symmetry

MDPI

Article

A Systematic Formulation into Neutrosophic Z Methodologies for Symmetrical and Asymmetrical Transportation Problem Challenges

Muhammad Kamran [1,2,†], Manal Elzain Mohamed Abdalla [3,*,†], Muhammad Nadeem [4,†], Anns Uzair [1,†], Muhammad Farman [5,6,†], Lakhdar Ragoub [7,†] and Ismail Naci Cangul [2,*,†]

[1] Department of Mathematics, Thal University Bhakkar, Bhakkar 30000, Pakistan; kamrankfueit@gmail.com (M.K.)
[2] Department of Mathematics, Uludag University, Bursa 16059, Turkey
[3] Department of Mathematics, College of Sciences and Arts (Muhyil), King Khalid University, Muhyil 61421, Saudi Arabia
[4] Department of Mathematics, Lahore Garrison University, Lahore 54810, Pakistan
[5] Department of Mathematics, Faculty of Arts and Sciences, Near East University, Nicosia 99138, Turkey
[6] Department of Computer Science and Mathematics, Lebanese American University, Beirut 11022801, Lebanon
[7] Mathematics Department, Prince Mugrin University, Al Madinah 42241, Saudi Arabia
* Correspondence: mabdalla@kku.edu.sa (M.E.M.A.); cangul@uludag.edu.tr (I.N.C.)
† These authors contributed equally to this work.

Abstract: This study formulates a multi-objective, multi-item solid transportation issue with parameters that are neutrosophic Z-number fuzzy variables such as transportation costs, supplies, and demands. This work covers two scenarios where uncertainty in the problem can arise: the fuzzy solid transportation problem and the interval solid transportation problem. The first scenario arises when we represent data problems as intervals instead of exact values, while the second scenario arises when the information is not entirely clear. We address both models when the uncertainty alone impacts the constraint set. In order to find a solution for the interval case, we generate an additional problem. Since this auxiliary problem is typical of solid transportation, we can resolve it using the effective techniques currently in use. In the fuzzy scenario, a parametric method is used to discover a fuzzy solution to the earlier issue. Parametric analysis identifies that the best parameterized approaches to complementary problems are characterized by the application of parametric analysis. We present a suggested algorithm for determining the stability set. Finally, we provide a numerical example and sensitivity analysis for the transportation problem, which is both symmetrical and asymmetrical.

Keywords: optimization; neutrosophics set; Z-numbers; transportation problem; optimal solution

Citation: Kamran, M.; Abdalla, M.E.M.; Nadeem, M.; Uzair, A.; Farman, M.; Ragoub, L.; Cangul, I.N. A Systematic Formulation into Neutrosophic Z Methodologies for Symmetrical and Asymmetrical Transportation Problem Challenges. *Symmetry* **2024**, *16*, 615. https://doi.org/10.3390/sym16050615

Academic Editors: Yunfei Fang, Peng Wu, Jian-Qiang Wang and Hsien-Chung Wu

Received: 6 April 2024
Revised: 30 April 2024
Accepted: 10 May 2024
Published: 15 May 2024

1. Introduction

The transportation problem (TP) was first stated by Hitchcock [1] and later by Balinski [2] as a linear programming problem (LPP). It is among operations research's most significant and useful application-based fields. The TP's goal is to reduce the cost of distributing goods from multiple sources or origins to multiple destinations. It is a specific kind of linear programming problem. The standard simplex method is unsuitable for handling transportation-related problems due to its distinctive framework. These issues call for a unique approach to resolution. A transportation issue first manifests itself at the point of shipment dispatch. A transportation issue originates at the point of conveyance of shipments. The price of moving a single unit of the shipment from its origin to its destination is known as the unit transportation cost [3]. We can divide transportation challenges into two categories: balanced and unbalanced. As is common knowledge, a transportation problem is a specific kind of linear programming problem (LPP) in which the objective is to minimize the overall cost of transportation by moving products from a set of origins to a set of destinations depending on the availability and demand of the sources and destinations. The transportation problem is considered balanced when the supply and demand are equal. An unbalanced TP is one where demand and supply are not equal. Depending on the situation, we introduce a dummy row or dummy column to ensure balance in this type of problem. Subsequently, we can handle the issue in a manner similar to that of a balanced problem.

If we describe the TPs as conventional LPPs, we can use the well-known simplex approach to solve them. There are multiple approaches to identifying a basic, feasible solution for the transportation issue. These techniques include Vogel's approximation approach, row minima, column minima, north-west corner, and matrix minima. In addition, there are other evolutionary methods available to address the transportation issue. An enhanced Vogel's approximation method for TPs was presented by Korukoglu and Balli [4]. It has long been believed that the decision-maker in traditional TPs is confident in the exact figures of the product's availability, demand, and transportation costs. However, the fluctuating economic landscape's instability makes it impossible to identify or address all aspects of the transportation problem. Therefore, TP estimates are the product of imprecise information and uncertainty. Due to economic uncertainties brought on by changing conditions, the decision-maker typically expects to have sufficient resources, requests, and transportation costs to meet transportation restrictions, even though not all transportation information is accurate or up-to-date. Since information is partial and uncertainty exists, the TP measure is an approximation. Fuzzy sets (FS) [5–7] are a crucial tool that academics in the domains of engineering and business have introduced to address the uncertainties that were first proposed by Zadeh [8]. It is true that FS has been defined as ambiguous and capable of handling hesitancy when it comes to real-world issues [9,10].

Atanassov [11] expanded fuzzy set to intuitionistic fuzzy set (IFS) by including a degree of non-membership. Currently, IFS is a useful method for handling both uncertainty and undesirables. The neutrosophic set (NS) introduces an expansion of the IFS that offers more regions for handling uncertainty; this is briefly explored in [12,13]. Additionally, Wang et al. [14] introduced the single-valued neutrosophic set, which is the NS's extension. It has been applied recently by a number of researchers in numerous technical and management sectors [15–18]. Furthermore, the lack of a clear definition for membership and non-membership degrees results in an inadequate representation of doubt and hesitancy in real-life situations. As a result, we admit a certain amount of additional ambiguity. Zimmermann [19] demonstrated that fuzzy linear programming consistently yields optimal and effective solutions. A fuzzy linear programming (LP) approach was presented by Chanas et al. [20] to address transportation issues involving precise costs, fuzzy supply, and fuzzy demand. The concept of the ideal solution for the transportation issue with fuzzy coefficients represented with the help of fuzzy numbers was put forth by Chanas and Kuchta [21], who also created an algorithm for locating the ideal solution. Subsequently, a number of scholars have put the transportation issues into distinct fuzzy contexts. Ishii and Tada [22] developed a widely known transportation problem that is frequently described using fuzzy environments as a bipartite network that has two node-sets, i.e., sets that include supply (or factory) and demand (or warehouse) nodes. Hashmi et al. [23] developed and examined a multi-objective approach for the two-stage fixed charge transportation strategy problem. Transportation is considered to occur from manufacturing facilities to distributors, and finally from distributors to customers. We regard the capacity of distributors, client demand, and manufacturing plant availability as fuzzy metrics. Celik, Erkan, and Emre Akyuz [24] created Interval type-2 fuzzy sets that are used in maritime transportation to choose the right shiploader type. Singh et al. [25] formulated a transportation problem with triangular intuitionistic fuzzy integers as expenses. Numerous other scholars extend the TPs in other fuzzy environments in an effort to minimize uncertainty and lower transportation costs [26–32].

The current manuscript addresses the TPs known as neutrosophic Z-numbers using an innovative approach. Z-numbers with neutrosophic information [33,34] encompass not only accurate values but are also exceptionally competent at capturing complex and uncertain information; yet, the previously utilized approaches have some limits. Their distinctiveness makes them ideal for TPs. Neutrosophic Z-numbers offer a thorough measure of uncertainty, making it possible to analyze travel conditions in greater detail. By offering decision-makers a greater range of options and results, this strategy improves decision making. We are embracing neutrosophic Z-numbers to open the door to more resilient and adaptable solutions for navigation issues in dynamic and unpredictable contexts.

1.1. Key Points of the Study

The following list includes the key points.

- This research work presents a novel approach for solving navigation problems using neutrosophic Z-numbers, which provides a unique approach to dealing with uncertainties.
- It acknowledges many unknowns inherent in travel data, including irregular travel patterns and changing demands, which are often overlooked by conventional methods.
- It shows a significant improvement in the ability to deal with uncertainties compared to traditional methods, leading to efficient and reliable solutions to transportation problems.
- We provide a practical demonstration of the proposed method by applying the developed algorithm to numerical examples, demonstrating its effectiveness in real-world situations.

1.2. Main Contributions

In this manuscript, we use neutrosophic Z-numbers to solve transportation problems. When it comes to transportation-related problems, such as irregular visitation patterns and fluctuating demand, there are many unknowns. Because traditional approaches cannot control these uncertainties to the required degree, they usually yield less-than-ideal results. However, by adding neutrosophic Z-values to the issue-solving method, we considerably increase the anomaly tolerance.

1. The goal of this research is to examine how neutrosophic Z-numbers (NZNs) adapt and function in a variety of domains when faced with unpredictable situations in transportation difficulties.
2. Explain the fundamental roles of algorithms in the context of NZNs and gain an understanding of the underlying ideas.
3. Development of a new algorithm specifically designed for the derived set that increases the efficiency of transportation problem solving.
4. Describe and analyze the algorithm in NZNs in detail, taking into account the most important aspects to obtain relevant data.
5. Creation of MATLAB code to facilitate the implementation of the proposed framework, providing a user-friendly tool for researchers and professionals in the field.
6. Application of the developed algorithm in mathematical models, demonstration of its practical efficacy and potential for real-world application.
7. Comprehensive solutions to balanced and unbalanced transport problems, offering comprehensive strategies for dealing with a variety of situations.

Motivation

Solving navigation problems using fuzzy set theory provides a powerful approach with practical applications in real-life situations. Consider, for example, a city's public transport system that aims to optimize bus routes based on uncertainties such as changes in passenger needs, heterogeneous traffic conditions, and subjective measurement methods struggle to capture these uncertainties accurately, resulting in poor road design. But the use of fuzzy set theory enables decision-makers to represent passenger needs and traffic congestion as fuzzy sets, providing a more nuanced understanding of system dynamics. This approach enables subjective preferences to be synthesized so that small changes in passenger numbers or traffic patterns do not significantly hinder the optimization process. In this context, leveraging fuzzy set theory enables transport authorities to design efficient and flexible bus routes that meet passenger needs, thus operating more efficiently and improving overall system performance.

In 2011, Zadeh introduced a concept which significantly expedited the use of fuzzy numbers [35]. Z-numbers' exact method of combining ordered pairs with fuzzy numbers allows them to evaluate the reliability and limitations of human knowledge. This novel method improves the reliability tier representation in modern fuzzy units, which is very helpful for making judgments about uncertainty. Z-numbers have been used extensively and in a variety of circumstances. They are essential for uncertainty assessments, as [36], for example, shows, especially when dealing with missing or inaccurate data. This component's Z-statistic, with varying degrees of confidence across multiple sources, can provide a comprehensive understanding of modified real-world scenarios and identify which ones require filtering. The use of Z-numbers can be very useful for navigation problems, which are highly complex and include many sources of uncertainty, such as dynamic traffic conditions, unforeseen problems, and changing demand. But travel planners and decision-makers can use Z-numbers to gain useful new insights into the dependence of their data inputs and even greater insight into the strengths of their solutions of the proposed. Moreover, the flexibility of Z-numbers goes beyond transportation and finds applications in various areas where uncertainty is widespread, e.g., Z-numbers can help supply chain managers implement supply chain management strategies more effectively used to account for uncertainty related to timing, clients and demand forecasts. Consideration of available resources can also contribute to optimizing patient referral strategies in health care. Z-numbers are incredibly useful tools for decision-makers negotiating the complexity of uncertain settings owing to their comprehensive character. Organizations can produce more robust and flexible approaches that are better suited for real-world dynamics by embracing and leveraging the inherent uncertainties instead of trying to eliminate them. Neutrosophic Z-numbers have the potential to transform the formulation of decisions in several fields and lead to more reliable and strong results in an uncertain world. This potential will certainly grow as research in this area continues to advance.

Advantages: Numerous benefits that significantly improve modelling, analysis, and decision-making in the field come from the use of neutrosophic Z-values in mobility analysis. These include incompleteness, ambiguity, and uncertainty regarding the mobility dynamic processing of measurements and information in travel data and metrics. Accurate risk assessment and sensitivity analysis are made possible by well-captured neutrosophic Z-values, which empower stakeholders to make informed decisions in the face of uncertainty. Additionally, the adaptability of neutrosophic Z-numbers facilitates the incorporation of many viewpoints and stakeholder views, promoting openness, interpretation, and collaborative decision-making in the planning and management of transportation. Through the promotion of research frontiers and innovation, neutrosophic Z-numbers facilitate the development of sustainable and efficient transport solutions that address the changing demands and obstacles of contemporary civilizations.

The remainder of the document is organised as follows: Section 2 presents some basic definitions based on Z-numbers and the neutrosophic set. Section 3 discusses the Crisp Transportation mathematical model with annotations. The Proposed Models in the NZN Environment for Transportation are introduced in Section 4. Sensitivity analysis is explained in detail in Section 6 and transportation problems are addressed with examples in Section 5. A description of limitations is provided in Section 7. Section 8 provides conclusions and prospective research scopes.

2. Preliminaries

We are dealing with neutrosophic Z-numbers in this research work, which are a further extension, or hybrid form, of Z-numbers and neutrosophic numbers. Therefore, before we begin working with neutrosophic Z-numbers, let us take a quick look at Z-numbers and neutrosophic numbers to understand what we are working with, where our working methods came from, and which numbers we are looking into in the future.

Definition 1. *Let us take into consideration an elementary set $\mathfrak{B}^Y$ that is not empty. An FS [8] $\mathbb{Z}^Y$ in $\mathfrak{B}^Y$ is defined formally with a mapping $T_{\mathbb{Z}^Y} : \mathfrak{B}^Y \to [0,1]$, where $\mathfrak{D}_{\mathbb{Z}^Y}(\curlywedge)$ is known as MD of $\curlywedge$, i.e.,*

$$\mathbb{Z}^Y = \left\{ \langle \curlywedge, T_{\mathbb{Z}^Y}(\curlywedge) \rangle | \curlywedge \in \mathfrak{B}^Y \right\},$$

Definition 2. *An IFS [11] A^Y in δ is mathematically represented as:*

$$A^Y = \left\{ \left\langle \curlywedge, T^Y(\curlywedge), F^Y(\curlywedge) \right\rangle | \curlywedge \in \delta \right\},$$

here $\curlywedge \in \delta, T^Y(\curlywedge)$ is known as MD and $F^Y(\curlywedge)$ is NMD for IFS A, where $(T^Y(\curlywedge), F^Y(\curlywedge)) \in [0,1]$, satisfying

$$0 \le (T^Y(\curlywedge) + F^Y(\curlywedge)) \le 1.$$

Definition 3. *Let us take a universal set "A". Now a "K^A" is a Z-number [35] that is in fact an order pair restraining fuzzified numbers, such that it is of the following form explained in [37],*

$$K^A = [(P^A, Q^A)(a_i)]$$

and the set of Z-numbers can be written as,

$$S = [< ai, (P^A, Q^A)(a_i) > | a_i \in A]$$

where "P^A" is the fuzzy value while "Q^A" is the value of reliability measure of "P". Here a_i shows that it is arbitrary and can behave for any element of the set A. Also the numeric values $P^A \in [0,1]$ and $Q^A \in [0,1]$.

Definition 4. *Suppose we have a universal set "A". Then "M^A" is a neutrosophic number [38], which is an ordered triple of three fuzzified numbers from the set "A", of the form given below as,*

$$M^A = [(T^A, I^A, F^A)(a_i)]$$

Also, "N" is the set of various neutrosophic numbers which can be shown mathematically as,

$$N = [< a_i, (T^A, I^A, F^A)(a_i) | a_i \in A]$$

For instance, the numeric values $T^A \in [0,1]$, $I^A \in [0,1]$ and $F^A \in [0,1]$. Where T^A shows the membership degree of truth, I^A denotes the membership degree of indeterminacy and lastly F^A denotes the membership degree of falsity. Also for every a_i existing in A we have the constraints for necessary for the existence of a neutrosophic number that is

$$0 \le T^A + I^A + F^A \le 3.$$

We now have a clear understanding of what Z-numbers and neutrosophic numbers are, thanks to the two definitions we explored prior to this. This means that we can fully comprehend neutrosophic Z-numbers, thus letting us inquire about what NZNs are.

Definition 5. *Assume we have a universal set "A". The neutrosophic Z-number [37] "W^A" is of the form demonstrated mathematically below:*

$$W^A = [T^A(P^A, Q^A)(a_i), I^A(P^A, Q^A)(a_i),$$
$$F^A(P^A, Q^A)(a_i)] = [(T_{P^A}, T_{Q^A})(a_i), (I_{P^A}, I_{Q^A})(a_i), (F_{P^A}, F_{Q^A})(a_i)]$$

However, the set of copious neutrosophic Z-numbers "Y" can be presented as

$$Y = [< a_i, T^A(P^A, Q^A)(a_i), I^A(P^A, Q^A)(a_i), F^A(P^A, Q^A)(a_i) | a_i \in A].$$

Additionally, the numeric values are $T_{P^A} \in [0,1]$, $T_{Q^A} \in [0,1]$, $I_{P^A} \in [0,1]$, $I_{Q^A} \in [0,1]$, $F_{P^A} \in [0,1]$ *and* $F_{Q^A} \in [0,1]$. *Here,* T^A *symbolizes the value of truth membership,* I^A *denotes the value of indeterminacy membership and* F^A *expresses the value of falsity membership in the set "A". Also,* P^A *is the fuzzified number from the set "A" and* Q^A *is the measure of reliability of* P^A. *Additionally, for every* a_i *existing in A we have the following necessary conditions for the existence of NZNs given below:*

$$0 \leq T_{P^A} + I_{P^A} + F_{P^A} \leq 3 \quad \text{and} \quad 0 \leq T_{Q^A} + I_{Q^A} + F_{Q^A} \leq 3.$$

Definition 6. *Suppose that*

$$W_1^A = [T_1^A(P^A, Q^A), I_1^A(P^A, Q^A), F_1^A(P^A, Q^A)] = [(T_{P^A 1}, T_{Q^A 1}), (I_{P^A 1}, I_{Q^A 1}), (F_{P^A 1}, F_{Q^A 1})]$$

and

$$W_2^A = [T_2^A(P^A, Q^A), I_2^A(P^A, Q^A), F_2^A(P^A, Q^A)] = [(T_{P^A 2}, T_{Q^A 2}), (I_{P^A 2}, I_{Q^A 2}), (F_{P^A 2}, F_{Q^A 2})]$$

be two NZNs and $\varrho > 0$. *Then we define the relations associated with NZNs [39] as follows:*

1. $W_1^A \subseteq W_2^A$ *iff*
   $T_{P^A 1} \leq T_{P^A 2}$, $T_{Q^A 1} \leq T_{Q^A 2}$, $I_{P^A 1} \geq I_{P^A 2}$, $I_{Q^A 1} \geq I_{Q^A 2}$, $F_{P^A 1} \geq F_{P^A 2}$ *and* $F_{Q^A 1} \geq F_{Q^A 2}$.
2. $W_1^A = W_2^A$ *iff,*
   $W_1^A \subseteq W_2^A$ *and* $W_1^A \supseteq W_2^A$
3. $W_1^A \cup W_2^A = [(T_{P^A 1} \vee T_{P^A 2}, T_{Q^A 1} \vee T_{Q^A 2}), (I_{P^A 1} \wedge I_{P^A 2}, I_{Q^A 1} \wedge I_{Q^A 2}), (F_{P^A 1} \wedge F_{P^A 2}, F_{Q^A 1} \wedge F_{Q^A 2})]$.
4. $W_1^A \cap W_2^A = [(T_{P^A 1} \wedge T_{P^A 2}, T_{Q^A 1} \wedge T_{Q^A 2}), (I_{P^A 1} \vee I_{P^A 2}, I_{Q^A 1} \vee I_{Q^A 2}), (F_{P^A 1} \vee F_{P^A 2}, F_{Q^A 1} \vee F_{Q^A 2})]$
5. $(W_1^A)^C = [(F_{P^A 1}, F_{Q^A 1}), (1 - I_{P^A 1}, 1 - I_{Q^A 1}), (T_{P^A 1}, T_{Q^A 1})]$ *(Complement of* W_1^A*)*
6. $W_1^A \oplus W_2^A = [(T_{P^A 1} + T_{P^A 2} - T_{P^A 1} T_{P^A 2}, T_{Q^A 1} + T_{Q^A 2} - T_{Q^A 1} T_{Q^A 2}), (I_{P^A 1} I_{P^A 2}, I_{Q^A 1} I_{Q^A 2}), (F_{P^A 1} F_{P^A 2}, F_{Q^A 1} F_{Q^A 2})]$
7. $W_1^A \otimes W_2^A = [(T_{P^A 1} T_{P^A 2}, T_{Q^A 1} T_{Q^A 2}), (I_{P^A 1} + I_{P^A 2} - I_{P^A 1} I_{P^A 2}, I_{Q^A 1} + I_{Q^A 2} - I_{Q^A 1} I_{Q^A 2}), (F_{P^A 1} + F_{P^A 2} - F_{P^A 1} F_{P^A 2}, F_{Q^A 1} + F_{Q^A 2} - F_{Q^A 1} F_{Q^A 2})]$
8. $\varrho W_1^A = [(1 - (1 - T_{P^A 1})^{\varrho}, 1 - (1 - T_{Q^A 1})^{\varrho}), ({I_{P^A 1}}^{\varrho}, {I_{Q^A 1}}^{\varrho}), ({F_{P^A 1}}^{\varrho},, {F_{Q^A 1}}^{\varrho})]$
9. ${W_1^A}^{\varrho} = [({T_{P^A 1}}^{\varrho}, {T_{Q^A 1}}^{\varrho}), (1 - (1 - I_{P^A 1})^{\varrho}, 1 - (1 - I_{Q^A 1})^{\varrho}), (1 - (1 - F_{P^A 1})^{\varrho}, 1 - (1 - F_{Q^A 1})^{\varrho})]$

Definition 7. *For the comparison of NZNs,*

$$W_i^A = [T_i^A(P^A, Q^A), I_i^A(P^A, Q^A), F_i^A(P^A, Q^A)] = [(T_{P^A i}, T_{Q^A i}), (I_{P^A i}, I_{Q^A i}), (F_{P^A i}, F_{Q^A i})]$$

we introduce the score function as

$$L_{(W_i^A)} = \frac{2 + T_{P^A i} T_{Q^A i} - I_{P^A i} I_{Q^A i} - F_{P^A i} F_{Q^A i}}{3}$$

for $L_{(W_i^A)} \in [0,1]$. *Therefore, we can say that if* $L_{(W_1^A)} \leq L_{(W_2^A)}$, *then there is the order* $W_1^A \leq W_2^A$.

3. Existing Model in Crisp Transportation

Let there be $\mathfrak{O}$ sources and $\mathfrak{G}$ destinations. In these transportation problems, our goal is to minimize the cost of fulfilling the demands of various destinations (like markets, shops, distribution centers, etc.) from multiple origins/sources (like powerplants, factories, farms, etc.) in order to maintain financial benefits for the sources. There is a limited supply (maximum quantity that can be produced) of each source while there is a demand (minimum quantity that is needed to be transported to it) to be satisfied of each destination. But

the point is that there are some assumptions for the demand and supply of product that all the restrictions should be crisp. Here
$\mathfrak{O}$ How many sources are there?
$\mathfrak{G}$ How many destinations are there?
i The index of origin for all $\mathfrak{O}$.
j The index of destination for all $\mathfrak{G}$.
q_{ij} The quantity of product that we have to transport from the point of origin to the destination.
T_{ij}^N The cost in neutrosophic Z-numbers per unit quantity that we will carry from the ith origin to the jth destination.
T_{ij} The cost per unit quantity when it is expressed in the form of crisp numbers.
m_{ij} The quantity which is available for supply from each source in crisp environment.
m_{ij}^N The quantity which is available for supply from each source in NZN environment.
n_{ij} The quantity which is to be demanded from each destination in crisp environment.
n_{ij}^N The quantity which is to be demanded from each destination in NZN environment.

Then, the transportation problem in crisp environment is as follows:

$$Min = \sum_{i=0}^{\mathfrak{O}} \sum_{j=0}^{\mathfrak{G}} q_{ij}.T_{ij}$$

Subject to
$$\sum_{j=0}^{\mathfrak{G}} q_{ij} = m_i = Supply, here, i = 1, 2, ..., \mathfrak{O}$$
$$\sum_{i=0}^{\mathfrak{O}} q_{ij} = n_j = Demand, here, j = 1, 2, ..., \mathfrak{G}$$
$$q_{ij} \geq 0 \; \forall \; i, j.$$

4. Proposed Models in NZN Environment for Transportation

In this section, if we interchange the parameter T_ij into neutrosophic Z-number (NZN) parameters T_ij^N, then the updated model that we obtain is called Type I neutrosophic Z-number transportation problem (T1 NZNTP) and its mathematical interpretation is as follows:

$$Min = \sum_{i=0}^{\mathfrak{O}} \sum_{j=0}^{\mathfrak{G}} q_{ij}.T_{ij}^N$$

Subject to
$$\sum_{j=0}^{\mathfrak{G}} q_{ij} = m_i = Supply, here, i = 1, 2, ..., \mathfrak{O}$$
$$\sum_{i=0}^{\mathfrak{O}} q_{ij} = n_j = Demand, here, j = 1, 2, ..., \mathfrak{G}$$
$$q_{ij} \geq 0 \; \forall \; i, j.$$

4.1. Main Algorithm

Step 1 To begin solving the NZN transportation issue, select any model.
In the transportation problem of Type 1 NZN, we have the cost value of the transportation as the neutrosophic Z-numbers while supply and demands are in crisp numbers. In this case, we will apply the score function to find the score value of each neutrosophic Z-number given in the problem either in the form of transportation cost, supply or demand.

Step 2 In this step, we will check whether the transportation problem is balanced or not.
For this, we have to show that $\sum a_i = \sum b_j$

i.e, demand = supply.
If the transportation is unbalanced, then we have to add a dummy row or column to balance the transportation problem.

Step 3 We are going to use Algorithm 1 to find the feasible solution of the given transportation problem.

Step 4 Write a clear and concise formulation of the transportation issue.

Step 5 For the goal function, replace all x_{ij} to obtain the transit cost.
End.

4.2. Algorithm 1

Step I We will use the table values from the first phase of the main algorithm in this stage.

Step II In this step, we will find the difference between the least and next to the least transportation cost and show it in a new column and row as penalty of that column or row.

Step III Find the maximum penalty and allocate the appropriate row or column of the maximum penalty to the cell with the lowest transportation cost.

Step IV May the highest penalty be the same for
Case 1: If there are multiple rows, choose the top row;
Case 2: If there are many columns, choose the column on the left.
Repeat steps III and IV until there is no supply left to fulfill and no demand left to satisfy.

The MATLAB code for the problem is given in Figure 1.

```
Cost = input[a11 a12 a13; a21 a22 a23; a31 a32 a33]
S = input[s1 s2 s3]
D = input[d1 d2 d3]
if sum(S)==sum(D)
   fprintf('Given Transportation Problem is Balanced\n');
else
   fprintf('Given Transportation Problem is UnBalanced\n');
   if sum(S)<sum(D)
      Cost(end+1,:)=zeros(1,size(S,2));
      S(end+1)=sum(D)-sum(S);
   elseif sum(D)<sum(S)
      Cost(:,end+1)=zeros(1,size(S,2));
      D(end+1)=sum(S)-sum(D);
   end
end
ICost = Cost;
A = zeros(size(Cost));
[m,n] = size(Cost);
BFS = m+n-1;
for i=1:m*n
   Col=sort(Cost,1);
   Row=sort(Cost,2);
   pRow=Row(:,2)-Row(:,1);
   pCol=Col(2,:)-Col(1,:);
Col = sort(Cost,1);      %Ascending order
Row = sort(Cost,2);
pRow = Row(:,2)-Row(:,1);%Penalty Row
pCol = Col(2,:)-Col(1,:);          %Penalty Column
R = max(pRow);
%Find maximum penalty row.
C = max(pCol);
%Find maximum penalty column.
Rmax = find(pRow==max(R,C));
%Maximum penalty value
Cmax = find(pCol==max(R,C));
Cr = Cost(Rmax,:);      %Penalty rows.
Cc = Cost(:,Cmax);      %Penalty columns.
if max(pRow)~=max(pCol)
   if max(pRow)>max(pCol)
[rowind,colind]=find(min(min(Cr))==Cost(Rmax,:));
      row1 = Rmax(rowind);        %find row index
      col1= colind;               %preserve column
   else
      [rowind,colind]=find(min(min(Cc))==Cost(:,Cmax));
      row1 = rowind;          %preserve row
      col1= Cmax(colind);      %find column index
   end
   a11 = min(S(row1),D(col1));
[val,ind]=max(a11);      %find max allotment
ii = row1(ind);          %identify row position
jj = col1(ind);          %identify column position
   else
[rowind1,colind1]=find(min(min(Cr))==Cost(Rmax,:));
      row1 = Rmax(rowind1);      %find row index
      col1= colind1;                  %preserve column
      C1 = Cost(row1,col1);      %allocated cost
      [rowind2,colind2]=find(min(min(Cc))==Cost(:,Cmax));
      row2 = rowind2;            %preserve row
      col2= Cmax(colind2);    %find column index
C2 = Cost(row2,col2);                  %allocated cost
if C1<C2
   a11 = min(S(row1),D(col1));
   [val,ind]=max(a11);        %find max allotment
   ii = row1(ind);         %identify row position
   jj = col1(ind);        %identify column position
else
    a11 = min(S(row2),D(col2));
   [val,ind]=max(a11);        %find max allotment
   ii = row2(ind);         %identify row position
   jj = col2(ind);        %identify column position
end
end                                  %End of if loop
b11 = min(S(ii),D(jj));              %Find the value
a(ii,jj)=b11;                      %Assign allocation
S(ii)=S(ii)-b11                  %Reduced row value
D(jj)=D(jj)-b11           %Reduced Column value
Cost(ii,jj)=inf;                       %Cell covered
End
                         %Computation of Initial cost
InitialCost = sum(sum(ICost.*X));
fprintf('Initial BFS Cost = %d\n',InitialCost);
                       %Print Initial Feasible solution
fprintf('Initial BFS = \n');
IB = array2table(X);
disp(IB);
```

- Transportation Cost in Type I and III is in NZN, so we will put their score values here.
- Supply and demand in Type II and III is NZN, so we will put their score values here.
- In this step we are going to examine whether our transportation problem is balanced or unbalanced.
- Now we will see how many allocations are going to exist in our current example.
- In this section we are going to arrange rows and column in ascending order.
- Now we will see the penalty rows and columns by finding difference between least and next to least.
- Next we will investigate whether the penalty row has maximum value or column penalty has and identify which one value is it.
- After identifying the maximum penalty, we will allot the least element in that row or column the amount of supply and demand according to the given values.
- This loop (process) will continue until all the supply and demands are thoroughly exhausted.
- In the next step we will find the initial transportation cost of the allotments and add them to find the Initial feasible solution.

Figure 1. MATLAB code for transportation problem in NZNs.

5. Transportation Problems

Any nation's economic development is heavily dependent on the movement of products. There are many obstacles that the transportation industry must overcome to prevent the efficient and economical transfer of commodities. This article will discuss the main obstacles to the efficient transportation of goods in the global economy, which include non-customs under-rate fuel, inadequate infrastructure, ineffective supply chain management, security issues, regulatory barriers, low freight and limited load availability, and the use of extremely old and non-compliant vehicles. Additionally, it will suggest possible ways to deal with these issues and enhance the nation's entire system for moving commodities. The nation can realise the full potential of its transportation sector by addressing the issues that negatively impact sustainable transportation, such as inadequate infrastructure, ineffective supply chain management, security concerns, regulatory obstacles, low freight rates, limited load availability, and the use of extremely old and non-compliant vehicles, as illustrated in Figure 2. Here, we will discuss different types of transportation problems in which we are going to use neutrosophic Z-numbers as transportation cost, supply, and demand, with different types defined below to address the challenges.

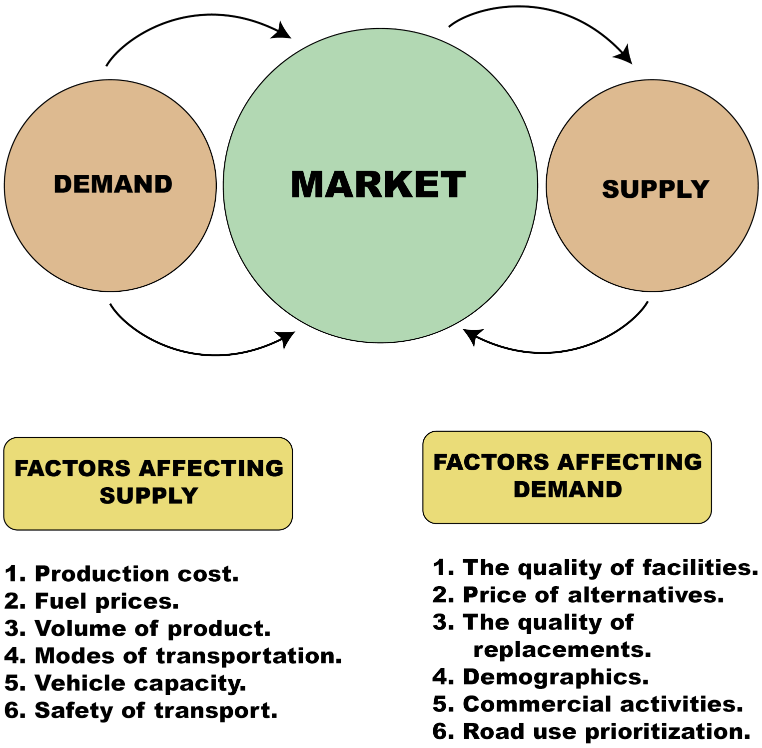

Figure 2. Transportation difficulties.

5.1. Illustrative Examples

In this section, we are going to use two algorithms to find the optimal solution of the given transportation problem. Where the neutrosophic Z-numbers are used as cost, supply and demand for the given transportation problem.

5.2. Balanced Transportation Problem

When there is an equal supply and demand, the transportation problem is said to be balanced.

Type 1 NZN Model

In this type, we are going to study neutrosophic Z-number as transportation cost.

Example 1. *A sugar corporation manufactures sugar in three plants in different areas in the countryside, which is subsequently shipped to three distribution sites by means of several roads. Each factory monthly output volume, each distribution center demand and the associated transportation cost per quintal are provided in Table 1. Where the supply of the first plant is 100 tonnes, supply of second plant is 300 tonnes and supply of third plant is 200 tonnes. Demand of first distribution center is 400 tonnes, demand of second distribution center is 50 tonnes and demand of third distribution center is 150 tonnes. This problem is shown graphically in Figure 3.*

Also "$\mathfrak{S}$" denotes destination of transportation while "$\mathfrak{O}$" denotes origin of the transportation. The transportation cost is given in the form of neutrosophic Z-numbers as

$$W_i^A = [T_i^A(P^A, Q^A), I_i^A(P^A, Q^A), F_i^A(P^A, Q^A)].$$

Keep in mind that the P^A is the neutrosophic value of the given set and Q^A are the components of measures of reliability for P^A. Where $T_i^A(P^A, Q^A)$ shows the value of truth, $I_i^A(P^A, Q^A)$ shows the value of indeterminacy and $F_i^A(P^A, Q^A)$ shows the value of falsity.

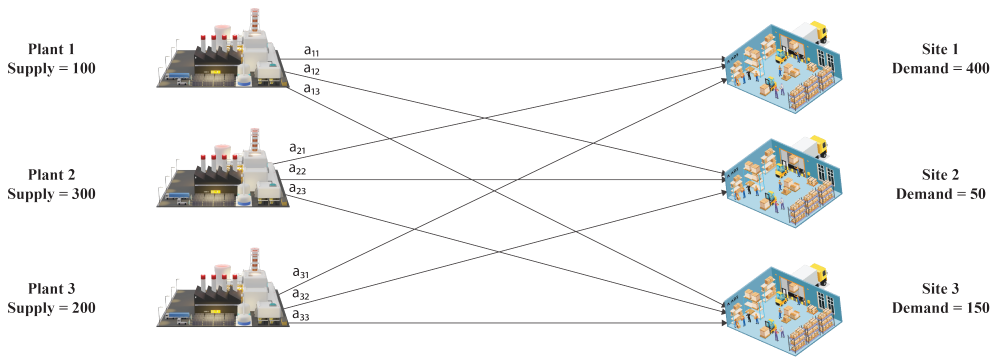

Figure 3. Graphical Representation of the Transportation Problem Given in Example 1.

Table 1. Input neutrosophic Z-numbers for transportation problem.

$\mathfrak{S}$	$\mathfrak{S}_1$	$\mathfrak{S}_2$	$\mathfrak{S}_3$	**Supply**
$\mathfrak{O}_1$	(0.1,0.2),(0.3,0.4),(0.5,0.6)	(0.5,0.9),(0.4,0.8),(0.3,0.7)	(0.9,0.2),(0.8,0.3),(0.7,0.4)	100
$\mathfrak{O}_2$	(0.8,0.4),(0.8,0.8),(0.2,0.9)	(0.1,0.9),(0.9,0.5),(0.5,0.6)	(0.6,0.4),(0.4,0.2),(0.2,0.8)	300
$\mathfrak{O}_3$	(0.8,0.5),(0.5,0.2),(0.2,0.9)	(0.9,0.8),(0.8,0.1),(0.1,0.7)	(0.7,0.53),(0.29,0.15),(0.6,0.4)	200
Demand	400	50	150	600

Step 1 *Firstly, we will find the score value of each neutrosophic Z-number cost given in the following Table 1 and substitute them with their score values as shown in Table 2.*

Table 2. Score Values of neutrosophic Z-numbers.

$\mathfrak{S}$	$\mathfrak{S}_1$	$\mathfrak{S}_2$	$\mathfrak{S}_3$	**Supply**
$\mathfrak{O}_1$	0.533	0.64	0.553	100
$\mathfrak{O}_2$	0.5	0.4466	0.666	300
$\mathfrak{O}_3$	0.706	0.856	0.6958	200
Demand	400	50	150	600

Step 2 *In this step, we are going to check whether the transportation is balanced or not.*

$$\sum a_i = 100 + 300 + 200 = 600 \quad (1)$$

and

$$\sum b_i = 400 + 50 + 150 = 600. \tag{2}$$

Hence, we can say that the given transportation problem is a balanced transportation problem. Table 3 shows the penalties and the very first allotment assigned in Example 1. When Tables 4–6 show the sequel allotments, Table 7 shows the complete allotment of the given input data in Table 1.

Table 3. Example 1: Initial allotment with penalties.

$\mathfrak{S}$	$\mathfrak{S}_1$	$\mathfrak{S}_2$	$\mathfrak{S}_3$	**Supply**	**Penalty**
M_1	0.533	0.64	0.553	100	0.02
M_2	0.5	$0.4466^{(50)}$	0.666	300/250	0.0534
M_3	0.706	0.856	0.6958	200	0.0102
Demand	400	50/0	150	600	
Penalty	0.033	0.1934	0.113		

Table 4. Second allotment in Example 1 with penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	0.533	0.64	0.553	100	0.02
$\mathfrak{D}_2$	$0.5^{(250)}$	$0.4466^{(50)}$	0.666	300/250/0	0.166
$\mathfrak{D}_3$	0.706	0.856	0.6958	200	0.0102
Demand	400/150	50/0	150	600	
Penalty	0.033	-	0.113		

Table 5. The penalties for the third allotment in Example 1.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	$0.533^{(100)}$	0.64	0.553	100/0	0.02
$\mathfrak{D}_2$	$0.5^{(250)}$	$0.4466^{(50)}$	0.666	300/250/0	-
$\mathfrak{D}_3$	0.706	0.856	0.6958	200	0.0102
Demand	400/150/50	50/0	150	600	
Penalty	0.173	-	0.1428		

Table 6. Example 1's fourth allotment with penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	$0.533^{(100)}$	0.64	0.553	100/0	-
$\mathfrak{D}_2$	$0.5^{(250)}$	$0.4466^{(50)}$	0.666	300/250/0	-
$\mathfrak{D}_3$	$0.706^{(50)}$	0.856	0.6958	200/150	0.0102
Demand	400/150/50/0	50/0	150	600	
Penalty	0.173	-	0.1428		

Table 7. Complete Allocation in Example 1 with Penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	Supply	Penalty
$\mathfrak{O}_1$	$0.533^{(100)}$	0.64	0.553	100/0	-
$\mathfrak{O}_2$	$0.5^{(250)}$	$0.4466^{(50)}$	0.666	300/250/0	-
$\mathfrak{O}_3$	$0.706^{(50)}$	0.856	$0.6958^{(150)}$	200/150/0	0.0102
Demand	400/150/50/0	50/0	150/0	600	
Penalty	-	-	0.1428		

Step 3 *In Vogel's approximation method for finding the feasible solutions, we have used the Algorithm 1 already defined above. After the complete allotments in Table 7, we obtain the feasible solution. Therefore, the initial feasible solution of the given data in Example 1 is as follows:*
$(\mathfrak{O}_1, \mathfrak{G}_1) = a_{11} = 100$, $(\mathfrak{O}_2, \mathfrak{G}_1) = a_{21} = 250$,
$(\mathfrak{O}_2, \mathfrak{G}_2) = a_{22} = 50$, $(\mathfrak{O}_3, \mathfrak{G}_1) = a_{31} = 50$,
$(\mathfrak{O}_3, \mathfrak{G}_3) = a_{33} = 150$
As mentioned above in the main algorithm, we can find the minimum cost of the given transportation problem using NZNs as follows:
$Min = 0.533 \times 100 + 0.5 \times 250 + 0.4466 \times 50 + 0.706 \times 50 + 0.6958 \times 150$ $Min = 53.3 + 125 + 22.33 + 35.3 + 104.37$ $Min = 340.3$
Now we will test whether the solution is optimal or not. For this, we will use the sensitivity analysis.

5.3. Unbalanced Transportation Problem

Here, we will discuss the cases where the transportation problem is unbalanced, i.e, $supply \neq demand$.

Type 1 NZN Model

In this section, we will discuss the unbalanced transportation problem in which cost is in the form of neutrosophic Z-numbers. On the other hand, demand and supply are not in the form of neutrosophic Z-numbers.

Example 2. *Pakistan steel mill has three factories which supply steel to three different sites. Each factory's monthly output volume, each site's demand and the associated transportation cost per quintal are provided in Table 8. When the supply of the first plant is 249 tonnes, the supply of the second plant is 135 tonnes and supply of the third plant is 141 tonnes. The demand of the first distribution centre is 200 tonnes, demand of second distribution centre is 250 tonnes and demand of third distribution centre is 100 tonnes. This problem is shown graphically in Figure 4. Also, "$\mathfrak{G}$" denotes the destination of transportation while $\mathfrak{O}$ denotes origin of the transportation. The transportation cost is given in the form of neutrosophic Z-numbers as*

$$W_i^A = [T_i^A(P^A, Q^A), I_i^A(P^A, Q^A), F_i^A(P^A, Q^A)].$$

Keep in mind that the P^A are the neutrosophic value of the given set and Q^A are the components of measures of reliability for P^A. When $T_i^A(P^A, Q^A)$ shows the value of truth, $I_i^A(P^A, Q^A)$ shows the value of indeterminacy and $F_i^A(P^A, Q^A)$ shows the value of falsity.

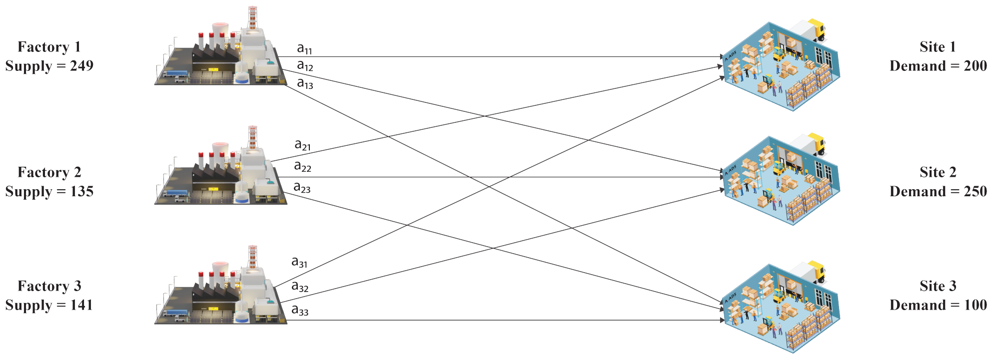

Figure 4. Graphical representation of the transportation problem given in Example 2.

Table 8. Input neutrosophic Z-numbers for unbalanced transportation problem in Example 2.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**
$\mathfrak{D}_1$	(0.69,0.81),(0.31,0.54),(0.63,0.29)	(0.71,0.19),(0.98,0.37),(0.17,0.28)	(0.55,0.89),(0.71,0.35),(0.43,0.241)	249
$\mathfrak{D}_2$	(0.43,0.21),(0.03,0.1),(0.9,0.87)	(0.05,0.97),(0.7,0.143),(0.3,0.5)	(0.879,0.71),(0.91,0.678),(0.61,0.93)	135
$\mathfrak{D}_3$	(0.08,0.13),(0.24,0.35),(0.05,0.64)	(0.7,0.01),(0.897,0.34),(0.87,0.05)	(0.09,0.1),(0.2,0.256),(0.03,0.35)	141
Demand	200	250	100	645

Step 1 *Firstly, we will find the score value of each neutrosophic Z-number cost given in Table 8 and substitute them with their score values.*

Since Vogel's transportation method is applicable for balanced transportation problems, we have to check whether the transportation problem is balanced or unbalanced. For this,
$\sum a_i = 249 + 135 + 141 = 525$
and
$\sum b_j = 200 + 250 + 100 = 550$
since $\sum a_i \neq \sum b_j$
So we will add a dummy row in Table 9 to balance the demand and supply and obtained the Table 10.

Table 9. Score Values of neutrosophic Z-numbers.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**
$\mathfrak{D}_1$	0.7362	0.5749	0.7124	249
$\mathfrak{D}_2$	0.4347	0.5994	0.4799	135
$\mathfrak{D}_3$	0.631	0.5528	0.649	141
Demand	200	250	100	

Table 10. Addition of a dummy row to balance the supply and demand.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**
$\mathfrak{D}_1$	0.7362	0.5749	0.7124	249
$\mathfrak{D}_2$	0.4347	0.5994	0.4799	135
$\mathfrak{D}_3$	0.631	0.5528	0.649	141
$\mathfrak{D}_4$	0	0	0	25
Demand	200	250	100	550

Now we can say that the given transportation problem is a balanced transportation problem. Table 11 shows the penalties and the very first allotment assigned in Example 2. Tables 12–15 show the sequel allotments; Table 16 shows the complete allotment of the given input data in Table 8.

Table 11. First allocation with penalties of Example 2.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	0.7362	0.5749	0.7124	249	0.1375
$\mathfrak{D}_2$	0.4347	0.5994	0.4799	135	0.0452
$\mathfrak{D}_3$	0.631	0.5528	0.649	141	0.0782
$\mathfrak{D}_4$	0	$0^{(25)}$	0	25/0	0
Demand	200	250/225	100	550	
Penalty	0.4347	0.5528	0.4799		

Table 12. Example 2: Second allotment with penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	0.7362	0.5749	0.7124	249	0.1375
$\mathfrak{D}_2$	$0.4347^{(135)}$	0.5994	0.4799	135/0	0.0452
$\mathfrak{D}_3$	0.631	0.5528	0.649	141	0.0782
$\mathfrak{D}_4$	0	$0^{(25)}$	0	25/0	-
Demand	200/65	250/225	100	550	
Penalty	0.1963	0.0221	0.1691		

Table 13. Example 2's third allotment with penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	0.7362	$0.5749^{(225)}$	0.7124	249/24	0.1375
$\mathfrak{D}_2$	$0.4347^{(135)}$	0.5994	0.4799	135/0	-
$\mathfrak{D}_3$	0.631	0.5528	0.649	141	0.0782
$\mathfrak{D}_4$	0	$0^{(25)}$	0	25/0	-
Demand	200/65	250/225/0	100	550	
Penalty	0.1052	0.0221	0.0634		

Table 14. Example 2's fourth allotment with penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	**Supply**	**Penalty**
$\mathfrak{D}_1$	0.7362	$0.5749^{(225)}$	0.7124	249/24	0.0238
$\mathfrak{D}_2$	$0.4347^{(135)}$	0.5994	0.4799	135/0	-
$\mathfrak{D}_3$	$0.631^{(65)}$	0.5528	0.649	141/76	0.018
$\mathfrak{D}_4$	0	$0^{(25)}$	0	25/0	-
Demand	200/65/0	250/225/0	100	550	
Penalty	0.1052	-	0.0634		

Table 15. Fifth allotment with Example 2 penalties.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	Supply	Penalty
$\mathfrak{O}_1$	0.7362	$0.5749^{(225)}$	$0.7124^{(24)}$	249/24/0	0.7124
$\mathfrak{O}_2$	$0.4347^{(135)}$	0.5994	0.4799	135/0	-
$\mathfrak{O}_3$	$0.631^{(65)}$	0.5528	0.649	141/76	0.649
$\mathfrak{O}_4$	0	$0^{(25)}$	0	25/0	-
Demand	200/65/0	250/225/0	100/76	550	
Penalty	-	-	0.0634		

Table 16. Complete allotment with penalties of Example 2.

$\mathfrak{S}$	$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	Supply	Penalty
$\mathfrak{O}_1$	0.7362	$0.5749^{(225)}$	$0.7124^{(24)}$	249/24/0	-
$\mathfrak{O}_2$	$0.4347^{(135)}$	0.5994	0.4799	135/0	-
$\mathfrak{O}_3$	$0.631^{(65)}$	0.5528	$0.649^{(76)}$	141/76/0	0.649
$\mathfrak{O}_4$	0	$0^{(25)}$	0	25/0	-
Demand	200/65/0	250/225/0	100/76/0	550	
Penalty	-	-	0.649		

In Vogel's approximation method for finding the feasible solutions, we have used the Algorithm 1 as defined above. After the complete allotments in Table 16, we obtain the feasible solution. Therefore, the initial feasible solution of the given data in Example 2 is as follows:
$(\mathfrak{O}_1, \mathfrak{G}_2) = a_{12} = 225$, $(\mathfrak{O}_1, \mathfrak{G}_3) = a_{13} = 24$,
$(\mathfrak{O}_2, \mathfrak{G}_1) = a_{21} = 135$, $(\mathfrak{O}_3, \mathfrak{G}_1) = a_{31} = 65$,
$(\mathfrak{O}_3, \mathfrak{G}_3) = a_{33} = 76$, $(\mathfrak{O}_4, \mathfrak{G}_2) = a_{42} = 25$
As mentioned above in the main algorithm, we can find the minimum cost of the given transportation problem using NZNs as follows:
$Min = 0.5749 \times 225 + 0.7124 \times 24 + 0.4347 \times 135 + 0.631 \times 65 + 0.649 \times 76 + 0 \times 25$ $Min = 129.3525 + 17.0976 + 58.6845 + 41.015 + 49.324 + 0$ $Min = 295.47$
So the minimum value is 295.47.

6. Sensitivity Analysis

Here, we test whether our algorithm results are optimal or not. For this purpose, we first define the testing algorithm to test the optimality. Subsequently, we test our results for both balanced and unbalanced problems.

6.1. Algorithm 2

Step I In the first step of algorithm, we will identify the locations where no allocation has been made and obtain the initial feasible solution.

Step II Starting from a vacant cell to occupied cells, draw a close loop, such that only the initial vacant cell and occupied cells are permitted locations to change direction with 90^o angle in this closed path. Insert the (+) and (−) signs one after another at every location, beginning with the (+) at first empty cell. Sum up the transportation costs of every cell traced by this closed loop. The resultant value is known as net cost change. Repeat the process for every location's transportation cost where no allotments are assigned.
Note: The first positive transportation cost is the only one with no allocations, afterwards all of them which have the sign (+) or (−) are the location's where allotments are assigned.

Step III If all the net cost changes are positive then the solution is optimal. Otherwise, draw a closed loop from the vacant cell bearing the largest negative net cost change.

Step IV On this closed loop, choose the cell having $(-)$ sign and the minimum allotted value. Allot this value to the vacant cell and it becomes the occupied cell. Subtract the same value from all allocations of cells traced on this path having $(-)$ sign and likewise add this value to the allotments of cells traced on the closed loop. From this, we will obtain a new table containing new allotments.

Step V Repeat Steps II to IV until all the net cost changes we obtain are positive and hence at that moment we will achieve our optimal solution.

After finding the optimal solution, repeat Steps 4 and 5 of the main algorithm to obtain the minimum value.
End.

6.1.1. Optimality Test for Example 1

In this part, our goal is to verify that feasible solution as the optimal solution, we are going to use Algorithm 2 and find the net cost change of each unoccupied cell.
$(\mathfrak{O}_1, \mathfrak{D}_2) = a_{12} = 0.64 - 0.4466 + 0.5 - 0.533 = 0.1604$
$(\mathfrak{O}_1, \mathfrak{D}_3) = a_{13} = 0.553 - 0.6958 + 0.706 - 0.533 = 0.0302$
$(\mathfrak{O}_2, \mathfrak{D}_3) = a_{23} = 0.666 - 0.6958 + 0.706 - 0.5 = 0.1762$
$(\mathfrak{O}_3, \mathfrak{D}_2) = a_{32} = 0.856 - 0.706 + 0.5 - 0.4466 = 0.2034$

Since all the net cost changes are positive, we can say that the solution we found is the optimal solution of the given transportation problem. So we have a verified conclusion that 340.3 is the minimum value we can obtain from the transportation problem at hand.

6.1.2. Optimality Test for Example 2

In this particular section, we will utilize Algorithm 2 to calculate the net cost change for every vacant cell and examine whether the feasible solution we have found is the optimal solution or not.
$(\mathfrak{O}_1, \mathfrak{D}_1) = a_{11} = 0.7362 - 0.7124 + 0.649 - 0.631 = 0.0418$
$(\mathfrak{O}_2, \mathfrak{D}_2) = a_{22} = 0.5994 - 0.5749 + 0.7124 - 0.649 + 0.631 - 0.4347 = 0.2842$
$(\mathfrak{O}_2, \mathfrak{D}_3) = a_{23} = 0.4799 - 0.4347 + 0.631 - 0.649 = 0.0272$
$(\mathfrak{O}_3, \mathfrak{D}_2) = a_{32} = 0.5528 - 0.5749 + 0.7124 - 0.649 = 0.0413$
$(\mathfrak{O}_4, \mathfrak{D}_1) = a_{41} = 0 - 0 + 0.5749 - 0.7124 + 0.649 - 0.631 = -0.1195$
$(\mathfrak{O}_4, \mathfrak{D}_3) = a_{43} = 0 - 0 + 0.5749 - 0.7124 = -0.1375$

Since there are negative net cost changes, we can say that 295.47 is not the optimal solution. In this case, we will change the allotments by utilizing Step IV of Algorithm 2, that is the vacant cell having the maximum negative value is $(\mathfrak{O}_4, \mathfrak{D}_3)$, so we will draw close loop from it and further proceed accordingly obtaining a new Table 17 of new allotments.

Table 17. Altered table for optimal solution of Example 2.

S	$\mathfrak{D}_1$	$\mathfrak{D}_2$	$\mathfrak{D}_3$	Supply
$\mathfrak{O}_1$	0.7362	$0.5749^{(249)}$	0.7124	249
$\mathfrak{O}_2$	$0.4347^{(135)}$	0.5994	0.4799	135
$\mathfrak{O}_3$	$0.631^{(65)}$	0.5528	$0.649^{(76)}$	141
$\mathfrak{O}_4$	0	$0^{(1)}$	$0^{(24)}$	25
Demand	200	250	100	550

Now to check whether the solution by the allotments in Table 17 is optimal, we will repeat the procedure according to algorithm 2.
$(\mathfrak{O}_1, \mathfrak{D}_1) = a_{11} = 0.7362 - 0.631 + 0.649 - 0 + 0 - 0.5749 = 0.1793$
$(\mathfrak{O}_1, \mathfrak{D}_3) = a_{13} = 0.7124 - 0 + 0 - 0.5749 = 0.1375$

$(\mathfrak{O}_2, \mathfrak{D}_2) = a_{22} = 0.5994 - 0 + 0 - 0.649 + 0.631 - 0.4347 = 0.1467$
$(\mathfrak{O}_2, \mathfrak{D}_3) = a_{23} = 0.4799 - 0.4347 + 0.631 - 0.649 = 0.0272$
$(\mathfrak{O}_3, \mathfrak{D}_2) = a_{32} = 0.5528 - 0 + 0 - 0.649 = -0.0962$
$(\mathfrak{O}_4, \mathfrak{D}_1) = a_{41} = 0 - 0 + 0.649 - 0.631 = 0.018$

We can conclude that the solution by the allotments of Table 17 is also not an optimal solution because there are negative net cost changes. Using Step IV of Algorithm 2, we will then alter the allotments. Specifically, we will draw a close loop from $(\mathfrak{O}_3, \mathfrak{D}_2)$, the unoccupied cell with the negative value of net cost change, and proceed accordingly to obtain a new table of allotments.

We will now repeat the process using algorithm 2 to see whether the preceding solution by the allocations of Table 18 is optimal or not.
$(\mathfrak{O}_1, \mathfrak{D}_1) = a_{11} = 0.7362 - 0.631 + 0.5528 - 0.5749 = 0.0831$
$(\mathfrak{O}_1, \mathfrak{D}_3) = a_{13} = 0.7124 - 0.5749 - 0.5528 - 0.649 = 0.0413$
$(\mathfrak{O}_2, \mathfrak{D}_2) = a_{22} = 0.5994 - 0.5528 + 0.631 - 0.4347 = 0.2429$
$(\mathfrak{O}_2, \mathfrak{D}_3) = a_{23} = 0.4799 - 0.4347 + 0.631 - 0.649 = 0.0272$
$(\mathfrak{O}_4, \mathfrak{D}_1) = a_{41} = 0 - 0 + 0.649 - 0.631 = 0.018$
$(\mathfrak{O}_4, \mathfrak{D}_2) = a_{42} = 0 - 0 + 0.649 - 0.5528 = 0.0962$

Table 18. Altered table for optimal solution of Example 2 using algorithm 2 .

S	$\mathfrak{D}_1$	$\mathfrak{D}_2$	$\mathfrak{D}_3$	Supply
$\mathfrak{O}_1$	0.7362	$0.5749^{(249)}$	0.7124	249
$\mathfrak{O}_2$	$0.4347^{(135)}$	0.5994	0.4799	135
$\mathfrak{O}_3$	$0.631^{(65)}$	$0.5528^{(1)}$	$0.649^{(75)}$	141
$\mathfrak{O}_4$	0	0	$0^{(25)}$	25
Demand	200	250	100	550

Since every value is positive, we are able to determine that the solution by the allotments assigned in Table 18, we encountered is the optimal one for the particular transportation problem. So to find the minimum transportation cost, we will employ Step 4 and Step 5 of main algorithm on Table 18.
$Min = 0.5749 \times 249 + 0.4347 \times 135 + 0.631 \times 65 + 0.5528 \times 10.649 \times 75 + 0 \times 25$
$Min = 143.1501 + 58.6845 + 41.015 + 0.5528 + 48.675 + 0$
$Min = 292.0774$

So the most minimum and optimal solution of the Example 2 is 292.0774. The result obtained from our algorithm is 295.47, which is very close to the optimal solution.

7. Limitations

- Our research focuses primarily on the application of neutrosophic group theory and Zadeh Z-numbers to navigation problems, which may limit its generalizability in other fields.
- The numerical methods used in our study can be prone to numerical complications when solving numerically, especially when dealing with large transport systems. However, researchers can minimize complexities and easily obtain the solution by using the MATLAB software.
- Although our approach provides promising results, its implementation may require significant computational resources and expertise, placing challenges on resource-limited personnel.
- Relying on numerical models and simulations to validate our methods may not fully capture the complexity and nuances of real-world navigation systems.
- The efficiency of our approach may be affected by data quality and availability, as well as by external factors such as regulatory restrictions and market trends.

8. Conclusions

Our research presents a pioneering approach to solving navigation problems by exploiting the power of neutrosophic Z-numbers. We demonstrated the effectiveness of our method in increasing the accuracy of the solution and reducing the uncertainty in cases of navigation. We examine the fuzzy transportation problem, wherein fuzzy quantities and fuzzy transportation cost per unit at the moment must be delivered.
Further details are given below:

- The approach provides a solution for complicated optimization problems.
- Method can manage determining the best option for many suppliers and locations.
- The study demonstrates the excellent accuracy of a suggested technique called Z-statistics.
- This strategy addresses multiple issues and uncertainty that numerical approaches for optimal solutions neglect.
- Our results undergo testing and verification, which demonstrate the dependability of our findings.
- By using cutting-edge rigorous verification techniques, we have ensured that the solutions we provide are not only correct but nearly accurate, notwithstanding some uncertainties.
- The numerical examples presented throughout verify the effectiveness of our method and highlight its practical application.
- The use of MATLAB codes provides additional accessibility and efficiency, facilitating greater adoption of our technique.
- To increase the reliability of our solutions and gain a better understanding of how uncertainty affects them, more research is required to create more reliable techniques for determining uncertainty and conducting sensitivity analyses. Work together with stakeholders and industry partners to integrate our techniques into current logistical processes so that quick and effective decision-making is possible. Examine how cutting-edge technologies like blockchain and the Internet of Things (IoT) can be combined to improve the visibility and trackability of transportation routes and to personalize and streamline our solutions.

Author Contributions: Conceptualization, A.U.; Data curation, L.R. and I.N.C.; Formal analysis, M.N., M.E.M.A., M.F., L.R. and I.N.C.; Funding acquisition, M.E.M.A., L.R. and I.N.C.; Investigation, M.N.; Methodology, M.K.; Project administration, M.A.; Resources, I.N.C.; Software, M.F.; Supervision, I.N.C.; Validation, M.F., L.R. and I.N.C.; Visualization, M.E.M.A.; Writing—original draft, M.K. and A.U.; Writing—review and editing, M.K., M.N. and I.N.C. All authors have read and agreed to the published version of the manuscript.

Funding: The authors extend their appreciation to Deanship of Scientific Research at King Khalid University for funding this work through Large Groups (Project under grant number (RGP2/32/44).

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Acknowledgments: The authors extend their appreciation to Deanship of Scientific Research at King Khalid University for funding this work through Large Groups (Project under grant number (RGP2/32/44).

Conflicts of Interest: The authors declare that they have no conflicts of interest regarding the publication of the research article.

References

1. Hitchcock, F.L. The distribution of a product from several sources to numerous localities. *J. Math. Phys.* **1941**, *20*, 224–230.
2. Balinski, M.L. Fixed-cost transportation problems. *Nav. Res. Logist. Q.* **1961**, *8*, 41–54.
3. Ghosh, S.; Roy, S.K.; Verdegay, J.L. Fixed-charge solid transportation problem with budget constraints based on carbon emission in neutrosophic environment. *Soft Comput.* **2022**, *26*, 11611–11625.

4. Korukoglu, S.; Balli, S. An improved Vogel's approximation method for the transportation problem. *Math. Comput. Appl.* **2011**, *16*, 370–381.
5. Meethom, W.; Koohathongsumrit, N. A decision support system for road freight transportation route selection with new fuzzy numbers. *Foresight* **2020**, *22*, 505–527.
6. Dou, C.; Woldt, W.; Bogardi, I.; Dahab, M. Numerical solute transport simulation using fuzzy sets approach. *J. Contam. Hydrol.* **1997**, *27*, 107–126.
7. Singh, A.; Das, A.; Bera, U.K.; Lee, G.M. Prediction of transportation costs using trapezoidal neutrosophic fuzzy analytic hierarchy process and artificial neural networks. *IEEE Access* **2021**, *9*, 103497–103512.
8. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353.
9. Shukla, A.K.; Prakash, V.; Nath, R.; Muhuri, P.K. Type-2 intuitionistic fuzzy TODIM for intelligent decision-making under uncertainty and hesitancy. *Soft Comput.* **2023**, *27*, 13373–13390.
10. Arora, H.D.; Naithani, A. A new definition for quartic fuzzy sets with hesitation grade applied to multi-criteria decision-making problems under uncertainty. *Decis. Anal. J.* **2023**, *7*, 100239.
11. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96.
12. Majumder, P.; Das, S.; Das, R.; Tripathy, B.C. Identification of the most significant risk factor of COVID-19 in economy using cosine similarity measure under SVPNS-environment. *Neutrosophic Sets Syst.* **2021**, *46*, 112–127.
13. Broumi, S.; Bakali, A.; Bahnasse, A. Neutrosophic sets: An overview. *Infin. Study* **2018** *2*, 403–434.
14. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single valued neutrosophic sets. *Infin. Study* **2010**, *12*, 20110.
15. Das, S.; Das, R.; Tripathy, B.C. Multi-criteria group decision making model using single-valued neutrosophic set. *Infin. Study* **2020** *16*, 421–429.
16. Ye, J.; Du, S.; Yong, R. Multi-criteria decision-making model using trigonometric aggregation operators of single-valued neutrosophic credibility numbers. *Inf. Sci.* **2023**, *644*, 118968.
17. Kamran, M.; Ismail, R.; Al-Sabri, E.H.A.; Salamat, N.; Farman, M.; Ashraf, S. An optimization strategy for MADM framework with confidence level aggregation operators under probabilistic neutrosophic hesitant fuzzy rough environment. *Symmetry* **2023**, *15*, 578.
18. Das, R.; Mukherjee, A.; Tripathy, B.C. Application of neutrosophic similarity measures in COVID-19. *Ann. Data Sci.* **2022**, *9*, 55–70.
19. Zimmermann, H.J. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets Syst.* **1978**, *1*, 45–55.
20. Chanas, S.; Kolodziejczyk, W.; Machaj, A. A fuzzy approach to the transportation problem. *Fuzzy Sets Syst.* **1984**, *13*, 211–221.
21. Chanas, S.; Kuchta, D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. *Fuzzy Sets Syst.* **1996**, *82*, 299–305.
22. Tada, M.; Ishii, H. An integer fuzzy transportation problem. *Comput. Math. Appl.* **1996**, *31*, 71–87.
23. Hashmi, N.; Jalil, S.A.; Javaid, S. A model for two-stage fixed charge transportation problem with multiple objectives and fuzzy linguistic preferences. *Soft Comput.* **2019**, *23*, 12401–12415.
24. Celik, E.; Akyuz, E. An interval type-2 fuzzy AHP and TOPSIS methods for decision-making problems in maritime transportation engineering: The case of ship loader. *Ocean Eng.* **2018**, *155*, 371–381.
25. Singh, S.K.; Yadav, S.P. A new approach for solving intuitionistic fuzzy transportation problem of type-2. *Ann. Oper. Res.* **2016**, *243*, 349–363.
26. Giri, B.K.; Roy, S.K. Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem. *Int. J. Mach. Learn. Cybern.* **2022**, *13*, 3089–3112.
27. Arora, J. An algorithm for interval-valued fuzzy fractional transportation problem. *Ski. Res. J.* **2018**, *8*, 71–75.
28. Bharati, S.K. Transportation problem with interval-valued intuitionistic fuzzy sets: Impact of a new ranking. *Prog. Artif. Intell.* **2021**, *10*, 129–145.
29. Pratihar, J.; Kumar, R.; Edalatpanah, S.A.; Dey, A. Modified Vogels approximation method for transportation problem under uncertain environment. *Complex Intell. Syst.* **2021**, *7*, 29–40.
30. Mondal, A.; Roy, S.K.; Midya, S. Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step fixed-charge solid transportation problem. *J. Ambient. Intell. Humaniz. Comput.* **2023**, *14*, 6975–6999.
31. Bagheri, M.; Ebrahimnejad, A.; Razavyan, S.; Hosseinzadeh Lotfi, F.; Malekmohammadi, N. Fuzzy arithmetic DEA approach for fuzzy multi-objective transportation problem. *Oper. Res.* **2022**, *22*, 1479–1509.
32. Akram, M.; Shah, S.M.U.; Al-Shamiri, M.M.A.; Edalatpanah, S.A. Extended DEA method for solving multi-objective transportation problem with Fermatean fuzzy sets. *Aims Math.* **2023**, *8*, 924–961.
33. Borah, G.; Dutta, P. Aggregation operators of quadripartitioned single-valued neutrosophic Z-numbers with applications to diverse COVID-19 scenarios. *Eng. Appl. Artif. Intell.* **2023**, *119*, 105748.
34. Kamran, M.; Salamat, N.; Hameed, M.S. Sine Trigonometric Aggregation Operators with Single-Valued Neutrosophic Z-Numbers: Application in Business Site Selection. *Neutrosophic Sets Syst.* **2024**, *63*, 18.
35. Zadeh, L.A. A note on Z-numbers. *Inf. Sci.* **2011**, *181*, 2923–2932.
36. Ren, Z.; Liao, H.; Liu, Y. Generalized Z-numbers with hesitant fuzzy linguistic information and its application to medicine selection for the patients with mild symptoms of the COVID-19. *Comput. Ind. Eng.* **2020**, *145*, 106517.
37. Xian, S.; Chai, J.; Li, T.; Huang, J. A ranking model of Z-mixture-numbers based on the ideal degree and its application in multi-attribute decision making. *Inf. Sci.* **2021**, *550*, 145–165.

38. Ye, J. Neutrosophic number linear programming method and its application under neutrosophic number environments. *Soft Comput.* **2018**, *22*, 4639–4646.
39. Du, S.; Ye, J.; Yong, R.; Zhang, F. Some aggregation operators of neutrosophic Z-numbers and their multicriteria decision making method. *Complex Intell. Syst.* **2021**, *7*, 429–438.