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On Neutrosophic Topological Spaces Generated by Single Value Neutrosophic Graph

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Abstract

The concept of a single-valued neutrosophic graph SVNS-G is recently studied, the bond between neutrosophic graph N-G and neutrosophic topological graph NT-G was my goal in this research, I try to find a relation on the vertices of the SVN-G to structure NT-G add some theorems and corollaries.

Keywords: Neutrosophic sets (NSs); a single-valued neutrosophic sets (SVNSs); neutrosophic topological space; neutrosophic graph N-G.

1. Introduction

Smarandache [1,2] submission on Neutrosophic sets (NSs) it is aid for managing with deficient, and clarity input in our true univers. (NSs) are extension of the hypothesis of fuzzy sets [3], intuitionistic fuzzy sets [4,5] and interval valued intuitionistic fuzzy sets [6]. The NSs is defined according to three distinct functions : the function of truth-membership write as (t), the function of an indeterminacy-membership write as (i), the function of a falsity membership write as (f), each operation independently within the proper criterion or noncriterion unit interval $]^{-}0, 1^{+}[$. These function aim to address true universe challenges effectively . Try to solve the problems in a true our world, (SVNSs) is a concept introduced by Wang et al at 2012. [7], a subclass of NSs. From a previous definition of SVNSs we can notice that it is a generalization to the term of intuitionistic fuzzy logic, since we defined three independent membership as a functions. Graph theory [14] has wide applications in various sciences, like operations research, computer science, physics, Biology, number theory and other science branches, If we have uncertain information associated with the set of edges or vertices or both set, our model embody a fuzzy graph. For more detail on fuzzy graphs and intuitionistic fuzzy graphs [17, 18, 19, 20, 21] regarded both vertex and edge sets represent like a fuzzy and intuitionistic fuzzy sets. In our research, we encounter situations where the relationships that exist between point or nods are uncertain. Traditional fuzzy graphs and intuitionistic fuzzy graphs often struggle to address this ambiguity effectively. To tackle this issue, Smarandache [22, 23, 24] introduced four fundamental types of neutrosophic graphs. Among these, two types, namely the (t, i, f)-Edge neutrosophic graph and the (t, i, f)-Vertex neutrosophic graph, are structured around the components (t, i, f) to better capture and represent indeterminate relationships. Smarandache's work [8] provides insights into these innovative graph structures. Smarandache [8], Lupianez [9,10] Salama [11] studied topology on neutrosophic sets and state the basics principles of it. N-G submit by S. Broumi, M. Talea, F. Smarandache [12,13]. The relationship between graph theory and topological space studied by many researches tried to solve some problems in our life by transforming the problem into a graph and then generating a topological space, the same relation can be found between N-G [12,15]. Imran et al. [25-27] gave fresh insights into the ideas of topological groups, generalized alpha generalized continuity and generalized semi generalized closed sets that are weakly neutrosophic. A. K. Abed et al. [28] established the interpretation of Neutrosophic D–Topological Spaces. Finally, the senses of neutrosophic generalized alpha generalized separation axioms were informed by Abdulkadhim et al. [29]. In the beginning of this article we referred to a basics definition

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of a NSs [2], adding all definition and basics properties of a N-G with some types of this graph, in third part we called the concept of a neighborhood of a vertex in a N-G defined in [13], we take a subset of neighborhood of some vertices in a strong SVNS-G to great a base for a strong SVNS-G then generated a NT-G then we stated some theorems. In fourth part to study a topological properties we define a new term a neighborhood complement of a subset of a strong SVNS-G then we define NNCs, state some theorem related to NNCs.

2. Preliminaries

In the first section, we mentioned several basics definitions, that we need to present this search.

Definition 2.1: [1]

A space consist from points "objects" write as X with generic elements represent as x, defined the NSs set k is an topic introduced like the following way

$$k = \{< x: T_k(x), I_k(x), F_k(x)>, x \in X\}$$

We mean by the functions T, I, F: X→ $]^{-}0,1^{+}[$ according to three distinct functions, function of truth-membership, function of indeterminacy -membership , function of a falsity -membership, for the element x ∈ X to the set k with the following condition

$$^{-}0 \leq T_k(x)+ I_k(x)+ F_k(x) \leq 3^{+} \tag{1}$$

Definition 2.2: [7]

A space consist from points "objects" write as X with generic elements represent as x, SVNSs k is specifying by a function of truth- membership represent as $T_k(x)$, function of indeterminacy - membership represent as $I_A(x)$, and a function of a falsity – membership represent as $F_k(x)$.

For every element x belong to X, $T_k(x), I_k(x), F_k(x) \in [0, 1]$. A SVNSs k can be express as following way

$$k = \{< x: T_k(x), I_k(x), F_k(x)>, x \in X\} \tag{2}$$

Definition 2.3: [12]

consider that $k = (T_k, I_k , F_k)$ and $l = (T_l , I_l , F_l)$ be SVNSs on a set X. If $l = (T_l, I_l, F_l)$ is a single value neutrosophic relation on a set X, then $k = (T_k, I_k , F_k)$ is named a single-valued neutrosophic relation on $l = (T_l, I_l, F_l)$ if

$T_l (x, y) \leq \min(T_k(x), T_k(y))$

$I_l (x, y) \geq \max(I_k(x), I_k(y))$ and

$F_l (x, y) \geq \max(F_kx), F_k(y))$ ∀ x, y ∈ X.

Definition 2.4:[13]

Let $\breve{G}$ = (V,E) a graph, (SVN-G) is a pair of two set represented as $G = (k, l)$ such that

1. The functions T_k:V→[0, 1], I_k:V→[0, 1] and F_k:V→[0, 1] indicate to the degree of truth- membership d_T(v), degree of indeterminacy-membership d_I(v), and falsity-membership d_F(v), of the vertices v_i ∈ V, and

$$0\leq d_T(v)+ d_I(v)+d_F(v) \leq 3 \ \forall v_i \in V \ (i=1, 2, \ldots,n)$$

2. define the following functions T_l: E ⊆ V x V →[0, 1], I_l:E ⊆ V x V →[0, 1] and F_l: E ⊆ V x V →[0, 1] as the following

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$T_l(\{v_i, v_j\}) \leq \min\,[T_k(v_i), T_k(v_j)]$,

$I_l(\{v_i, v_j\}) \geq \max\,[I_k(v_i), I_k(v_j)]$ and

$F_l(\{v_i, v_j\}) \geq \max\,[F_k(v_i), F_k(v_j)]$

indicate to the degree specified to truth-membership d_T(v), the degree specified to indeterminacy-membership d_I(v) and the degree specified to falsity- membership d_F(v) of the edge $(v_i, v_j) \in$ E, since

$0 \leq T_l(\{v_i, v_j\}) + I_l(\{v_i, v_j\}) + F_l(\{v_i, v_j\}) \leq 3$ for every $(v_i, v_j) \in$ E (i, j = 1, 2,…, n).

We point out to k as SVNSs vertex of V , l the SVNSs edge of E , respectively, observe that l is a symmetric SVN -relation on k. We refer (v_i, v_j) to the objects of E such that, $G = (k, l)$ denoted to SVN-G of $\breve{G}$= (V, E).

Example 2.5:
For an example the graph $\breve{G}$ consider that the vertices set denoted by $V = \{u_1, u_2, u_3, u_4\}$, and the edges set represent as $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$. Let k be SVNS- subset of V such that l be a SVNS- subset of E written as

	u_1	u_2	u_3	u_4
T_A	0.4	0.7	0.2	0.5
I_A	0.3	0.1	0.5	0.1
F_A	0.5	0.1	0.6	0.4

	u_1u_2	u_2u_3	u_3u_4	u_4u_1
T_B	0.4	0.1	0.3	0.3
I_B	0.4	0.6	0.3	0.4
F_B	0.6	0.6	0.7	0.5

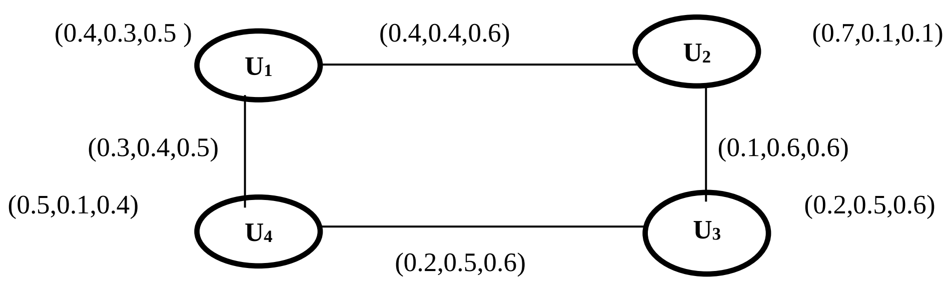

Figure 1: Single- valued neutrosophic graph

in figure 1

1: (u_1,0.4, 0.3,0.5) SVNS-vertex is refer to SVNs vertex

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2: (u_1u_4, 0.3, 0.4, 0.5) SVNS-edge is refer to SVNs edge

3:(u_3, 0.2, 0.5, 0.6) and (u_4, 0.5 ,0.1, 0.4) are SVNS- adjacent vertices.

4:(u_2u_3, 0.1, 0.6, 0.6) and (u_3u_4, 0.3, 0.3, 0.7) are SVNS- adjacent edge .

Observe that if this situation T_{lij} = I_{lij} = F_{lij} for some i and j, in this situation when there exist no line 'edge' between v_i and v_j those two vertices in other situation will be an edge between the vertices v_i and v_j.

Definition 2.6:[14]

Adjacent vertices is a term called for any two vertices if the following rules satisfying

$T_l(u_i, u_j)$ = min [$T_k(u_i)$, $T_k(u_j)$],

$I_l(u_i, u_j)$ = max [$I_k(u_i)$, $I_k(u_j)$] and

$F_l(u_i, u_j)$ = max [$F_k(u_i)$, $F_k(u_j)$]. In this situation the vertices u_i and u_j are the open neighbors and (u_i, u_j) is incident at u_i and u_j also.

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Definition 2.7:[13]

A simple SVNS-G is a term given to SVNS-G, $G = (k, l)$ when the graph has no loop or a multiple edge.

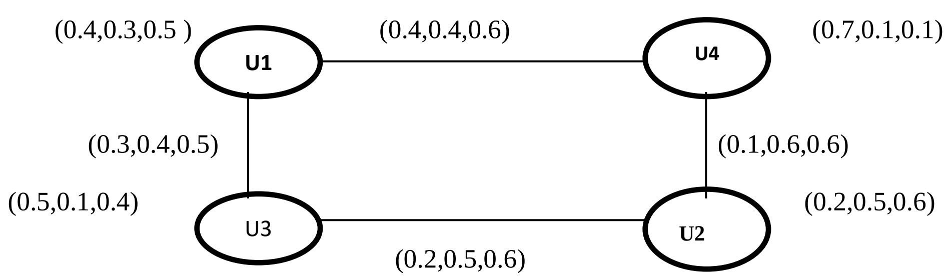

Figure 2: Simple SVNS-G

Definition 2.8:[13]

A sequence of difference vertices $u_0, u_1, .., u_n$, of SVN-G, $G = (k, l)$ is called a path write as P such that the following functions is satisfying T_k (u_i, u_{i-1})≥ 0 , I_k (u_i, u_{i-1})≥ 0 , F_k (u_i, u_{i-1})≥ 0 for 0≤ i≤ n . n denote to the length of this sequence (path) P. The sequential of two vertices are called line or arc consist this path "P" closed path will be a cycle if $u_0 = u_n$ for all value of $n \geq 3$.

Definition 2.9:[13]

The connectedness in SVNS-G , $G = (k, l)$ it mean that every pair in the set of vertices of the graph G has at least one SVN-path between them, in other situation it called disconnected.

Definition 2.10:[13]

The accumulate of degree of truth-membership, accumulate of degree of indeterminacy- membership and accumulate of degree of falsity-membership of any vertex u in SVNS-G, $G = (k, l)$, is of the edges in the graph such that incident on vertex u represent as d(u)= (d_T(u), d_I(u), d_F(u)) where

$d_T(u)=\sum_{v\neq u} T_l(v, u)$ truth-membership degree of a vertex.

$d_I(u)=\sum_{v\neq u} I_l(v,u)$. indeterminacy- membership degree of a vertex

$d_F(u)=\sum_{v\neq u} F_l(v, u)$ falsity-membership degree of a vertex.

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Example 2.11:
Suppose that $G = (k, l)$ is SVNS-G of $\breve{G}$= (V,E) where V={$u1, u2, u3, u4$} and E={$u1u4, u4u2, u2u3, u3u1$}, d($u1$)= (0.7, 0.8, 0.6), d($u2$)= (0.3, 0.9, 0.8), d($u3$)= (0.5, 0.9, 1), d($u4$)= (0.5, 0.8, 0.4)

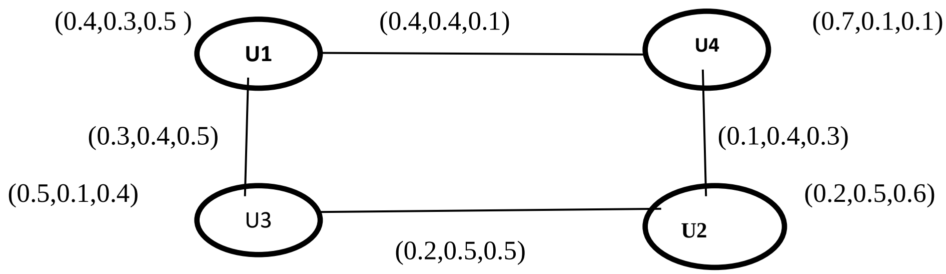

Figure 3: Single- valued neutrosophic graph

Definition 2.12:[13]
A strong single valued neutrosophic graph is a concept of SVNS-G, $G = (k, l)$ of $\breve{G}$ = (V, E) if

$T_l(u_i, u_j) = \min [T_k(u_i), T_k(u_j)]$
$I_l(u_i, u_j) = \max [I_k(u_i), I_k(u_j)]$
$F_l(u_i, u_j) = \max [F_k(u_i), F_k(u_j)]$
For each (ui, uj) involve to E.

Example 2.13:
Let we have a graph $\breve{G}$, set of vertices is V= {u_1, u_2, u_3, u_4}, the set of edges set write as E={u_1u_2, u_2u_3, u_3u_4, u_4u_1}. Suppose that k be SVNS-subset of V and let l SVNS-G subset of E represent like this table

	u_1	u_2	u_3	u_4
T_A	0.4	0.7	0.2	0.5
I_A	0.3	0.1	0.5	0.1
F_A	0.5	0.1	0.6	0.4

	u_1u_2	u_2u_3	u_3u_4	u_4u_1
T_B	0.4	0.2	0.2	0.4
I_B	0.3	0.5	0.5	03
F_B	0.5	0.6	0.6	0.5

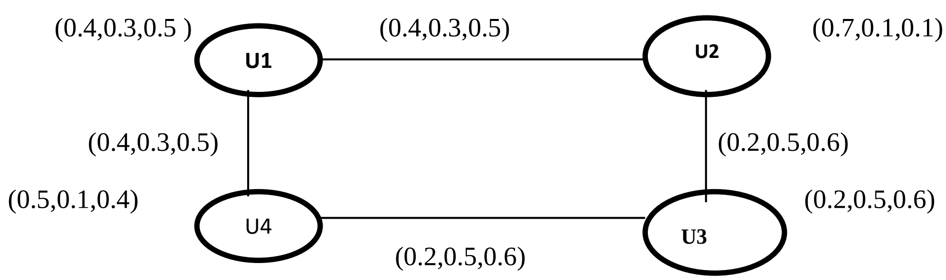

Figure 4: Strong single- valued neutrosophic graph

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Definition 2.14:[9]

Consider the subset of a SVNSs on a space X , written as (X,τ) is named a neutrosophic topological space if the following conditions state

i. $\tilde{X}$ and $\tilde{\emptyset} \in \tau$,

ii. If a neutrosophic set involve in τ then the union of any set must involve in the subset τ .

iii. If two neutrosophic set involve in τ Then intersection of two sets must be in τ .

We called the pair (X,τ) SVNS-T "single value neutrosophic topology" on X. notice that, any elements of τ are called open NSs in X. A is called closed NSs if $A^c \in \tau$, since $A \in N(X)$.

Remark 2.15:

By Definition 2.14, $\tilde{X}$ should contain a complete information. Hence, a degree of indeterminacy is 0 and a degree of non- membership is **0** and a degree of membership is **1**. In the same way $\tilde{\emptyset}$ should have a complete uncertainty. Such that , a degree of indeterminacy is 1 and degree of non-membership is **1** and degree of membership is **0**.

Definition 2.16:[16]

The equality of two SVNSs k and l represent as $k = l$, satisfying if and only if $k \subseteq l$ and $l \subseteq k$.

Definition 2.17:

We can find the the union of two SVNSs k and l that it is SVNSs C, represent as $C = k \cup l$, if we define the function of truth-membership, function of indeterminacy-membership and function of falsity- membership are related to the set k and l by the following relations.

$T_C(x) = max(T_k(x), T_l(x))$

$I_C(x) = max(I_k(x), I_l(x))$

$F_C(x) = max(F_k(x), F_l(x))$

Definition 2.18:

We can find the the intersection of two SVNSs k and l have the set C, represent as $C = k \cap l$

$T_C(x) = min(T_k(x), T_l(x))$

$I_C(x) = min(I_k(x), I_l(x))$

$F_C(x) = min(F_k(x), F_l(x))$

Example 2.19:

Suppose that k is SVNSs of X defined by $k = \{(0.3, 0.4,0.5), (0.5, 0.2,0.3), (0.7, 0.2,0.2)\}$ and suppose that l a SVNSs of X written as by l = [illegible]

$k \cup l$ = {(0.6,0.4,0.2), (0.5,0.2,0.3), (0.7,0.2,0.2)}

$k \cap l$ = {(0.3,0.4,0.5),(0.3,0.2,0.6),(0.4,0.2,0.5)}.

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3. Neutrosophic Topological Graph

Take a graph $\breve{\boldsymbol{G}}$ the set of vertices $\boldsymbol{V} = \{v_1, v_2, v_3, v_4\}$, and the set of edges is $\boldsymbol{E}=\{v_1v_2, v_2v_3, v_3v_4, \boldsymbol{v_4v_1}\}$. Let $\boldsymbol{k}$ be SVN- subset of $\boldsymbol{V}$ and let $\boldsymbol{l}$ be SVN- subset of $\boldsymbol{E}$, $\boldsymbol{G} = (\boldsymbol{k}, \boldsymbol{l})$ is a strong SVNS-G. Let $\boldsymbol{NE_v} = \boldsymbol{S}$ be the neighborhood of the vertex v for every $\boldsymbol{v_i} \in \boldsymbol{V}$ of a strong SVN-G, we build a sub base for a topology by $S_G=\{\boldsymbol{NE_{vi}} = \boldsymbol{Si}\text{:}\ \boldsymbol{v_i} \in \boldsymbol{V}(\boldsymbol{G}), \boldsymbol{i} \in \boldsymbol{I}\}$ then we formed a base $\boldsymbol{\beta}_G$ by employ the finite intersection of elements of S_G, from the arbitrary union of elements of $\boldsymbol{\beta}_G$, we obtain the topological structure $\boldsymbol{\tau}$ on $\boldsymbol{G}$. In the following theorem, we construct a neutrosophic topological graph which given by definition 2.14 from a strong single value neutrosophic graph $\boldsymbol{G}$ we deal with simple connected strong single value neutrosophic graph in our study.

Theorem 3.1:
Each neutrosophic topological structure τ on SVNS-G, $G = (k, l)$ is neutrosophic topological graph.
Proof:
Let τ be a neutrosophic topological structure for a single valued neutrosophic graph $G = (k, l)$. Now, we must state that τ is a neutrosophic topological graph.
(i) Let $Ai \in \tau$, $Ai = \cup_{\boldsymbol{i\in I}} Bi$, where $Bi \in \boldsymbol{\beta_G}$, $Bi = \cap_{\boldsymbol{i=1}}^{\boldsymbol{n}} Si$, where $Si \in S_G, Si = NE_{v_i}, v_j \in Ai$, then $Ai = \cup_{i\in I} (\cap_{\boldsymbol{i=1}}^{\boldsymbol{n}} NE_{v_i})$, then $NE_{v_i} = (\cap_{\boldsymbol{j\neq i}}^{\boldsymbol{n}} vj)$, where vj ={all neutrosophic neighborhood of v_i, $i\in$ I} for all $v_i \in G$, NE_{v_i}called neutrosophic open neighborhood of v_i ,and then $\cup_{\boldsymbol{i\in I}} ((\cap_{\boldsymbol{j\neq i}}^{\boldsymbol{n}} vj), \in \boldsymbol{\tau}, v_j \in Ai$ and $\cup_{\boldsymbol{i\in I}} Ai \in \tau$.
(ii) Let Ai, $C_i \in \tau$, $Ai = \cup_{\boldsymbol{i\in I}} Bi_i$, where $Bi \in \boldsymbol{\beta}_G$, $Bi = \cap_{\boldsymbol{i=1}}^{\boldsymbol{n}} Si$, where $Si \in SG, Si = NE_{v_i}$, $vi \in Ai$, then $Ai = \cup_{\boldsymbol{i\in I}} (\cap_{\boldsymbol{j=1}}^{\boldsymbol{n}} NE_{v_i})$. Then $Ci = \cup_{\boldsymbol{i\in I}} (\cap_{\boldsymbol{i=1v}}^{\boldsymbol{n}} NE_{v_i})$, there are two cases :
Case I: If there are no elements in intersection, i.e, $Ai \cap Ci = \tilde{\emptyset}$, since $\tilde{\emptyset} \in \tau$, then $A_i \cap C_i \in \tau$.
Case II: suppose that there
is an elements in the intersection $Ai \cap Ci$, then we denote it $\{y_n\text{:}\ n \in N\}$. So $\{y_n\} \in NE_{v_i}$, $Ai = (\cup_{\boldsymbol{i\in I}} (\cap_{\boldsymbol{j=1}}^{\boldsymbol{n}} NE_{v_i})$ and $Ci = \cup_{\boldsymbol{i\in I}} NE_{v_i}$ So $\{y_n\text{:}\ n \in N\}$ one of these sets.
Therefore $\{y_n\text{:}\ n \in N\} \in \tau$. Let us give an examples to explain the above theorem.

Example 3.2:
Let $G = (k, l)$ be a strong SVNS-G , as in example 2.13**,** We construct a neutrosophic topological space as follows: let NE_v be the neutrosophic open neighborhood for every $v_i \in V$ in G so we have the following sets: $NE_{v_1}=\{v_4 , v_2\}$, $NE_{v_2}=\{v_1, v_3\}$, $NE_{v_3} =\{v_4, v_2\}$, $NE_{v_4} = \{v_1, v_3\}$.
We note that $NE_{v_1} = NE_{v_3}$
$NE_{v_1} = \{(0.4,0.2,0.5), (0.6,0.3,0.2)\}$, $NE_{v_2} = \{(0.5,0.1,0.4), (0.2,0.3,0.4)\}$, $NE_{v_3} = \{(0.4,0.2,0.5), (0.6,0.3,0.2)\}$, $NE_{v_4} = \{(0.5,0.1,0.4), (0.2,0.3,0.4)\}$
$S_G = \{\{(0.4,0.2,0.5) , (0.6,0.3,0.2)\}, \{(0.5,0.1,0.4), (0.2,0.3,0.4)\}\}$
$\beta_G = \{\{(0.4,0.2,0.5), (0.6,0.3,0.2)\}, \{(0.5,0.1,0.4), (0.2,0.3,0.4), \}\{(0.4,0.2,0.5)(0.2,0.3,0.4)\}\}$
$\tau = \{ \tilde{X}, \tilde{\emptyset}, \{(0.4,0.2,0.5) , (0.6,0.3,0.2)\}, \{(0.5,0.1,0.4), (0.2,0.3,0.4)\}\{(0.4,0.2,0.5)(0.2,0.3,0.4)\},$
$\{(0.5,0.1,0.4), (0.6,0.3,0.2)\}\}$.
We can take another example by taking one types of SVNS-G that we write it above

Definition 3.3:[13]
SVNS-G, $G = (k, l)$ of $\breve{G} = (V, E)$ named complete SVNS-G by state the following statements

$T_l(v_i, v_j) = \min [T_K(v_i), T_K(v_j)]$
$I_l(v_i, v_j) = \max [I_K(v_i), I_K(v_j)]$
$F_l(v_i, v_j) = \max [F_K(v_i), F_K(v_j)]$ consider that v_i, v_j are vertices in the set of vertex V of our graph G.

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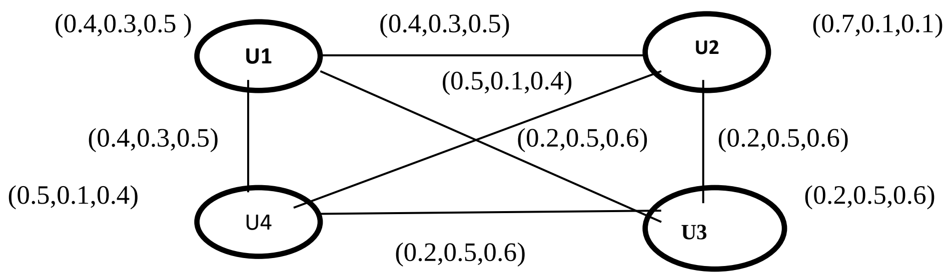

Figure 5: Complete- single valued neutrosophic − graph

Corollary 3.4:
Any complete SVNS-G is a strong SVNS-G.
Proof:
The proof directly from definition 2.12, 3.3.

Corollary 3.5:
Suppose G is complete -SVNS-G then (G, τ) is NT-G.
Proof:
From theorem 3.1 that if G is SSVN-G then (G, τ) is NT-G and by corollary 3.4, G is strong SVNS-G then (G, τ) is NT-G.

Theorem 3.6:[12]
If G is a complete SVNS-G then it is a regular SVNS-G.

Corollary 3.7:
If $\boldsymbol{G}$ is complete SVNS-G then $(\boldsymbol{G}, \boldsymbol{\tau})$ is a regular NT-G.
Proof:
From theorem 3.6, $\boldsymbol{G}$ is a regular SVNS-G and from corollary 3.5 then $(\boldsymbol{G}, \boldsymbol{\tau})$ is NT-G.

Theorem 3.8:
Let $\boldsymbol{k_n}$ be a complete SVNS-G with n vertices, where $\boldsymbol{V}(\boldsymbol{k_n}) = \{\boldsymbol{v_i}\}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{n}$. Then the neutrosophic topological space generated by $\boldsymbol{k_n}$, ($\boldsymbol{n} \neq \boldsymbol{2}$) is non-discrete neutrosophic topological space on $\boldsymbol{k_n}$.

Proof:
According to definition 2.9, the vertices neutrosophic neighborhoods relation of a set $\boldsymbol{V}(\boldsymbol{k_n}) = \{\boldsymbol{v_i}\}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{n}$, is defined as $\boldsymbol{NE_{vi}} = \{(\boldsymbol{v_{i+1}}, \boldsymbol{v_{i+2}}, \ldots, \boldsymbol{v_n})\}$ $\forall\, \boldsymbol{v} \in$ V($\boldsymbol{k_n}$), therefore $\boldsymbol{v_{j \neq i}} \in N$E($\boldsymbol{v_i}$), $\forall\, \boldsymbol{v} \in$ V($\boldsymbol{k_n}$), then for any $1 \leq i \leq n$, $\boldsymbol{s_{k_n}} = \{\boldsymbol{NE_{vi}}\}$therefore $(\cap_{\boldsymbol{j=1 j \neq i}}^{\boldsymbol{n}} \boldsymbol{NE_{vj}}) \cap (\cap_{\boldsymbol{i=1 j \neq i}}^{\boldsymbol{n}} \boldsymbol{NE_{vi}}) \neq \{\boldsymbol{v_i}\} \in \boldsymbol{\beta_{k_n}}$.
Then $\cup$ ($(\cap_{\boldsymbol{j=1 j \neq i}}^{\boldsymbol{n}} \boldsymbol{NE_{vj}}) \cap (\cap_{\boldsymbol{i=1 j \neq i}}^{\boldsymbol{n}} \boldsymbol{NE_{vi}})$) $\neq \{\boldsymbol{v_i}\} \in \boldsymbol{\beta_{k_n}}$. So $\{\boldsymbol{v_i}\} \notin \boldsymbol{\beta_{k_n}} \forall\, \boldsymbol{v} \in$ V($\boldsymbol{k_n}$), then $\{\boldsymbol{v_i}\} \notin \tau$.

For a following corollary we need the following definitions and proposition.

Definition 3.9:[13]
The complement of complete SVNS-G , $G = (k, l)$ on SVNS-G , $\breve{G}$
1.$k = \bar{k}$
2.$\overline{T_k}(vi)=T_K(vi)$, $\overline{I_k}(vi)=I_k(vi)$, $\overline{F_k}(vi)= F_K(vi)$ for all vi $\in G$
3. $\overline{T_k}(vi, vj)=min[T_K(v_i),T_K(v_j)]- T_l(v_i, vj)$
$\overline{I_k}(vi.vj)=max[I_k(vi), I_k(vj)]- I_l(vi, vj)$
$\overline{F_k}(vi, vj) = max[F_K(vi), F_k(vj)]- F_K(vi, vj)$ for all $vi, vj \in G$.

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Definition 3.10:[13]
Suppose that $G = (k, l)$ is SVNS-G . We obtain from Definition (3.9) , that $\bar{\bar{G}}$ that given by the SVNS-G $\bar{\bar{G}} = (\bar{V}, \bar{\bar{E}})$, $k = \bar{\bar{k}}$.
$\overline{\overline{T_k}}(vi, vj)$=$min[T_K(v_i), T_K(v_j),]- T_l(v_i, vj)$
$\overline{\overline{I_k}}$ $(vi.vj)$=$max[I_k(vi), I_k(vj)]- I_l(vi, vj)$
$\overline{\overline{F_k}}$ (vi, vj) = $max[F_K(vi), F_k(vj)]- F_K(vi, vj)$ such that vi and vj are vertices in NT-G .

Proposition 3.11:[13]
If $\boldsymbol{G}$ is a strong SVNS-G then $\boldsymbol{G = \bar{\bar{G}}}$ **.**

From above we have the following corollary.

Corollary 3.12:
Providing that (G, τ) be SVNS-topological graph then $(\bar{\bar{G}}, \tau)$ called NT-G.
Proof:
From the definition 3.9, 3.10 and proposition 3.11 therefore $G = \bar{\bar{G}}$ then $(\bar{\bar{G}}, \tau)$ called a NT-G.

Definition 3.13:[13]
Providing that (G, τ) be NT-G put Y as a non-empty subset of G. That we have Y as a neutrosophic relative topology we defined it as τ_Y =A∩$\bar{Y}$, $A \in \tau$.

$$\boldsymbol{\bar{Y}(x)} = \begin{cases} (\mathbf{1}, \mathbf{0}, \mathbf{0}) & \boldsymbol{if\ x \in Y} \\ (\mathbf{0}, \mathbf{1}, \mathbf{1}) & \boldsymbol{otherwis} \end{cases}$$

The following example show the neutrosophic subspaces of a SVN-topological graph in example 3.2.

Example 3.14:
Let $\boldsymbol{G = (k, l)}$ be strong SVNS-G, as in example 2.11, and providing that $(\boldsymbol{G}, \boldsymbol{\tau})$ be a NT-G.
$\tau = \{ \tilde{\boldsymbol{X}}, \tilde{\emptyset}$, {{(0.4,0.2,0.5), (0.6,0.3,0.2)}, {(0.5,0.1,0.4), (0.2,0.3,0.4)}{(0.4,0.2,0.5), (0.2,0.3,0.4)}, {(0.5,0.1,0.4), (0.6,0.3,0.2)}}. Let Y ={(0.5,0.1,0.4),(0.6,0.3,0.2)}. Let A_i ∈ τ.
$\boldsymbol{A_1} \cap \boldsymbol{\bar{Y}}$ = $\{(\mathbf{0.4}, \mathbf{0.2}, \mathbf{0.5}), (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.2})\} = \{(\mathbf{0}, \mathbf{1}, \mathbf{1}), (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.2})\}$
$\boldsymbol{A_2} \cap \boldsymbol{\bar{Y}} = \{(\mathbf{0.5}, \mathbf{0.1}, \mathbf{0.4}), (\mathbf{0.2}, \mathbf{0.3}, \mathbf{0.4})\} = \{(\mathbf{0.5}, \mathbf{0.1}, \mathbf{0.4}), (\mathbf{0}, \mathbf{1}, \mathbf{1})\}$
$\boldsymbol{A_3} \cap \boldsymbol{\bar{Y}} = \{(\mathbf{0.4}, \mathbf{0.2}, \mathbf{0.5}), (\mathbf{0.2}, \mathbf{0.3}, \mathbf{0.4})\} = \{(\mathbf{0}, \mathbf{1}, \mathbf{1}), (\mathbf{0}, \mathbf{1}, \mathbf{1})\}$
$\boldsymbol{A_4} \cap \boldsymbol{\bar{Y}}$=$\{(\mathbf{0.5}, \mathbf{0.1}, \mathbf{0.4}), (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.2})\} = \{ (\mathbf{0.5}, \mathbf{0.1}, \mathbf{0.4}), (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.2})\}$.
τ_Y ={$\bar{Y}$, $\tilde{\emptyset}$, {(0,1,1), (0.6,0.3,0.2) }, {(0.5,0.1,0.4), (0,1,1)}, { (0.5,0.1,0.4), (0.6,0.3,0.2)}}.
Then (Y, τ_Y) is neutrosophic subspace of a SVNS-topological graph.

Theorem 3.15:
Let (Y, τ_Y) be a neutrosophic subspace of a SVNS - topological graph (G, τ) if S is open-NS subset in τ_Y and $\bar{Y}$ is open-NSs in τ then S is open-NSs in τ.
Proof:
Providing that (Y, τ_Y) be a neutrosophic subspace from (G,τ), SVNS-topological graph. Let S be an open–NSs of Y which implies that $S \in \tau_Y$, that is $\bar{Y}$ an open-NSs of τ that $\bar{Y} \in \tau$, we need to show that s is an open-NSs of G, since S is open-NSs of Y, S = $A_i \cap \bar{Y}$ for $A_i \in \tau$, since A_i and $\bar{Y}$ are open-NSs, by third axiom of NT-G their intersection is neutrosophic open in τ. That is $S = A_i \cap \bar{Y} \in \tau$. Hence, S is open NSs in G.

4. Neutrosophic Neighborhood Closed Set (NNCs)

Definition 4.1:
Let O be a subset of vertices of N-G, and NE_O denote the vertices of V(G)\O which has a neutrosophic neighbourhood in O. Then a neutrosophic neighborhood complement of O, denoted by $\bar{O}$, is the complement of $N(O) \cup O$ in V(G).

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Definition 4.2:
A subset A in a NT-G, (G, τ) is NNCs if its neutrosophic neighborhood complement $\overline{O}$ of a N-G is open N-G in τ.

Theorem 4.3:
The $\widetilde{\emptyset}$, $\overline{Y}$ in a subspace N-G of a SVNS-topological graph (G, τ) are NNCs.
Proof:
From definition 4.1 neutrosophic neighborhood complement of $\widetilde{\emptyset}$ is $\tilde{X}$, $\tilde{X} \in \tau$ so by definition 4.2, $\widetilde{\emptyset}$ is NNCs.
By the same way we can prove that $\overline{Y}$ is NNCs.

Theorem 4.4:
Every subset of a complete NT-G is NNCs.

Proof:
Providing that O be any subset of a complete SVNS-G.
Then a neutrosophic neighborhood complement of O denoted by $\overline{O} = \tilde{\emptyset}$ and $\tilde{\emptyset} \in \tau$ then O is NNCs.

Theorem 4.5:
Intersection of two NNCs O, P is NNCs if $\overline{O} \cap \overline{P} = \overline{O \cap P}$.
Proof:
Suppose we have O, P two NNCs. Then a neutrosophic neighborhood complement of O, P denoted by $\overline{O}$, $\overline{P} \in \tau$.
Then $\overline{O} \cap \overline{P} \in \tau$ so $\overline{O \cap P} \in \tau$ then $O \cap P$ is NNCs.

Theorem 4.6:
If we have two NNCs O, P then their union is NNCs if $\overline{O} \cup \overline{P} = \overline{O \cup P}$.
Proof:
We have two NNCs. Then a neutrosophic neighborhood complement of O, P represented as $\overline{O}$, $\overline{P} \in \tau$.
Then $\overline{O} \cup \overline{P} \in \tau$ so $\overline{O \cup P} \in \tau$ then $O \cup P$ is NNCs.

5.Conclusions

In this paper we find the structure of a neutrosophic topological graph $(\boldsymbol{G}, \boldsymbol{\tau})$ from the base consist from the set of a neutrosophic neighborhoods of some vertices of a strong single valued neutrosophic graph generated a neutrosophic topological graph $(\boldsymbol{G}, \boldsymbol{\tau})$, then we applied this idea on some types of a neutrosophic graph and the complement of those graph to state some results. for first time we define a new term NNCs and we state the relation of the union and intersection of two NNCs.

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