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Solving Bi-Level Multi-Objective Quadratic Programming Problem with Neutrosophic Parameters in Objective Functions and Constraints Using Fuzzy Approach

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Abstract: This paper aims to present the neutrosophic technique and fuzzy approach to determine the preferred compromise solution for bi-level multi-objective quadratic programming problem with neutrosophic parameters in both the objective functions and constraints. The introduced technique starts firstly to convert the neutrosophic parameters that exist in the objective functions to crisp numbers then convert the neutrosophic parameters that exist in the constraints to crisp numbers. After that, we applied the fuzzy approach to determine the optimal solution for our bi-level multi-objective quadratic programming problem (BLMOQP).

Keywords: Bi-level programming; Multi-objective; Neutrosophic set; Trapezoidal neutrosophic number; Fuzzy approach; Quadratic programming

1 Introduction

In a bi-level programming problem (BLPP), decisions are made within an optimization system at two separate levels; first, the upper level is known as the leader, and second, the lower level is known as the follower. [1].
In the bi-level programming problem, there are two optimization problems, named respectively the upper level (optimization) problem and the lower level (optimization) problem [2].
The upper-level problem is a compounded optimization problem containing the lower-level variables, and the lower level problem is an optimization problem taking the upper-level variables as parameters [2].
Bi-level programming (BLP) is a subset of a multi-level programming problem that is known as a problem of mathematical programming that tackles hierarchical planning problems in a two-level or hierarchical organization with two decision-makers (DMs) [3].
Multi-objective programming is a category of optimization problems that over a set of constraints have more than one objective function [4].
Multi-objective (multi-criteria) optimization is a technique used to solve problems when multiple objective functions have to be optimized simultaneously [4].
Multiple objective linear programming is the method of optimizing a series of objective functions simultaneously and systematically [5].
The goal of Quadratic Programming is to reduce (maximize) a quadratic objective function according to a collection of linear constraints. If the coefficients in the objective function are precisely understood to be crisp, these models can be solved by classical methods and algorithms [3].
Quadratic programming (QP) is a problem of optimization that objective function of which is a quadratic function and the constraints are linear equivalences or inequalities. QP is the problem of maximizing an objective function and is one of the simplest types of non-linear programming [6].
The decision-making process is a very challenging challenge for the decision-maker due to the complex diversity and ambiguity of the existing socio-economic system [7].

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The presence of a fuzzy set, and then an intuitive fuzzy set, made it possible to manage such a situation smoothly. These provide information about the membership and non-membership values of the object to the set. However, the reality, e.g. the sporting outcome, the decision-making process, the casting of votes, etc., where tri-component outcomes can arise, cannot be clarified by these sets. A third part can occur between affirmation and negation of a possible event. Smarandache's Neutrosophic Set (Ns) calls the "indeterminacy". Here, decision-makers are given the chance to include their reluctance in evaluating certain facts more precisely. Thus, the neutrosophic theory is a more generalized view of the intuitionistic fuzzy theory [7].

Neutrosophic introduced by Smarandache as a modern branch of philosophy, with ancient origins, concerned with "the origin, nature, and scope of neutrality, as well as their interactions with different spectra of ideas" [8].

The fundamental thesis of neutrosophy is that each concept has not only a certain degree of truth, as is commonly believed in many-valued logic contexts, but also a falsity and indeterminacy value, which must be regarded independently of each other. Smarandache seems to recognize such "indeterminacy" in both a subjective and an empirical context, i.e. ambiguity as well as imprecision, vagueness, mistake, doubtfulness, etc. [8].

Neutrosophic Set (NS) seems to be a generalization of fuzzy set and intuitionistic fuzzy set and, can interact with uncertain, indeterminate and incongruous knowledge where indeterminacy is clearly quantified, truth membership and false membership are completely separate [8].

The fundamental principle of fuzzy programming approaches assumes that the Lower-Level Decision Maker (LLDM) optimizes its objective function, taking into account the objective or choice of the Upper-Level Decision Maker (ULDM). In the decision-making process, considering the membership functions of the fuzzy objectives for the decision variables of the ULDM, the LLDM solves the FP problem with the collection of constraints on the total satisfaction level of the ULDM. If the proposed solution is not acceptable to the ULDM, the solution search will be resumed by redefining the proposed membership functions until a satisfactory solution is found [9].

The fuzzy approach uses the principle of tolerance membership to build a fuzzy max-min decision model for producing Pareto's optimal (satisfactory) three-level programming problem; the first-level decision-maker (FLDM) defines his optimization methods and decisions with potential tolerances defined by the membership functions of the fuzzy set theory. Then the second level decision-maker (SLDM) determines his objective functions, and decisions, in the perspective of FLDM, with potential tolerances defined by the membership functions of the fuzzy set theory. Finally, the third-level decision-maker (TLDM) uses the choice details for the upper-level decision that decision-maker subject to the upper-level decision [10].

In [10] T.I. Sultan et al. proposed a fuzzy approach for solving a three-planner model and a solution method for solving this problem. Each level aims to optimize its problem independently as a large-scale programming problem by using a decomposition strategy to control optimization by a set of sub-problems that can be quickly overcome.

In [11] Emam et al. proposed a solution for Bi-Level Multi-Objective Large Scale Integer Quadratic Programming (BLMOLSIQP) problem, where all the decision parameters in the objective functions are symmetric trapezoidal fuzzy numbers, and have block angular structure of the constraints. The proposed algorithm based on a-level sets of fuzzy numbers, the weighting approach, the Taylor method, the Decomposition technique, and the Branch and Bound technique is used to find a compromised solution to the problem under consideration. In [9] M. A. Abdel-Fattah et al. presented a fuzzy approach for solving a three-decision maker's model and proposed how to find the solution of three-level chance constraints quadratic programming problem.

In [12] Emam et al. focused on solving a hierarchical structure problem that consist of two stages, first stage decision-maker and second stage decision maker. Each level has multiple objectives need to reach and achieve them under same constrains with neutrosophic parameter in these objective functions. The algorithm starts with converting the trapezoidal neutrosophic parameter to crisp then we will use first-order Taylor method to convert the quadratic form to linear form. After that, we will use the weight technique to make the problem as single objective function for each stage, and in the final phase, we will use the interactive approach to deal with the problem of bi-level linear programming.

This paper arranged as follows: In Section 2, we will introduce the problem formulation and some preliminary discussions. In Section 3, we will discuss how the bi-level multi-objective quadratic programming problem with neutrosophic parameters in constrains and objective functions (BLLSMOQPP) will be converted to a bi-level multi-objective quadratic programming problem. In Section 4, we will introduce the fuzzy approach concept to solve the bi-level multi-objective quadratic programming problem. In Section 5, we will apply our algorithm on a numerical example. Lastly, the findings, and recommendations for future works are given in Section 6.

2 Problem Formulation and Preliminary Discussion

In this section, we discuss the structure, formulation of the bi-level multi-objective quadratic programming problem with neutrosophic parameter in both the objective functions and constraints also we recall some important de?nitions related to neutrosophic set and bi-level multi-objective problem.

2.1 Problem Formulation:

The bi-level multi-objective quadratic programming problem with neutrosophic parameter in the objective functions and constrains can represent as follows:
[Upper Level]

$$\max_{z_1,x_2} F_1(x) = \max_{X_1,X_2}(f_{11}(x), f_{12}(x), \ldots, f_{1u}(x)) \quad (1)$$

Where x_3, x_4 solves
[Lower Level]

$$\max_{x_3,x_4} F_2(x) = \max_{X_3,X_4}(f_{21}(x), f_{22}(x), \ldots, f_{2n}(x)) \quad (2)$$

Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i(3) \quad (3)$$

And where

$$f_{ik} = c_{nk}x_+ \frac{1}{2}x^T L_k^i x \ , \ (n = 1,2), (k = 1,2,\ldots,n_i) \quad (4)$$

Let the functions F_1 and F_2 are quadratic objective functions defined on R^n and (L^1, L^2) are $m \times n$ matrices describing the coefficients of the quadratic terms, c_{nk} are $1 \times m$ matrices and trapezoidal neutrosophic numbers in the above problem (1), (2), (4).
Let x_1, x_2, x_3, x_4 be actual vector variables representing the choice of the first level and the second decision level. In addition, the decision-maker at the upper-level has x_1, x_2 indicating the option of the first level and the decision maker at lower-level has x_3, x_4 indicating level choice, $b = (\tilde{b}_0, \ldots, \tilde{b}_p)^T$ are trapezoidal neutrosophic numbers, $b = (\tilde{b}_0, \ldots, \tilde{b}_p)^T$ is an $(p+1)$ vector, and $a_{01}, \ldots, a_{0p}, d_1, \ldots, d_p$ are constants.

2.2 Preliminaries:

In this section, we write the important definitions and preliminaries for our problem:
Definition 1. For any $(x_1, x_2 \in H_1 = \{x_1, x_2 | (x_1, x_2, x_3, \ldots, x_p) \in H\})$ given by the first level, if the decision-making variable $(x_3, x_4 \in H_2 = \{x_3, x_4 | (x_1, x_2, x_3, \ldots, x_p) \in H\})$ is the Pareto optimal solution of the second level, then (x_1, x_2, x_3, x_4) is a feasible solution of (BLMONQPP).
Definition 2. If $x^* \in R^m$ is a feasible solution of the (BLMONQPP), no other feasible solution $x \in H$ exists, such that $F_1(x^*) \leq F_1(x)$ so x^* is the Pareto optimal solution of the (BLMOQPP).
Definition 3. [13] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is defined by a function of membership of fact T_A, a function of membership of indeterminacy I_A and a function of falsity F_A.$F_A(x), T_A(x), I_A(x)$ are real standard or a non-standard subset of $]({}^-0, {}^+1[$. There is no limitation for the submission of $F_A(x), T_A(x), I_A(x)$.
Definition 4. [14] The trapezoidal neutrosophic number $\tilde{Z}$ is a neutrosophic set in R with the following T, I and, F membership functions:

$$T_{\tilde{Z}}(x) = \begin{cases} \propto_{\tilde{Z}} \left(\frac{x-z_1}{z_2-z_1}\right) & (z_1 \leq x \leq z_2) \\ \propto_{\tilde{Z}} & (z \leq x \leq z_3) \\ \propto_{\tilde{Z}} & (z_3 \leq x \leq z_4) \\ 0 & otherwise \end{cases} \quad (5)$$

$$I_{\tilde{Z}}(x) = \begin{cases} \frac{(z_2 - x + \theta_{\tilde{Z}}(x - z_1'))}{(Z_2 - Z_1')} & (z_1' \leq x \leq z_2) \\ \theta_{\tilde{Z}} & (z_2 \leq x \leq z_3) \\ \frac{(x - z_3 + \theta_{\tilde{Z}}(z_4' - x))}{(Z_4' - Z_3)} & (z_3 \leq x \leq z_4') \\ 1 & otherwise \end{cases} \quad (6)$$

$$F_{\tilde{Z}}(x) = \begin{cases} \frac{(z_2 - x + \beta_{\tilde{Z}}(X - z"_1)}{(Z_2 - Z_1')} & (z"_1 \leq x \leq z_2) \\ \beta_{\tilde{Z}} & (z_2 \leq x \leq z_3) \\ \frac{(x - z_3 + \beta_{\tilde{Z}}(z"_4 - x))}{(Z"_4 - Z_3)} & (z_3 \leq x \leq z"_4) \\ 1 & otherwise \end{cases} \quad (7)$$

where $\propto_{\tilde{Z}}, \theta_{\tilde{Z}}$ and $\beta_{\tilde{Z}}$ represent the maximum truthiness degree, minimum indeterminacy degree, and minimum falsity degree, sequentially; $\propto_{\tilde{Z}}, \theta_{\tilde{Z}}$; and $\beta_{\tilde{Z}} \in [0,1]$. Additionally, $z"_1 \leq z_1 \leq z'_1 \leq z_2 \leq z_3 \leq z'_4 \leq z_4 \leq z"_4$. The membership functions of trapezoidal neutrosophic numbers are presented in Fig. 1.

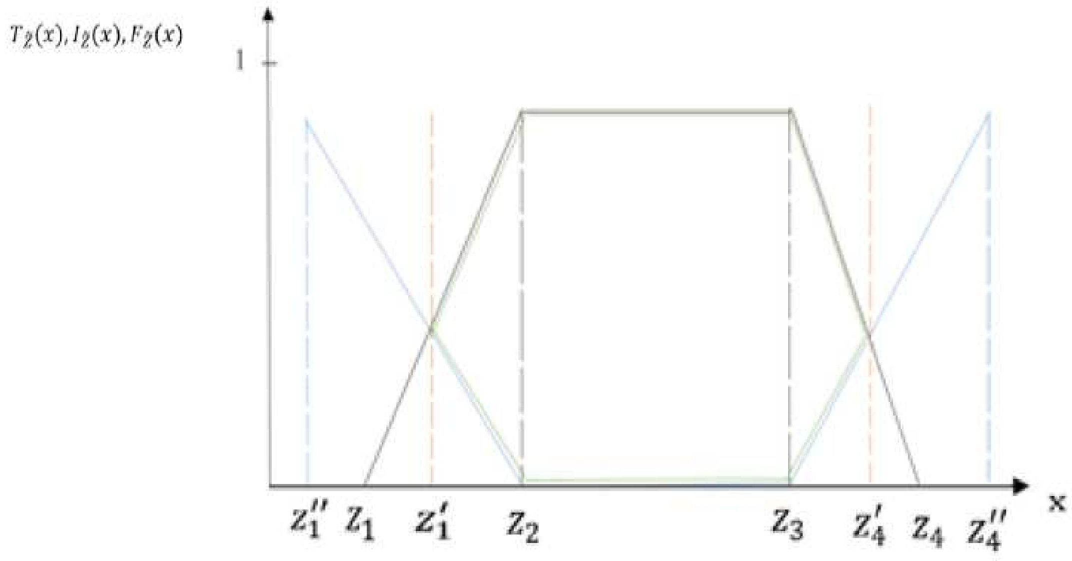

Fig. 1: The T, I, and F membership functions of the trapezoidal neutrosophic number

3 Solution Concepts:

In this section, we will discuss our solution strategies and concepts to solve our problem.

3.1 Ranking Method:

We will use the ranking method to transform the neutrosophic number that exists in the objective functions and constraints to crisp number.

In the following type, the trapezoidal neutrosophic number presented [15]:

$(\tilde{Z} = z^l, z^{m1}, z^{m2}, z^u; T_{\tilde{Z}}, I_{\tilde{Z}}, F_{\tilde{Z}})$ where $\tilde{Z}$ is a trapezoidal neutrosophic number and z^l, z^{m1}, z^{m2}, z^u are the lower bound, first and second median value and upper bound for trapezoidal neutrosophic number, respectively. In addition, $F_{\tilde{Z}}, T_{\tilde{Z}}, I_{\tilde{Z}}$ represent the falsity degrees of the trapezoidal number, truth degrees of the trapezoidal number and finally the indeterminacy degrees of the trapezoidal number.

In case the objective function or the problem is a case of maximization state, at that point the ranking function for this trapezoidal neutrosophic number can be expressed as the following [15]:

$$R(\tilde{Z}) = \left| \left(\frac{-\frac{1}{3}(3z^l - 9z^u) + 2(Z^{m1} - Z^{m2})}{2} \right) * (T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}}) \right| \tag{8}$$

However, if the objective function or the problem is a case of minimization, the ranking method for such a trapezoid is as following [15]

$$R(\tilde{Z}) = \left| \left(\frac{(z^l - z^u) - 3(Z^{m1} - g^{m2})}{-4} \right) * (T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}}) \right| \tag{9}$$

In case the author works with a symmetric neutrosophic trapezoidal number that has the following form: $\tilde{Z} = \langle (z^{m1}, z^{m2}); \alpha, \beta \rangle$, where $\alpha = \beta$ and $\beta > 0$, The ranking function will then be described as follows for the neutrosophic number [15].

$$R(\tilde{Z}) = \left(\frac{(Z^{m1} + Z^{m2}) + (\alpha + \beta)}{2} \right) * T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}} \tag{10}$$

3.2 Fuzzy approach:

The neutrosophic parameters in the objective functions and constraints removed using EQ (8) so the problem transformed to deterministic bi-level multi-objective quadratic programming problem.

Now we will use the fuzzy approach to solve BLMOQPP that depend on the tolerance between the first level decision maker and second level decision maker.

We will start with getting the satisfactory solution that is acceptable to the FLDM, and then give the FLDM decision variables and goals with tolerance to the SLDM for him/her to seek the satisfactory solution that nearest to FLDM optimal solution. We will discuss how to solve our problem using the fuzzy approach:

First step:

FLDM solves his problem individually and finds the best solution and worst solution $(f_{11}^*, f_{11}^-, f_{12}^*, f_{12}^-)$. Then FLDM build his/her membership functions using these values as follows:

$$\mu_{f_{11}}[f_{11}(x)] = \begin{cases} 1 & if\ f_{11}(x) > f_{11}^*, \\ \frac{f_{11}(x) - f_{11}^-}{f_{11}^* - f_{11}^-} & if\ f_{11}^- \leq f_{11}(x) \leq f_{11}^* \\ 0 & if\ f_{11}^- \geq f_{11}(x). \end{cases} \tag{11}$$

$$\mu_{f_{12}}[f_{12}(x)] = \begin{cases} 1 & if\ f_{12}(x) > f_{12}^*, \\ \frac{f_{12}(x) - f_{12}^-}{f_{12}^* - f_{12}^-} & if\ f_{12}^- \leq f_{12}(x) \leq f_{12}^* \\ 0 & if\ f_{12}^- \geq f_{12}(x). \end{cases} \tag{12}$$

Now FLDM will take the membership functions and add them as constraints belong with the normal problem constrain then build the following Tchebycheff problem:

$$\max \delta \tag{13}$$

Subject to

$x \in G,$

$\mu_{f_{11}}[f_{11}(x)] \geq \delta,$

$\mu_{f_{12}}[f_{12}(x)] \geq \delta,$

$\delta \in [0,1].$

After solving the upper Tchebycheff we will get this solution $(x_1^F, x_2^F, x_3^F, x_4^F, f_1 1^F, f_1 2^F, \delta^F)$ for the first-level decision-maker.

Second step:

SLDM solves his problem individually and finds the best solution and worst solution $(f_2 1^*, f_2 1^-, f_2 2^*, f_2 2^-)$. Then SLDM build his/her membership functions using these values as follows:

$$\mu_{f_{21}}[f_{21}(x)] = \begin{cases} 1 & if\ f_{21}(x) > f_{21}^*, \\ \frac{f_{21}(x) - f_{21}^-}{f_{21}^* - f_{21}^-} & if\ f_{21}^- \leq f_{21}(x) \leq f_{21}^* \\ 0 & if\ f_{21}^- \geq f_{21}(x). \end{cases} \tag{14}$$

$$\mu_{f_{22}}[f_{22}(x)] = \begin{cases} 1 & if\ f_{22}(x) > f_{22}^*, \\ \frac{f_{22}(x) - f_{22}^-}{f_{22}^* - f_{22}^-} & if\ f_{22}^- \leq f_{22}(x) \leq f_{22}^* \\ 0 & if\ f_{22}^- \geq f_{22}(x). \end{cases} \tag{15}$$

The SLDM will take the membership functions and add them as constraints belong with the normal problem constrain then builds the Tchebycheff.

SLDM get this solution $(x_1^s, x_2^s, x_3^s, x_4^s, f_{21}^s, f_{22}^s, \delta^s)$ after solving the Tchebycheff.

The FLDM recognizes that it is not realistic to use the optimal decisions x_1^F, x_2^F as a control factors for the SLDM. It is more realistic to have some flexibility that helps the SLDM to look for the ideal solution and to reduce the time of search or interactions. Therefore, we need some tolerance that gives the SLDM an extent feasible region to search for his/her optimal solution and reduce the search time. Therefore, we will calculate new decision variable x_1, x_2 that should be around x_1^F, x_2^F with maximum tolerance t_{11}, t_{12} and the following membership function specify x_1^F, x_2^F as:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_{11})}{t_{11}} & x_1^F - t_{11} \le x_1 \le x_1^F, \\ \frac{(x_1^F+t_{11})-x_1}{t_{11}} & x_1^F \le x_1 \le x_1^F + t_{11}, \end{cases} \tag{16}$$

$$\mu(x_2) = \begin{cases} \frac{x_2-(x_2^F-t_{12})}{t_{12}} & x_2^F - t_{12} \le x_2 \le x_2^F, \\ \frac{(x_2^F+t_{12})-x_2}{t_{12}} & x_2^F \le x_2 \le x_2^F + t_{12}, \end{cases} \tag{17}$$

Third step

FLDM goals may reasonably consider all $f_{1k} \ge f_{1k}^F, k = 1,2,.....N$ are completely acceptable and all $f_{1k} < f'_{1k} = f_{1k}(x_1^S, x_2^S, x_3^S, x_4^S),\ k = 1,2,.....N$ are completely unacceptable, and that the preference with $[f_{1k}^F, f'_{1k}]$ is linearly increasing. Because of the fact that the SLDM got the optimum at $(x_1^S, x_2^S, x_3^S, x_4^S)$, which in turn provides the FLDM the objective function values f'_{1k}, makes any $f_{1k} \le f'_{1k}$ unattractive in practice so we make new membership functions for FLDM as follow:

$$\mu'_{f_{1k}}[f_{1k}(x)] = \begin{cases} 1 & if\ f_{1k}(x) > f_{1k}^F, \\ \frac{f_{1k}(x)-f_{1k}^-}{f_{1k}^F-f_{1k}^-} & if\ f_{1k}^- \le f_{1k}(x) \le f_{1k}^F, \\ 0 & if\ f_{1k}(x) \le f_{1k}^-. \end{cases} \tag{18}$$

Fourth step: The SLDM goals may reasonably consider all $f_{2r} \ge f_{2r}^S$, $r = 1,2,\dots Q$ are acceptable and all $f_{2r} < f'_{2r} = f_{2r}(x_1^F, x_2^F, x_3^F, x_4^F)$ are unacceptable, and that the preference with $[f_{2r}^S, f'_{2r}]$ are linearly increasing. Because of SLDM got the optimum at $(x_1^S, x_2^S, x_3^S, x_4^S)$, which in turn provides the FLDM the objective function values f'_{2r}, makes any $f_{2r} \le f'_{2r}$ unattractive in practice so we make new membership functions for the SLDM as follow:

$$\mu'_{f_{2r}}[f_{2r}(x)] = \begin{cases} 1 & if\ f_{2r}(x) > f_{2r}^S, \\ \frac{f_{2r}(x)-f_{2r}^-}{f_{2r}^S-f_{2r}^-} & if\ f_{2r}^- \le f_{2r}(x) \le f_{2r}^S, \\ 0 & if\ f_{2r}(x) \le f_{2r}^-. \end{cases} \tag{19}$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision - makers, we can solve the Tchebycheff problem for all decision-makers:

$$\begin{aligned} &\max \delta \\ &\text{Subject to} \\ &x \in G, \\ &\frac{x_1-(x_1^F-t_{11})}{t_{11}} \ge \delta I, \\ &\frac{(x_1^F+t_{11})-x_1}{t_{11}} \ge \delta I, \\ &\frac{x_2-(x_2^F-t_{12})}{t_{12}} \ge \delta I, \\ &\frac{(x_2^F+t_{12})-x_2}{t_{12}} \ge \delta I, \\ &\mu'_{f_{1k}}[f_{1k}(x)] \ge \delta I, \\ &\mu'_{f_{2r}}[f_{2r}(x)] \ge \delta I, \\ &x_1, x_2, x_3, x_4 \in G, \\ &t_{11}, t_{12} > 0, \\ &\delta \in [0,1]. \end{aligned} \tag{20}$$

4 Numerical example

[First Level]

$$\max_{X_1,X_2} F_1 \approx [(2,4,5,9)X_1 + (3,5,9,11)X_2^2 + (1,2,5,8)X_4, (3,6,7,9)X_1^2 + (1,4,6,10)X_2^2 + (2,5,6,7)X_3]$$

Where x_3, x_4 solves
[Second Level]

$$\max_{X_3,X_4} F_2 \approx [(1,3,4,6)X_1 + (4,5,7,10)X_3^2 + (2,5,8,12)X_4^2, (3,7,9,12)X_2 + (8,11,12,16)X_3^2 + (2,6,7,12)X_4^2]$$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} (58,70,120,186)$,
$6x_1 + 4x_2 \tilde{\leq} (50,60,110,150)$,
$x_3 + x_4 \tilde{\leq} (18,20,45,56)$,
$x_1, x_2, x_3, x_4 \geq 0$.
In this example, we solve a BLMOQPP problem with trapezoidal neutrosophic numbers in both objective functions and constrains. The order of element for trapezoidal neutrosophic numbers is as follows: lower, ?rst median value, second median value and ?nally the upper bound. The decision makers' decide the degree about each value of trapezoidal neutrosophic number is (0.9, 0.3, and 0.2) for the first level, second level and constraints.
First step:
Because this BLMOQPP with neutrosophic parameters and the problem is a maximization, then each trapezoidal number will convert to its equivalent crisp number by using the below equation (eq.8).

$$R(\tilde{a}) = \left| \left(\frac{-\frac{1}{3}(3a^l - 9a^u) + 2(a^{m_1} - a^{m_2})}{2} \right) * (T_{\tilde{a}} - I_{\tilde{a}-F_{\tilde{a}}}) \right|$$

Then, the crisp model of the previous problem will be as follows:
[First Level]

$$\max_{X_1,X_2} F_1 = 4.6X_1 + 4.4X_2^2 + 3.4X_4, 4.4X_1^2 + 5X_2^2 + 1.6X_3$$

Where x_3, x_4 solves
[Second Level]

$$\max_{X_3,X_4} F_2 = 3X_1 + 4.4X_3^2 + 5.6X_4^2, 5.8X_2 + 7.6X_3^2 + 6.4X_4^2$$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} 80$,
$6x_1 + 4x_2 \tilde{\leq} 60$,
$x_3 + x_4 \tilde{\leq} 20$,
$x_1, x_2, x_3, x_4 \geq 0$.
Second step:
We will use Fuzzy Approach to solve bi-level multi-objectives quadratic programming problem.
A) We will get the best and worst solution for each objective function at the first level, so we will solve the below equations individual:

$$\max_{X_1,X_2} F_{11} = 4.6X_1 + 4.4X_2^2 + 3.4X_4$$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} 80$,

$6x_1 + 4x_2 \tilde{\leq} 60,$
$x_3 + x_4 \tilde{\leq} 20,$
$x_1, x_2, x_3, x_4 \geq 0.$

$$\max_{X_1,X_2} F_{12} = 4.4X_1^2 + 5X_2^2 + 1.6X_3$$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} 80,$
$6x_1 + 4x_2 \tilde{\leq} 60,$
$x_3 + x_4 \tilde{\leq} 20,$
$x_1, x_2, x_3, x_4 \geq 0.$
The best and worst solution for the objective functions of the first level:
$f_{11}^* = 1058$ $\overline{f_{11}} = 0$
$f_{12}^* = 1157$ $\overline{f_{12}} = 0$
B) We will build the membership functions using the value of $f_{11}^*, \overline{f_{11}}, f_{12}^*, \overline{f_{12}}$ then using the membership functions of fuzzy set theory (2.15) and (2.16) we will get the solution for the first level:
$\max \delta,$
Subject to
$x \in G,$
$4.6X_1 + 4.4X_2^2 + 3.4X_4 - 1058\delta \geq 0,$
$4.4X_1^2 + 5X_2^2 + 1.6X_3 - 1157\delta \geq 0,$
$\delta \in [0, 1].$
So, the solution is:
$(x_1^f, x_2^f, x_3^f, x_4^f) = (6.266, 5.6, .46, 13.166),\ (F_{11}^f, F_{12}^f) = (211.6, 330.33)\ \delta = 0.2$
C) We will get the best and worst solution for each objective function at the second level, so we will solve the below equations individual:

$$\max_{X_1,X_2} F_{21} = 3X_1 + 4.4X_3^2 + 5.6X_4^2$$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} 80,$
$6x_1 + 4x_2 \tilde{\leq} 60,$
$x_3 + x_4 \tilde{\leq} 20,$
$x_1, x_2, x_3, x_4 \geq 0.$

$$\max_{X_1,X_2} F_{22} = 5.8X_2 + 7.6X_3^2 + 6.4X_4^2$$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} 80,$
$6x_1 + 4x_2 \tilde{\leq} 60,$
$x_3 + x_4 \tilde{\leq} 20,$
$x_1, x_2, x_3, x_4 \geq 0.$
The best and worst solution for the objective functions of the first level:
$f_{21}^* = 2270$ $\overline{f_{21}} = 0$
$f_{22}^* = 3127$ $\overline{f_{22}} = 0$
D) We will build the membership functions using the value of $f_{21}^*, \overline{f_{21}}, f_{22}^*, \overline{f_{22}}$ then using the membership functions of fuzzy set theory (2.15) and (2.16) we will get the solution for the first level:
$\max \delta,$
Subject to

$x \in G,$
$3X_1 + 4.4X_3^2 + 5.6X_4^2 - 2270\beta \geq 0,$
$5.8X_2 + 7.6X_3^2 + 6.4X_4^2 - 3127\beta \geq 0,$
$\delta \in [0,1].$
So, the solution is:
$(x_1^s, x_2^s, x_3^s, x_4^s) = (1.2346, 1, 235, 18.8, 1.235), (F_{21}^f, F_{22}^f) = (1567.381, 2703.068)\beta = 0.1$

The first level knows that using the optimal decision (x_1^f, x_2^f) as control variables for the second level are not practical. So we need some tolerance that gives the second level an extent feasible region to search for his/her optimal solution and also reduce the search time.

So we will calculate new x_1, x_2 that will be around with (x_1^f, x_2^f) with maximum tolerance.

In order to generate the satisfactory solution with all satisfaction for all DMs. We can have the following Tchebycheff problem:

$\max \delta,$

Subject to
$x_1 + x_2 + x_3 + x_4 \tilde{\leq} 80,$
$6x_1 + 4x_2 \tilde{\leq} 60,$
$x_3 + x_4 \tilde{\leq} 20,$
$x_1 - \delta \tilde{\geq} 5.266,$
$x_1 + \delta \tilde{\leq} 7.266,$
$x_2 - \delta \tilde{\geq} 4.6,$
$x_2 + \delta \tilde{\leq} 6.6,$
$4.6X_1 + 4.4X_2^2 + 3.4X_4 - 195.011\delta \geq 16.5889,$
$4.4X_1^2 + 5X_2^2 + 1.6X_3 - 285.918\delta \geq 44.4123,$
$3X_1 + 4.4X_3^2 + 5.6X_4^2 - 576.918\delta \geq 0,$
$5.8X_2 + 7.6X_3^2 + 6.4X_4^2 - 1559.579\delta \geq 0,$
$x_1, x_2, x_3, x_4, \delta \geq 0,$
$\delta \leq 1.$

The compromise solution is $X_0 = (6.21957, 5.670645, 0, 20)$; and $\delta = 0.9293551$ (overall satisfaction for all DMs).

5 Conclusions and Suggestions for Future Research

This research paper proposed an algorithm for solving the BLMOQPP with neutrosophic parameters in the objective functions and constraints. The technique that used to solve our problem depended on minimizing the problem difficulty by converting the neutrosophic nature of the problem in both objective functions and constraints into corresponding crisp model. Then we used fuzzy approach to get the solution for BLMOQPP. We proposed a numerical example to explain and demonstrate our algorithm steps.

However, a variety of topics remain subject to potential discussion and can explored by regular, bi-level neutrosophic optimization:

1. Bi-level fractional programming problems with neutrosophic parameters in both objective function and constraints.
2. Bi-level fractional multi-objective decision-making problems with neutrosophic parameters in both objective functions and constraints with integrity conditions.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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