^t P_7 + \tau_8^t P_8 + \tau_9^t P_9 + \tau_{10}^t P_{10} + \tau_{11}^t P_{11} + \tau_{12}^t P_{12} + \tau_{13}^t P_{13} + \tau_{14}^t P_{14} + \tau_{15}^t P_{15} + \tau_{16}^t P_{16} + \tau_{17}^t P_{17} + \tau_{18}^t P_{18}.$$

$$\begin{aligned} \det \tau^t &= \det(\tau_0^t) + [\det(\sum_{i=0}^1 \tau_i^t) - \det(\tau_0^t)]P_1 + [\det(\sum_{i=0}^2 \tau_i^t) - \det(\sum_{i=0}^1 \tau_i^t)]P_2 + [\det(\sum_{i=0}^3 \tau_i^t) - \\ &\det(\sum_{i=0}^2 \tau_i^t)]P_3 + [\det(\sum_{i=0}^4 \tau_i^t) - \det(\sum_{i=0}^3 \tau_i^t)]P_4 + [\det(\sum_{i=0}^5 \tau_i^t) - \det(\sum_{i=0}^4 \tau_i^t)]P_5 + [\det(\sum_{i=0}^6 \tau_i^t) - \\ &\det(\sum_{i=0}^5 \tau_i^t)]P_6 + [\det(\sum_{i=0}^7 \tau_i^t) - \det(\sum_{i=0}^6 \tau_i^t)]P_7 + [\det(\sum_{i=0}^8 \tau_i^t) - \det(\sum_{i=0}^7 \tau_i^t)]P_8 + \\ &[\det(\sum_{i=0}^9 \tau_i^t) - \det(\sum_{i=0}^8 \tau_i^t)]P_9 + [\det(\sum_{i=0}^{10} \tau_i^t) - \det(\sum_{i=0}^9 \tau_i^t)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i^t) - \\ &\det(\sum_{i=0}^{10} \tau_i^t)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i^t) - \det(\sum_{i=0}^{11} \tau_i^t)]P_{12} + [\det(\sum_{i=0}^{13} \tau_i^t) - \det(\sum_{i=0}^{12} \tau_i^t)]P_{13} + \\ &[\det(\sum_{i=0}^{14} \tau_i^t) - \det(\sum_{i=0}^{13} \tau_i^t)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i^t) - \det(\sum_{i=0}^{14} \tau_i^t)]P_{15} + [\det(\sum_{i=0}^{16} \tau_i^t) - \\ &\det(\sum_{i=0}^{15} \tau_i^t)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i^t) - \det(\sum_{i=0}^{16} \tau_i^t)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i^t) - \det(\sum_{i=0}^{17} \tau_i^t)]P_{18} = \det(\tau_0) + \\ &[\det(\sum_{i=0}^1 \tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^2 \tau_i) - \det(\sum_{i=0}^1 \tau_i)]P_2 + [\det(\sum_{i=0}^3 \tau_i) - \det(\sum_{i=0}^2 \tau_i)]P_3 + \\ &[\det(\sum_{i=0}^4 \tau_i) - \det(\sum_{i=0}^3 \tau_i)]P_4 + [\det(\sum_{i=0}^5 \tau_i) - \det(\sum_{i=0}^4 \tau_i)]P_5 + [\det(\sum_{i=0}^6 \tau_i) - \det(\sum_{i=0}^5 \tau_i)]P_6 + \\ &[\det(\sum_{i=0}^7 \tau_i) - \det(\sum_{i=0}^6 \tau_i)]P_7 + [\det(\sum_{i=0}^8 \tau_i) - \det(\sum_{i=0}^7 \tau_i)]P_8 + [\det(\sum_{i=0}^9 \tau_i) - \det(\sum_{i=0}^8 \tau_i)]P_9 + \\ &[\det(\sum_{i=0}^{10} \tau_i) - \det(\sum_{i=0}^9 \tau_i)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i) - \det(\sum_{i=0}^{10} \tau_i)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i) - \det(\sum_{i=0}^{11} \tau_i)]P_{12} + \\ &[\det(\sum_{i=0}^{13} \tau_i) - \det(\sum_{i=0}^{12} \tau_i)]P_{13} + [\det(\sum_{i=0}^{14} \tau_i) - \det(\sum_{i=0}^{13} \tau_i)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i) - \det(\sum_{i=0}^{14} \tau_i)]P_{15} + \\ &[\det(\sum_{i=0}^{16} \tau_i) - \det(\sum_{i=0}^{15} \tau_i)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i) - \det(\sum_{i=0}^{16} \tau_i)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i) - \det(\sum_{i=0}^{17} \tau_i)]P_{18} = \\ &\det \tau. \end{aligned}$$

3). we have:

$$\begin{aligned} \tau.Y &= \tau_0 Y_0 + [\sum_{i=0}^1 \tau_i \sum_{i=0}^1 Y_i - \tau_0 Y_0]P_1 + [\sum_{i=0}^2 \tau_i \sum_{i=0}^2 Y_i - \sum_{i=0}^1 \tau_i \sum_{i=0}^1 Y_i]P_2 + [\sum_{i=0}^3 \tau_i \sum_{i=0}^3 Y_i - \\ &\sum_{i=0}^2 \tau_i \sum_{i=0}^2 Y_i]P_3 + [\sum_{i=0}^4 \tau_i \sum_{i=0}^4 Y_i - \sum_{i=0}^3 \tau_i \sum_{i=0}^3 Y_i]P_4 + [\sum_{i=0}^5 \tau_i \sum_{i=0}^5 Y_i - \sum_{i=0}^4 \tau_i \sum_{i=0}^4 Y_i]P_5 + \\ &[\sum_{i=0}^6 \tau_i \sum_{i=0}^6 Y_i - \sum_{i=0}^5 \tau_i \sum_{i=0}^5 Y_i]P_6 + [\sum_{i=0}^7 \tau_i \sum_{i=0}^7 Y_i - \sum_{i=0}^6 \tau_i \sum_{i=0}^6 Y_i]P_7 + [\sum_{i=0}^8 \tau_i \sum_{i=0}^8 Y_i - \\ &\sum_{i=0}^7 \tau_i \sum_{i=0}^7 Y_i]P_8 + [\sum_{i=0}^9 \tau_i \sum_{i=0}^9 Y_i - \sum_{i=0}^8 \tau_i \sum_{i=0}^8 Y_i]P_9 + [\sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} Y_i - \sum_{i=0}^9 \tau_i \sum_{i=0}^9 Y_i]P_{10} + \\ &[\sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} Y_i - \sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} Y_i]P_{11} + [\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} Y_i - \sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} Y_i]P_{12} + [\sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} Y_i - \\ &\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} Y_i]P_{13} + [\sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} Y_i - \sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} Y_i]P_{14} + [\sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} Y_i - \sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} Y_i]P_{15} + \\ &[\sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} Y_i - \sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} Y_i]P_{16} + [\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} Y_i - \sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} Y_i]P_{17} + [\sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} Y_i - \\ &\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} Y_i]P_{18}. \\ \det(\tau.Y) &= \det(\tau_0 Y_0) + [\det(\sum_{i=0}^1 \Delta_i \sum_{i=0}^1 Y_i) - \det(\Delta_0 Y_0)]P_1 + [\det(\sum_{i=0}^2 \Delta_i \sum_{i=0}^2 Y_i) - \\ &\det(\sum_{i=0}^1 \tau_i \sum_{i=0}^1 Y_i)]P_2 + [\det(\sum_{i=0}^3 \tau_i \sum_{i=0}^3 Y_i) - \det(\sum_{i=0}^2 \tau_i \sum_{i=0}^2 Y_i)]P_3 + [\det(\sum_{i=0}^4 \tau_i \sum_{i=0}^4 Y_i) - \\ &\det(\sum_{i=0}^3 \tau_i \sum_{i=0}^3 Y_i)]P_4 + [\det(\sum_{i=0}^5 \tau_i \sum_{i=0}^5 Y_i) - \det(\sum_{i=0}^4 \tau_i \sum_{i=0}^4 Y_i)]P_5 + [\det(\sum_{i=0}^6 \tau_i \sum_{i=0}^6 Y_i) - \\ &\det(\sum_{i=0}^5 \tau_i \sum_{i=0}^5 Y_i)]P_6 + [\det(\sum_{i=0}^7 \tau_i \sum_{i=0}^7 Y_i) - \det(\sum_{i=0}^6 \tau_i \sum_{i=0}^6 Y_i)]P_7 + [\det(\sum_{i=0}^8 \tau_i \sum_{i=0}^8 Y_i) - \\ &\det(\sum_{i=0}^7 \tau_i \sum_{i=0}^7 Y_i)]P_8 + [\det(\sum_{i=0}^9 \tau_i \sum_{i=0}^9 Y_i) - \det(\sum_{i=0}^8 \tau_i \sum_{i=0}^8 Y_i)]P_9 + [\det(\sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} Y_i) - \\ &\det(\sum_{i=0}^9 \tau_i \sum_{i=0}^9 Y_i)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} Y_i) - \det(\sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} Y_i)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} Y_i) - \\ &\det(\sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} Y_i)]P_{12} + [\det(\sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} Y_i) - \det(\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} Y_i)]P_{13} + [\det(\sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} Y_i) - \\ &\det(\sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} Y_i)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} Y_i) - \det(\sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} Y_i)]P_{15} + [\det(\sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} Y_i) - \\ &\det(\sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} C_i)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} Y_i) - \det(\sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} C_i)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} Y_i) - \\ &\det(\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} C_i)]P_{18} = \det(\tau_0) \det(C_0) + [\det(\sum_{i=0}^j \tau_i) \cdot \det(\sum_{i=0}^j Y_i) - \det(\tau) \cdot \det(\sum_{i=1}^{j-1} Y_{i-1})]P_i = \\ &\det(\tau) \det(Y); 1 \leq j \leq 18. \end{aligned}$$

Definition.

Let $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a symbolic 18-plithogenic real square matrix, then:

τ is called orthogonal if and only if $\tau^t = \tau^{-1}$.

Theorem:

τ is orthogonal if and only if $\sum_{i=0}^j \tau_i; 0 \leq j \leq 18$ are orthogonal.

Proof

$\tau^t = \tau^{-1}$, hence:

$$\begin{aligned} \tau_0^t + \sum_{i=1}^{18} \tau_i^t P_i &= \tau_0^{-1} + [(\sum_{i=0}^1 \tau_i)^{-1} - \tau_0^{-1}] P_1 + [(\sum_{i=0}^2 \tau_i)^{-1} - (\sum_{i=0}^1 \tau_i)^{-1}] P_2 + [(\sum_{i=0}^3 \tau_i)^{-1} - \\ &(\sum_{i=0}^2 \tau_i)^{-1}] P_3 + [(\sum_{i=0}^4 \tau_i)^{-1} - (\sum_{i=0}^3 \tau_i)^{-1}] P_4 + [(\sum_{i=0}^5 \tau_i)^{-1} - (\sum_{i=0}^4 \tau_i)^{-1}] P_5 + [(\sum_{i=0}^6 \tau_i)^{-1} - \\ &(\sum_{i=0}^5 \tau_i)^{-1}] P_6 + [(\sum_{i=0}^7 \tau_i)^{-1} - (\sum_{i=0}^6 \tau_i)^{-1}] P_7 + [(\sum_{i=0}^8 \tau_i)^{-1} - (\sum_{i=0}^7 \tau_i)^{-1}] P_8 + [(\sum_{i=0}^9 \tau_i)^{-1} - \\ &(\sum_{i=0}^8 \tau_i)^{-1}] P_9 + [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^9 \tau_i)^{-1}] P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}] P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - \\ &(\sum_{i=0}^{11} \tau_i)^{-1}] P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}] P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}] P_{14} + [(\sum_{i=0}^{15} \tau_i)^{-1} - \\ &(\sum_{i=0}^{14} \tau_i)^{-1}] P_{15} + [(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}] P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}] P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - \\ &(\sum_{i=0}^{17} \tau_i)^{-1}] P_{18}, \text{ thus:} \end{aligned}$$

$$\left\{ \begin{array}{l} \tau_0^t = \tau_0^{-1} \\ \tau_1^t = \left(\sum_{i=0}^1 \tau_i \right)^{-1} - \tau_0^{-1} \\ \tau_2^t = \left(\sum_{i=0}^2 \tau_i \right)^{-1} - \left(\sum_{i=0}^1 \tau_i \right)^{-1} \\ \tau_3^t = \left(\sum_{i=0}^3 \tau_i \right)^{-1} - \left(\sum_{i=0}^2 \tau_i \right)^{-1} \\ \tau_4^t = \left(\sum_{i=0}^4 \tau_i \right)^{-1} - \left(\sum_{i=0}^3 \tau_i \right)^{-1} \\ \tau_5^t = \left(\sum_{i=0}^5 \tau_i \right)^{-1} - \left(\sum_{i=0}^4 \tau_i \right)^{-1} \\ \tau_6^t = \left(\sum_{i=0}^6 \tau_i \right)^{-1} - \left(\sum_{i=0}^5 \tau_i \right)^{-1} \\ \tau_7^t = \left(\sum_{i=0}^7 \tau_i \right)^{-1} - \left(\sum_{i=0}^6 \tau_i \right)^{-1} \\ \tau_8^t = \left(\sum_{i=0}^8 \tau_i \right)^{-1} - \left(\sum_{i=0}^7 \tau_i \right)^{-1} \\ \tau_9^t = \left(\sum_{i=0}^9 \tau_i \right)^{-1} - \left(\sum_{i=0}^8 \tau_i \right)^{-1} \\ \tau_{10}^t = \left(\sum_{i=0}^{10} \tau_i \right)^{-1} - \left(\sum_{i=0}^9 \tau_i \right)^{-1} \\ \tau_{11}^t = \left(\sum_{i=0}^{11} \tau_i \right)^{-1} - \left(\sum_{i=0}^{10} \tau_i \right)^{-1} \\ \tau_{12}^t = \left[\left(\sum_{i=0}^{12} \tau_i \right)^{-1} - \left(\sum_{i=0}^{11} \tau_i \right)^{-1} \right] \\ \tau_{13}^t = \left(\sum_{i=0}^{13} \tau_i \right)^{-1} - \left(\sum_{i=0}^{12} \tau_i \right)^{-1} \\ \tau_{14}^t = \left(\sum_{i=0}^{14} \tau_i \right)^{-1} - \left(\sum_{i=0}^{13} \tau_i \right)^{-1} \\ \tau_{15}^t = \left(\sum_{i=0}^{15} \tau_i \right)^{-1} - \left(\sum_{i=0}^{14} \tau_i \right)^{-1} \\ \tau_{16}^t = \left(\sum_{i=0}^{16} \tau_i \right)^{-1} - \left(\sum_{i=0}^{15} \tau_i \right)^{-1} \\ \tau_{17}^t = \left(\sum_{i=0}^{17} \tau_i \right)^{-1} - \left(\sum_{i=0}^{16} \tau_i \right)^{-1} \\ \tau_{18}^t = \left(\sum_{i=0}^{18} \tau_i \right)^{-1} - \left(\sum_{i=0}^{17} \tau_i \right)^{-1} \end{array} \right.$$

This implies that:

$$\left\{ \begin{array}{l} \tau_0^t = \tau_0^{-1} \\ \sum_{i=0}^1 \tau_i^t = (\sum_{i=0}^1 \tau_i)^{-1} \\ \sum_{i=0}^2 \tau_i^t = (\sum_{i=0}^2 \tau_i)^{-1} \\ \sum_{i=0}^3 \tau_i^t = (\sum_{i=0}^3 \tau_i)^{-1} \\ \sum_{i=0}^4 \tau_i^t = (\sum_{i=0}^4 \tau_i)^{-1} \\ \sum_{i=0}^5 \tau_i^t = (\sum_{i=0}^5 \tau_i)^{-1} \\ \sum_{i=0}^6 \tau_i^t = (\sum_{i=0}^6 \tau_i)^{-1} \\ \sum_{i=0}^7 \tau_i^t = (\sum_{i=0}^7 \tau_i)^{-1} \\ \sum_{i=0}^8 \tau_i^t = (\sum_{i=0}^8 \tau_i)^{-1} \\ \sum_{i=0}^9 \tau_i^t = (\sum_{i=0}^9 \tau_i)^{-1}, \\ \sum_{i=0}^{10} \tau_i^t = (\sum_{i=0}^{10} \tau_i)^{-1} \\ \sum_{i=0}^{11} \tau_i^t = (\sum_{i=0}^{11} \tau_i)^{-1} \\ \sum_{i=0}^{12} \tau_i^t = (\sum_{i=0}^{12} \tau_i)^{-1} \\ \sum_{i=0}^{13} \tau_i^t = (\sum_{i=0}^{13} \tau_i)^{-1} \\ \sum_{i=0}^{14} \tau_i^t = (\sum_{i=0}^{14} \tau_i)^{-1} \\ \sum_{i=0}^{15} \tau_i^t = (\sum_{i=0}^{15} \tau_i)^{-1} \\ \sum_{i=0}^{16} \tau_i^t = (\sum_{i=0}^{16} \tau_i)^{-1} \\ \sum_{i=0}^{17} \tau_i^t = (\sum_{i=0}^{17} \tau_i)^{-1} \\ \sum_{i=0}^{18} \tau_i^t = (\sum_{i=0}^{18} \tau_i)^{-1} \end{array} \right.$$

Definition:

The square 19-plithogenic matrix is defined:

$\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$; $(\tau_i)_{n \times n}$ is square matrix of real entries.

Example.

Consider the symbolic 19-plithogenic matrix:

$$\Delta = \begin{pmatrix} -6 & -9 \\ 1 & 56 \end{pmatrix} + \begin{pmatrix} 1 & 12 \\ 65 & 31 \end{pmatrix} P_1 + \begin{pmatrix} 123 & -1 \\ 12 & 31 \end{pmatrix} P_2 + \begin{pmatrix} -96 & -8 \\ -19 & -16 \end{pmatrix} P_3 + \begin{pmatrix} 38 & 65 \\ 116 & 31 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \\ \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 42 & -41 \\ 2 & -8 \end{pmatrix} P_{12} + \\ \begin{pmatrix} 41 & -1 \\ 21 & -9 \end{pmatrix} P_{13} + \begin{pmatrix} 4 & -11 \\ 21 & -8 \end{pmatrix} P_{14} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{15} + \begin{pmatrix} 41 & -31 \\ 32 & -8 \end{pmatrix} P_{16} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{17} + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{18} + \\ \begin{pmatrix} 42 & -41 \\ 2 & -8 \end{pmatrix} P_{19}.$$

Definition.

Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a symbolic 19-plithogenic matrix of size $n \times n$, hence:

$$\det \tau = \det(\tau_0) + [\det(\sum_{i=0}^1 \tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^2 \tau_i) - \det(\sum_{i=0}^1 \tau_i)]P_2 + [\det(\sum_{i=0}^3 \tau_i) - \det(\sum_{i=0}^2 \tau_i)]P_3 + [\det(\sum_{i=0}^4 \tau_i) - \det(\sum_{i=0}^3 \tau_i)]P_4 + [\det(\sum_{i=0}^5 \tau_i) - \det(\sum_{i=0}^4 \tau_i)]P_5 + [\det(\sum_{i=0}^6 \tau_i) - \det(\sum_{i=0}^5 \tau_i)]P_6 + [\det(\sum_{i=0}^7 \tau_i) - \det(\sum_{i=0}^6 \tau_i)]P_7 + [\det(\sum_{i=0}^8 \tau_i) - \det(\sum_{i=0}^7 \tau_i)]P_8 + [\det(\sum_{i=0}^9 \tau_i) - \det(\sum_{i=0}^8 \tau_i)]P_9 + [\det(\sum_{i=0}^{10} \tau_i) - \det(\sum_{i=0}^9 \tau_i)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i) - \det(\sum_{i=0}^{10} \tau_i)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i) - \det(\sum_{i=0}^{11} \tau_i)]P_{12} + [\det(\sum_{i=0}^{13} \tau_i) - \det(\sum_{i=0}^{12} \tau_i)]P_{13} + [\det(\sum_{i=0}^{14} \tau_i) - \det(\sum_{i=0}^{13} \tau_i)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i) - \det(\sum_{i=0}^{14} \tau_i)]P_{15} + [\det(\sum_{i=0}^{16} \tau_i) - \det(\sum_{i=0}^{15} \tau_i)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i) - \det(\sum_{i=0}^{16} \tau_i)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i) - \det(\sum_{i=0}^{17} \tau_i)]P_{18} + [\det(\sum_{i=0}^{19} \tau_i) - \det(\sum_{i=0}^{18} \tau_i)]P_{19}.$$

Theorem

Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a symbolic 19-plithogenic matrix of size $n \times n$, hence:

1. τ is invertible if and only if $\det \tau$ is an invertible symbolic 19-plithogenic real number.
2. $\tau^{-1} = \tau_0^{-1} + [(\sum_{i=0}^1 \tau_i)^{-1} - \tau_0^{-1}]P_1 + [(\sum_{i=0}^2 \tau_i)^{-1} - (\sum_{i=0}^1 \tau_i)^{-1}]P_2 + [(\sum_{i=0}^3 \tau_i)^{-1} - (\sum_{i=0}^2 \tau_i)^{-1}]P_3 + [(\sum_{i=0}^4 \tau_i)^{-1} - (\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [(\sum_{i=0}^5 \tau_i)^{-1} - (\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [(\sum_{i=0}^6 \tau_i)^{-1} - (\sum_{i=0}^5 \tau_i)^{-1}]P_6 + [(\sum_{i=0}^7 \tau_i)^{-1} - (\sum_{i=0}^6 \tau_i)^{-1}]P_7 + [(\sum_{i=0}^8 \tau_i)^{-1} - (\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [(\sum_{i=0}^9 \tau_i)^{-1} - (\sum_{i=0}^8 \tau_i)^{-1}]P_9 + \dots$

$$\begin{aligned}
& (\sum_{i=0}^8 \tau_i)^{-1} P_9 + [(\sum_{i=1}^{10} \tau_i)^{-1} - (\sum_{i=0}^9 \tau_i)^{-1}] P_{10} + [(\sum_{i=1}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}] P_{11} + [(\sum_{i=1}^{12} \tau_i)^{-1} - \\
& (\sum_{i=0}^{11} \tau_i)^{-1}] P_{12} + [(\sum_{i=1}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}] P_{13} + [(\sum_{i=1}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}] P_{14} + [(\sum_{i=1}^{15} \tau_i)^{-1} - \\
& (\sum_{i=0}^{14} \tau_i)^{-1}] P_{15} + [(\sum_{i=1}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}] P_{16} + [(\sum_{i=1}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}] P_{17} + [(\sum_{i=1}^{18} \tau_i)^{-1} - \\
& (\sum_{i=0}^{17} \tau_i)^{-1}] P_{18} + [(\sum_{i=1}^{19} \tau_i)^{-1} - (\sum_{i=0}^{18} \tau_i)^{-1}] P_{19}.
\end{aligned}$$

Proof

1). Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$, then τ is invertible if and only if there exists $I = I_0 + \sum_{i=1}^{19} I_i P_i$ such that:
 $\tau \times I = U_{n \times n}$, hence:

$$\left\{ \begin{array}{l} \tau_0 I_0 = U_{n \times n} \\ \sum_{i=0}^1 \tau_i \sum_{i=0}^1 I_i - \tau_0 I_0 = O_{n \times n} \\ \sum_{i=0}^2 \tau_i \sum_{i=0}^2 I_i - \sum_{i=0}^1 \tau_i \sum_{i=0}^1 I_i = O_{n \times n} \\ \sum_{i=0}^3 \tau_i \sum_{i=0}^3 I_i - \sum_{i=0}^2 \tau_i \sum_{i=0}^2 I_i = O_{n \times n} \\ \sum_{i=0}^4 \tau_i \sum_{i=0}^4 I_i - \sum_{i=0}^3 \tau_i \sum_{i=0}^3 I_i = O_{n \times n} \\ \sum_{i=0}^5 \tau_i \sum_{i=0}^5 I_i - \sum_{i=0}^4 \tau_i \sum_{i=0}^4 I_i = O_{n \times n} \\ \sum_{i=0}^6 \tau_i \sum_{i=0}^6 I_i - \sum_{i=0}^5 \tau_i \sum_{i=0}^5 I_i = O_{n \times n} \\ \sum_{i=0}^7 \tau_i \sum_{i=0}^7 I_i - \sum_{i=0}^6 \tau_i \sum_{i=0}^6 I_i = O_{n \times n} \\ \sum_{i=0}^8 \tau_i \sum_{i=0}^8 I_i - \sum_{i=0}^7 \tau_i \sum_{i=0}^7 I_i = O_{n \times n} \\ \sum_{i=0}^9 \tau_i \sum_{i=0}^9 I_i - \sum_{i=0}^8 \tau_i \sum_{i=0}^8 I_i = O_{n \times n} \\ \sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} I_i - \sum_{i=0}^9 \tau_i \sum_{i=0}^9 I_i = O_{n \times n} \\ \sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} I_i - \sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} I_i = O_{n \times n} \\ \sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} I_i - \sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} I_i = O_{n \times n} \\ \sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} I_i - \sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} I_i = O_{n \times n} \\ \sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} I_i - \sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} I_i = O_{n \times n} \\ \sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} I_i - \sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} I_i = O_{n \times n} \\ \sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} I_i - \sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} I_i = O_{n \times n} \\ \sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} I_i - \sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} I_i = O_{n \times n} \\ \sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} I_i - \sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} I_i = O_{n \times n} \\ \sum_{i=0}^{19} \tau_i \sum_{i=0}^{19} I_i - \sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} I_i = O_{n \times n} \end{array} \right.$$

This implies that:

$$\begin{cases} \tau_0 I_0 = U_{n \times n} \\ \sum_{i=0}^r \tau_i \sum_{i=0}^r I_i = U_{n \times n} ; 1 \leq r \leq 19 \end{cases}$$

Hence $\det(\sum_{i=0}^r \tau_i) \neq 0$ for all $1 \leq r \leq 19$, so that $\det(\tau)$ is invertible in $19 - SP_R$.

2). $\sum_{i=0}^r I_i = (\sum_{i=0}^r \tau_i)^{-1}$ for $1 \leq r \leq 19$, therefore

$$\begin{aligned} \tau^{-1} &= \tau_0^{-1} + [(\sum_{i=0}^1 \tau_i)^{-1} - \tau_0^{-1}]P_1 + [(\sum_{i=0}^2 \tau_i)^{-1} - (\sum_{i=0}^1 \tau_i)^{-1}]P_2 + [(\sum_{i=0}^3 \tau_i)^{-1} - (\sum_{i=0}^2 \tau_i)^{-1}]P_3 + \\ &+ [(\sum_{i=0}^4 \tau_i)^{-1} - (\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [(\sum_{i=0}^5 \tau_i)^{-1} - (\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [(\sum_{i=0}^6 \tau_i)^{-1} - (\sum_{i=0}^5 \tau_i)^{-1}]P_6 + \\ &+ [(\sum_{i=0}^7 \tau_i)^{-1} - (\sum_{i=0}^6 \tau_i)^{-1}]P_7 + [(\sum_{i=0}^8 \tau_i)^{-1} - (\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [(\sum_{i=0}^9 \tau_i)^{-1} - (\sum_{i=0}^8 \tau_i)^{-1}]P_9 + \\ &+ [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^9 \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + \\ &+ [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + [(\sum_{i=0}^{15} \tau_i)^{-1} - (\sum_{i=0}^{14} \tau_i)^{-1}]P_{15} + \\ &+ [(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}]P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - (\sum_{i=0}^{17} \tau_i)^{-1}]P_{18} + \\ &+ [(\sum_{i=0}^{19} \tau_i)^{-1} - (\sum_{i=0}^{18} \tau_i)^{-1}]P_{19}. \end{aligned}$$

Definition.

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{19} \varepsilon_i P_i$ be a symbolic 19-plithogenic real number and $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a symbolic 19-plithogenic square real matrix, then ε is called symbolic 18-plithogenic eigen value if and only if $\tau X = \varepsilon X$.

X is called symbolic 19-plithogenic eigenvector.

Theorem:

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{19} \varepsilon_i P_i \in 19 - SP_R$, $X = X_0 + \sum_{i=1}^{19} X_i P_i$ be a symbolic 19-plithogenic vector, then ε is eigen value of $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ with X as the corresponding eigen vector if and only if:

$\sum_{i=0}^j \varepsilon_i$ is eigen value of $\sum_{i=0}^j \tau_i$ with $\sum_{i=0}^j X_i$ as eigen vector with $0 \leq j \leq 19$.

Proof:

It is clear that r is an eigen value of τ with X as an eigen vector if and only if:

$\tau X = \varepsilon X$, which is equivalent to:

$$\begin{cases} \tau_0 X_0 = \varepsilon_0 X_0 \\ \sum_{i=0}^j \tau_i \sum_{i=0}^j X_i = \sum_{i=0}^j \varepsilon_i \sum_{i=0}^j X_i ; 1 \leq j \leq 19 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^j \varepsilon_i$ is an eigen value of $\sum_{i=0}^j \tau_i$ with $\sum_{i=0}^j X_i$ as an eigen vector for all $1 \leq j \leq 19$.

Theorem:

$$\begin{aligned} \tau^n &= \tau_0^n + P_1 \left[\left(\sum_{i=0}^1 \tau_i \right)^n - \tau_0^n \right] + \left[\left(\sum_{i=0}^2 \tau_i \right)^n - \left(\sum_{i=0}^1 \tau_i \right)^n \right] P_2 + \left[\left(\sum_{i=0}^3 \tau_i \right)^n - \left(\sum_{i=0}^2 \tau_i \right)^n \right] P_3 \\ &+ \left[\left(\sum_{i=0}^4 \tau_i \right)^n - \left(\sum_{i=0}^3 \tau_i \right)^n \right] P_4 + \left[\left(\sum_{i=0}^5 \tau_i \right)^n - \left(\sum_{i=0}^4 \tau_i \right)^n \right] P_5 + \left[\left(\sum_{i=0}^6 \tau_i \right)^n - \left(\sum_{i=0}^5 \tau_i \right)^n \right] P_6 \\ &+ \left[\left(\sum_{i=0}^7 \tau_i \right)^n - \left(\sum_{i=0}^6 \tau_i \right)^n \right] P_7 + \left[\left(\sum_{i=0}^8 \tau_i \right)^n - \left(\sum_{i=0}^7 \tau_i \right)^n \right] P_8 + \left[\left(\sum_{i=0}^9 \tau_i \right)^n - \left(\sum_{i=0}^8 \tau_i \right)^n \right] P_9 \\ &+ \left[\left(\sum_{i=0}^{10} \tau_i \right)^n - \left(\sum_{i=0}^9 \tau_i \right)^n \right] P_{10} + \left[\left(\sum_{i=0}^{11} \tau_i \right)^n - \left(\sum_{i=0}^{10} \tau_i \right)^n \right] P_{11} + \left[\left(\sum_{i=0}^{12} \tau_i \right)^n - \left(\sum_{i=0}^{11} \tau_i \right)^n \right] P_{12} \\ &+ \left[\left(\sum_{i=0}^{13} \tau_i \right)^n - \left(\sum_{i=0}^{12} \tau_i \right)^n \right] P_{13} + \left[\left(\sum_{i=0}^{14} \tau_i \right)^n - \left(\sum_{i=0}^{13} \tau_i \right)^n \right] P_{14} + \left[\left(\sum_{i=0}^{15} \tau_i \right)^n - \left(\sum_{i=0}^{14} \tau_i \right)^n \right] P_{15} \\ &+ \left[\left(\sum_{i=0}^{16} \tau_i \right)^n - \left(\sum_{i=0}^{15} \tau_i \right)^n \right] P_{16} + \left[\left(\sum_{i=0}^{17} \tau_i \right)^n - \left(\sum_{i=0}^{16} \tau_i \right)^n \right] P_{17} + \left[\left(\sum_{i=0}^{18} \tau_i \right)^n - \left(\sum_{i=0}^{17} \tau_i \right)^n \right] P_{18} \\ &+ \left[\left(\sum_{i=0}^{19} \tau_i \right)^n - \left(\sum_{i=0}^{18} \tau_i \right)^n \right] P_{19} \end{aligned}$$

Theorem:

Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a square 19-plithogenic invertible real matrix, then:

- 1). $\det(\tau^{-1}) = (\det \tau)^{-1}$
- 2). $\det \tau^t = \det \tau$
- 3). $\det(\tau.Y) = \det \tau \cdot \det Y$; $Y = Y_0 + \sum_{i=1}^{19} Y_i P_i$.

Proof:

- 1). $\det \tau^{-1} = \det(\tau_0^{-1}) + P_1[\det(\sum_{i=0}^1 \tau_i)^{-1} - \det(\tau_0^{-1})] + [\det(\sum_{i=0}^2 \tau_i)^{-1} - \det(\sum_{i=0}^1 \tau_i)^{-1}]P_2 +$
 $[\det(\sum_{i=0}^3 \tau_i)^{-1} - \det(\sum_{i=0}^2 \tau_i)^{-1}]P_3 + [\det(\sum_{i=0}^4 \tau_i)^{-1} - \det(\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [\det(\sum_{i=0}^5 \tau_i)^{-1} -$
 $\det(\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [\det(\sum_{i=0}^6 \tau_i)^{-1} - \det(\sum_{i=0}^5 \tau_i)^{-1}]P_6 + [\det(\sum_{i=0}^7 \tau_i)^{-1} - \det(\sum_{i=0}^6 \tau_i)^{-1}]P_7 +$
 $[\det(\sum_{i=0}^8 \tau_i)^{-1} - \det(\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [\det(\sum_{i=0}^9 \tau_i)^{-1} - \det(\sum_{i=0}^8 \tau_i)^{-1}]P_9 + [\det(\sum_{i=0}^{10} \tau_i)^{-1} -$
 $\det(\sum_{i=0}^9 \tau_i)^{-1}]P_{10} + [\det(\sum_{i=0}^{11} \tau_i)^{-1} - \det(\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [\det(\sum_{i=0}^{12} \tau_i)^{-1} - \det(\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} +$
 $[\det(\sum_{i=0}^{13} \tau_i)^{-1} - \det(\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [\det(\sum_{i=0}^{14} \tau_i)^{-1} - \det(\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + [\det(\sum_{i=0}^{15} \tau_i)^{-1} -$
 $\det(\sum_{i=0}^{14} \tau_i)^{-1}]P_{15} + [\det(\sum_{i=0}^{16} \tau_i)^{-1} - \det(\sum_{i=0}^{15} \tau_i)^{-1}]P_{16} + [\det(\sum_{i=0}^{17} \tau_i)^{-1} - \det(\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} +$
 $[\det(\sum_{i=0}^{18} \tau_i)^{-1} - \det(\sum_{i=0}^{17} \tau_i)^{-1}]P_{18} + [\det(\sum_{i=0}^{19} \tau_i)^{-1} - \det(\sum_{i=0}^{18} \tau_i)^{-1}]P_{19} = (\det \tau)^{-1}$.
- 2). $\tau^t = \tau_0^t + \tau_1^t P_1 + \tau_2^t P_2 + \tau_3^t P_3 + \tau_4^t P_4 + \tau_5^t P_5 + \tau_6^t P_6 + \tau_7^t P_7 + \tau_8^t P_8 + \tau_9^t P_9 + \tau_{10}^t P_{10} + \tau_{11}^t P_{11} +$
 $\tau_{12}^t P_{12} + \tau_{13}^t P_{13} + \tau_{14}^t P_{14} + \tau_{15}^t P_{15} + \tau_{16}^t P_{16} + \tau_{17}^t P_{17} + \tau_{18}^t P_{18} + \tau_{19}^t P_{19}$.
 $\det \tau^t = \det(\tau_0^t) + [\det(\sum_{i=0}^1 \tau_i^t) - \det(\tau_0^t)]P_1 + [\det(\sum_{i=0}^2 \tau_i^t) - \det(\sum_{i=0}^1 \tau_i^t)]P_2 + [\det(\sum_{i=0}^3 \tau_i^t) -$
 $\det(\sum_{i=0}^2 \tau_i^t)]P_3 + [\det(\sum_{i=0}^4 \tau_i^t) - \det(\sum_{i=0}^3 \tau_i^t)]P_4 + [\det(\sum_{i=0}^5 \tau_i^t) - \det(\sum_{i=0}^4 \tau_i^t)]P_5 + [\det(\sum_{i=0}^6 \tau_i^t) -$
 $\det(\sum_{i=0}^5 \tau_i^t)]P_6 + [\det(\sum_{i=0}^7 \tau_i^t) - \det(\sum_{i=0}^6 \tau_i^t)]P_7 + [\det(\sum_{i=0}^8 \tau_i^t) - \det(\sum_{i=0}^7 \tau_i^t)]P_8 +$
 $[\det(\sum_{i=0}^9 \tau_i^t) - \det(\sum_{i=0}^8 \tau_i^t)]P_9 + [\det(\sum_{i=0}^{10} \tau_i^t) - \det(\sum_{i=0}^9 \tau_i^t)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i^t) -$
 $\det(\sum_{i=0}^{10} \tau_i^t)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i^t) - \det(\sum_{i=0}^{11} \tau_i^t)]P_{12} + [\det(\sum_{i=0}^{13} \tau_i^t) - \det(\sum_{i=0}^{12} \tau_i^t)]P_{13} +$
 $[\det(\sum_{i=0}^{14} \tau_i^t) - \det(\sum_{i=0}^{13} \tau_i^t)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i^t) - \det(\sum_{i=0}^{14} \tau_i^t)]P_{15} + [\det(\sum_{i=0}^{16} \tau_i^t) -$
 $\det(\sum_{i=0}^{15} \tau_i^t)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i^t) - \det(\sum_{i=0}^{16} \tau_i^t)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i^t) - \det(\sum_{i=0}^{17} \tau_i^t)]P_{18} +$
 $[\det(\sum_{i=0}^{19} \tau_i^t) - \det(\sum_{i=0}^{18} \tau_i^t)]P_{19} = \det(\tau_0) + [\det(\sum_{i=0}^1 \tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^2 \tau_i) -$
 $\det(\sum_{i=0}^1 \tau_i)]P_2 + [\det(\sum_{i=0}^3 \tau_i) - \det(\sum_{i=0}^2 \tau_i)]P_3 + [\det(\sum_{i=0}^4 \tau_i) - \det(\sum_{i=0}^3 \tau_i)]P_4 + [\det(\sum_{i=0}^5 \tau_i) -$
 $\det(\sum_{i=0}^4 \tau_i)]P_5 + [\det(\sum_{i=0}^6 \tau_i) - \det(\sum_{i=0}^5 \tau_i)]P_6 + [\det(\sum_{i=0}^7 \tau_i) - \det(\sum_{i=0}^6 \tau_i)]P_7 + [\det(\sum_{i=0}^8 \tau_i) -$
 $\det(\sum_{i=0}^7 \tau_i)]P_8 + [\det(\sum_{i=0}^9 \tau_i) - \det(\sum_{i=0}^8 \tau_i)]P_9 + [\det(\sum_{i=0}^{10} \tau_i) - \det(\sum_{i=0}^9 \tau_i)]P_{10} + [\det(\sum_{i=0}^{11} \tau_i) -$
 $\det(\sum_{i=0}^{10} \tau_i)]P_{11} + [\det(\sum_{i=0}^{12} \tau_i) - \det(\sum_{i=0}^{11} \tau_i)]P_{12} + [\det(\sum_{i=0}^{13} \tau_i) - \det(\sum_{i=0}^{12} \tau_i)]P_{13} + [\det(\sum_{i=0}^{14} \tau_i) -$
 $\det(\sum_{i=0}^{13} \tau_i)]P_{14} + [\det(\sum_{i=0}^{15} \tau_i) - \det(\sum_{i=0}^{14} \tau_i)]P_{15} + [\det(\sum_{i=0}^{16} \tau_i) - \det(\sum_{i=0}^{15} \tau_i)]P_{16} + [\det(\sum_{i=0}^{17} \tau_i) -$
 $\det(\sum_{i=0}^{16} \tau_i)]P_{17} + [\det(\sum_{i=0}^{18} \tau_i) - \det(\sum_{i=0}^{17} \tau_i)]P_{18} + [\det(\sum_{i=0}^{19} \tau_i) - \det(\sum_{i=0}^{18} \tau_i)]P_{19} = \det \tau$.
- 3). we have:
 $\tau.Y = \tau_0 Y_0 + [\sum_{i=0}^1 \tau_i \sum_{i=0}^1 Y_i - \tau_0 Y_0]P_1 + [\sum_{i=0}^2 \tau_i \sum_{i=0}^2 Y_i - \sum_{i=0}^1 \tau_i \sum_{i=0}^1 Y_i]P_2 + [\sum_{i=0}^3 \tau_i \sum_{i=0}^3 Y_i -$
 $\sum_{i=0}^2 \tau_i \sum_{i=0}^2 Y_i]P_3 + [\sum_{i=0}^4 \tau_i \sum_{i=0}^4 Y_i - \sum_{i=0}^3 \tau_i \sum_{i=0}^3 Y_i]P_4 + [\sum_{i=0}^5 \tau_i \sum_{i=0}^5 Y_i - \sum_{i=0}^4 \tau_i \sum_{i=0}^4 Y_i]P_5 +$
 $[\sum_{i=0}^6 \tau_i \sum_{i=0}^6 Y_i - \sum_{i=0}^5 \tau_i \sum_{i=0}^5 Y_i]P_6 + [\sum_{i=0}^7 \tau_i \sum_{i=0}^7 Y_i - \sum_{i=0}^6 \tau_i \sum_{i=0}^6 Y_i]P_7 + [\sum_{i=0}^8 \tau_i \sum_{i=0}^8 Y_i -$
 $\sum_{i=0}^7 \tau_i \sum_{i=0}^7 Y_i]P_8 + [\sum_{i=0}^9 \tau_i \sum_{i=0}^9 Y_i - \sum_{i=0}^8 \tau_i \sum_{i=0}^8 Y_i]P_9 + [\sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} Y_i - \sum_{i=0}^9 \tau_i \sum_{i=0}^9 Y_i]P_{10} +$
 $[\sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} Y_i - \sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} Y_i]P_{11} + [\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} Y_i - \sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} Y_i]P_{12} + [\sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} Y_i -$
 $\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} Y_i]P_{13} + [\sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} Y_i - \sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} Y_i]P_{14} + [\sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} Y_i - \sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} Y_i]P_{15} +$
 $[\sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} Y_i - \sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} Y_i]P_{16} + [\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} Y_i - \sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} Y_i]P_{17} + [\sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} Y_i -$
 $\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} Y_i]P_{18} + [\sum_{i=0}^{19} \tau_i \sum_{i=0}^{19} Y_i - \sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} Y_i]P_{19}$.
 $\det(\tau.Y) = \det(\tau_0) \det(C_0) + [\det(\sum_{i=0}^j \tau_i) \cdot \det(\sum_{i=0}^j Y_i) - \det(\tau) \cdot \det(\sum_{i=1}^{j-1} Y_{i-1})]P_i = \det(\tau) \det(Y)$; $1 \leq j \leq 19$.

Definition.

Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a symbolic 19-plithogenic real square matrix, then:

τ is called orthogonal if and only if $\tau^t = \tau^{-1}$.

Theorem:

τ is orthogonal if and only if $\sum_{i=0}^j \tau_i$; $0 \leq j \leq 19$ are orthogonal.

Proof

$\tau^t = \tau^{-1}$, hence:

$$\tau_0^t + \sum_{i=1}^{19} \tau_i^t P_i = \tau_0^{-1} + [(\sum_{i=0}^1 \tau_i)^{-1} - \tau_0^{-1}]P_1 + [(\sum_{i=0}^2 \tau_i)^{-1} - (\sum_{i=0}^1 \tau_i)^{-1}]P_2 + [(\sum_{i=0}^3 \tau_i)^{-1} -$$

 $(\sum_{i=0}^2 \tau_i)^{-1}]P_3 + [(\sum_{i=0}^4 \tau_i)^{-1} - (\sum_{i=0}^3 \tau_i)^{-1}]P_4 + [(\sum_{i=0}^5 \tau_i)^{-1} - (\sum_{i=0}^4 \tau_i)^{-1}]P_5 + [(\sum_{i=0}^6 \tau_i)^{-1} -$
 $(\sum_{i=0}^5 \tau_i)^{-1}]P_6 + [(\sum_{i=0}^7 \tau_i)^{-1} - (\sum_{i=0}^6 \tau_i)^{-1}]P_7 + [(\sum_{i=0}^8 \tau_i)^{-1} - (\sum_{i=0}^7 \tau_i)^{-1}]P_8 + [(\sum_{i=0}^9 \tau_i)^{-1} -$
 $(\sum_{i=0}^8 \tau_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^9 \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} -$

$$\begin{aligned}
 & (\sum_{i=0}^{11} \tau_i)^{-1} P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}] P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}] P_{14} + [(\sum_{i=0}^{15} \tau_i)^{-1} - \\
 & (\sum_{i=0}^{14} \tau_i)^{-1}] P_{15} + [(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}] P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}] P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - \\
 & (\sum_{i=0}^{17} \tau_i)^{-1}] P_{18} + [(\sum_{i=0}^{19} \tau_i)^{-1} - (\sum_{i=0}^{18} \tau_i)^{-1}] P_{19}, \text{ thus:}
 \end{aligned}$$

$$\left\{ \begin{array}{l}
 \tau_0^t = \tau_0^{-1} \\
 \tau_1^t = \left(\sum_{i=0}^1 \tau_i \right)^{-1} - \tau_0^{-1} \\
 \tau_2^t = \left(\sum_{i=0}^2 \tau_i \right)^{-1} - \left(\sum_{i=0}^1 \tau_i \right)^{-1} \\
 \tau_3^t = \left(\sum_{i=0}^3 \tau_i \right)^{-1} - \left(\sum_{i=0}^2 \tau_i \right)^{-1} \\
 \tau_4^t = \left(\sum_{i=0}^4 \tau_i \right)^{-1} - \left(\sum_{i=0}^3 \tau_i \right)^{-1} \\
 \tau_5^t = \left(\sum_{i=0}^5 \tau_i \right)^{-1} - \left(\sum_{i=0}^4 \tau_i \right)^{-1} \\
 \tau_6^t = \left(\sum_{i=0}^6 \tau_i \right)^{-1} - \left(\sum_{i=0}^5 \tau_i \right)^{-1} \\
 \tau_7^t = \left(\sum_{i=0}^7 \tau_i \right)^{-1} - \left(\sum_{i=0}^6 \tau_i \right)^{-1} \\
 \tau_8^t = \left(\sum_{i=0}^8 \tau_i \right)^{-1} - \left(\sum_{i=0}^7 \tau_i \right)^{-1} \\
 \tau_9^t = \left(\sum_{i=0}^9 \tau_i \right)^{-1} - \left(\sum_{i=0}^8 \tau_i \right)^{-1} \\
 \tau_{10}^t = \left(\sum_{i=0}^{10} \tau_i \right)^{-1} - \left(\sum_{i=0}^9 \tau_i \right)^{-1} \\
 \tau_{11}^t = \left(\sum_{i=0}^{11} \tau_i \right)^{-1} - \left(\sum_{i=0}^{10} \tau_i \right)^{-1} \\
 \tau_{12}^t = \left[\left(\sum_{i=0}^{12} \tau_i \right)^{-1} - \left(\sum_{i=0}^{11} \tau_i \right)^{-1} \right] \\
 \tau_{13}^t = \left(\sum_{i=0}^{13} \tau_i \right)^{-1} - \left(\sum_{i=0}^{12} \tau_i \right)^{-1} \\
 \tau_{14}^t = \left(\sum_{i=0}^{14} \tau_i \right)^{-1} - \left(\sum_{i=0}^{13} \tau_i \right)^{-1} \\
 \tau_{15}^t = \left(\sum_{i=0}^{15} \tau_i \right)^{-1} - \left(\sum_{i=0}^{14} \tau_i \right)^{-1} \\
 \tau_{16}^t = \left(\sum_{i=0}^{16} \tau_i \right)^{-1} - \left(\sum_{i=0}^{15} \tau_i \right)^{-1} \\
 \tau_{17}^t = \left(\sum_{i=0}^{17} \tau_i \right)^{-1} - \left(\sum_{i=0}^{16} \tau_i \right)^{-1} \\
 \tau_{18}^t = \left(\sum_{i=0}^{18} \tau_i \right)^{-1} - \left(\sum_{i=0}^{17} \tau_i \right)^{-1} \\
 \tau_{19}^t = \left(\sum_{i=0}^{19} \tau_i \right)^{-1} - \left(\sum_{i=0}^{18} \tau_i \right)^{-1}
 \end{array} \right.$$

This implies that:

$$\left\{ \begin{array}{l} \tau_0^t = \tau_0^{-1} \\ \sum_{i=0}^1 \tau_i^t = (\sum_{i=0}^1 \tau_i)^{-1} \\ \sum_{i=0}^2 \tau_i^t = (\sum_{i=0}^2 \tau_i)^{-1} \\ \sum_{i=0}^3 \tau_i^t = (\sum_{i=0}^3 \tau_i)^{-1} \\ \sum_{i=0}^4 \tau_i^t = (\sum_{i=0}^4 \tau_i)^{-1} \\ \sum_{i=0}^5 \tau_i^t = (\sum_{i=0}^5 \tau_i)^{-1} \\ \sum_{i=0}^6 \tau_i^t = (\sum_{i=0}^6 \tau_i)^{-1} \\ \sum_{i=0}^7 \tau_i^t = (\sum_{i=0}^7 \tau_i)^{-1} \\ \sum_{i=0}^8 \tau_i^t = (\sum_{i=0}^8 \tau_i)^{-1} \\ \sum_{i=0}^9 \tau_i^t = (\sum_{i=0}^9 \tau_i)^{-1} \\ \sum_{i=0}^{10} \tau_i^t = (\sum_{i=0}^{10} \tau_i)^{-1}, \\ \sum_{i=0}^{11} \tau_i^t = (\sum_{i=0}^{11} \tau_i)^{-1} \\ \sum_{i=0}^{12} \tau_i^t = (\sum_{i=0}^{12} \tau_i)^{-1} \\ \sum_{i=0}^{13} \tau_i^t = (\sum_{i=0}^{13} \tau_i)^{-1} \\ \sum_{i=0}^{14} \tau_i^t = (\sum_{i=0}^{14} \tau_i)^{-1} \\ \sum_{i=0}^{15} \tau_i^t = (\sum_{i=0}^{15} \tau_i)^{-1} \\ \sum_{i=0}^{16} \tau_i^t = (\sum_{i=0}^{16} \tau_i)^{-1} \\ \sum_{i=0}^{17} \tau_i^t = (\sum_{i=0}^{17} \tau_i)^{-1} \\ \sum_{i=0}^{18} \tau_i^t = (\sum_{i=0}^{18} \tau_i)^{-1} \\ \sum_{i=0}^{19} \tau_i^t = (\sum_{i=0}^{19} \tau_i)^{-1} \end{array} \right.$$

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