A Pert model based on the Dampster & Shafer's Theory of Evidence - application to product development

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Abstract
The epistemic uncertainty affecting the estimates of lead times in new product development may have significant side effects over the management of the said projects, and nowadays an effective tool to manage this kind of uncertainty is still missing for new product development projects. In this paper we propose a new Project Evaluation and Review Technique (PERT) model based on lead times defined by the Theory of Evidence. The main advantages of our proposal come from the inclusion and measurement of the epistemic uncertainty lying into lead time estimates along the PERT network, which is further analysed by means of Monte Carlo simulation. The work is completed by applying this model to a real instance of new product development process within a company, and the results so obtained are analysed by investigating how different combination and transformation rules affect the probability distribution of the activity lead times.

Keywords: PERT; Dempster & Shafer Theory; New product development; Uncertainty management; Project management

1 INTRODUCTION

Completing projects within deadlines, while reducing costs, has become a critical activity for companies' competitiveness (Pinha and Ahluwalia, 2019) as well as delivering right quantities at the right time is a key aspect of current competitiveness in business environment (Sundar, Balaji and Satheesh Kumar, 2019). Given a project scheduling, trying to deal with uncertainty in operations management (Spalanzani and Samuel, 2007) and with constrained resources (Ganji, Alinaghian and Sajadi, 2016) has been a topic for decades (Ma, 2012).

By representing projects as sequences of the activities, Critical Path Method (CPM) and PERT techniques are among the most widespread tools for these purposes (Rose, 2013) (Bangun Suryono, 2020). By using the critical path algorithm, the PERT method is adopted worldwide to: i) evaluate possible evolutions of the project (both from temporal and an economic perspective); ii) avoid bad allocations of the resources; iii) identify the most critical activities (on which project duration and costs depend); and iv) check the progress of the entire project day by day, in order to discover deviations from the objectives (Kerzner, 2017). Despite the merits of PERT, its goodness is strictly affected by the precision of the activity-lead times estimations.

The pioneering 3-points-estimation through Beta distribution has been replaced over years by different methods, since it was criticized to introduce biases and underestimations (i.e., the so-called “Jensen gap”) (Banerjee & Paul, 2008; Trietsch & Baker, 2012), but calibrating the PERT in order to validate results and probability distributions is still an open topic (Salas-Morera *et al.*, 2018). Some researchers introduced the log-normal distribution coming from historical data (Trietsch and Baker, 2012); others studied the effects of representing lead times using different probability distributions under a stochastic PERT framework (Hajdu and Bokor, 2014). In these researches, Monte Carlo simulation then showed there is not a substantial difference on the total duration of the project in using different types of distribution mainly due to the central limit theorem that with many activities assures that the sum of the durations converges to a normal distribution, while there are instead much more significant variations following deviations of +/- 10% in the values of the 3-point-estimation (Hajdu and Bokor, 2016). Indeed, a significant contribution to overcome and analyse the weaknesses of PERT, made possible by the progress of information technology, comes with the simulation. It has been proven that integrating PERT with Monte Carlo simulation produces much more reliable results in the scheduling of project activities (Wyrozębski and Wyrozębska, 2013). Several researchers suggest that this approach leads to much more robust conclusions about timing and risks, providing a completer and more realistic picture of projects evolution (Na, Wuliang, & Hua, 2014; Deshmukh & Rajhans, 2018). The more accentuated the divergences are, the more there is no "critical path dominance" marked in the network instance.

However, since the activities lead times often come from expert’s judgments, if no historical data are available, epistemic uncertainty is the bias that more affects these estimations (Fink and Pinchovski, 2020).

In order to report their generic nature, some researchers used fuzzy numbers to represent lead times (Morovatdar et al., 2011; Saadoon et al., 2014; Mazlum & Güneri, 2015). The discussion then expanded including earliest/latest start and finish times too as fuzzy numbers, even coming to review the concept of activities-criticality in such a perspective (Saadoon *et al.*, 2014). Even though fuzzy theory allows to represent the uncertainty beneath PERT projects, it remained little used in practice for several reasons, in particular the definition of a membership function, the use of fuzzy algebra and the final de-fuzzification of results are very often complex and unintuitive steps. Moreover, these operations involve further manipulations on data that are already unprecise (Talon and Curt, 2017).

Despite all these attempts to represent uncertainty in PERT models, researchers have not yet found a definitive method either to measure the epistemic uncertainty, nor to treat it according to the situations, nor to study the consequences onto the research of critical paths, and consequently, on project expected completion times. This lack is critical for new product development projects, as it leads to low revenue due to market share losses, higher risks for the quality of the final product, and higher costs due to process inefficiencies. If delays regularly occur, then coordination between activities can be improved through a better representation of the available knowledge in order to not generate misalignments and immediately discover where problems could arise. This is linked to the goodness of information estimates, and it is therefore the ideal context where the here proposed PERT model can be applied.

Exploring fields outside of project management, it can be noticed that belief structures, which are the basis of the Dempster & Shafer Theory of Evidence, are being used in modelling environment with a high uncertainty content (e.g., Denœux, Younes, & Abdallah, 2010; Smets, 2000; Onera & Smarandache, 2012, Jing & Tang, 2021), to such an extent that has been either synthetized or hybridized with fuzzy logic (e.g., Dymova, Sevastianov, & Bartosiewicz, 2010; Du, Wang, & Wang, 2019) and soft set (Vijayabalaji and Ramesh, 2019) to deal with uncertainty effectively. The Dempster & Shafer Theory of Evidence is considered, thanks to its basic concepts, a very effective and helpful tool in representing, studying and measuring epistemic uncertainty affecting systems parameters. In new product development projects, these features could provide several advantages to achieve activities lead times estimations as far as possible consistent with the available information (Kukulies and Schmitt, 2018).

However, to the best of our knowledge, analysing early and recent contributions from the PERT literature, we did not find any contribution proposing a PERT model based on the Dempster & Shafer Theory of Evidence. In order to bridge this gap, we present and apply a new PERT model of this type.

The structure of the paper is as follows. Section 2 contains all the preliminaries of the Dempster & Shafer Theory of Evidence that are necessary to model and validate our proposal, which is explained in Section 3. The experimental analysis has been performed on a real case study (Section 4), while Section 5 contains the conclusions and some ideas for the further research agenda.

2 DEMPSTER & SHAFER THEORY OF EVIDENCE

Dempster (Dempster, 1968) and later Shafer (Smith and Shafer, 1976) introduced the Dempster and Shafer Theory of Evidence (DSTE) as a framework to model uncertainty through the evidences within a system. The novelty of this theory lies in the ability to assign a probability to sets of events, instead of just single events (singletons). Each set of events is called hypothesis, and at the centre of DSTE is the definition of Power Set, or Frame of Discernment (FoD), that is the set containing all the hypotheses that a probability is given to, as mutually-exclusive elements: $\theta = \{\theta_1, \theta_2, \dots, \theta_n\}$. Another peculiarity is that hypothesis probabilities are represented as interval values, delimited by a lower and an upper bound, rather than as point values, in order to model the uncertainty situation.

The three functions used to build the so called "belief functions" are reported below.

The *Basic Probability Assignment* function (BPA) assigns a point probability ("*mass*") between [0;1] to some hypothesis in the FoD. Hypothesis with non-zero mass are called *Focal elements* for the FoD and constitute the *Body of Evidences* for the system. For each sub-set A in the FoD, BPA formulation is given as follows:

$$
\begin{aligned}
&m(A): \theta \to [0; 1] \\
&m(\emptyset) = 0 \qquad (1) \\
&\textstyle\sum_{A \in \theta} m(A) = 1
\end{aligned}
$$

While original DSTE is based on a Closed World Assumption (CWA) where $m(\emptyset) = 0$, an Open World Assumption (OWA) with $m(\emptyset) > 0$ means that the truth may also live in elements not specified in the body of evidence (Daniel, 2016). An OWA implies DSTE to flow into the Transferable Belief Model (TBM), that is considered its extension (Smets, 2000).

The *Belief* function represents the credibility of a hypothesis *A* in the face of the evidence provided. It measures the intensity with which evidences support a given hypothesis, then it constitutes the lower bound LB of the probability interval for that hypothesis:

$$Bel(A) = \textstyle\sum_{B \in \theta \mid B \subseteq A} m(B) \qquad (2)$$

The *Plausibility* function represents the intensity with which evidences do not contrast a given hypothesis, and constitutes the upper bound UB of the probability interval for that hypothesis:

$$Pl(A) = \textstyle\sum_{B \in \theta \mid (B \cap A) \neq \emptyset} m(B) \qquad (3)$$

Despite non-additivity of these functions, Belief structures can be updated by introducing new evidences into the system. This implies the distribution of the masses in the universal set to be modified, then belief functions of each hypothesis will change. This feature makes DSTE also suitable for control applications, allowing to study new outcomes of the scenario resulting from the new evidences introduced (Liu *et al.*, 2014).

2.1 Rules of combination in DSTE

In the common case where the evidence comes from multiple sources it is necessary to aggregate them in order to combine and summarize the body of knowledge available upon the same FoD. The aim is to obtain a structure that is as lean, manageable and interpretable as possible. Aggregation rules are used to represent the resultant of the masses defined by several sources of information. These rules are summed up into 3 categories, depending on the degree of reliability of information sources, each of them using different set operators to drive the aggregation (e.g., Dubois & Prade, 1992; Yang et al., 2020): *Conjunctive* rules are used when sources are supposed to be fully-reliable, and use AND logic operators; *Disjunctive* rules suppose at least one of the sources to be unreliable, then OR logic operators are used; *Trade-off* rules are used in hybrid situation, with a mixed usage of both operators. Some of the most well-known and used aggregation functions are here summarised:

- *Dempster's Rule of combination*: conceived as a generalization of the Bayes theorem, this is the original conjunctive rule which DSTE is born with. Given two masses (evidence) m_1 and m_2 for the same hypothesis A of the power set X, resulting m_{12} is calculated:

$$m_{12}(A) = \frac{\sum_{B \cap C = A} m_1(B) * m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B) * m_2(C)} \ , \forall \ A \subseteq X \qquad (4)$$

  Denominator is a normalization factor, and its second member, denoted as k, represents the amount of conflict between the information sources, varying between 0 and 1. The higher this value, the more the sources of evidence are contrasting (Martin, 2012). However, the higher k is, the more the aggregation results will be counterintuitive. Hence, since its introduction, this rule has been severely criticized because of its pitfalls, in particular in case of dependent (e.g. Fu & Yang, 2014) and conflicting sources (e.g., Fu & Yang, 2011; Yu, Yang, Yang, Ma, & Min, 2015). For a recent criticism to the Dempster's rule of combination in case of conflicting source, the reader can refer to Silva & de Almeida-Filho (2018).

- *Yager's Rule*: it avoids the conflict-based issues due to the normalization factor by making use of the quasi-associative operator called Ground Probability Assignment (GPA) (Yager, 1987). GPA can also be updated every time a new evidence is introduced in the system. GPA formula of each of the n evidences for hypothesis A is given:

$$q(A) = \sum_{\cap_{i=1}^{n} A_i = A} m_1(A_i) * m_2(A_2) * \dots * m_n(A_n) \tag{5}$$
$$q(\emptyset) = \sum_{\cap_{i=1}^{n} A_i = \emptyset} m_1(A_i) * m_2(A_2) * \dots * m_n(A_n)$$

Starting from GPA, combined mass for m_1 and m_2 from sources 1 and 2 are defined through the Ground Probability Mass Assignment (GPMA):

$$GPMA \{ \; q(A) = \sum_{B,C \in \theta \,|\, B \cap C = A} m_1(B) * m_2(C) \;\; ; \;\; q(\emptyset) = \sum_{B,C \in \theta \,|\, B \cap C = \emptyset} m_1(B) * m_2(C) \tag{6}$$

Final masses for each element of the FoD are then calculated as:

$$m^Y(\emptyset) = 0$$
$$m^Y(X) = q(X) + q(\emptyset) \tag{7}$$
$$m^Y(A) = q(A)$$

Yager assigns the mass of the conflict $q(\emptyset)$ to the universal set, so that conflict is interpreted as a lack of information, suggesting that further investigations should be made to enrich the knowledge of the system. This assignment also enlarges Plausibility of all the elements in the FoD, widening their uncertainty range. However, the higher the mass of the universal set (hence the conflict), the more all other hypotheses will be discredited, appearing to be almost likely. In this sense Yager's rule shows to be more cautious, since it gives great credit to uncertainty.

- *Inagaki's Rule*: it uses Yager's GPA to define a continuous class of combination operators, incorporating the Dempster's rule and the Yager's rule as cases (Inagaki, 1991). It expresses the combined mass of evidence from multiple sources as:

$$m(A) = q(A) + f(A) * q(\emptyset) \tag{8}$$
$$where\ A \neq \emptyset, and: \sum_{A \subseteq X, A \neq \emptyset} f(A) = 1$$

Given $k = f(A)/q(A)$, aggregated masses for hypothesis are defined as follows:

$$m^U(A) = [1 + k * q(\emptyset)] * q(A),\ \ with\ A \neq X, \emptyset$$
$$m^U(X) = [1 + k * q(\emptyset)] * q(X) + [1 + k * q(\emptyset) - k] * q(\emptyset) \tag{9}$$
$$0 \leq k \leq \frac{1}{1 - q(\emptyset) - q(X)}$$

By setting k, evidences are filtered on the conflict, allowing to oscillate between Dempster and Yager rules (Inagaki, 1991) (Abdul Rahman, 2014). In case of indecision on which aggregation rule to use, this rule can be used, but in order to avoid distortions, k value should be kept low in the event of a high conflict, since the scale parameter is greater as the conflict between the sources is be marked $(k = 1/\left(1 - q(\emptyset)\right)$.

- *Dubois & Prade's rule*: this is a disjunctive combination rule considering the union of the subsets, which does not generate conflict and not discard any evidence provided. The aggregated mass for a hypothesis A is given by:

$$m^{DP}(A) = \sum_{B,C \in \theta\ |\ (B \cup C) = A} m_1(B) * m_2(C) \tag{10}$$

This rule is used when at least one information sources is not considered reliable (Dubois and Prade, 1992). Moreover, if two hypotheses are expressed as range form (sets with cardinality > 1), the resulting disjunctive aggregation will be an interval having the lower bound of the two as lower bound, and the upper bound the greater of the two (Sentz and Ferson, 2002). Dubois & Prade's rule, by assigning the conflict to the union of the masses that generated it, supports all the hypotheses in the union. This feature is useful in case of high conflict between the sources, but the result may be inaccurate, since hypotheses reported in interval form wide the uncertainty spectrum.

- *Proportional Conflict Redistribution rules*: in this case conjunctive operators are used, but it aims to redistribute the mass of the conflict (partial or total) to the masses (non-empty sets) that generate it. This group includes 5 rules (named from PCR1 to PCR5), all of which are variants of the original, but differ in the way the conflict is allocated (Smarandache and Dezert, 2005). Below we present the PCR5, the most sophisticated and considered to be the most correct from the mathematical and conceptual point of view (Onera and Smarandache, 2012):

$$m_{PCR5}(A) = \sum_{x_1,x_2 \in X \backslash : x_1 \cap x_2 \neq \emptyset} m_1(x_1) + m_2(x_2) + \sum_{B \in X \backslash \{A\} \backslash : A \cap B = \emptyset} [\frac{m_1^2(A) * m_2(B)}{m_1(A)+m_2(B)} + \\ + \frac{m_2^2(A) * m_1(B)}{m_2(A)+m_1(B)}] \quad (11)$$

Pairs of elements conflicting in the aggregation will see their masses increase proportionally to the conflict, while the mass assigned to the FoD (X) remains unvaried. It is the opposite of the Yager's rule. In fact, PCR5 aims to maximize the information content of the aggregated masses, leaving less room for uncertainty and attributing more weight to the evidence available in the system.

- *Other rules of combination*: the aim of this paper is not to provide a survey of DSTE combination rules, as there is a plethora of them, all with different purposes. For more rules see for example Yang & Fu (2009) and Fan, Song, Lei, Wang, & Bai (2018).

2.2 Rules of transformation in DSTE

Although DSTE can represent epistemic uncertainty, today there seems to be no definitive way to use mass-form data decision making reasoning. Belief functions are not easily interpretable to describe the behaviour of the modelled system, and transforming belief structures into probability distributions seems to be the most common way to use these data (Deng, Li and Deng, 2012). The choice of the transformation rule directly impacts on such probability distributions. There are several rules proposed to move from a belief universe to a probabilistic one, some of them are reported below.

- *Pignistic Transformation*: given an element of the FoD “x”, $BetP_m(x)$ shares its mass in equal parts among the singletons that compose it (Smets, 2005). It is a precautionary transformation method which, aiming to spread the masses of non-singletones subsets equally, keeps the entropy of the information represented high, and consequently lowers the Probabilistic Information Content (PIC). It can be viewed as an extension of Laplace indifference principle, according to which equally possible outcomes have equal probability (Dubois, Prade and Smets, 2008). Given Ω_s as the singletons set, the formulation of pignistic transformation is:

$$BetP_m(x) = \sum_{a \subseteq \Omega_s \setminus : x \in a} \frac{m(a)}{|a|} \tag{12}$$

- *Plausibility Transformation*: it is also defined by Dempster himself “approximation of Bayesian theory”, as the correct way to deal with the evidences (Cobb and Shenoy, 2006). It is always consistent with the Dempster’s transformation rule, and is expressed as follows:

$$Pl_{P_m(x)} = \frac{\sum\{Pl_m(\{x\}) | x \in \Omega_s\}}{Pl_m(\{x\})} \tag{13}$$

- *Generalized Pignistic Transformation*: it uses subsets cardinality and aims to maximize the PIC. This goal is the dual of maximizing the concept of entropy proposed by Shannon (Onera and Smarandache, 2012). Its formulation is given as:

$$GPT(x) = m(x) + (m(x) + \varepsilon) * \sum_{Y \in X, \setminus : x \in Y, |Y| \geq 2} \frac{m(Y)}{\sum_{Z \in X \setminus : Z \subseteq Y, |Z| \geq 2} m(Z) + \varepsilon * |Y|} \tag{14}$$

where ε is a very small and positive number that can be arbitrarily set to make the maximization effect more pronounced.

- *Shannon’s Entropy Minimization*: it is based on the link between uncertainty (degree of uniformity between the probabilities assigned to hypothesis), Shannon Entropy (E_H) and PIC (defined as in Information Theory). Since PIC $= 1 - E_H$, and that the uncertainty coming from transformation is directly proportional to E_H, then uncertainty can be minimized by minimizing E_H directly (thus maximizing the PIC) (Han, Dezert and Duan, 2016):

$$
\begin{aligned}
& min_{\{P(x)\backslash : x\in\Omega_s\}}\{\textstyle\sum_{x\in X} P(x) * log_2(P(x))\} \\
& s.t.: \\
& Bel(B) \leq \textstyle\sum_{x\in X} P(x) \leq Pls(B) \\
& 0 \leq P(x) \leq 1 \;\;,\;\; for\ each\ x \in \Omega_s \\
& \textstyle\sum_{x\in\Omega_s} P(x) = 1
\end{aligned}
\tag{15}
$$

A numerical example comparing rules results is given (Table 1), considering the following BPAs: $m(1) = 0.61$; $m(2) = 0.1$; $m(\theta) = 0.29$.

	Transformation rules			
Lead times	$BetP_m(x)$	$Pl_{P_m(x)}$	$GPT(x)$	$Deng_t(x)$
1	0.755	0.697	0.836	0.61
2	0.245	0.303	0.164	0.39

Table 1. An example of comparison of some rules of transformation.

It can be noticed from Table 1 that, in fact, $GPT(x)$ tends to minimize the entropy of the resulting distribution, while other transformation rules such as $Deng_t(x)$ (Han, Dezert and Duan, 2016) aim to maximise it. $Pl_{P_m(x)}$ and $BetP_m(x)$ are somehow included between these two extremes.

2.3 Total uncertainty representation

According to the Shannon's Theory of Information, the entropy of an information source is closely related to the amount of information a message contains. Shannon therefore defined the entropy E_H of an information source, and this value was soon employed to quantify the overall degree of uncertainty (i.e. TU) within a system modelled by DSTE. This topic has aroused such great interest because, during the combination and transformation of the BPAs into probability distributions, information leaks may occur. Many measures were introduced over years as reworks of the original entropy proposed by Shannon (Pan *et al.*, 2019). However, TU can always be described as the sum of 2 components: *Conflict*

(information associated to empty sets) and *Non-Specificity* (associated to sets having cardinality>1). Keeping both components separated in the calculus of TU allows to understand the way in which such uncertainty comes to be formed among the BPAs system, and consequently to understand how to go about widening our knowledge by acquiring new evidences. Finally, the practical goodness of an entropy definition is currently measured by 8 "desirable" properties (Jiroušek and Shenoy, 2018).
The entropy defined by Jirousek and Shenoy (H_{js}) is reported, as an example of those considered by us more useful for our purposes and to illustrate the structure of a sophisticated measure that satisfies many properties:

$$H_{js}(m) = \sum_{x \in X} Pt(x) * \log\left[\frac{1}{Pt(x)}\right] + \sum_{A \in 2^X} m(A) * \log(|A|) \tag{16}$$

The first addendum is Shannon's entropy ($Pt(x)$ is the probability resulting from the plausibility transformation of x), which measures the conflict component; the second addendum is the entropy of Dubois and Prade for non-specificity. Since the range property is satisfied, each component is always found within [0; $log_2(|X|)$]. This is useful to compare different uncertainty situations on the same system.
By applying these measures also in the lead time estimation, a way to represent, measure and break down epistemic uncertainty into its components has been found. This allows to create new analysis for the PERT network and the project itself. To do this, the same entropy measure must be used for all the activities, and it must satisfy:
a) total uncertainty as sum of conflicting and non-specificity components;
b) monotonicity, so that the measurement is sensitive to drops and increases in the information when updated.
The entropy expression proposed by Jiroušek and Shenoy (H_{js}) is a valid option, as well as its variation (H_{pq}) proposed by Pan, Zhou, Tang, Li, & Huang (2019).
For each activity, following data can be obtained:

Entropy measure	Conflict	Non-specificity	TU
H_{js}	$\sum_{x \in X} Pt(x) * \log\left[\frac{1}{Pt(x)}\right]$	$\sum_{A \in 2^X} m(A) * \log(\lvert A \rvert)$	Conflict + Non-specificity

H_{pq}	$\sum_{A \in 2^X} m(A) * \log\left[\frac{1}{Pm(A)}\right]$	$\sum_{A \in 2^X} m(A) * \log(\lvert A \rvert)$	Conflict + Non-specificity

Table 1. Total uncertainty representation.

Note that these values must be considered in relation to the minimum and maximum of TU values $[0; 2 * log_2(|X|)]$, then a clearer representation in the percent value (%), so that comparison between different activities can be made. Thanks to this analysis, a project manager can assess to: a) continue his/her management with the data available at that time; b) reject the experts' proposals and require further investigations, in order to reduce uncertainty rate. Actually, other evaluations should be made. For example, greater attention should be paid to the uncertainty values of high criticality activities, or to those activities responsible alone of a good portion of the overall duration of the project. Further indexes of analysis have been developed according with this purpose, dealing with epistemic uncertainty.

3 A PERT MODEL BASED ON THE DSTE

Once the basic activities of the project have been identified through the work breakdown structure (WBS), the pert network is constructed on the basis of the precedence constraints. Then, the determination of the activity lead times is the heart of the calculus of the critical path in the PERT model. To do this through the DSTE, the evidences that support certain hypotheses about the evolution of a system are needed first. Secondly, the tools of combination (in case of multiple sources of information), transformation (to get data usable in practice) and updating (as over time new evidence will be added, correcting the previous ones) will have to be studied. The final outputs of the PERT project are then analysed using well-known analysis parameters for reticular techniques but, as shown later, some features of the belief structures can be exploited to construct new parameters. When historical data or parameters for the activities lead times do not exist, in the practice it is often used the judgment of sector experts (Rose, 2013). Experts are expressing on which hypotheses may be probable for these values, providing evidence for the system on the basis of their knowledge or data. A mathematical structure to represent the evidences is now illustrated.

Judgements are provided in form of the so called "Frame of Masses" (*FoM*): a mass can be assigned to non-singleton hypothesis, even to the universal set Θ, and for each *FoM* the sum of all the masses is 1. This structure models the degree of uncertainty for each expert to his/her hypothesis.

Activity name	Expert 1		
LT_hypothesis	$h1$	$h2$	Θ
Masses BPA	$m(h1)_1$	$m(h2)_1$	$m(\theta)_1$

Table 3. Structure of a Frame of masses (*FoM*), coming from a generic "Expert 1" for a generic "Activity" possible lead time.

Since the use of single *FoM* for different experts is not at all practical, the *FoMs* will be combined to obtain an aggregate information which is a summary of the degree of knowledge of the involved actors. There are several other reasons to do so: i) individual information is the result of the concurrency of certain conditions in a situation, that rarely can tell much more about that situation under different conditions; ii) the aggregation of the evidence allows extension phenomena (different information from different sources are included in the same model); and iii) through an aggregate BPA, conflict between information sources can be studied. Under a CWA, aggregation of *FoMs* from multiple sources can be made through the DSTE combination rules. In order to combine the *FoMs* from two different experts, the combination rules are given, as reported in Table 4.

	Expert 1	$h1$	$h2$	Θ
Expert 2	MASSES	$m(h1)_1$	$m(h2)_1$	$m(\theta)_1$
$h1$	$m(h1)_2$	$m(h1)_1 * m(h1)_2$	$\boldsymbol{m(h1)_2 * m(h2)_1}$	$m(h1)_2 * m(\theta)_1$
$h3$	$m(h3)_2$	$\boldsymbol{m(h3)_2 * m(h1)_1}$	$\boldsymbol{m(h3)_2 * m(h2)_1}$	$m(h3)_2 * m(\theta)_1$
Θ	$m(\theta)_2$	$m(\theta)_2 * m(h1)_1$	$m(\theta)_2 * m(h2)_1$	$m(\theta)_2 * m(\theta)_1$

Table 4. The combination rules.

Empty intersections between experts' hypothesis, as combinations of hypotheses that are not compatible (none of the two is contained in the other), are reported in bold. The parameter k is the degree of conflict between the sources (experts), given as:

$$k = \sum_{(i=1..n\,;\,j=1..m\,|\,h_i \cap h_j = \emptyset)} m(h_i)_{Expert\ 2} * m(h_j)_{Expert\ 1} \qquad (17)$$

Applying a combination rule among those presented in Section 2.1, for each combined mass ($m_{RULE}(h)$) is possible to calculate its Belief (Bel), Plausibility (Pl) and Uncertainty functions:

Combined masses		Belief	Plausibility	Uncertainty [%]
$h1$	$m_{RULE}(h1)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \subseteq h1} m_{RULE}(h_i)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \cap h1 \neq \emptyset} m_{RULE}(h_i)$	$[Pl(h1) - Bel(h1)] * 100$
$h2$	$m_{RULE}(h2)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \subseteq h2} m_{RULE}(h_i)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \cap h2 \neq \emptyset} m_{RULE}(h_i)$	$[Pl(h2) - Bel(h2)] * 100$
$h3$	$m_{RULE}(h3)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \subseteq h3} m_{RULE}(h_i)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \cap h3 \neq \emptyset} m_{RULE}(h_i)$	$[Pl(h3) - Bel(h3)] * 100$
Θ	$m_{RULE}(\theta)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \subseteq \theta} m_{RULE}(h_i)$	$\sum_{i=1\ldots\theta\,;\,h_i \mid h_i \cap \theta \neq \emptyset} m_{RULE}(h_i)$	$[Pl(\theta) - Bel(\theta)] * 100$

Table 5. Belief, Plausibility and Uncertainty% of each combined hypothesis.

According with the DSTE, hypotheses can be expressed as intervals, enriching the amount of possible case studies. The number of combined masses $m_{RULE}(h)$ depends on the rule adopted, and on the hypothesis forms and values. Disjunctive or conjunctive operators must be used in the previous masses-cross matrix depending on the type of the combination rule (see Section 2.1).

As PERT is also a project control tool, this model allows to update lead-time estimates after new evidence. In fact, updating existing BPAs after the introduction of new evidence in the system is always possible. Thanks to the associative property of the Dempster's rule and by the quasi-associativity of many other rules, a new *Fom* that represents a new proof coming into the system is combined with the already existing *Fom*, redistributing by this way the masses among the focal elements, and making some hypotheses more probable than they were before.

However, the hypotheses expressed as sets of events, with probabilities expressed as BPAs, are not ready to be used properly in the PERT. This information, to be used, are needed to be transformed into probability distribution for activity lead times. By using transformation rules among those previously described, resulting distributions will reflect:

a) epistemic uncertainty related to the evidence provided by the sources;

b) features given by the aggregation mode used;

c) amount of conflict between the sources;
d) manager's aversion, neutrality, or propension to risk reflected by the choice of the transformation rule.

3.1 A new method to calculate starting BPAs

As no definitive framework for assigning BPAs exists (Jing and Tang, 2021), scientists are still proposing innovative methods linked to their use cases (e.g., Fei et al., 2019; Xu et al., 2013). Since, to the best of our knowledge, a PERT model based on the DSTE has not yet been treated in the literature, to complete the project framework we propose a method also for the BPA determination, inspired to the AHP (Analytic Hierarchy Process) method for weights assignment in decision-making processes, as follows.

a) Each expert provides the hypotheses about his/her activity lead times.
b) Each expert pairwise compares the entire set of hypotheses ($(n^2 - 1) / 2$ comparisons). For each couple ($i;\ j$) he/she gives a judgment from 1 to 9, with step 2, on "how the hypothesis i is more probable than the hypothesis j?" (fundamental scale).
c) Judgments will fill the square matrix of comparisons for the expert, and the AHP engine (ref. https://bpmsg.com/) will give in output the weight vector normalized for the hypotheses, and the consistency index CI of the matrix (Saaty, 1980).
d) The obtained weights will correspond to the BPAs for the hypothesis.
e) In AHP method, CI represents the inconsistency index of the user, due to his/her specific unknowledge for the comparisons. CI measures the variance of the error between the final estimation of the weights and the indications provided by the user (Saaty, 1980). In the case of a perfectly consistent matrix, CI is equal to zero, which means the final weight estimate will exactly match the judgment given by the user. On the contrary, as the value of CI increases, we approach the situation where the user provides completely randomly judgments: situation that undoubtedly reflects total absence of information. It is straightforward that this value is likely to model the expert’s epistemic uncertainty, as it represents “how much the user is reliable”. We used then CI to represent BPA for the hypothesis *Θ*, i.e. the universal set of possibilities.
f) As hypothesis *Θ* is introduced (with BPA=CI), another normalization is needed.

3.2 Monte Carlo simulation and critical path research

It has been shown that the critical path research, through mathematical model or exact heuristic algorithms, is a low computational effort operation, and Monte Carlo simulation has become a tool used in conjunction with critical path, as a method both of analysis and of validation (e.g, Wyrozębski & Wyrozębska, 2013; Tysiak, 2011). In this paper something similar is proposed, in order to test how the steps involved in DSTE for activity lead times drive the results of PERT and on its analysis. A Monte Carlo method is used to supply a critical path algorithm, while simulation records the results of the critical paths research at each iteration. Any type of analysis on the recorded results can then be carried out. This is very similar to stochastic PERT situation, since activities lead times have been estimated through probability distributions. Each distribution is inclusive of all the possibilities supported so far by the evidences expressed as BPAs. The iterative process underlying this Monte Carlo simulation is reported below.

1. For each iteration, a random number in the [0; 1] range is generated for each activity. Based on its probability distribution, a lead time is associated with the activity by means of the standard Monte Carlo approach (see Table 6). By using this rule, uniform distribution between limits is assumed. Due to the Central Limit Theorem, normality can be assumed in the long-run.

Lead Time Hypothesis	$h1$	$\dots$	hn
Hypothesis probability	$P(h1)$	$\dots$	$P(hn)$
Lead Time	$if\big(Random\ Value < P(h1)\big)$ $then\ (Lead\ Time = h1)$ $else\ (Lead\ Time = 0)$	$\dots$	$if(Random\ Value < P(hn))$ $and\ \Big(Random\ Value > P\big(h(n-1)\big)\Big)$ $then\ (Lead\ Time = hn)$ $else\ (Lead\ Time = 0)$

Table 6. Pseudo-random generation of lead times.

Non-zero value in the 3rd raw corresponds to the activity lead time in this iteration.

2. The obtained lead time is used in the critical path research for this iteration. For this purpose, an algorithm that compares the Early Start (ES) and the Late Start (LS) times

for each activity, on the basics of predecessors-successors relationship, is used, identifying which activities have Slack time = 0. In alternative, the mathematical model for critical path research can be used. At this point also the GANTT diagram for the project in the iteration is obtained, reporting critical activities maximum slack of non-critical ones. The overall duration of the project coincides with the late finish of the last milestone.

3. The steps for the assignment of the activity time and the calculation of the critical path are repeated for a number of times that assures a total deviation less than 5%; this number can be calculated using the expression reported in (Bukaçi *et al.*, 2016) or alternatively the iterative method described in (Sellitto, 2020), and the results of each iteration are recorded.

4. The analysis based on the recorded results is performed.

3.3 The analysis

The analysis based on the results of the simulation is a key tool to: a) a greater understanding of the activities network; b) predict possible evolutions for the project; c) measure and monitor project parameters (i.e., completion time); d) get probability estimations for aggregated events. Some useful analysis indexes for the PERT are now reported:

1. Percentage Criticality Index ($CI\%$): how many times among the iterations an activity turns out to be critical.
2. Average slack time of non-critical activities; it measures the average slippage of which a non-critical activity could count on.
3. Minimum, Maximum and Average project duration; given by single values, but when the number of iterations is high, a distribution curve can be tracked for the time completion of the project within a confidence interval (e.g., 90%).
4. Forecast on deadlines; same as point 3, but for project milestones.
5. Activities average impact on the project duration ($AI\%$): to which critical activities is better to speed up, in order to obtain the most significant benefits for the project, in terms of time and costs. In fact, this aspect is not linear, but depends on the parallelism between activities and their slack times.

6. "What if" and sensitivity analysis: with the arbitrary modification of one of the parameters of the system, or with the introduction of new data.

Since all these analyses are typical for all stochastic PERT approaches, new parameters are here proposed when applying the DSTE to deal with epistemic uncertainty scenarios. More specifically, thanks to the PIC analysis, we can reconnect the simulation outputs and their variances to the overall or uncertainty of the project activities that generated them, by the definition of new parameters suitable to deal with the uncertainty underlying the project data. In particular, they are:

1. Critical Uncertainty (CU) tells in which measure the activity criticality is linked with its uncertainty:

$$CU = CI\% * TU\% \tag{18}$$

2. Uncertainty Impact (UI) measures the impact of a lead time estimate's uncertainty on the total duration of the project:

$$UI = AI\% * TU\% \tag{19}$$

By splitting TU into its components (conflict and non-specificity), analogous index can be created.

4 APPLICATION TO A PRODUCT DEVELOPMENT CASE

In this section, through an instance coming from a real context, the consequences of the choices made in each of the points previously described are being studied, as well as the resulting impact on the final representation of the problem. The proposed PERT model based on DSTE is applied to a real new product development process of a structured international company that produces garden machines. The product development process follows the Advanced Product Quality Planning standard (APQP), that is a structured approach aimed at ensuring customer satisfaction through product and process design. By using this tool, the timing with which development phases are carried out, comes to be

very relevant for the company. However, due to the numerous planning and control activities, as well as to the correctness of the information on activity lead times, projects often miss the deadlines set for the builds of each phase. This feature always leads to:
a) missed revenues, caused by the late arrival on the market's seasonality;
b) greater risks undertaken during the development process;
c) higher costs sustained during the phases affected by delay (Rose, 2013).
In this context, the focus has been restricted to the Finding Assembly Materials phase and the activities which is compose it, as it is the longest and most problematic phase. Activities necessary to sample and supply components for definitive-build machines have been defined, and then the PERT network for this phase has been built, on the basics of logical-technical precedence constraints. The durations of each network activity were estimated as described in Section 3, and then Monte Carlo simulation was performed. Analysis on the results were conducted to test the properties of this model.
A simplified version of the real network (see Figure 1) is used in this paper to summarize the dynamics of the model for this case study:

Activity ID	Project Activities:	Predecessors
10	Start	
20	Engine DRW (definitive drawing release)	10
30	Suction unit DRW	20
40	Shell group DRW	30
50	Engine MLT (Mold construction lead time)	20
60	Engine 1BLT (first batch samples lead time)	50
70	Suction unit RFQ (request for purchasing quotation)	30
80	Suction unit 1BLT	70
90	Shell group RFQ	40
100	Shell group MLD	90
110	Shell group 1BLT	100
120	Finish (GAMMA BUILD)	60; 80; 110

Table 7. Matrix of the PERT project activities.

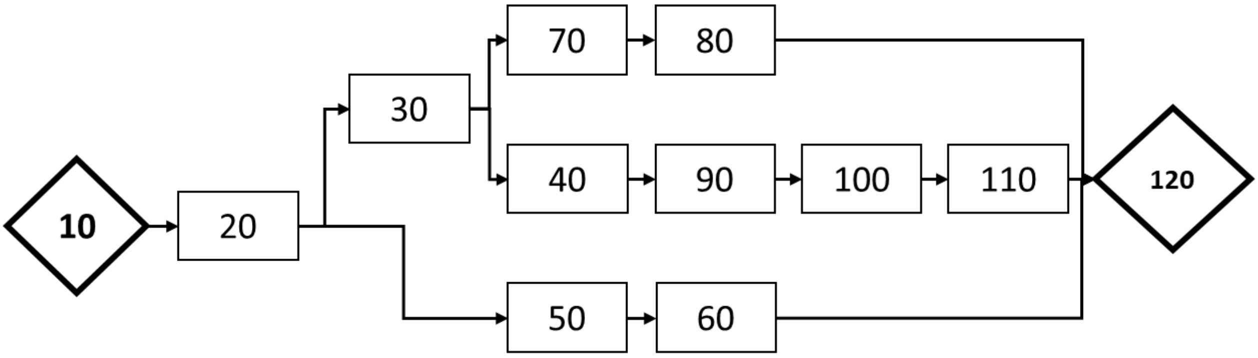

Figure 1. Project activities Graph.

Activity ID:	Expert 1	Expert 2
10	{(4; 0.2); (5; 0.5); ([5;6]; 0.2); (Ө; 0.1)}	{([5;6]; 0.1); (7; 0.3); (9;0.4); (Ө; 0.2)}
20	{([1;2]; 0.4); ([2;3];0.3); (3;0.1); (Ө; 0.2)}	{(1; 0.2); (2;0.2); ([3;4]; 0.4); (4;0.1)}
30	{(6; 0.3); (7; 0.4); ([7;8]; 0.1); (Ө; 0.2)}	{(5; 0.3); ([6;7]; 0.5); (8;0.1); (Ө; 0.1)}
40	{(6; 0.3); (7; 0.4); ([7;8]; 0.1); (Ө; 0.2)}	{(5; 0.3); ([6;7]; 0.5); (8; 0.1); (Ө; 0.1)}
50	{(9; 0.3); (11; 0.4); (12; 0.2); (Ө; 0.1)}	{(8; 0.6); (10; 0.1); ([11;12]; 0.1); (13; 0.1); (Ө; 0.1)}
60	{(2; 0.6); (3; 0.3); (Ө; 0.1)}	{(2; 0.7) ;(3; 0.1); (Ө; 0.2)}
70	{(4; 0.4); (5; 0.5); (Ө; 0.1)}	{(3; 0.2); (4; 0.7); (Ө; 0.1)}
80	{(2; 0.8); (3; 0.1); (Ө; 0.1)}	{(2;0.3); (4;0.1); (Ө; 0.6)}
90	{(5; 0.4); ([5;6]; 0.2); (6; 0.1); (Ө; 0.3)}	{(6; 0.2); (7;0.3); ([5;6]; 0.4); (Ө; 0.1)}
100	{(6; 0.2); (7; 0.3); ([7;8]; 0.4); (Ө; 0.1)}	{(5; 0.1); ([6;7]; 0.1); (8; 0.7); (Ө; 0.1)}
110	{(9; 0.3); (10; 0.4); ([10;11]; 0.1); (Ө; 0.2)}	{(10; 0.1); ([9;10]; 0.5); (11; 0.3); (Ө; 0.1)}

Table 8. Starting BPAs given by two experts for each hypothesis on project activity lead times. (“hypothesis lead time [weeks]”; “BPA for the hypothesis").

4.1 Three approaches to apply the model

It has been shown how different combination and transformation choices affect the probability distribution of the activity lead times. However, the macro outcomes of the simulation on a given network could not be so sensitive to these variations, due to path-dominance phenomena issues. Since it is very difficult to identify analogies to predict the consequences of combination and transformation mixes on different networks, it is more appropriate to focus on the mixes affecting the same reference instance, considering each project as a stand-alone case. Three approaches, according with the manager’s risk propension, were analysed. It is worth of remarking that there are no “right” aggregation and transformation rules, but just more or less useful methods to represent epistemic uncertainty:

1. *Cautious decision maker, low risk propension*: if the manager is used to take decisions only in possess of reliable information, he/she should decide to elaborate BPAs through an aggregation rule that gives importance to both conflict and non-specificity between the sources (e.g. Yager, 1987), and then use high-entropy-likely transformation methods ($Deng_t(x)$). By doing so, the Total Uncertainty among the estimations grows, causing Monte Carlo simulation to return project values affected by higher variance. The manager will then be able to study the corrective actions and face different evolutions of the project and concentrate where more information should be collected.

2. *Daring decision maker, high risk propension*: if a decision maker needs very fast and expressive data, he/she should decide to significantly validate the evidence at his/her disposal, addressing his/her plans based on the scenario that is most likely to date. Despite all, even new evidences that will arrive will be evaluated to the maximum, so it would be wise for the manager to also build flexible plans in order to adapt to the sudden change. PCR5 rule together with the $Deng_t(x)$ should be used to conserve the level of entropy, or the $GPT(x)$ to further enhance the evidence, while the Shannon's Entropy Minimization could be used to maximize it.

3. *Neutrality*: designed for situation in which the manager does not want to stray into hasty judgments, but not even be too slow in drawing up an action plan. We suggest the use of the Inagaki's rule, in order to set the conflict scale factor, and subsequently the $PlP_m(x)$ for a coherent transformation. To get instead fast and faithful outcomes, the Yager and $BetP_m(x)$ pair works well.

Below, a numerical summary comparison of some outcomes for the project after the simulation is reported, obtained using two of the illustrated approaches. In the columns you can find values corresponding to the index of analysis defined in Section 3, while in row the used approach.

Approach	Activity ID	Avg Slack	TU%	Conflict Unc%	Non-specificity Unc%	Avg activity duration	AI%	UI%	Avg project duration	Std-deviation project duration
Yager's + Pla(x)	20	0	89%	56%	44%	6.1	16%	14%	38.04	3.116978594
	30	0	57%	69%	31%	2.5	7%	4%		
	40	0	44%	69%	31%	6.5	17%	8%		
	50	18.7	84%	59%	41%	10.5	28%	23%		
	60	18.77	59%	75%	25%	2.3	6%	4%		
	70	23.2	66%	71%	29%	3.2	8%	6%		
	80	23	47%	81%	19%	2.5	6%	3%		
	90	0	50%	68%	32%	6.1	16%	8%		
	100	0	73%	67%	33%	6.8	18%	13%		
	110	0	43%	62%	38%	9.9	26%	11%		
PCR5 + GPT(x)	20	0	49%	92%	8%	6.1	16%	8%	38.16	2.259525292
	30	0	44%	87%	13%	2.6	7%	3%		
	40	0	49%	91%	9%	6.4	17%	8%		
	50	20.34	46%	98%	2%	9.9	26%	12%		
	60	20.34	41%	98%	2%	2.2	6%	2%		
	70	24.3	40%	96%	4%	3.2	8%	3%		
	80	24.3	30%	90%	10%	2.3	6%	2%		
	90	0	39%	84%	16%	5.8	15%	6%		
	100	0	42%	95%	5%	7.3	19%	8%		
	110	0	29%	85%	15%	9.9	26%	8%		

Table 9. Comparison of the project PERT results using two opposite approaches: Yager's rule of transformation + Plausibility transformation; PCR5 + Generalized Pignistic Transformation.

The standard deviation of the project duration ($\mathrm{Std}-\mathrm{deviation}$) is calculated by using the outcomes of the simulation on project completion time at each iteration.
Assuming normality in the time to completion estimates, the confidence intervals ($\mathrm{CI} = \mathrm{Avg} \pm 2 * \mathrm{Std} - \mathrm{deviation}$) can be computed; the results for the two approaches are reported in Table 10.

Approach	CI
Yager's + Pla(x)	[37.84; 38.23]
PCR5 + GPT(x)	[38.02; 38.29]

Table 10. Confidence intervals calculated for both the outcomes of approaches reported in Table 9.

Using the outcomes of both the approaches, it is possible to compare the probabilities to complete the project within a certain number of weeks (see Table 11).

Number of weeks to complete the project	Yager's + Pla(x)	PCR5 + GPT(x)
35	16%	8%
36	26%	17%

37	37%	30%
38	49%	47%
39	62%	65%
40	74%	79%

Table 11. Comparison of the probabilities to complete the project within a certain number of weeks using the outcomes of the approaches reported in Table 9.

In Table 9 there are no relevant changes about criticality of activities, because of the path dominance situation within the project instance. However, it is evident that the proposed indexes of analysis are strongly influenced by the approach used. Differences can also be found in the variances of the slack times. The way in which $TU\%$ is conditioned is closely related to the type of approach chosen to estimate activity lead times, as well as the percentage distribution of its own components (Conflict% and Non-specificity%). As a result, the proposed method, even in the presence of a dominant critical path, can represent epistemic uncertainty coherently with the user's needs and risk likely. For example, in front of these data, a manager could decide to take actions to reduce the uncertainty connected to the critical activities of the project, as well as to those with greater impact on the evolution of the project.

5 CONCLUSIONS

In this paper, a PERT model based on the Dempster & Shafer Theory of Evidence was for the first time conceived and applied to a real context to show its properties and utility.
In particular, the model proves to be a good tool for representing and managing those projects affected by a significant epistemic uncertainty component. In fact, the main advantages come from the possibility of including and measuring the epistemic non-knowledge lying within the estimates for the activity lead times. Estimates are generated and updated starting from the evidence available in the system and then are elaborated with the rules set forth in the Theory of Evidence. Subsequently, the resulting PERT network is built and analysed using a Monte Carlo simulation, combined with an iterative critical path research process, in order to be studied through the several parameters for the planning and the management of the operations.
The manager of a project will therefore be further supported in the decision-making process, by receiving indication on the evolution of the project based on the proof

(evidence) available. All this is aimed at taking appropriate corrective actions to avoid project delays, which would result in higher costs and risks for the organization.
The future research agenda will concern the extension of Dempster & Shafer Theory of Evidence to the Transferable Belief model (TBM), in order to study which novelties could arise by adopting an Open World Assumption in the representation of epistemic uncertainty for this PERT model.

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