Handbook of Research on Generalized and Hybrid Set Structures and Applications for Soft Computing

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Chapter 14
Neutrosophic Soft Sets and Their Properties

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ABSTRACT

Soft set plays an important role in the theory of approximations, as parameterized family of subsets in the universe of discourse. On the other hand, neutrosophic set is based on the neutrosophic philosophy, which states that: Every idea A has an opposite anti(A) and its neutral neut(A). This is the main theme of neutrosophic sets and logics. This chapter is about the hybrid structure called neutrosophic soft set, i.e. a soft set defined over a neutrosophic set. This chapter begins with the introduction of soft sets and neutrosophic sets. The notions of neutrosophic soft sets are defined and their properties studied. Then the algebraic structures associated with neutrosophic soft sets are debated. After that, the mappings on soft classes are studied with some of their properties. Finally, the notion of intuitionistic neutrosophic soft sets is taken into consideration.

1. INTRODUCTION

The data associated with real world problems like engineering problems, social problems, economic problems, computer problems, decision making, medical diagnosis etc. are often uncertain or imprecise. All data are not necessarily crisp, precise, and of deterministic nature because of their fuzziness. Different theories and approaches have been proposed to solve these problems associated with real data. In 1965, Zadeh proposed fuzzy set sets to overcome these problems. A huge amount of research has been conducted on different aspects of fuzzy sets such interval valued fuzzy sets, intuitionistic fuzzy sets, type-2 fuzzy sets and so on. Fuzzy sets have been successfully applied in several areas like signal processing, knowledge representation, decision making, stock markets etc.

A Russian mathematician, Molodstov in 1999, proposed a new mathematical framework so called soft sets. A soft set is a parameterized collection of subsets of the crisp set. It is completely a new approach

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to modeling vagueness and uncertain data and therefore soft set theory is free from the hurdles which affect the existing methods. In other words, soft sets are neighborhood systems and thus are a special case of context dependent fuzzy sets. A lot of research has been done on soft sets. There are a lot of hybrid structures that have been introduced such as fuzzy soft sets, intuitionistic fuzzy soft sets, rough soft sets and so on. Soft sets are more general which have more capabilities to handling uncertainty. The theory of soft sets have been successfully applied in several fields such as smoothness of functions, game theory, operation research, Riemann integration, Parron integration, decision making, probability and so on. Kharal studies mappings on soft classes and fuzzy soft classes.

Smarandache in 1995, introduced the theory of neutrosophic sets under the neutrosophy which is a new branch of philosophy that study the origin, nature, and scope of neutralities as well as their interaction with ideational spectra. A neutosophic set is basically characterized by a truth membership function T, an indeterminacy membership function I and falsity membership function F. Neutrosophic set is a generalized framework to several other theories such as fuzzy sets, intuitionistic fuzzy sets, paraconsistent set and so on. Neutrosophic sets can handle uncertainty, inconsistent, incomplete, indeterminate etc. The neutrosophic set and set theoretic operators need to be specified from scientific or engineering point of view. Indeterminacy in neutrosophic sets are quantified explicitly, and T, I, and F are complementally independent, which is very significant in several applications such as information fusion, physics, computer, networking, decision making, information theory etc.

In this chapter, we present the notions of neutrosophic soft sets, which is a parameterized family of subsets of the neutrosophic set. In section 1, we give a brief introduction. In section 2, we present some of the basic definitions and notions of soft sets, neutosophic sets and intuitionistic neutrosophic sets. Then, we study neutrosophic soft sets with some of their basic properties in section 3. In the next section, the algebraic structures associated with neutrosophic soft sets are introduced, such as semirings, semigroups, lattices etc. In section 5, the mappings on neutrosophic soft classes are studied. Section 6 explores the notions of intuitionistic neutrosophic soft sets. We also study some of their properties with an application in section 7.

2. NEUTROSOPHIC SOFT SETS

In this section the neutrosophic soft set is introduced and studied some of their basic properties with illustrative examples.

Definition 2.1: Let U be a universe of discourse and E is a set of parameters and $A \subset E$. Let $P(U)$ denote the power set of all neutrosophic sets of U. A pair (F,A) is called a neutrosophic soft set over U where $F$is a mapping given by $F: A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F,A), or as the set of a-approximate elements of the soft set.

For illustration, consider the following example.

Example 2.2: Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider

E={beautiful, wooden, costly, very costly, moderate, green souroundings, in good repair, in bad repair, cheap, expensive}.

In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surrounding and so on. Suppose that there are five houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{a_1, a_2, a_3, a_4\}$, where a_1 stands for the parameter 'beautiful', a_2 stands for the parameter 'wooden', a_3 stands for the parameter 'costly', and a_4 stands for the parameter 'moderate'. Suppose that,

$F(\text{beautiful}) = \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$,

$F(\text{wooden}) = \{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\}$,

$F(\text{costly}) = \{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}$,

$F(\text{moderate}) = \{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}$.

The neutrosophic soft set (F,E) is a parameterized family $\{F(a_j), j=1,2,\ldots 10\}$ of all neutrosophic sets of U and describe a collection of approximation of an object. The mapping F here is 'house(.)', where dot(.) is to be filled up by a parameter $a \in E$. Therefore, $F(a_1)$ means 'houses(beautiful)' whose functional-value is the neutrosophic set

$\{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$.

Thus the neutrosophic soft set (F,A) can be viewed as a collection of approximation as below:

$(F,A) = \{F(\text{beautiful}) = \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$,

$F(\text{wooden}) = \{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\}$,

$F(\text{costly}) = \{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}$,

$F(\text{moderate}) = \{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}\}$,

where each approximation has two parts: (1) a predicate p, and (2) an approximate value-set v. For example, for the approximation "beautiful houses"

$= \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$,

we have

1. The predicate name beautiful houses, and
2. The approximate value-set is

$\{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$.

Thus the neutosophi soft set (F,E) can be viewed as a collection of approximation like $(F,A) = \{p_1=v_1, p_2=v_2, \ldots, p_{10}=v_{10}\}$.

The class of all value sets of a neutrosophic soft set (F,E) is termed as value-class of the neutrosophic soft set and is denoted by $C_{(F,E)}$. Clearll $C_{(F,E)}\subset P(U)$. For two neutrosophic soft sets (F,A) and (G,B) over a common universe U, (F,A) is said to be a neutrosophic soft subset of (G,B) if $A\subseteq B$ and $T_{F(a)}(x)\leq T_{G(a)}(x)$, $I_{F(a)}(x)\leq I_{G(a)}(x)$, and $F_{F(a)}(x)\leq F_{G(a)}(x)$ for all $a\in A$, $x\in U$. It is denoted by $(F,A)\subseteq (G,B)$. Similarly (F,A) is called neutrosophic soft super set of (G,B) if (G,B) is a neutrosophic soft subset of (F,A), which is denoted as $(F,A)\supseteq (G,B)$. Two neutrosophic soft sets (F,A) and (G,B) are said to be equal if (F,A) is a neutrosophic soft subset of (G,B) and (G,B) is a soft subsets of (F,A). This is denoted by $(F,A)= (G,B)$.

Definition 2.3: Let E be a set of parameters. The NOT set of E, denoted by $\rceil E$ and is defined by $\rceil E=\{\rceil e_j,\forall j\}$ where $\rceil e_j=note_j$.

Definition 2.4: The complement of a neutrosophic soft set (F,A) is denoted by $(F,A)^C$ and is defined by $(F,A)^C=(F^C, \rceil A)$ where $F^C: \rceil A\rightarrow P(U)$ is a mapping given by $F^C(a)=$ neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$, and $F_{F^c(x)} = T_{F(x)}$, for all $a\in A$. It is clear that $((F,A)^C)^C=(F,A)$.

Definition 2.5: A neutosophic soft set (H,A) over the universe of discourse U is termed to be empty or null neutrosophic soft set with respect to the parameter set A if $T_{H(a)}(m)=0$, $F_{H(a)}(m)=0$, and $I_{H(a)}(m)=0$ for all $m\in U$ and $a\in A$. The null neutrosophic soft set is denoted by Φ_A.

Definition 2.6: The union of two neutrosophic soft sets (F,A) and (G,B) over a universe U is a neutrosophic soft set (H,C) which is denoted as $(F,A)\cup(G,B)=(H,C)$ where $C=A\cup B$ and for all $c\in C$, the truth-membership, indeterminacy-membership and falsity-membership of (H,C) are as follows:

$$T_{H(c)}(\mathrm{m}) = \begin{cases} T_{F(c)}(\mathrm{m}), & if \quad c\in A-B, \\ T_{G(c)}(\mathrm{m}), & if \quad c\in B-A, \\ \max\left(T_{F(c)}(m), T_{G(c)}(m)\right), & if \quad c\in A\cap B. \end{cases}$$

$$I_{H(c)}(\mathrm{m}) = \begin{cases} I_{F(c)}(\mathrm{m}), & if \quad c\in A-B, \\ I_{G(c)}(\mathrm{m}), & if \quad c\in B-A, \\ \dfrac{I_{F(c)}(m)+I_{G(c)}(m)}{2}, & if \quad c\in A\cap B. \end{cases}$$

$$F_{H(c)}(\mathrm{m}) = \begin{cases} F_{F(c)}(\mathrm{m}), & if \quad c\in A-B, \\ F_{G(c)}(\mathrm{m}), & if \quad c\in B-A, \\ \min\left(F_{F(c)}(m), F_{G(c)}(m)\right), & if \quad c\in A\cap B. \end{cases}$$

Definition 2.7: The intersection of two neutrosophic soft sets (F,A) and (G,B) over U is a neutrosophic soft set (H,C) which is denoted as $(F,A)\cap(G,B)=(H,C)$ where $C=A\cup B$ and for all $c\in C$, the truth-membership, indeterminacy membership and falsity-membership of (H,C) are as following:

$$T_{H(c)}(\mathrm{m}) = \begin{cases} T_{F(c)}(\mathrm{m}), & if \quad c \in A - B, \\ T_{G(c)}(\mathrm{m}), & if \quad c \in B - A, \\ \min\left(T_{F(c)}(m), T_{G(c)}(m)\right), & if \quad c \in A \cap B. \end{cases}$$

$$I_{H(c)}(\mathrm{m}) = \begin{cases} I_{F(c)}(\mathrm{m}), & if \quad c \in A - B, \\ I_{G(c)}(\mathrm{m}), & if \quad c \in B - A, \\ \dfrac{I_{F(c)}(m) + I_{G(c)}(m)}{2}, & if \quad c \in A \cap B. \end{cases}$$

$$F_{H(c)}(\mathrm{m}) = \begin{cases} F_{F(c)}(\mathrm{m}), & if \quad c \in A - B, \\ F_{G(c)}(\mathrm{m}), & if \quad c \in B - A, \\ \max\left(F_{F(c)}(m), F_{G(c)}(m)\right), & if \quad c \in A \cap B. \end{cases}$$

Proposition 2.8: For any two neutrosophic soft sets $(F,A)$and (G,B), the following holds.

1. $(F,A) \cup (F,A) = (F,A)$,
2. $(F,A) \cup (G,B) = (G,B) \cup (F,A)$,
3. $(F,A) \cap (F,A) = (F,A)$,
4. $(F,A) \cap (G,B) = (G,B) \cap (F,A)$,
5. $(F,A) \cup \Phi_A = (F,A)$,
6. $(F,A) \cap \Phi_A = \Phi_A$.

Proposition 2.9: For any three neutrosophic soft sets (F,A), (G,B) and, we have

1. $(F,A) \cup [(G,B) \cup (H,C)] = [(F,A) \cup (G,B)] \cup (H,C)$,
2. $(F,A) \cap [(G,B) \cap (H,C)] = [(F,A) \cap (G,B)] \cap (H,C)$,
3. $(F,A) \cup [(G,B) \cap (H,C)] = [(F,A) \cup (G,B)] \cap [(F,A) \cup (H,C)]$,
4. $(F,A) \cap [(G,B) \cup (H,C)] = [(F,A) \cap (G,B)] \cup [(F,A) \cup (H,C)]$.

The proofs are simple and thus omitted.

Definition 2.10: Let (F,A) and (G,B) be two neutrosophic soft sets over a common universe U. Their 'AND' operation, denoted as $(F,A)\wedge (G,B)$, and is defined by $(F,A)\wedge (G,B)=(H, A\times B)$, where the truth membership, indeterminacy membership and falsity membership of $(H, A\times B)$ are as follows:

$T_{H(a,b)}(m) = \min(T_{F(a)}(m), T_{G(b)}(m))$,

$$I_{H(a,b)}(m) = \frac{I_{F(a)}(m) + I_{G(b)}(m)}{2},$$

$F_{H(a,b)}(m) = \max(F_{F(a)}(m), F_{G(b)}(m))$

for all $(a,b) \in A \times B$.

Definition 2.11: The 'OR' operation (F,A) and (G,B) denoted as $(F,A) \vee (G,B)$, and is defined by $(F,A) \vee (G,B) = (H, A \times B)$, where the truth membership, indeterminacy membership and falsity membership of $(H, A \times B)$ are as follows:

$T_{H(a,b)}(m) = \max(T_{F(a)}(m), T_{G(b)}(m))$,

$$I_{H(a,b)}(m) = \frac{I_{F(a)}(m) + I_{G(b)}(m)}{2},$$

$F_{H(a,b)}(m) = \min(F_{F(a)}(m), F_{G(b)}(m))$

for all $(a,b) \in A \times B$.

Proposition 2.12: For two neutrosophic soft sets, the following are true.

1. $[(F,A) \vee (G,B)]^c = (F,A)^c \wedge (G,B)^c$,
2. $[(F,A) \wedge (G,B)]^c = (F,A)^c \vee (G,B)^c$.

3. DISTRIBUTIVE LAWS FOR NEUTROSOPHIC SOFT SETS

In this section we present distributive laws on the collection of neutrosophic soft set. It is interesting to see that the equality does not hold in some assertions and counter example is given to show it.

Let U be an initial universe and E be the set of parameters. We denote the collection as follows:

$NSS(U)^E$: The collection of all neutrosophic soft sets over U.
$NSS(U)_A$: The collection of all those neutrosophic soft sets over U with a fixed parameter set A.
Theorem 3.1: Let (H,A) and (G,B) be two neutrosophic soft sets over the common universe U. Then

1. $(H,A) \cup_R (H,A) = (H,A)$ and $(H,A) \cap_R (H,A) = (H,A)$,
2. $(H,A) \cap_R \Phi_A = \Phi_A$,
3. $(H,A) \cup_R \Phi_A = (H,A)$,
4. $(H,A) \cup_R \bigcup_A = \bigcup_A$,
5. $(H,A) \cap_R \bigcup_A = (H,A)$,
6. $((H,A) \cup_R (G,B))^C = (H,A)^C \cap_R (G,B)^C$.
7. $((H,A) \cap_R (G,B))^C = (H,A)^C \cup_R (G,B)^C$

Table 1. Distributive Laws for neutrosophic soft sets

	$\cap_E$	$\cup_E$	$\cap_R$	$\cup_R$
$\cap_E$	1	0	0	1
$\cup_E$	0	1	1	0
$\cap_R$	1	1	1	1
$\cup_R$	1	1	1	1

Proof: Straightforward.

Remark 3.2: Let $\alpha, \beta \in \{\cup_R, \cap_R, \cup_E, \cap_E\}$, if $(H,A)\ \alpha\ ((G,B)\ \beta\ (K,C)) = ((H,A)\ \alpha\ (G,B))\ \beta\ ((H,A)\ \alpha\ (K,C))$ holds, then we have 1, otherwise 0 in Table 1.

Proofs in the cases where equality holds can be followed by definition of respective operations. For which the equality does not hold, see the following example.

Example 3.3: Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word. Consider $U = \{h_1, h_2, h_3, h_4, h_5\}$ and E={beautiful, wooden, costly, green surroundings, good repair, cheap, expensive} .

Suppose that A={beautiful, wooden, costly, green surroundings}, B={costly, good repair, green surroundings}, and C={costly, good repair, beautiful}. Let (F,A), (G,B), and (H,C) be the neutrosophic soft sets over U, which are defined in Boxes 1, 2, and 3.

Let $(F,A) \cup_E ((G,B) \cup_R (H,C)) = (I, A \cup (B \cap C))$ and $((F,A) \cup_E (G,B)) \cup_R ((F,A) \cup_E (H,C)) = (J, (A \cup B) \cap (A \cup C))$. Then we get Boxes 4 and 5

Thus,

$$(F,A) \cup_E ((G,B) \cup_R (H,C)) \neq ((F,A) \cup_E (G,B)) \cup_R ((F,A) \cup_E (H,C)).$$

Similarly we can show that,

$$(F,A) \cap_E ((G,B) \cap_R (H,C)) \neq ((F,A) \cap_E (G,B)) \cap_R ((F,A) \cap_E (H,C)).$$

and

$$(F,A) \cup_E ((G,B) \cap_E (H,C)) \neq ((F,A) \cup_E (G,B)) \cap_E ((F,A) \cup_E (H,C)),$$

$$(F,A) \cap_E ((G,B) \cup_E (H,C)) \neq ((F,A) \cap_E (G,B)) \cup_E ((F,A) \cap_E (H,C)).$$

We now initiate the study of algebraic structures associated with single and double binary operations, for the set of all neutrosophic soft sets over the universe U, and the set of all neutrosophic soft sets with a fixed set of parameters. Recall that, let U be an initial universe and E be the set of parameters. Then we have: $NSS(U)^E$: The collection of all neutrosophic soft sets over U. $NSS(U)_A$ The collection of all those neutrosophic soft sets over U with a fixed parameter set A.

Commutative Monoids

From Theorem 4.1, it is clear that $(NSS(U)^E, \alpha)$ are idempotent, commutative, semigroups for $\alpha \in \{\cup_R, \cap_R, \cup_E, \cap_E,\}$.

1. $(NSS(U)^E, \cup_R)$ is a monoid with Φ_E as an identity element, $(NSS(U)_A, \cup_R)$ is a subsemigroup of $(NSS(U)^E, \cup_R)$.
2. $(NSS(U)^E, \cap_R)$ is a monoid with Φ_E as an identity element, $(NSS(U)_A, \cap_R)$ is a subsemigroup of $(NSS(U)^E, \cap_R)$.
3. $(NSS(U)^E, \cup_E)$ is a monoid with Φ_Φ as an identity element, $(NSS(U)_A, \cup_E)$ is a subsemigroup of $(NSS(U)^E, \cup_E)$.
4. $(NSS(U)^E, \cap_E)$ is a monoid with Φ_Φ as an identity element, $(NSS(U)_A, \cap_E)$ is a subsemigroup of $(NSS(U)^E, \cap_E)$

Semirings

1. $(NSS(U)^E, \cup_R, \cap_R)$ is a commutative, idempotent semiring with $\bigcup_E$ as an identity element.
2. $(NSS(U)^E, \cup_R, \cup_E)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.
3. $(NSS(U)^E, \cup_R, \cap_E)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.
4. $(NSS(U)^E, \cap_R, \cup_R)$ is a commutative, idempotent semiring with Φ_E as an identity element.
5. $(NSS(U)^E, \cap_R, \cup_E)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.
6. $(NSS(U)^E, \cap_R, \cap_E)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.
7. $(NSS(U)^E, \cup_E, \cap_R)$ is a commutative, idempotent semiring with Φ_E as an identity element.
8. $(NSS(U)^E, \cap_E, \cup_R)$ is a commutative, idempotent semiring with Φ_E as an identity element.

Lattices

Remark 3.4: Let $\alpha,\beta \in \{\cup_R, \cap_R, \cup_E, \cap_E,\}$. If the absorption law $(F,A)\ \alpha\ ((F,A)\ \beta\ (G,B)) = (F,A)$ holds we write 1 otherwise 0 in Box 6.

1. $(NSS(U)^E, \Phi_\Phi, \bigcup_E, \cup_R, \cap_E)$ and$(NSS(U)^E, \Phi_\Phi, \bigcup_E, \cap_R, \cup_E)$ are lattices with $(NSS(U)_A, \Phi_A, \bigcup_A, \cup_R, \cap_E)$ and $(NSS(U)_A, \Phi_A, \bigcup_A, \cap_R, \cup_E)$ as their sublattices respectively.
2. $(NSS(U)^E, \Phi_\Phi, \bigcup_E, \cup_E, \cap_R)$ and $(NSS(U)^E, \Phi_\Phi, \bigcup_E, \cap_R, \cup_E)$ are lattices with $(NSS(U)_A, \Phi_A, \bigcup_A, \cup_E, \cap_R)$ and $(NSS(U)_A, \Phi_A, \bigcup_A, \cap_R, \cup_E)$ as their sublattices respectively.

The above mentioned lattices and sublattices are bounded distributive lattices.

Proposition 3.5: For the lattice of neutrosophic soft set $(NSS(U)^E, \Phi_\Phi, \bigcup_E, \cup_R, \cap_E)$ for any (H,A) and $(G,B) \in NSS(U)^E$, then

1. $(H,\ A) \widetilde{\subset} (G,\ B)$ if and only if $(H,A) \cup_R (G,B) = (H,A)$
2. $(H,\ A) \widetilde{\subset} (G,\ B)$ if and only if $(H,A) \cap_E (G,B) = (G,B)$

Proof: Straightforward.

Proposition 3.6: For the lattice of neutrosophic soft set ($NSS(U)^E$, Φ_Φ, $\bigcup_E$, $\cup_E$, $\cap_R$) for any (H,A) and $(G,B) \in NSS(U)^E$, then

1. $(H,A) \subset (G,B)$ if and only if $(H,A) \cup_E (G,B) = (G,B)$
2. $(H,A) \subset (G,B)$ if and only if $(H,A) \cap_R (G,B) = (H,A)$

Proof: Straightforward.

4. MAPPINGS ON NEUTROSOPHIC SOFT CLASSES

In this section, we introduce the notion of mapping on neutrosophic soft classes. Neutrosophic soft classes are collections of neutrosophic soft sets. We also define and study the properties of neutrosophic soft images and neutrosophic soft inverse images of neutrosophic soft sets, and support them with example and theorems.

Definition 4.1: Let X be a universe and E be a set of parameters. Then the collection of all neutrosophic soft sets over X with parameters from E is called a neutrosophic soft class and is denoted as $(\widetilde{X,E})$.

Definition 4.2: Let $(\widetilde{X,E})$ and $(\widetilde{Y,E'})$ be neutrosophic soft classes. Let r: $X \rightarrow Y$ and $s: E \rightarrow E'$ be mappings. Then a mapping $f:(\widetilde{X,E}) \rightarrow (\widetilde{Y,E'})$ is defined as follows:

For a neutrosophic soft set (F,A) in $(\widetilde{X,E})$, $f(F,A)$ is a neutrosophic soft set in $(\widetilde{Y,E'})$ obtained as follows:

$$f(F,A)(\beta)(y) = \begin{cases} \underset{x \in r^{-1}(y)}{\vee} \left(\underset{a}{\vee} F(\alpha) \right), \\ \textit{if}\; r^{-1}(y) \neq \varnothing \;\; \textit{and} \qquad s^{-1}(\beta) \cap A \neq \varnothing, \\ (0,0,0) \quad \textit{otherwise.} \end{cases}$$

for $\beta \in s(E) \subseteq E'$, $y \in Y$ and $\forall \alpha \in s^{-1}(\beta) \cap A$. $f(F,A)$ is called a neutrosophic soft image of the neutrosophic soft set (F,A).

Definition 4.3: Let $(\widetilde{X,E})$ and $(\widetilde{Y,E'})$ be neutrosophic soft classes. Let r: $X \rightarrow Y$ and $s: E \rightarrow E'$ be mappings. Then a mapping $f^{-1}:(\widetilde{X,E}) \rightarrow (\widetilde{Y,E'})$ is defined as follows:

For a neutrosophic soft set (G,B) in $(\widetilde{Y,E'})$, $f^{-1}(G,B)$ is a neutrosophic soft set in $(\widetilde{X,E})$ obtained as follows:

$$f^{-1}(G,B)(\alpha)(x) = \begin{cases} G(s(\alpha))(r(x),) & \textit{if}\; s(\alpha) \in B, \\ (0,0,0) & \textit{otherwise.} \end{cases}$$

for $\alpha \in s^{-1}(B) \subseteq E$ and $x \in X$. $f^{-1}(G,B$ is called a neutrosophic soft inverse image of the neutrosophic soft set (G,B).

Example 4.4: Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$ and let $E = \{e_1, e_2, e_3\}$ and $E' = \{e_1', e_2', e_3'\}$. Suppose that $(\widetilde{X,E})$ and $(\widetilde{Y,E'})$ are neutrosophic soft classes. Define $r: X \to Y$ and $s: E \to E'$ as follows:

$r(x_1)=y_1$, $r(x_2)=y_2$, $r(x_3)=y_3$, $s(e_1)=e_1'$, $s(e_2)=e_3'$, $s(e_3)=e_2'$.

Let (F,A) and (G,B) be two neutrosophic soft sets over X and Y respectively such that

$$(F,A) = \{(e_1, \{\langle x_1, 0.4, 0.2, 0.3\rangle, \langle x_2, 0.7, 0.3, 0.4\rangle, \langle x_3, 0.5, 0.2, 0.2\rangle\}), (e_2, \{\langle x_1, 0.2, 0.2, 0.7\rangle, \langle x_2, 0.3, 0.1, 0.8\rangle, \langle x_3, 0.2, 0.3, 0.6\rangle\}), (e_3, \{\langle x_1, 0.8, 0.2, 0.1\rangle, \langle x_2, 0.9, 0.1, 0.1\rangle, \langle x_3, 0.1, 0.4, 0.5\rangle\})\},$$

$$(G,B) = \{(e_1', \{\langle y_1, 0.2, 0.4, 0.5\rangle, \langle y_2, 0.1, 0.2, 0.6\rangle, \langle y_3, 0.2, 0.5, 0.3\rangle\}), (e_2', \{\langle y_1, 0.8, 0.1, 0.1\rangle \langle y_2, 0.5, 0.5, 0.5\rangle, \langle y_3, 0.3, 0.4, 0.4\rangle\}), (e_3', \{\langle y_1, 0.7, 0.3, 0.3\rangle, \langle y_2, 0.9, 0.2, 0.1\rangle, \langle y_3, 0.8, 0.2, 0.1\rangle\})\}.$$

Then we define a mapping $f : (\widetilde{X,E}) \to (\widetilde{Y,E'})$ as follows:

For a neutrosophic soft set (F,A) in $(\widetilde{X,E})$, $f(F,A)$ is a neutrosophic soft set in $(\widetilde{Y,E'})$ and is obtained as follows:

$$f(F,A)(e_1')(y_1) = \left(\underset{x \in r^{-1}(y_1)}{\vee}\left(\underset{a}{\vee} F(\alpha)\right)\right) = \left(\underset{x \in \{x_1\}}{\vee}\left(\underset{\alpha \in \{e_1\}}{\vee} F(\alpha)\right)\right)$$
$$= \left(\underset{x \in \{x_1\}}{\vee}\left(\{\langle x_1, 0.4, 0.2, 0.3\rangle, \langle x_2, 0.7, 0.3, 0.4\rangle, \langle x_3, 0.5, 0.2, 0.2\rangle\}\right)\right) = (0.4, 0.2, 0.3).$$

$$f(F,A)(e_1')(y_2) = \left(\underset{x \in r^{-1}(y_2)}{\vee}\left(\underset{a}{\vee} F(\alpha)\right)\right) = (0,0,0) \text{ as } r^{-1}(y_2) = \varnothing.$$

$$f(F,A)(e_1')(y_3) = \left(\underset{x \in r^{-1}(y_3)}{\vee}\left(\underset{a}{\vee} F(\alpha)\right)\right) = \left(\underset{x \in \{x_2, x_3\}}{\vee}\left(\underset{\alpha \in \{e_1\}}{\vee} F(\alpha)\right)\right)$$
$$= \left(\underset{x \in \{x_2, x_3\}}{\vee}\left(\{\langle x_1, 0.4, 0.2, 0.3\rangle, \langle x_2, 0.7, 0.3, 0.4\rangle, \langle x_3, 0.5, 0.2, 0.2\rangle\}\right)\right).$$
$$= \left(\max(0.7, 0.5), \frac{0.3+0.2}{2}, \min(0.4, 0.2)\right) = (0.7, 0.25, 0.2)$$

By similar calculations, consequently, we get

$$\left(f\left(F,A\right),B\right)=\left\{\left(e_1',\left\{\left\langle y_1,0.4,0.2,0.3\right\rangle,\ \left\langle y_2,0,0,0\right\rangle,\ \left\langle y_3,0.7,0.25,0.2\right\rangle\right\}\right),\ \left(e_2',\left\{\left\langle y_1,0.8,0.2,0.1\right\rangle,\right.\right.\right.$$
$$\left.\left.\left.\left\langle y_2,0,0,0\right\rangle,\ \left\langle y_3,0.9,0.25,0.1\right\rangle\right\}\right),\left(e_3',\left\{\left\langle y_1,0.2,0.2,0.7\right\rangle,\left\langle y_2,0,0,0\right\rangle,\left\langle y_3,0.3,0.2,0.6\right\rangle\right\}\right)\right\}.$$

Next for the neutrosophic soft inverse images, the mapping $f^{-1}:(\widetilde{X,E})\to(\widetilde{Y,E'})$ is defined as follows:

For a neutrosophic soft set (G,B) in $(\widetilde{Y,E'})$, $(f^{-1}(G,B),A)$ is a neutrosophic soft set in $(\widetilde{X,E})$ obtained as follows:

$$f^{-1}\left(G,B\right)\left(e_1\right)\left(x_1\right)=\left(G\left(s\left(e_1\right)\right)\left(r\left(x_1\right)\right)\right)=\left(G\left(e_1'\right)\left(y_1\right)\right)=\left(0.2,0.4,0.5\right),$$
$$f^{-1}\left(G,B\right)\left(e_1\right)\left(x_2\right)=\left(G\left(s\left(e_1\right)\right)\left(r\left(x_2\right)\right)\right)=\left(G\left(e_1'\right)\left(y_3\right)\right)=\left(0.2,0.5,0.3\right),$$
$$f^{-1}\left(G,B\right)\left(e_1\right)\left(x_3\right)=\left(G\left(s\left(e_1\right)\right)\left(r\left(x_3\right)\right)\right)=\left(G\left(e_1'\right)\left(y_3\right)\right)=\left(0.2,0.5,0.3\right).$$

By similar calculations, consequently, we get

$$\left(f^{-1}\left(G,B\right),A\right)=\left\{\left(e_1,\left\{\left\langle x_1,0.2,0.4,0.5\right\rangle,\ \left\langle x_2,0.2,0.5,0.3\right\rangle,\ \left\langle x_3,0.2,0.5,0.3\right\rangle\right\}\right),\ \left(e_2,\left\{\left\langle x_1,0.7,0.3,0.3\right\rangle,\right.\right.\right.$$
$$\left.\left.\left.\left\langle x_2,0.8,0.2,0.1\right\rangle,\ \left\langle x_3,0.8,0.2,0.1\right\rangle\right\}\right),\ \left(e_3,\left\{\left\langle x_1,0.8,0.1,0.1\right\rangle,\ \left\langle x_2,0.3,0.4,0.4\right\rangle,\ \left\langle x_3,0.3,0.4,0.4\right\rangle\right\}\right)\right\}.$$

Definition 4.5: Let $f:(\widetilde{X,E})\to(\widetilde{Y,E'})$ be a mapping and (F,A) and (G,B) neutrosophic soft sets in $(\widetilde{X,E})$. Then for $\beta\in E'$, $y\in Y$, the neutrosophic soft union and intersection of neutrosophic soft images (F,A) and (G,B) are defined as follows:

$$\left(f\left(F,A\right)\tilde{\vee}f\left(G,B\right)\right)\left(\beta\right)\left(y\right)=f\left(F,A\right)\left(\beta\right)\left(y\right)\vee f\left(G,B\right)\left(\beta\right)\left(y\right).$$
$$\left(f\left(F,\mathrm{A}\right)\tilde{\wedge}f\left(G,B\right)\right)(\beta)(y)=f\left(F,A\right)\left(\beta\right)\left(y\right)\wedge f\left(G,B\right)\left(\beta\right)\left(y\right).$$

Definition 4.6: Let $f:(\widetilde{X,E})\to(\widetilde{Y,E'})$ be a mapping and (F,A) and (G,B) neutrosophic soft sets in $\left(X,E'\right)$. Then for $\alpha\in E$, $x\in X$, the neutrosophic soft union and intersection of neutrosophic soft inverse images (F,A) and (G,B) are defined as follows:

$$\left(f^{-1}\left(F,A\right)\tilde{\vee}f^{-1}\left(G,B\right)\right)\left(\alpha\right)\left(x\right)=f^{-1}\left(F,A\right)\left(\alpha\right)\left(x\right)\vee f^{-1}\left(G,B\right)\left(\alpha\right)\left(x\right).$$
$$\left(f^{-1}\left(F,A\right)\tilde{\wedge}f^{-1}\left(G,B\right)\right)\left(\alpha\right)\left(x\right)=f^{-1}\left(F,A\right)\left(\alpha\right)\left(x\right)\wedge f^{-1}\left(G,B\right)\left(\alpha\right)\left(x\right).$$

Theorem 4.7: Let $f:(\widetilde{X,E})\rightarrow(\widetilde{Y,E'})$ be a mapping. Then for neutrosophic soft sets (*F,A*) and (*G,B*) in the neutrosophic soft class $(\widetilde{X,E})$,

a. $f(\varnothing)=\varnothing$.
b. *f*(*X*) ⊆ *Y*.
c. $f\left((F,A)\tilde{\vee}(G,B)\right)=f(F,A)\tilde{\vee} f(G,B)$.
d. $f\left((F,A)\tilde{\wedge}(G,B)\right)\subseteq f(F,A)\tilde{\wedge} f(G,B)$.
e. If (*F,A*) ⊆ (*G,B*), then *f*(*F,A*) ⊆ *f*(*G,B*).

Proof: For (a), (b) and (e) the proof is trivial, so we just give the proof of (c) and (d).

c. For $\beta\in E'$ and *y*∈*Y*, we want to prove that

$$f\left((F,A)\tilde{\vee}(G,B)\right)(\beta)(y)=f(F,A)(\beta)(y)\tilde{\vee} f(G,B)(\beta)(y)$$

For left hand side, consider $f\left((F,A)\tilde{\vee}(G,B)\right)(\beta)(y)=f(H,A\cup B)(\beta)(y)$. Then

$$f(H,A\cup B)(\beta)(y)=\begin{cases}\underset{x\in r^{-1}(y)}{\vee}\left(\underset{a}{\vee}H(\alpha)\right), \\ \text{if } r^{-1}(y)\neq\varnothing \text{ and } s^{-1}(\beta)\cap(A\cup B)\neq\varnothing, \\ (0,0,0) \quad \text{otherwise.}\end{cases} \tag{1}$$

where *H*(α) = ∪(*F*(α),*G*(α)).

Considering only the non-trivial case, then Equation 1 becomes:

$$f(H,A\cup B)(\beta)(y)=\underset{x\in r^{-1}(y)}{\vee}\left(\vee\cup\left(F(\alpha),G(\alpha)\right)\right) \tag{2}$$

For right hand side and by using Definition 3.4, we have

$$\begin{aligned}\left(f(F,A)\tilde{\vee} f(G,B)\right)(\beta)(y)&=f(F,A)(\beta)(y)\vee f(G,B)(\beta)(y)\\ &=\left(\underset{x\in r^{-1}(y)}{\vee}\left(\underset{\alpha\in s^{-1}(\beta)\cap A}{\vee}F(\alpha)\right)\right)\vee\left(\underset{x\in r^{-1}(y)}{\vee}\left(\underset{\alpha\in s^{-1}(\beta)\cap B}{\vee}G(\alpha)\right)\right)\\ &=\left(\underset{x\in r^{-1}(y)}{\vee}\ \underset{\alpha\in s^{-1}(\beta)\cap(A\cup B)}{\vee}\left(F(\alpha)\vee G(\alpha)\right)\right)=\underset{x\in r^{-1}(y)}{\vee}\left(\vee\cup\left(F(\alpha),G(\alpha)\right)\right)\end{aligned} \tag{3}$$

From Equations 2 and 3, we get (c).

d. For $\beta\in E'$ and *y*∈*Y*, and using Definition 3.4, we have

$$f\left(\left(F,A\right)\tilde{\wedge}\left(G,B\right)\right)\left(\beta\right)\left(y\right)=f\left(H,A\cup B\right)\left(\beta\right)\left(y\right)=\underset{x\in r^{-1}\left(y\right)}{\vee}\left(\underset{\alpha\in s^{-1}\left(\beta\right)\cap\left(A\cup B\right)}{\vee}H\left(\alpha\right)\right)$$
$$=\underset{x\in r^{-1}\left(y\right)}{\vee}\left(\underset{\alpha\in s^{-1}\left(\beta\right)\cap\left(A\cup B\right)}{\vee}F\left(\alpha\right)\wedge G\left(\alpha\right)\right)=\underset{x\in r^{-1}\left(y\right)}{\vee}\left(\underset{\alpha\in s^{-1}\left(\beta\right)\cap\left(A\cup B\right)}{\vee}F\left(\alpha\right)\wedge G\left(\alpha\right)\right)$$
$$\subseteq\left(\underset{x\in r^{-1}\left(y\right)}{\vee}\left(\underset{\alpha\in s^{-1}\left(\beta\right)\cap A}{\vee}F\left(\alpha\right)\right)\right)\wedge\left(\underset{x\in r^{-1}\left(y\right)}{\vee}\left(\underset{\alpha\in s^{-1}\left(\beta\right)\cap B}{\vee}G\left(\alpha\right)\right)\right)$$
$$=f\left(\left(F,A\right)\right)\left(\beta\right)\left(y\right)\wedge\left(\left(G,B\right)\right)\left(\beta\right)\left(y\right)=\left(f\left(F,A\right)\tilde{\wedge}f\left(G,B\right)\right)\left(\beta\right)\left(y\right).$$

This gives (d).

Theorem 4.7: Let $f:(\widetilde{X,E})\rightarrow(\widetilde{Y,E'})$ be mapping. Then for neutrosophic soft sets (*F,A*), (*G,B*) in the neutrosophic soft class $\left(X,E'\right)$, we have:

1. $f^{-1}\left(\varnothing\right)=\varnothing$.
2. $f^{-1}(Y)=X$.
3. $f^{-1}\left(\left(F,A\right)\tilde{\vee}\left(G,B\right)\right)=f^{-1}\left(F,A\right)\tilde{\vee}f^{-1}\left(G,B\right)$.
4. $f^{-1}\left(\left(F,A\right)\tilde{\wedge}\left(G,B\right)\right)=f^{-1}\left(F,A\right)\tilde{\wedge}f^{-1}\left(G,B\right)$.
5. If $(F,A)\subseteq(G,B)$, then $f^{-1}(F,A)\subseteq f^{-1}(G,B)$.

5. INTUITIONISTIC NEUTROSOPHIC SOFT SET

In this section, we will initiate the study on hybrid structure involving both intuitionistic neutrosophic set and soft set theory.

Definition 5.1: Let U be an initial universe set and A ⊂E be a set of parameters. Let N(U) denotes the set of all intuitionistic neutrosophic sets of U. The collection (F,A) is termed to be the soft intuitionistic neutrosophic set over U, where F is a mapping given by F: A → N(U).

Remark 5.2: We will denote the intuitionistic neutrosophic soft set defined over an universe by INSS.

Let us consider the following example.

Example 5.3: Let U be the set of blouses under consideration and E is the set of parameters (or qualities). Each parameter is a intuitionistic neutrosophic word or sentence involving intuitionistic neutrosophic words. Consider

E = {Bright, Cheap, Costly, very costly, Colorful, Cotton, Polystyrene, long sleeve, expensive}.

In this case, to define a intuitionistic neutrosophic soft set means to point out Bright blouses, Cheap blouses, Blouses in Cotton and so on. Suppose that, there are five blouses in the universe U given by, U = $\{b_1,b_2,b_3,b_4,b_5\}$ and the set of parameters A = $\{a_1,a_2,a_3,a_4\}$, where each a_i is a specific criterion for blouses:

a_1 stands for 'Bright',
a_2 stands for 'Cheap',
a_3 stands for 'costly',
a_4 stands for 'Colorful',

Suppose that,

F(Bright) = $\{\langle b_1, 0.5, 0.6, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2\rangle, \langle b_3, 0.6, 0.2, 0.3\rangle, \langle b_4, 0.7, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3\rangle\}$.

F(Cheap) = $\{\langle b_1, 0.6, 0.3, 0.5\rangle, \langle b_2, 0.7, 0.4, 0.3\rangle, \langle b_3, 0.8, 0.1, 0.2\rangle, \langle b_4, 0.7, 0.1, 0.3\rangle, \langle b_5, 0.8, 0.3, 0.4\}$.

F(Costly) = $\{\langle b_1, 0.7, 0.4, 0.3\rangle, \langle b_2, 0.6, 0.1, 0.2\rangle, \langle b_3, 0.7, 0.2, 0.5\rangle, \langle b_4, 0.5, 0.2, 0.6\rangle, \langle b_5, 0.7, 0.3, 0.2\rangle\}$.

F(Colorful) = $\{\langle b_1, 0.8, 0.1, 0.4\rangle, \langle b_2, 0.4, 0.2, 0.6\rangle, \langle b_3, 0.3, 0.6, 0.4\rangle, \langle b_4, 0.4, 0.8, 0.5\rangle, \langle b_5, 0.3, 0.5, 0.7\rangle\}$.

The intuitionistic neutrosophic soft set (INSS) (F, E) is a parameterized family $\{F(e_i), i = 1,\cdots,10\}$ of all intuitionistic neutrosophic sets of U and describes a collection of approximation of an object. The mapping F here is 'blouses (.)', where dot(.) is to be filled up by a parameter $e_i \in E$. Therefore, $F(e_1)$ means 'blouses (Bright)' whose functional-value is the intuitionistic neutrosophic set

$\{\langle b_1, 0.5, 0.6, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2\rangle, \langle b_3, 0.6, 0.2, 0.3\rangle, \langle b_4, 0.7, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3\rangle\}$.

Thus we can view the intuitionistic neutrosophic soft set (INSS) (F, A) as a collection of approximation as below:

(F, A) = {Bright blouses = $\{\langle b_1, 0.5, 0.6, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2\rangle, \langle b_3, 0.6, 0.2, 0.3\rangle, \langle b_4, 0.7, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3\rangle\}$, Cheap blouses = $\{\langle b_1, 0.6, 0.3, 0.5\rangle, \langle b_2, 0.7, 0.4, 0.3\rangle, \langle b_3, 0.8, 0.1, 0.2\rangle, \langle b_4, 0.7, 0.1, 0.3\rangle, \langle b_5, 0.8, 0.3, 0.4\rangle\}$, costly blouses = $\{\langle b_1, 0.7, 0.4, 0.3\rangle, \langle b_2, 0.6, 0.1, 0.2\rangle, \langle b_3, 0.7, 0.2, 0.5\rangle, \langle b_4, 0.5, 0.2, 0.6\rangle, \langle b_5, 0.7, 0.3, 0.2\rangle\}$, Colorful blouses = $\{\langle b_1, 0.8, 0.1, 0.4\rangle, \langle b_2, 0.4, 0.2, 0.6\rangle, \langle b_3, 0.3, 0.6, 0.4\rangle, \langle b_4, 0.4, 0.8, 0.5\rangle, \langle b_5, 0.3, 0.5, 0.7\rangle\}$},

where each approximation has two parts: (i) a predicate p, and (ii) an approximate value-set v (or simply to be called value-set v).

For example, for the approximation

'Bright blouses = $\{\langle b_1, 0.5, 0.6, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2\rangle, \langle b_3, 0.6, 0.2, 0.3\rangle, \langle b_4, 0.7, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3\rangle\}$'.

we have (i) the predicate name 'Bright blouses', and (ii) the approximate value-set is

$\{\langle b_1, 0.5, 0.6, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2\rangle, \langle b_3, 0.6, 0.2, 0.3\rangle, \langle b_4, 0.7, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3\rangle\}$.

Thus, an intuitionistic neutrosophic soft set (F, E) can be viewed as a collection of approximation like $(F, E) = \{p_1 = v_1, p_2 = v_2, \cdots, p_{10} = v_{10}\}$. In order to store an intuitionistic neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the intuitionistic neutrosophic soft set in the above example). In this table, the entries are c_{ij} corresponding to the blouse b_i and the parameter e_j, where c_{ij} = (true-membership value of b_i, indeterminacy-membership value of b_i, falsity membership value of b_i) in $F(e_j)$. The Table 2 represent the intuitionistic neutrosophic soft set (F, A) described above.

Remark 5.4: An intuitionistic neutrosophic soft set is not an intuituionistic neutrosophic set but a parametrized family of an intuitionistic neutrosophic subsets.

Definition 5.5: For two intuitionistic neutrosophic soft sets (F, A) and (G, B) over the common universe U. We say that (F, A) is an intuitionistic neutrosophic soft subset of (G, B) if and only if

1. $A \subset B$.
2. F(a) is an intuitionistic neutrosophic subset of G(a).

Or $T_{F(a)}(x) \leq T_{G(a)}(x), I_{F(a)}(x) \leq I_{G(a)}(x), F_{F(a)}(x) \geq F_{G(a)}(x), \forall a \in A, x \in U$.

We denote this relationship by $(F, A) \subseteq (G, B)$.

(F, A) is said to be intuitionistic neutrosophic soft super set of (G, B) if (G, B) is an intuitionistic neutrosophic soft subset of (F, A). We denote it by $(F, A) \supseteq (G, B)$.

Table 2. Tabular form of the INSS (F, A)

U	Bright	Cheap	Costly	Colorful
b_1	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7, 0.4, 0.3)	(0.8, 0.1, 0.4)
b_2	(0.4, 0.7, 0.2)	(0.7, 0.4, 0.3)	(0.6, 0.1, 0.2)	(0.4, 0.2, 0.6)
b_3	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.5)	(0.3, 0.6, 0.4)
b_4	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5, 0.2, 0.6)	(0.4, 0.8, 0.5)
b_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.4)	(0.7, 0.3, 0.2)	(0.3, 0.5, 0.7)

Table 3. Tabular form of the INSS (F, A)

U	Small	Large	Colorful
O_1	(0.4, 0.3, 0.6)	(0.3, 0.1, 0.7)	(0.4, 0.1, 0.5)
O_2	(0.3, 0.1, 0.4)	(0.4, 0.2, 0.8)	(0.6, 0.3, 0.4)
O_3	(0.6, 0.2, 0.5)	(0.3, 0.1, 0.6)	(0.4, 0.3, 0.8)
O_4	(0.5, 0.1, 0.6)	(0.1, 0.5, 0.7)	(0.3, 0.3, 0.8)
O_5	(0.3, 0.2, 0.4)	(0.3, 0.1, 0.6)	(0.5, 0.2, 0.4)

Table 4. Tabular form of the INSS (G, B)

U	Small	Large	Colorful	Very Smooth
O1	(0.6, 0.4, 0.3)	(0.7, 0.2, 0.5)	(0.5, 0.7, 0.4)	(0.1, 0.8, 0.4)
O2	(0.7, 0.5, 0.2)	(0.4, 0.7, 0.3)	(0.7, 0.3, 0.2)	(0.5, 0.7, 0.3)
O3	(0.6, 0.3, 0.5)	(0.7, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.2,,0.9, 0.4)
O4	(0.8, 0.1, 0.4)	(0.3, 0.6, 0.4)	(0.4, 0.5, 0.7)	(0.4, 0.4, 0.5)
O5	(0.5, 0.4, 0.2)	(0.4, 0.1, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.8, 0.3)

Example 5.6: Let (F,A) and (G,B) be two INSSs over the same universe $U = \{o_1,o_2,o_3,o_4,o_5\}$. The INSS (F,A) describes the sizes of the objects whereas the INSS (G, B) describes its surface textures. Consider the tabular representation of the INSS (F, A) is shown in Table 3.

The tabular representation of the INSS (G, B) is given by Table 4.
Clearly, by definition 3.5 we have $(F, A) \subset (G, B)$.

Definition 5.7: Two INSSs (F, A) and (G, B) over the common universe U are said to be intuitionistic neutrosophic soft equal if (F, A) is an intuitionistic neutrosophic soft subset of (G, B) and (G, B) is an intuitionistic neutrosophic soft subset of (F, A) which can be denoted by (F, A)= (G, B).

Definition 5.8: The complement of an intuitionistic neutrosophic soft set (F, A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c=(F^c,\rceil A)$, where $F^c:\rceil A \to N(U)$ is a mapping given by $F^c(\alpha)$ = intutionistic neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}, I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

Example 5.9: As an illustration consider the example presented in the example 3.2. the complement $(F,A)^c$ describes the 'not attractiveness of the blouses'. is given below.

F(not bright) = $\{\langle b_1, 0.3, 0.6, 0.5\rangle, \langle b_2, 0.2, 0.7, 0.4\rangle, \langle b_3, 0.3, 0.2, 0.6\rangle, \langle b4, 0.2, 0.3, 0.7\rangle, \langle b_5, 0.3, 0.2, 0.8\rangle\}$.

F(not cheap) = $\{\langle b_1, 0.5, 0.3, 0.6\rangle, \langle b_2, 0.3, 0.4, 0.7\rangle, \langle b_3, 0.2, 0.1, 0.8\rangle, \langle b_4, 0.3, 0.1, 0.7\rangle, \langle b_5, 0.4, 0.3, 0.8\rangle\}$.

F(not costly) = $\{\langle b_1, 0.3, 0.4, 0.7\rangle, \langle b_2, 0.2, 0.1, 0.6\rangle, \langle b_3, 0.5, 0.2, 0.7\rangle, \langle b_4, 0.6, 0.2, 0.5\rangle, \langle b_5, 0.2, 0.3, 0.7\rangle\}$.

F(not colorful) = $\{\langle b_1, 0.4, 0.1, 0.8\rangle, \langle b_2, 0.6, 0.2, 0.4\rangle, \langle b_3, 0.4, 0.6, 0.3\rangle, \langle b_4, 0.5, 0.8, 0.4\rangle, \langle b_5, 0.7, 0.5, 0.3\rangle\}$.

Definition 5.10: An intuitionistic neutrosophic soft set (F,A) over U is said to be empty or null intuitionistic neutrosophic soft (with respect to the set of parameters) denoted by Φ_A or (Φ,A) if $T_{F(e)}(m) = 0, F_{F(e)}(m) = 0$ and $I_{F()}(m) = 0, \forall m \in U, \forall a \in A$.

Example 5.11: Let $U = \{b_1,b_2,b_3,b_4,b_5\}$, the set of five blouses be considered as the universal set and A = {Bright, Cheap, Colorful } be the set of parameters that characterizes the blouses. Consider the intuitionistic neutrosophic soft set (F, A) which describes the cost of the blouses and

F(bright) = $\{\langle b_1, 0, 0, 0\rangle, \langle b_2, 0, 0, 0\rangle, \langle b_3, 0, 0, 0\rangle, \langle b_4, 0, 0, 0\rangle, \langle b_5, 0, 0, 0\rangle\}$, F(cheap) = $\{\langle b_1, 0, 0, 0\rangle, \langle b_2, 0, 0, 0\rangle, \langle b_3, 0, 0, 0\rangle, \langle b_4, 0, 0, 0\rangle, \langle b_5, 0, 0, 0\rangle\}$,

F(colorful) = $\{\langle b_1, 0, 0, 0\rangle, \langle b_2, 0, 0, 0\rangle, \langle b_3, 0, 0, 0\rangle, \langle b_4, 0, 0, 0\rangle, \langle b_5, 0, 0, 0\rangle\}$.

Here the NINSS (F, A) is the null intuitionistic neutrosophic soft set.

Definition 5.12: Let (F,A) and (G,B) be two INSSs over the same universe U.Then the union of (F,A) and (G,B) is denoted by '(F,A)∪(G,B)' and is defined by (F,A)∪(G,B)=(K,C), where C=A∪B and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$T_{K(e)(}m) = T_{F(e)}(m)$, if $e \in A - B$,
$= T_{G(e)}(m)$, if $e \in B - A$,
$= \max(T_{F(e)}(m), T_{G(e)}(m))$, if $e \in A \cap B$.

$I_{K(e)(}m) = I_{F(e)}(m)$, if $e \in A - B$,
$= I_{G(e)}(m)$, if $e \in B - A$,
$= \min(I_{F(e)}(m), I_{G(e)}(m))$, if $e \in A \cap B$.

$F_{K(e)(}m) = F_{F(e)}(m)$, if $e \in A - B$,
$= F_{G(e)}(m)$, if $e \in B - A$,
$= \min(F_{F(e)}(m), F_{G(e)}(m))$, if $e \in A \cap B$.

Example 5.13: Let (F, A) and (G, B) be two INSSs over the common universe U. Consider the tabular representation of the INSS (F, A) is shown in Table 5.

The tabular representation of the INSS (G, B) is shown in Table 6.
Using definition 3.12 the union of two INSS (F, A) and (G, B) is (K, C) can be represented into Table 7.

Table 5. Tabular form of the INSS (F, A)

	Bright	Cheap	Colorful
b_1	(0.6, 0.3, 0.5)	(0.7, 0.3, 0.4)	(0.4, 0.2, 0.6)
b_2	(0.5, 0.1, 0.8)	(0.6, 0.1, 0.3)	(0.6, 0.4, 0.4)
b_3	(0.7, 0.4, 0.3)	(0.8, 0.3, 0.5)	(0.5, 0.7, 0.2)
b_4	(0.8, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.8, 0.2, 0.3
b_5	(0.6, 0.3, 0.2)	(0.7, 0.3, 0.5)	(0.3, 0.6, 0.5

Table 6. Tabular form of the INSS (G, B)

U	Costly	Colorful
b_1	(0.6, 0.2, 0.3)	(0.4, 0.6, 0.2)
b_2	(0.2, 0.7, 0.2)	(0.2, 0.8, 0.3)
b_3	(0.3, 0.6, 0.5)	(0.6, 0.3, 0.4)
b_4	(0.8, 0.4, 0.1)	(0.2, 0.8, 0.3)
b_5	(0.7, 0.1, 0.4)	(0.5, 0.6, 0.4)

Table 7. Tabular form of the INSS (K, C)

U	Bright	Cheap	Colorful	Costly
b_1	(0.6, 0.3, 0.5)	(0.7, 0.3, 0.4)	(0.4, 0.2, 0.2)	(0.6, 0.2, 0.3)
b_2	(0.5, 0.1, 0.8)	(0.6, 0.1, 0.3)	(0.6, 0.4, 0.3)	(0.2, 0.7, 0.2)
b_3	(0.7, 0.4, 0.3)	(0.8, 0.3, 0.5)	(0.6, 0.3, 0.2)	(0.3, 0.6, 0.5)
b_4	(0.8, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.8, 0.2, 0.3)	(0.8, 0.4, 0.1)
b_5	(0.6, 0.3, 0.2)	(0.7, 0.3, 0.5)	(0.5, 0.6, 0.4)	(0.7, 0.1, 0.4)

Definition 5.14: Let (F,A) and (G,B) be two INSSs over the same universe U such that $A \cap B \neq 0$. Then the intersection of (F,A) and (G,B) is denoted by '$(F,A) \cap (G, B)$' and is defined by $(F, A) \cap (G, B) = (K,C)$, where $C = A \cap B$ and the truth-membership, indeterminacy membership and falsity-membership of (K, C) are related to those of (F,A) and (G,B) by:

$T_{K(e)(}m) = \min (T_{F(e)}(m), T_{G(e)}(m))$,

$I_{K(e)(}m) = \min (I_{F(e)}(m), I_{G(e)}(m))$,

$F_{K(e)(}m) = \max (F_{F(e)}(m), F_{G(e)}(m))$, for all $e \in C$.

Example 5.15: Consider the above example 3.15. The intersection of (F, A) and (G, B) can be represented into Table 8.

Proposition 5.16: If (F, A) and (G, B) are two INSSs over U and on the basis of the operations defined above, then:

1. Idempotency laws:

$(F,A) \cup (F,A) = (F,A)$ and $(F,A) \cap (F,A) = (F,A)$.

2. Commutative laws:

$(F,A) \cup (G,B) = (G,B) \cup (F,A)$ and $(F,A) \cap (G,B) = (G,B) \cap (F,A)$.

3. $(F,A) \cup \Phi = (F,A)$.
4. $(F,A) \cap \Phi = \Phi$.
5. $[(F,A)^c]^c = (F,A)$.

Proof: The proof of the propositions 1 to 5 are obvious.

Proposition 5.17: If (F, A), (G, B) and (K, C) are three INSSs over U,then:

1. $(F,A) \cap [(G,B) \cap (K,C)] = [(F,A) \cap (G,B)] \cap (K,C)$.
2. $(F,A) \cup [(G,B) \cup (K,C)] = [(F,A) \cup (G,B)] \cup (K,C)$.
3. Distributive laws: $(F,A) \cup [(G,B) \cap (K,C)] = [(F,A) \cup (G,B)] \cap [(F,A) \cup (K,C)]$.
4. $(F,A) \cap [(G,B) \cup (K,C)] = [(H,A) \cap (G,B)] \cup [(F,A) \cap (K,C)]$.

Example 5.18: Let (F,A) = {⟨b_1, 0.6, 0.3, 0.1⟩, ⟨b_2, 0.4, 0.7, 0.5⟩, ⟨b_3, 0.4, 0.1, 0.8⟩}, (G,B) = {⟨b_1, 0.2, 0.2, 0.6), ⟨b_2, 0.7, 0.2, 0.4⟩, ⟨b_3, 0.1, 0.6, 0.7⟩} and (K,C) = {⟨b_1, 0.3, 0.8, 0.2⟩, ⟨b_2, 0.4, 0.1, 0.6⟩, ⟨b_3, 0.9, 0.1, 0.2⟩} be three INSSs of U. Then,

(F,A) ∪ (G,B) = {⟨b_1, 0.6, 0.2, 0.1⟩, ⟨b_2, 0.7, 0.2, 0.4⟩, ⟨b_3, 0.4, 0.1, 0.7⟩}.

(F,A) ∪ (K,C) = {⟨b_1, 0.6, 0.3, 0.1⟩, ⟨b_2, 0.4, 0.1, 0.5⟩, ⟨b_3, 0.9, 0.1, 0.2⟩}.

(G,B) ∩ (K,C)] = {⟨b_1, 0.2, 0.2, 0.6⟩, ⟨b_2, 0.4, 0.1, 0.6⟩, ⟨b_3, 0.1, 0.1, 0.7⟩}.

(F,A) ∪ [(G,B) ∩ (K,C)] = {⟨b_1, 0.6, 0.2, 0.1⟩, ⟨b_2, 0.4, 0.1, 0.5⟩, ⟨b_3, 0.4, 0.1, 0.7⟩}.

[(F,A) ∪ (G,B)] ∩ [(F,A) ∪ (K,C)] = {⟨b_1, 0.6, 0.2, 0.1⟩, ⟨b_2, 0.4, 0.1, 0.5⟩, ⟨b_3, 0.4, 0.1, 0.7⟩}.

Hence distributive (3) proposition verified. Proof, can be easily proved from definition 3.14.and 3.16.

Definition 5.19: Let (F, A) and (G, B) be two INSSs over the same universe U. then (F, A) ''AND (G, B) denoted by '(F, A) ∧ (G, B)and is defined by (F, A) ∧ (G, B) = (K, A × B), where K(α, β)=F(α)∩ B(β) and the truth-membership, indeterminacy-membership and falsity-membership of (K, A×B) are as follows:

$T_{K(\alpha,\beta)}(m) = \min(T_{F(\alpha)}(m), T_{G(\beta)}(m))$,

$I_{K(\alpha,\beta)}(m) = \min(I_{F(\alpha)}(m), I_{G(\beta)}(m))$,

$F_{K(\alpha,\beta)}(m) = \max(F_{F(\alpha)}(m), F_{G(\beta)}(m))$,

$\forall \alpha \in A, \forall \beta \in B$.

Example 5.20: Consider the same example 3.15 above. Then the tabular representation of (F,A) AND (G, B) is shown in Table 9.

Table 8. Tabular form of the INSS (K, C)

U	Colorful
b_1	(0.4, 0.2, 0.6)
b_2	(0.2, 0.4, 0.4)
b_3	(0.6, 0.3, 0.4)
b_4	(0.8, 0.2, 0.3)
b_5	(0.3, 0.6, 0.5)

Definition 5.21: If (F,A) and (G,B) be two INSSs over the common universe U then '(F,A) OR(G,B)' denoted by $(F,A) \vee (G,B)$ is defined by $(F, A) \vee (G, B) = (O, A \times B)$, where, the truth-membership, indeterminacy membership and falsity-membership of $O(\alpha, \beta)$ are given as follows:

$T_O(\alpha,\beta)(m) = \max(T_F(\alpha)(m), T_G(\beta)(m))$,

$I_O(\alpha,\beta)(m) = \min(I_F(\alpha)(m), I_G(\beta)(m))$,

$F_O(\alpha,\beta)(m) = \min(F_F(\alpha)(m), F_G(\beta)(m))$,

$\forall \alpha \in A, \forall \beta \in B$.

Example 5.22: Consider the same example 3.14 above. Then the tabular representation of (F, A) OR (G, B) is shown in Table 10.

Proposition 5.23: If (F, A) and (G, B) are two INSSs over U, then

1. $[(F,A) \wedge (G,B)]^c = (F,A)^c \vee (G,B)^c$,
2. $[(F,A) \vee (G,B)]^c = (F,A)^c \wedge (G,B)^c$.

Proof:

1. Let $(F,A)=\{\langle b, T_{F(x)}(b), I_{F(x)}(b), F_{F(x)}(b)\rangle | b \in U\}$ and $(G,B) = \{\langle b, T_{G(x)}(b), I_{G(x)}(b), F_{G(x)}(b)\rangle | b \in U\}$ be two INSSs over the common universe U. Also let $(K, A \times B) = (F,A) \wedge (G,B)$, where, $K(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A \times B$ then $K(\alpha, \beta) = \{\langle b, \min(T_{F(\alpha)}(b), T_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \max(F_{F(\alpha)}(b), F_{G(\beta)}(b)) \rangle | b \in U\}$.

Therefore,

$[(F,A) \wedge (G,B)]^c = (K, A \times B) = \{\langle b, \max(F_{F(\alpha)}(b), F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \min(T_{F(\alpha)}(b), T_{G(\beta)}(b)) \rangle | b \in U\}$.

Table 9. Tabular representation of the INSS (K, A × B)

u	**(Bright, Costly)**	**(Bright, Colorful)**	**(Cheap, Costly)**
b_1	(0.6, 0.2, 0.5)	(0.4, 0.3, 0.5)	(0.6, 0.2, 0.4)
b_2	(0.2, 0.1, 0.8)	(0.2, 0.1, 0.8)	(0.2, 0.1, 0.3)
b_3	(0.3, 0.4, 0.5)	(0.6, 0.3, 0.4)	(0.3, 0.3, 0.5)
b_4	(0.8, 0.4, 0.1)	(0.2, 0.4,0.3)	(0.6, 0.3, 0.2)
b_5	(0.6, 0.1, 0.4)	(0.5, 0.3, 0.4)	(0.7, 0.1, 0.5)
u	**(Cheap, Colorful)**	**(Colorful, Costly)**	**(Colorful, Colorful)**
b_1	(0.4, 0.3, 0.4)	(0.4, 0.2, 0.6)	(0.4, 0.2, 0.6)
b_2	(0.2, 0.1, 0.3)	(0.2, 0.4, 0.4)	(0.2, 0.4, 0.4)
b_3	(0.6, 0.3, 0.5)	(0.3, 0.6, 0.5)	(0.5, 0.3, 0.4)
b_4	(0.2, 0.3, 0.3)	(0.8, 0.2, 0.3)	(0.2, 0.2, 0.3)
b_5	(0.5, 0.3, 0.5)	(0.3, 0.1, 0.5)	(0.3, 0.6, 0.5)

Table 10. Tabular representation of the INSS (O, A × B)

u	**(Bright, Costly)**	**(Bright, Colorful)**	**(Cheap, Costly)**
b_1	(0.6, 0.2, 0.3)	(0.6, 0.3, 0.2)	(0.7, 0.2, 0.3)
b_2	(0.5, 0.1, 0.2)	(0.5, 0.1, 0.3)	(0.6, 0.1, 0.2)
b_3	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.3)	(0.8, 0.3, 0.5)
b_4	(0.8, 0.4, 0.1)	(0.8, 0.4, 0.1)	(0.8, 0.3, 0.1)
b_5	(0.7, 0.1, 0.2)	(0.6, 0.3, 0.4)	(0.7, 0.1, 0.4)
u	**(Cheap, Colorful)**	**(Colorful, Costly)**	**(Colorful, Colorful)**
b_1	(0.7, 0.3, 0.2)	(0.6, 0.2, 0.3)	(0.4, 0.2, 0.2)
b_2	(0.6, 0.1, 0.3)	(0.6, 0.4, 0.2)	(0.6, 0.4, 0.3)
b_3	(0.8, 0.3, 0.4)	(0.5, 0.6, 0.2)	(0.5, 0.7, 0.2)
b_4	(0.6, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.8, 0.2, 0.3)
b_5	(0.7, 0.3, 0.4)	(0.7, 0.1, 0.4)	(0.5, 0.6, 0.4)

Again

$(F,A)^c \vee (G,B) = \{\langle b, \max(F^c_{F(\alpha)}(b)), F^c_{G(\beta)}(b)), \min(I^c_{F(\alpha)}(b), I^c_{G(\beta)}(b)), \min(T^c_{F(\alpha)}(b), T^c_{G(\beta)}(b))\rangle | b \in U\}$.

$= \{\langle b, \min(T_{F(\alpha)}(b), T_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \max(F_{F(\alpha)}(b), F_{G(\beta)}(b))\rangle | b \in U\}^c$.

$= \{\langle b, \max(F_{F(\alpha)}(b), F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \min(T_{F(\alpha)}(b), T_{G(\beta)}(b))\rangle | b \in U\}$.

It follows that $[(F,A) \wedge (G,B)]^c = (F,A)^c \vee (G,B)^c$.

2. Let $(F, A) = \{\langle b, T_{F(x)}(b), I_{F(x)}(b), F_{F(x)}(b)\rangle | b \in U\}$ and $(G,B) = \{\langle b, T_{G(x)}(b), I_{G(x)}(b), F_{G(x)}(b)\rangle | b \in U\}$ be two INSSs over the common universe U. Also let $(O,A \times B) = (F,A) \vee (G,B)$, where, $O(\alpha,\beta) = F(\alpha) \cup G(\beta)$ for all $(\alpha,\beta) \in A \times B$. then

$O(\alpha,\beta) = \{\langle b, \max(T_{F(\alpha)}(b), T_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \min(F_{F(\alpha)}(b), F_{G(\beta)}(b))\rangle | b \in U\}$.

$[(F,A) \vee (G,B)]^c = (O,A \times B)^c = \{\langle b, \min(F_{F(\alpha)}(b), F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \max(T_{F(\alpha)}(b), T_{G(\beta)}(b))\rangle | b \in U\}$.

Again

$(H,A)^c \wedge (G,B)^c = \{\langle b, \min(F^c_{F(\alpha)}(b), F^c_{G(\beta)}(b)), \min(I^c_{F(\alpha)}(b), I^c_{G(\beta)}(b)), \max(T^c_{F(\alpha)}(b), T^c_{G(\beta)}(b))\rangle | b \in U\}$.

$= \{\langle b, \max(T_{F(\alpha)}(b), T_{G(\beta)}(b)), \min(I^c_{F(\alpha)}(b), I^c_{G(\beta)}(b)), \min(F_{F(\alpha)}(b), F_{G(\beta)}(b))\rangle | b \in U\}^c$.

$= \{\langle b, \min(F_{F(\alpha)}(b), F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b), I_{G(\beta)}(b)), \max(T_{F(\alpha)}(b), T_{G(\beta)}(b))\rangle | \; b \in U\}.$

It follows that $[(F,A) \vee (G,B)]^c = (F,A)^c \wedge (G,B)^c$.

6. AN APPLICATION OF INTUITIONISTIC NEUTROSOPHIC SOFT SET IN A DECISION MAKING PROBLEM

For a concrete example of the concept described above, we revisit the blouse purchase problem in Example 3.3. So let us consider the intuitionistic neutrosophic soft set S = (F,P) (see also Table 10 for its tabular representation), which describes the "attractiveness of the blouses" that Mrs. X is going to buy. on the basis of her m number of parameters $(e_1,e_2,\ldots,e_m)$ out of n number of blouses$(b_1,b_2,\ldots,b_n)$. We also assume that corresponding to the parameter $e_j(j = 1,2,\cdots,m)$ the performance value of the blouse b_i $(i = 1,2,\cdots,n)$ is a tuple $p_{ij} = (T_{F(ej)}(b_i), I_{F(ej)}(b_i), T_{F(ej)}(b_i))$, such that for a fixed i that values p_{ij} $(j = 1,2,\cdots,m)$ represents an intuitionistic neutrosophic soft set of all the n objects. Thus the performance values could be arranged in the form of a matrix called the 'criteria matrix'. The more are the criteria values, the more preferability of the corresponding object is. Our problem is to select the most suitable object i.e. the object which dominates each of the objects of the spectrum of the parameters e_j. Since the data are not crisp but intuitionistic neutrosophic soft the selection is not straightforward. Our aim is to find out the most suitable blouse with the choice parameters for Mrs. X. The blouse which is suitable for Mrs. X need not be suitable for Mrs. Y or Mrs. Z, as the selection is dependent on the choice parameters of each buyer. We use the technique to calculate the score for the objects.

Definition 6.1: Comparison matrix. The Comparison matrix is a matrix whose rows are labelled by the object names of the universe such as $b_1,b_2,\cdots,b_n$ and the columns are labelled by the parameters $e_1,e_2,\cdots,e_m$.

The entries are $c_{ij,}$ where $c_{ij,}$ is the number of parameters for which the value of b_i exceeds or is equal to the value $b_{j.}$ The entries are calculated by $c_{ij} = a + d - c$, where 'a' is the integer calculated as 'how many times T_{bi} (e_j) exceeds or equal to T_{bk} (e_j)', for $b_i \neq b_k$, $\forall b_k \in U$, 'd'is the integer calculated as 'how many times $I_{bi(ej)}$ exceeds or equal to $I_{bk(ej)}$', for $b_i \neq b_k$, $\forall b_k \in U$ and 'c' is the integer 'how many times $F_{bi(ej)}$ exceeds or equal to $F_{bk}(e_j)$', for $b_i \neq b_k$, $\forall b_k \in U$.

Definition 6.2: Score of an object. The score of an object b_i is S_i and is calculated as:

$$S_i = \sum_j c_{ij}$$

Now the algorithm for most appropriate selection of an object will be as follows.

Algorithm

1. Input the intuitionistic Neutrosophic Soft Set (F, A).
2. Input P, the choice parameters of Mrs. X which is a subset of A.

Table 11. Tabular form of the INSS (F,P)

U	Bright	Costly	Polystyreneing	Colorful	Cheap
b_1	(0.6, 0.3, 0.4)	(0.5, 0.2, 0.6)	(0.5, 0.3, 0.4)	(0.8, 0.2, 0.3)	(0.6, 0.3, 0.2)
b_2	(0.7, 0.2, 0.5)	(0.6, 0.3, 0.4)	(0.4, 0.2, 0.6)	(0.4, 0.8, 0.3)	(0.8, 0.1, 0.2)
b_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.5)
b_4	(0.7, 0.5, 0.2)	(0.4, 0.8, 0.3)	(0.8, 0.2, 0.4)	(0.8, 0.3, 0.4)	(0.8, 0.3, 0.4)
b_5	(0.3, 0.8, 0.4)	(0.3, 0.6, 0.1)	(0.7, 0.3, 0.2)	(0.6, 0.2, 0.4)	(0.6, 0.4, 0,2)

Box 1. Neutrosophic soft set (F,A)

U	Costly	Good Repair	Green Surrounding
h_1	⟨0.6, 0.5, 0.4⟩	⟨0.7, 0.2, 0.5⟩	⟨0.6, 0.2, 0.7⟩
h_2	⟨0.7, 0.3, 0.6⟩	⟨0.6, 0.6, 0.8⟩	⟨0.4, 0.5, 0.2⟩
h_3	⟨0.7, 0.6, 0.3⟩	⟨0.1, 0.7, 0.5⟩	⟨0.6, 0.4, 0.7⟩

Box 2. Neutrosophic soft set (G,B)

U	Beautiful	Wooden	Costly	Green Surrounding
h_1	⟨0.7, 0.4, 0.5⟩	⟨0.6, 0.7, 0.8⟩	⟨0.8, 0.1, 0.7⟩	⟨0.5, 0.3, 0.7⟩
h_2	⟨0.8, 0.5, 0.3⟩	⟨0.2, 0.4, 0.7⟩	⟨0.5, 0.8, 0.9⟩	⟨0.6, 0.2, 0.3⟩
h_3	⟨0.4, 0.6, 0.8⟩	⟨0.8, 0.4, 0.6⟩	⟨1.0, 0.3, 0.4⟩	⟨0.7, 0.1, 0.5⟩

Box 3. Neutrosophic soft set (H,C)

U	Costly	Good Repair	Green Surrounding
h_1	⟨0.4, 0.4, 0.6⟩	⟨0.5, 0.7, 0.4⟩	⟨0.6, 0.8, 0.4⟩
h_2	⟨0.3, 0.7, 0.4⟩	⟨0.3, 0.7, 0.4⟩	⟨0.5, 0.3, 0.3⟩
h_3	⟨0.2, 0.4, 0.3⟩	⟨0.4, 0.5, 0.3⟩	⟨0.8, 0.5, 0.8⟩

Box 4. Neutrosophic soft set $(I, A \cup (B \cap C))$

U	Beautiful	Wooden	Costly	Green Surrounding	Good Repair
h_1	⟨0.7, 0.4, 0.5⟩	⟨0.6, 0.7, 0.8⟩	⟨0.8, 0.1, 0.7⟩	⟨0.5, 0.3, 0.7⟩	⟨0.7, 0.7, 0.4⟩
h_2	⟨0.8, 0.5, 0.3⟩	⟨0.2, 0.4, 0.7⟩	⟨0.5, 0.8, 0.9⟩	⟨0.6, 0.2, 0.3⟩	⟨0.6, 0.7, 0.4⟩
h_3	⟨0.4, 0.6, 0.8⟩	⟨0.8, 0.4, 0.6⟩	⟨1.0, 0.3, 0.4⟩	⟨0.7, 0.1, 0.5⟩	⟨1.0, 0.7, 0.3⟩

Box 5. Neutrosophic soft set (J, (A ∪ B) ∩ (A ∪ C))

U	Beautiful	Wooden	Costly	Green Surrounding	Good Repair
h_1	⟨0.7, 0.8, 0.4⟩	⟨0.6, 0.7, 0.8⟩	⟨0.8, 0.5, 0.7⟩	⟨0.6, 0.3, 0.7⟩	⟨0.7, 0.7, 0.4⟩
h_2	⟨0.8, 0.5, 0.3⟩	⟨0.2, 0.4, 0.7⟩	⟨0.7, 0.8, 0.4⟩	⟨0.6, 0.5, 0.2⟩	⟨0.6, 0.7, 0.4⟩
h_3	⟨0.8, 0.6, 0.8⟩	⟨0.8, 0.4, 0.6⟩	⟨1.0, 0.6, 0.3⟩	⟨0.7, 0.4, 0.5⟩	⟨1.0, 0.7, 0.3⟩

Box 6. Absorption law for neutrosophic soft sets

	$\cap_E$	$\cup_E$	$\cap_R$	$\cup_R$
$\cap_E$	0	0	0	1
$\cup_E$	0	0	1	0
$\cap_R$	0	1	0	0
$\cup_R$	1	0	0	0

Table 12. Comparison matrix of the INSS (F, P)

U	Bright	Costly	Polystyreneing	Colorful	Cheap
b_1	0	-2	3	0	2
b_2	-1	1	-2	2	2
b_3	3	5	0	4	-1
b_4	6	3	3	3	4
b_5	7	2	6	-1	3

Table 13. Score table

U	Score (S_i)
b_1	3
b_2	2
b_3	11
b_4	19
b_5	17

3. Consider the INSS (F, P) and write it in tabular form.
4. Compute the comparison matrix of the INSS (F, P).
5. Compute the score S_i of b_i,∀i.
6. Find S_k = maxi S_i
7. If k has more than one value then any one of b_i may be chosen.

To illustrate the basic idea of the algorithm, now we apply it to the intuitionistic neutrosophic soft set based decision making problem.

Suppose the wishing parameters for Mrs. X where

P = {Bright, Costly, Polystyreneing, Colorful, Cheap}.

Consider the INSS (F, P) presented into Table 11.

Clearly, the maximum score is the score 19, shown in Table 11 for the blouse b_4. Hence the best decision for Mrs. X is to select b_4, followed by b_5.

The comparison-matrix of the above INSS (F, P) is represented into Table 12.

Next we compute the score for each b_i as shown in Table 13.

7. CONCLUSION

Soft set theory is a mathematical framework to handle uncertain information. Several hybrid structures have been introduced since its appearance. In this chapter, the authors presented the study on neutrosophic soft set which is a hybridized structure of soft set and neutrosophic set. Some set theoretic operations are studied in this chapter such as union, intersection, AND operation, OR operations etc. The authors also studied the algebraic structures associated with neutrosophic sets such as semigroups, semirings, lattices, bounded lattices etc. Further, mappings on soft neutrosophic classes have been presented under discussion in this chapter to capture more properties of neutrosophic soft set. Intuitionistic neutrosophic soft set is also under consideration in this chapter to which studied neutrosophic soft set in a broader sense. At the end, a decision making application is presented to show the application practice of neutrosophic sets in the real life problems.

REFERENCES

Ali, I. M., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers and Mathematics with Applications, 57*, 1547-1553.

Ali, I. M., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers & Mathematics with Applications (Oxford, England)*, *61*(9), 2647–2654. doi:10.1016/j.camwa.2011.03.011

Alkhazaleh, S. & Salleh, R. A. (2012). Fuzzy soft multiset. Abstract and Applied Analysis.

Alkhazaleh, S., Salleh, R. A., & Hassan, N. (2011). Fuzzy parameterized interval-valued fuzzy soft set. *Applied Mathematical Sciences*, *5*(67), 3335–3346.

Alkhazaleh, S., Salleh, R. A., & Hassan, N. (2011). *Neutrosophic soft set*. Advances in Decision Sciences.

Alkhazaleh, S., Salleh, R. A., & Hassan, N. (2011). Soft multisets theory. *Applied Mathematical Sciences*, *5*(72), 3561–3573.

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. doi:10.1016/S0165-0114(86)80034-3

Bhowmik, M., & Pal, M. (2009). Intuitionistic neutrosophic set. *Journal of Information and Computing Science*, *2*(4), 142–152.

Broumi, S., & Smarandache, F. (2013). More on Intuitionistic Neutrosophic Soft Sets. *Computer Science and Information Technology*, *1*(4), 257–268.

Broumi, S., & Smarandache, F. (2014). On Neutrosophic Implications. *Neutrosophic Sets and Systems*, *2*, 9–17.

Broumi, S., & Smarandache, S. (2014). Intuitionistic Neutrosophic Soft Set. *Journal of Information and Computer Science*, *8*, 130–140.

Chen, D., Tsang, E. C. C., Yeung, D. S., & Wang, W. (2005). The parameterization reduction of soft sets and its application. *Computers & Mathematics with Applications (Oxford, England)*, *49*(5-6), 757–763. doi:10.1016/j.camwa.2004.10.036

Deli, I., & Broumi, S. (2015). Neutrosophic soft relations and some properties. *Annals of Fuzzy Mathematics and Informatics*, *9*(1), 169–182.

Deli, I., & Broumi, S. (n.d.). Neutrosophic Soft Matrices and NSM-decision Making. *Journal of Intelligent and Fuzzy Systems*.

Kharal, A., & Ahmad, B. (2001). Mapping on soft classes. *New Mathematics and Natural Computing*, *7*(3), 471–481. doi:10.1142/S1793005711002025

Kharal, A., & Ahmad, B. (2009). *Mappings on fuzzy soft classes*. Advances in Fuzzy Systems.

Maji, P. K. (2013). Neutrosophic soft set. *Annals of Fuzzy Mathematics and Informatics*, *5*, 157–168.

Maji, P. K., Biswas, R., & Roy, A. R. (2002). An application of soft sets in a decision making problem. *Computers & Mathematics with Applications (Oxford, England)*, *44*(8-9), 1077–1083. doi:10.1016/S0898-1221(02)00216-X

Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & Mathematics with Applications (Oxford, England)*, *45*(4-5), 555–562. doi:10.1016/S0898-1221(03)00016-6

Maji, P. K., Roy, A. R., & Biswas, R. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics*, *9*(3), 589–602.

Molodtsov, D. (1999). Soft set theory first results. *Computers & Mathematics with Applications (Oxford, England)*, *37*(4-5), 19–31. doi:10.1016/S0898-1221(99)00056-5

Roy, A. R., & Maji, P. K. (2007). A fuzzy soft set theoretic approach application of soft set in a decision making problem. *Computers & Mathematics with Applications (Oxford, England)*, *203*, 412–418.

Salleh, A. R., Alkhazaleh, S., Hassan, N., & Ahmad, A. G. (2012). Multiparameterized soft set. *Journal of Mathematics and Statistics*, *8*(1), 92–97. doi:10.3844/jmssp.2012.92.97

Shabir, M., & Naz, M. (2011). On soft topological spaces. *Compers and Mathematics with Applications*, *61*(7), 1786–1799. doi:10.1016/j.camwa.2011.02.006

Smarandache, F. (1998). *Neutrosophy*. Rehoboth,: Neutrosophic Probability, Set, and Logic, Amer. Res. Press.

Smarandache, F. (1999). *A Unifying Field in Logics.Neutrosophy: Neutrosophic Probability, Set and Logic*. Rehoboth: American Research Press.

Smarandache, F. (2005). Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, *24*, 287–297.

Turksen, I. (1986). Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems*, *20*(2), 191–210. doi:10.1016/0165-0114(86)90077-1

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. doi:10.1016/S0019-9958(65)90241-X

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-Part I. *Information Sciences, 7*, 199-249.

KEY TERMS AND DEFINITIONS

Neutrosophic Set: Let X be a universe of discourse and a neutrosophic set A on X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$ where $T, I, F: X \to]^-0,1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, neutrosophic set takes the value in the interval [0,1], because it is difficult to use neutrosophic set with value from real standard or non-standard subsets of $]^-0,1^+[$ in real life application like scientific and engineering problems.

Neutrosophic Subset: A neutrosophic set A is contained in another neutrosophic set B, if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \leq F_B(x)$ for all $x \in X$.

Significant Element: An element x of U is called significant with respect to neutrosophic set A of U if the degree of truth-membership or falsity-membership or indeterminacy-membership value, i.e.,$T_{A(x)}$ or $F_{A(x)}$or $I_{A(x)} \leq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity-membership all can not be significant. We define an intuitionistic neutrosophic set by $A = \{\langle x: T_{A(x),} I_{A(x),} F_{A(x)} \rangle, x \in U\}$,wheremin $\{ T_{A(x),} F_{A(x)} \} \leq 0.5$, min $\{ T_{A(x)}, I_{A(x)} \} \leq 0.5$, min $\{ F_{A(x)}, I_{A(x)} \} \leq 0.5$, for all $x \in U$, with the condition $0 \leq T_{A(x)} + I_{A(x)} + F_{A(x)} \leq 2$.

Soft Intersection: The extended intersection of two soft sets (F,A) and (G,B) over the common universe U is the soft set (H,C), where $C=A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A-B \\ G(e) & \text{if } e \in B-A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F,A) \cap_E (G,B) = (H,C)$.

Soft Set: Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denote the power set of U and let A be a non-empty subset of E. A pair (F,A) is called soft set over U, where F is mapping given by $F: A \to P(U)$.

Soft Subset: For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if (i) $A \subseteq B$, (ii) $F(e) \subseteq G(e) \forall e \in A$. We write $(F,A) \subseteq (G,B)$. Two soft sets (F,A) and (G,B) over a common universe $U$are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Soft Union: The union of two soft sets (*F,A*) and (*G,B*) over the common universe *U* is the soft set (*H,C*), where *C*=*A*∪*B* and for all *e*∈*C*, $H(e)=\begin{cases} F(e) & \text{if } e\in A-B \\ G(e) & \text{if } e\in B-A \\ F(e)\cup G(e) & \text{if } e\in A\cap B. \end{cases}$ We write (*F,A*) $\cup_E$ (*G,B*) = (*H,C*).