symmetry

Article

A Study on Some Properties of Neutrosophic Multi Topological Group

Bhimraj Basumatary [1,*], Nijwm Wary [1], Dimacha Dwibrang Mwchahary [2], Ashoke Kumar Brahma [3], Jwngsar Moshahary [4], Usha Rani Basumatary [1] and Jili Basumatary [1]

1 Department of Mathematical Sciences, Bodoland University, Kokrajhar 783370, India; nijwmwr0@gmail.com (N.W.); usharanibsty@gmail.com (U.R.B.); jilibasumatary@gmail.com (J.B.)
2 Department of Mathematics, Kokrajhar Government College, Kokrajhar 783370, India; ddmwchahary@kgc.edu.in
3 Department of Mathematics, Science College, Kokrajhar 783370, India; ashoke_brahma@rediffmail.com
4 Department of Agricultural Statistics, SCS College of Agriculture, AAU, Dhubri 783376, India; jwngsar.moshahary@aau.ac.in
* Correspondence: brbasumatary14@gmail.com

Abstract: In this paper, we studied some properties of the neutrosophic multi topological group. For this, we introduced the definition of semi-open neutrosophic multiset, semi-closed neutrosophic multiset, neutrosophic multi regularly open set, neutrosophic multi regularly closed set, neutrosophic multi continuous mapping, and then studied the definition of a neutrosophic multi topological group and some of their properties. Moreover, since the concept of the almost topological group is very new, we introduced the definition of neutrosophic multi almost topological group. Finally, for the purpose of symmetry, we used the definition of neutrosophic multi almost continuous mapping to define neutrosophic multi almost topological group and study some of its properties.

Keywords: neutrosophic multi continuous mapping; neutrosophic multi topological group; neutrosophic multi almost continuous mapping; neutrosophic multi almost topological group

check for updates

Citation: Basumatary, B.; Wary, N.; Mwchahary, D.D.; Brahma, A.K.; Moshahary, J.; Basumatary, U.R.; Basumatary, J. A Study on Some Properties of Neutrosophic Multi Topological Group. *Symmetry* **2021**, *13*, 1689. https://doi.org/10.3390/sym13091689

Academic Editors: Florentin Smarandache, Yanhui Guo and Alexei Kanel-Belov

Received: 3 July 2021
Accepted: 6 September 2021
Published: 13 September 2021

1. Introduction

Following the introduction of the fuzzy set (FS) [1], a variety of studies on generalisations of FS concepts were performed. In the sense that the theory of sets should have been a particular case of the theory of FSs, the theory of FSs is a generalisation of the classical theory of sets. Following the generalisation of FSs, many scholars used the theory of generalised FSs in a variety of fields in science and technology. Fuzzy topology (FT) was first introduced by Chang [2], and Intuitionistic fuzzy topological space (FITS) was defined by Coker [3]. Many researchers studied topology based on neutrosophic sets (NS), such as Lupianez [4–7] and Salama et al. [8]. Kelly [9] defined the concept of bitopological space (BTS) in 1963. Kandil et al. [10] studied the topic of fuzzy bitopological space (FBTS). Some characteristics of Intuitionistic Fuzzy Bitopological Space (IFBTS) were addressed by Lee et al. [11]. Garg [12] investigated how to rank interval-valued Pythagorean FSs using a modified score function. A Pythagorean fuzzy method for order of preference by similarity to ideal solution (TOPSIS) method based on Pythagorean FSs was discussed, which took the experts' preferences in the form of interval-valued Pythagorean fuzzy decision matrices. Moreover, different explorations of the theory of Pythagorean FSs can be seen in [13–19]. Yager [20] proposed the q-rung orthopair FSs, in which the sum of the qth powers of the membership (MS) and non-MS degrees is restricted to one [21]. Peng and Liu [22] studied the systematic transformation for information measures for q-rung orthopair FSs. Pinar and Boran [23] applied a q-rung orthopair fuzzy multi-criteria group decision-making method for supplier selection based on a novel distance measure.

Cuong et al. [24] proposed a picture FS as an extension of FS and Intuitionistic fuzzy set (IFS) that contains the concept of an element's positive, negative, and neutral MS degree. Cuong [25] investigated several picture FS characteristics and proposed distance measurements between picture FS. Phong et al. [26] investigated some picture fuzzy relation compositions. Cuong et al. [27] examined the basic fuzzy logic operators: negations, conjunctions, and disjunctions, as well as their implications on picture FSs, and also developed main operations for fuzzy inference processes in picture fuzzy systems. For picture FSs, Cuong et al. [28] demonstrated properties of an involutive picture negator and some related De Morgan fuzzy triples. Viet et al. [29] presented a picture fuzzy inference system based on MS graph, and Singh [30] studied correlation coefficients of picture FS. Garg [31] studied some picture fuzzy aggregation operations and their applications to multi-criteria decision-making. Quek et al. [32] used T-spherical fuzzy weighted aggregation operators to investigate the MADM problem. Garg [33] suggested interactive aggregation operators for T-spherical FSs and used the proposed operators to solve the MADM problem. Zeng et al. [34] studied on multi-attribute decision-making process with immediate probabilistic interactive averaging aggregation operators of T-spherical FSs and its application in the selection of solar cells. Munir et al. [35] investigated T-spherical fuzzy Einstein hybrid aggregation operators and how they could be applied in multi-attribute decision-making issues. Mahmood et al. [36] proposed the idea of a spherical FS and consequently a T-spherical FS.

Many researchers also studied FT and then generalised it in the IFS and then to the neutrosophic topology. Warren [37] studied the boundary of an FS in FT. Warren [37] studied some properties of the boundary of an FS and found that some properties are not the same as the properties of the crisp boundary of a set. Later, many authors studied the properties of the boundary of an FS. Tang [38] made heavy use of the notion of fuzzy boundary. Kharal [39] studied Frontier and Semifrontier in IFTSs. Salama et al. [40] studied generalised neutrosophic topological space (NTS), where they have discussed on properties of generalised closed sets. Azad [41] introduced the concepts of fuzzy semi-continuity (FSC), fuzzy almost continuity (FAC), and fuzzy weakly continuity (FWC) (FWC). Smarandache [42,43] suggested neutrosophic set (NS) theory, which generalised FST and IFST and incorporated a degree of indeterminacy as an independent component. Mwchahary et al. [44] studied on properties of the boundary of neutrosophic bitopological space (NBTS). Many authors studied the properties of the boundary of an FS by several methods (FS, IFS, and NS), but some of its properties are not the same as the properties of the crisp boundary of a set.

Blizard [45] traced multisets back to the very origin of numbers, arguing that in ancient times, the number was often represented by a collection of n strokes, tally marks, or units. The idea of fuzzy multiset (FMS) was introduced by Yager [46] as fuzzy bags. In the interest of brevity, we consider our attention to the basic concepts such as an open FMS, closed FMS, interior, closure, and continuity of FMSs. Yager, in [46], generalised the FS by introducing the concept of FMS (fuzzy bag), and he discussed a calculus for them in [47]. An element of an FMS can occur more than once with possibly the same or different MS values. If every element of an FMS can occur at most once, we go back to FSs [48]. In [49], Onasanya et al. defined the multi-fuzzy group (FMG), and in [50,51], the authors defined fuzzy multi-polygroups and fuzzy multi-Hv-ideals and studied their properties. In [52], Neutrosophic Multigroup (NMG) and their applications are observed. A new type of FS (FMS) was studied by Sebastian et al. [53]. This set makes use of ordered sequences of MS functions to express problems that are not covered by other extensions of FS theory, such as pixel colour. Dey et al. [54] were the first to establish the concept of multi-fuzzy complex numbers and multi-fuzzy complex sets. Over a distributive lattice, the authors [54] proposed multi fuzzy complex nilpotent matrices. Yong et al. [55] recently proposed the notion of the multi-fuzzy soft set, which is a more general fuzzy soft set, for its application to decision making.

Motivation

There is a lot of ambiguity information in the real world that crisp values cannot manage. The FS theory [1], proposed by Zadeh, is an age-old and excellent tool for dealing with uncertain information; however, it can only be used on random processes. As a result, Sebastian et al. [56] introduced FMSs, Atanassov [57] suggested the IFS theory, and Shinoj et al. [58] launched intuitionistic FMSs, all based on FS theory. The theories mentioned above have expanded in a variety of ways and have applications in a variety of fields, including algebraic structures. Some of the selected papers are those on FSs [59–61], FMSs [62–64], IFSs [65–72], and intuitionistic FMSs [73]. However, these theories are incapable of dealing with all forms of uncertainty, such as indeterminate and inconsistent data in various decision-making situations. To address this shortfall, Smarandache [74] proposed the NS theory, which makes Atanassov's [57] theory very practical and easy to apply. In this current decade, neutrosophic environments are mainly interested by different fields of researchers. In Mathematics, much theoretical research has also been observed in the sense of neutrosophic environment. A more theoretical study will be required to build a broad framework for decision-making and to define patterns for the conception and implementation of complex networks. Deli et al. [75] and Ye [76,77] proposed the notion of neutrosophic multiset (NMS) for modelling vagueness and uncertainty in order to improve the NS theory further. From the literature survey, it was noticed that precisely the properties of the neutrosophic multi topological group (NMTG) are not performed. Now, as an update for the research in NMS, we introduced the definition of a neutrosophic semi-open set, neutrosophic semi-closed set, neutrosophic regularly open set, neutrosophic regularly closed set, neutrosophic continuous mapping, neutrosophic open mapping, neutrosophic closed mapping, neutrosophic semi-continuous mapping, neutrosophic semi-open mapping, neutrosophic semi-closed mapping. Moreover, we tried to prove some of their properties and also cited some examples. We defined the neutrosophic multi almost topological group by using the definition of neutrosophic multi almost continuous mapping and investigate some properties and theorems of a neutrosophic multi almost topological group.

2. Materials and Methods

Definition 1 ([42]). *Let X be a non-empty fixed set. A neutrosophic set (NS)* A *is an object with the form* $\mathrm{A} = \{< x,\ \mu_A\ ,\sigma_A,\ \gamma_A > \ : x \in X\}$, *where* $T, I, F\ : X \longrightarrow [0,\ 1]$ *and* $0 \le \mu_A + \sigma_A + \gamma_A \le 3$ *and* $\mu_A\ (x)$, $\sigma_A(x)$, *and* $\gamma_A(x)$ *represents the degree of MS function, the degree indeterminacy, and the degree of non-MS function, respectively, of each element* $x \in X$ *to set* A.

Definition 2 ([78]). *A neutrosophic multiset (NMS) is a type of neutrosophic set (NS) in which one or more elements are repeated with the same or different neutrosophic components.*

Example 1. *Let* $X = \{a, b, c\}$ *then*

$$\mathcal{A} = \left\{ \begin{array}{c} < a, 0.6, 0.1, 0.2 >, < a, 0.5, 0.1, 0.3 >, < a, 0.4, 0.2, 0.4 >, \\ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\}$$

is an NMS, as the elementsa , b, care repeated.

However, $B = \{< a, 0.8, 0.3, 0.1 >,\ < b, 0.5, 0.3, 0.4 >, < c, 0.4, 0.4, 0.6 >\}$*is an NS and not an NMS.*

Definition 3 ([52]). *The Empty NMS is defined as* $0_{\mathrm{NM}} = \left\{ \mathrm{m} \in \mathrm{X}; < m_{(0,1,1)} > \right\}$, *where* m *can be repeated.*

Definition 4 ([52]). *The Whole NMS is defined as* $1_{\mathrm{NM}} = \left\{\mathrm{m} \in \mathrm{X}; < m_{(1,0,0)} >\right\}$, *where* m *can be repeated.*

Definition 5 ([52]). *Let* $X \neq \phi$, *and a neutrosophic multiset (NMS) A on X can be expressed as* $A = \left\{m \in X; \left(m_{<\mathfrak{T}_{A(m)}, \mathfrak{I}_{A(m)}, \mathfrak{F}_{A(m)}>}\right)\right\}$, *then the complement of A is defined as* $A^C = \left\{m \in X; \left(m_{<\mathfrak{F}_{A(m)}, 1-\mathfrak{I}_{A(m)}, \mathfrak{T}_{A(m)}>}\right)\right\}$. *where m can be repeated depending on its multiplicity, and the* $\mathfrak{T}, \mathfrak{I}, \mathfrak{F}$ *values may or may not be equal.*

Definition 6 ([52]). *Let* $X \neq \phi$ *and* $A = \left\{m \in X; \left(m_{<\mathfrak{T}_{A(m)}, \mathfrak{I}_{A(m)}, \mathfrak{F}_{A(m)}>}\right)\right\}$ *and* $B = \left\{m \in X; \left(m_{<\mathfrak{T}_{B(m)}, \mathfrak{I}_{B(m)}, \mathfrak{F}_{B(m)}>}\right)\right\}$ *are NMSs. Then*

(i) $A \Cap B = \left\{m \in X; m_{<\min(\mathfrak{T}_A(m), \mathfrak{T}_B(m)), \max(\mathfrak{I}_A(m), \mathfrak{I}_B(m)), \max(\mathfrak{F}_A(m), \mathfrak{F}_B(m))>}\right\}$;

(ii) $A \Cup B = \left\{m \in X; m_{<\max(\mathfrak{T}_A(m), \mathfrak{T}_B(m)), \min(\mathfrak{I}_A(m), \mathfrak{I}_B(m)), \min(\mathfrak{F}_A(m), \mathfrak{F}_B(m))>}\right\}$.

Definition 7 ([78]). *Let* $X \neq \phi$, *and a neutrosophic multiset topology (NMT) on X is a family* τ_X *of neutrosophic multi subsets of X if the following conditions hold:*

(i) $0_{NM}, 1_{NM} \in \tau_X$;
(ii) $G_1 \Cap G_2 \in \tau_X$ for $G_1, G_2 \in \tau_X$;
(iii) $\Cup G_i \in \tau_X$, $\forall \left\{G_{N_i} : i \in J\right\} \preccurlyeq \tau_X$.

Then (X, τ_X) *is known as a neutrosophic multi topological space (NMTS), and any NMS in* τ_X *is called a neutrosophic multi-open set (NMOS). The element of* τ_X *are said to be NMOSs, an NMS F is neutrosophic multi closed set (NMCoS) if* F^c *is NMOS.*

Definition 8 ([52]). *Let X be a classical group and A be a neutrosophic multiset (NMS) on X. Then A is said to be neutrosophic multi groupoid over X if*

(i) ${T_i}^G(mn) \geq {T_i}^G(m) \longrightarrow {T_i}^G(n)$;
(ii) ${I_i}^G(mn) \leq {I_i}^G(m) \longrightarrow {I_i}^G(n)$;
(iii) ${F_i}^G(mn) \leq {F_i}^G(m) \longrightarrow {F_i}^G(n)$, $\forall\, m, n \in X$ *and* $i = 1, 2, \ldots, P$.

Moreover, A is said to be neutrosophic multi-group (NMG) over X if the neutrosophic multi groupoid satisfies the following:

(i) ${T_i}^G\left(m^{-1}\right) \geq {T_i}^G(m)$;
(ii) ${I_i}^G\left(m^{-1}\right) \leq {I_i}^G(m)$;
(iii) ${F_i}^G\left(m^{-1}\right) \leq {F_i}^G(m)$, $\forall\, m \in X$ *and* $i = 1, 2, \ldots, P$.

Definition 9 ([52]). *Let* $\mathcal{G}$ *be an NMG in a group X, and e be the identity of X. We define the NMS* $\mathbb{G}_e$ *by*

$$\mathbb{G}_e = \{m \in X : \mathfrak{T}_{\mathbb{G}}(m) = \mathfrak{T}_{\mathbb{G}}(e),\ \mathfrak{I}_{\mathbb{G}}(m) = \mathfrak{I}_{\mathbb{G}}(e),\ \mathfrak{F}_{\mathbb{G}}(m) = \mathfrak{F}_{\mathbb{G}}(e)\}$$

We note for an NMG $\mathbb{G}$ *in a group X, for every* $m \in X :\ \mathfrak{T}_{\mathbb{G}}\left(m^{-1}\right) = \mathfrak{T}_{\mathbb{G}}(m),\ \mathfrak{I}_{\mathbb{G}}\left(m^{-1}\right) = \mathfrak{I}_{\mathbb{G}}(m)$ *and* $\mathfrak{F}_{\mathbb{G}}\left(m^{-1}\right) = \mathfrak{F}_{\mathbb{G}}(m)$. *Moreover, for the identity* $e \in X :\ \mathfrak{T}_{\mathbb{G}}(e) \succcurlyeq \mathfrak{T}_{\mathbb{G}}(m),\ \mathfrak{I}_{\mathbb{G}}(e) \succcurlyeq \mathfrak{I}_{\mathbb{G}}(m)$ *and* $\mathfrak{F}_{\mathbb{G}}(e) \preccurlyeq \mathfrak{F}_{\mathbb{G}}(m)$.

3. Results

Definition 10. *Let* (X, τ_X) *be NMTS. Then for an NMS* $A = \left\{< x, \mu_{N_i}, \sigma_{N_i}, \delta_{N_i} > : x \in X\right\}$, *the neutrosophic interior of A can be defined as* $NM \backsim Int\ (A) = \left\{< x, \Cup\mu_{N_i}, \Cap\sigma_{N_i}, \Cap\delta_{N_i} > :\ x \in X\right\}$.

Definition 11. *Let* (X, τ_X) *be NMTS. Then for an NMS* $A = \left\{< x, \mu_{N_i}, \sigma_{N_i}, \delta_{N_i} > : x \in X\right\}$, *the neutrosophic closure of A can be defined as* $NM \backsim Cl\ (A) = \left\{< x, \Cap\mu_{N_i}, \Cup\sigma_{N_i}, \Cup\delta_{N_i} > :\ x \in X\right\}$.

Definition 12. *Let $\mathbb{G}$ be an NMG on a group X. Let τ_X be a NMT on $\mathbb{G}$, then $(\mathbb{G}, \tau_X)$ is known as a neutrosophic multi topological group (NMTG) if it satisfies the given conditions:*

(i) $\alpha : (\mathbb{G}, \tau_X) \times (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ *defined by* $\alpha(m,n) = mn$, $\forall\, m,n \in X$, *is relatively neutrosophic multi continuous;*

(ii) $\beta : (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ *defined by* $\beta(m) = m^{-1}$, $\forall\, m \in X$, *is relatively neutrosophic multi continuous.*

Definition 13. *Let $\mathcal{A}$ be an NMS of an NMTS (X, τ_X), then $\mathcal{A}$ is called a neutrosophic multi semi-open set (NMSOS) of X if $\exists$ a $\mathcal{B} \in \tau_X$, such that $\mathcal{A} \preccurlyeq MN \backsim Int(MN \sim Cl(\mathcal{B}))$.*

Example 2. *Let* $X = \{a, b\}$:

$$\mathcal{A} = \left\{ \begin{array}{l} <a, 0.8, 0.1, 0.2>, <a, 0.7, 0.1, 0.3>, <a, 0.6, 0.2, 0.4>, \\ <b, 0.7, 0.2, 0.3>, <b, 0.6, 0.3, 0.4>, <b, 0.4, 0.2, 0.5> \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{l} <a, 0.9, 0.1, 0.1>, <a, 0.8, 0.1, 0.2>, <a, 0.7, 0.2, 0.3>, \\ <b, 0.8, 0.2, 0.2>, <b, 0.7, 0.2, 0.3>, <b, 0.5, 0.2, 0.4> \end{array} \right\}.$$

Then $\tau = \{0_X, 1_X, \mathcal{B}\}$ is neutrosophic multi topological space.
Then $Cl(\mathcal{B}) = 1_X$, $Int(Cl(\mathcal{B})) = 1_X$.
Hence, $\mathcal{B}$ is NMSOS.

Definition 14. *Let $\mathcal{A}$ be an NMS of an NMTS (X, τ_X), then $\mathcal{A}$ is called a neutrosophic multi semi-closed set (NMSCoS) of X if $\exists$ a $\mathcal{B}^c \in \tau_X$, such that $MN \backsim Cl(MN \sim Int(\mathcal{B})) \preccurlyeq \mathcal{A}$.*

Lemma 1. *Let $\phi : X \longrightarrow Y$ be a mapping and $\{\mathcal{A}_\alpha\}$ be a family of NMSs of Y, then (1) $\phi^{-1}(\Cup \mathcal{A}_\alpha) = \Cup\, \phi^{-1}(\mathcal{A}_\alpha)$ and (ii) $\phi^{-1}(\Cap \mathcal{A}_\alpha) = \Cap \phi^{-1}(\mathcal{A}_\alpha)$.*

Proof. Proof is straightforward. □

Lemma 2. *Let $\mathcal{A}$, $\mathcal{B}$ be NMSs of X and Y, then $1_X - \mathcal{A} \times \mathcal{B} = (\mathcal{A}^c \times 1_X) \Cup (1_X \times \mathcal{B}^c)$.*

Proof. Let (p,q) be any element of $X \times Y$, $(1_X - \mathcal{A} \times \mathcal{B})(p,q) = \max(1_X - \mathcal{A}(p), 1_X - \mathcal{B}(q)) = \max\{(\mathcal{A}^c \times 1_X)(p,q), (\mathcal{B}^c \times 1_X)(p,q)\} = \{(\mathcal{A}^c \times 1_X) \Cup (1_X \times \mathcal{B}^c)\}(p,q)$, for each $(p,q) \in X \times Y$. □

Lemma 3. *Let $\phi_i : X_i \longrightarrow Y_i$ and $\mathcal{A}_i$ be NMSs of Y_i, $i = 1,2$; we have $(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2) = {\phi_1}^{-1}(\mathcal{A}_1) \times {\phi_2}^{-1}(\mathcal{A}_2)$.*

Proof. For each $(p_1, p_2) \in X_1 \times X_2$, we have

$$\begin{aligned} (\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2)(p_1, p_2) &= (\mathcal{A}_1 \times \mathcal{A}_2)((\phi_1(p_1), \phi_2(p_2)) \\ &= \min\{\mathcal{A}_1\phi_1(p_1), \mathcal{A}_2\phi_2(p_2)\} \\ &= \min\{{\phi_1}^{-1}(\mathcal{A}_1)(p_1),\ {\phi_2}^{-1}(\mathcal{A}_2)(p_2)\} \\ &= \left({\phi_1}^{-1}(\mathcal{A}_1) \times {\phi_2}^{-1}(\mathcal{A}_2)\right)(p_1, p_2). \end{aligned}$$

□

Lemma 4. *Let $\psi : X \longrightarrow X \times Y$ be the graph of a mapping $\phi : X \longrightarrow Y$. Then, if $\mathcal{A}, \mathcal{B}$ is NMSs of X and Y, $\psi^{-1}(\mathcal{A} \times \mathcal{B}) = \mathcal{A} \Cap \phi^{-1}(\mathcal{B})$.*

Proof. For each $p \in X$, we have

$$\begin{aligned} \psi^{-1}(\mathcal{A} \times \mathcal{B})(p) = (\mathcal{A} \times \mathcal{B})\psi(p) &= (\mathcal{A} \times \mathcal{B})(p, \phi(p)) \\ &= \min\{\mathcal{A}(p), \mathcal{B}(\phi(p))\} \\ &= \left(\mathcal{A} \Cap \phi^{-1}(\mathcal{B})\right)(p). \end{aligned}$$

□

Lemma 5. *For a family $\{\mathcal{A}\}_\alpha$ of NMSs of NMTS (X, τ_X), $\Cup\ NM \backsim Cl(\mathcal{A}_\alpha) \preccurlyeq NM \backsim Cl(\Cup(\mathcal{A}_\alpha))$. In the case that $\mathcal{B}$ is a finite set, $\Cup NM \backsim Cl(\mathcal{A}_\alpha) \preccurlyeq NM \backsim Cl(\Cup(\mathcal{A}_\alpha))$. Moreover, $\Cup NM \backsim Int(\mathcal{A}_\alpha) \preccurlyeq NM \backsim Int(\Cup(\mathcal{A}_\alpha))$, where a subfamily $\mathcal{B}$ of (X, τ_X) is said to be subbase for (X, τ_X) if the collection of all intersections of members of $\mathcal{B}$ forms a base for (X, τ_X).*

Lemma 6. *For an NMS $\mathcal{A}$ of an NMTS (X, τ_X), (a) $1_{NM} - NM \backsim Int(\mathcal{A}) = NM \backsim Cl(1_{NM} - \mathcal{A})$, and (b) $1_{NM} - NM \backsim Cl(\mathcal{A}) = NM \backsim Int(1_{NM} - \mathcal{A})$.*

Proof. Proof is straightforward. □

Theorem 1. *The statements below are equivalent:*

(i) $\mathcal{A}$ *is an NMCoS;*
(ii) $\mathcal{A}^c$ *is an NMOS;*
(iii) $NM \backsim Int(NM \backsim Cl(\mathcal{A})) \preccurlyeq \mathcal{A}$;
(iv) $NM \backsim Cl(NM \backsim Int(\mathcal{A}^c)) \succcurlyeq \mathcal{A}^c$.

Proof. (i) and (ii) are equivalent follows from Lemma 6, since for an NMS $\mathcal{A}$ of an NMTS (X, τ_X) such that $1_{NM} - NM \backsim Int(\mathcal{A}) = NM \backsim Cl(1_{NM} - \mathcal{A})$ and $1_{NM} - NM \backsim Cl(\mathcal{A}) = NM \backsim Int(1_{NM} - \mathcal{A})$.

(i)⇒(iii). By definition, $\exists$ an NMCoS $\mathcal{B}$ such that $NM \backsim Int(\mathcal{B}) \preccurlyeq \mathcal{A} \preccurlyeq \mathcal{B}$; hence, $NM \backsim Int(\mathcal{B}) \preccurlyeq \mathcal{A} \preccurlyeq NM \backsim Cl(\mathcal{A}) \preccurlyeq \mathcal{B}$. Since $NM \backsim Int(\mathcal{B})$ is the largest NMOS contained in $\mathcal{B}$, we have $NM \backsim Int(NM \backsim Cl(\mathcal{B})) \preccurlyeq NM \backsim Int(\mathcal{B}) \preccurlyeq \mathcal{A}$;

(iii)⇒(i) follows by taking $\mathcal{B} = NM \backsim Cl(\mathcal{A})$;

(ii)⇔(iv) can similarly be proved. □

Theorem 2. *(i) Arbitrary union of NMSOSs is an NMSOS;*
(ii) Arbitrary intersection of NMSCoSs is an NMSCoS.

Proof. (i) Let $\{\mathcal{A}_\alpha\}$ be a collection of NMSOSs of an NMTS (X, τ_X). Then $\exists$ a $\mathcal{B}_\alpha \in \tau_X$ such that $\mathcal{B}_\alpha \preccurlyeq \mathcal{A}_\alpha \preccurlyeq NM \backsim Cl(\mathcal{B}_\alpha)$ for each α. Thus, $\Cap\ \mathcal{B}_\alpha \preccurlyeq \Cup \mathcal{A}_\alpha \preccurlyeq \Cup\ NM \backsim Cl(\mathcal{B}_\alpha) \preccurlyeq NM \backsim Cl(\Cup(\mathcal{B}_\alpha))$ (Lemma 5), and $\Cup \mathcal{B}_\alpha \in \tau_X$, this shows that $\Cup \mathcal{B}_\alpha$ is an NMSOS;

(ii) Let $\{\mathcal{A}_\alpha\}$ be a collection of NMSCoSs of an NMTS (X, τ_X). Then $\exists$ a $\mathcal{B}_\alpha \in \tau_X$ such that $NM \backsim Int(\mathcal{B}_\alpha) \preccurlyeq \mathcal{A}_\alpha \preccurlyeq \mathcal{B}_\alpha$ for each α. Thus, $NM \backsim Int(\Cap(\mathcal{B}_\alpha)) \preccurlyeq \Cap NM \backsim Int(\mathcal{B}_\alpha) \preccurlyeq \Cap \mathcal{A}_\alpha \preccurlyeq \Cap \mathcal{B}_\alpha$ (Lemma 5), and $\Cup \mathcal{B}_\alpha \in \tau_X$, this shows that $\Cap \mathcal{B}_\alpha$ is an NMSCoS. □

Remark 1. *It is clear that every NMOS (NMCoS) is an NMSOS (NMSCoS). The converse is not true.*

Example 3. *From Example 2, it is clear that $\mathcal{B}$ is a neutrosophic multi semi-open set, but $\mathcal{B}$ is not NMOS.*

Theorem 3. *If $(X,\ \tau_X)$ and $(Y,\ \tau_Y)$ are NMTSs, and X is a product related to Y. Then the product $\mathcal{A} \times \mathcal{B}$ of an NMSOS $\mathcal{A}$ of X and an NMSOS $\mathcal{B}$ of Y is an NMSOS of the neutrosophic multi-product space $X \times Y$.*

Proof. Let $\mathcal{P} \preccurlyeq \mathcal{A} \preccurlyeq NM \backsim Cl(\mathcal{P})$ and $\mathcal{Q} \preccurlyeq \mathcal{B} \preccurlyeq NM \backsim Cl(\mathcal{Q})$, where $\mathcal{P} \in \tau_X$ and $\mathcal{Q} \in \tau_Y$. Then $\mathcal{P} \times \mathcal{Q} \preccurlyeq \mathcal{A} \times \mathcal{B} \preccurlyeq NM \backsim Cl(\mathcal{P}) \times NM \backsim Cl(\mathcal{Q})$. For NMSs $\mathcal{P}$'s of X and $\mathcal{Q}$'s of Y, we have:

(a) $\inf\{\mathcal{P}, \mathcal{Q}\} = \min\{\inf \mathcal{P},\ \inf \mathcal{Q}\}$;
(b) $\inf\{\mathcal{P} \times 1_{NM}\} = (\inf \mathcal{P}) \times 1_{NM}$;
(c) $\inf\{1_{NM} \times \mathcal{Q}\} = 1_{NM} \times (\inf \mathcal{Q})$.

It is sufficient to prove $Nm \backsim Cl(\mathcal{A} \times \mathcal{B}) \succcurlyeq NM \backsim Cl(\mathcal{A}) \times NM \backsim Cl(\mathcal{B})$. Let $\mathcal{P} \in \tau_X$ and $\mathcal{Q} \in \tau_Y$. Then

$$\begin{aligned} NM \backsim Cl(\mathcal{A} \times \mathcal{B}) &= \inf\{(\mathcal{P} \times \mathcal{Q})^c \big| (\mathcal{P} \times \mathcal{Q})^c \succcurlyeq \mathcal{A} \times \mathcal{B} \\ &= \inf\{(\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c) | (\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c) \\ &\succcurlyeq \mathcal{A} \times \mathcal{B}\} \\ &= \inf\{(\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c) |\ \mathcal{P}^c \succcurlyeq \mathcal{A} \ or\ \mathcal{Q}^c \succcurlyeq \mathcal{B}\} \\ &= \min\begin{bmatrix} inf\ \{(\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c) | \mathcal{P}^c \succcurlyeq \mathcal{A}\}, \\ inf\ \{(\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c) |\ \mathcal{Q}^c \succcurlyeq \mathcal{B}\} \end{bmatrix} \end{aligned}$$

Since, $\inf\{(\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c)\ |\mathcal{P}^c \succcurlyeq \mathcal{A}\} \succcurlyeq \inf\{(\mathcal{P}^c \times 1_{NM})\ |\mathcal{P}^c \succcurlyeq \mathcal{A}\}$

$$= \inf\{\mathcal{P}^c |\ \mathcal{P}^c \succcurlyeq \mathcal{A}\} \times 1_{NM} = NM \backsim Cl(\mathcal{A}) \times 1_{NM}$$

and $\inf\{(\mathcal{P}^c \times 1_{NM}) \Cup (1_{NM} \times \mathcal{Q}^c)\ |\ \mathcal{Q}^c \succcurlyeq \mathcal{B}\} \succcurlyeq \inf\{(1_{NM} \times \mathcal{Q}^c)\ |\ \mathcal{Q}^c \succcurlyeq \mathcal{B}\}$

$$= 1_{NM} \times \inf\{\mathcal{Q}^c | \mathcal{Q}^c \succcurlyeq \mathcal{B}\} = 1_{NM} \times NM \backsim Cl(\mathcal{B})$$

we have, $NM \backsim Cl(\mathcal{A} \times \mathcal{B}) \succcurlyeq \min\{NM \backsim Cl(\mathcal{A}) \times 1_{NM},\ 1_{NM} \times NM \backsim Cl(\mathcal{B})\} = NM \backsim Cl(\mathcal{A}) \times NM \backsim Cl(\mathcal{B})$, hence the result. □

Definition 15. *An NMS $\mathcal{A}$ of an NMTS (X, τ_X) is called a neutrosophic multi regularly open set (NMROS) of (X, τ_X) if $NM \backsim Int(NM \backsim Cl(\mathcal{A})) = \mathcal{A}$.*

Example 4. *Let $X = \{a, b\}$* and

$$\mathcal{A} = \left\{ \begin{array}{l} < a, 0.4, 0.5, 0.5 >, < a, 0.3, 0.5, 0.6 >, < a, 0.2, 0.6, 0.7 >, \\ < b, 0.5, 0.7, 0.6 >, < b, 0.4, 0.5, 0.7 >, < b, 0.3, 0.5, 0.8 > \end{array} . \right\}$$

Then $\tau = \{0_X, 1_X, \mathcal{A}\}$ is neutrosophic multi topological space.
Clearly, $Cl(\mathcal{A}) = \mathcal{A}^C,\ Int(Cl(\mathcal{A})) = \mathcal{A}$.
Hence, $\mathcal{A}$ is NMROS.

Definition 16. *An NMS $\mathcal{A}$ of an NMTS (X, τ_X) is called a neutrosophic multi regularly closed set (NMRCoS) of (X, τ_X) if $NM \backsim Cl(NM \backsim Int(\mathcal{A})) = \mathcal{A}$.*

Theorem 4. *An NMS $\mathcal{A}$ of NMTS (X, τ_X) is an NMRO if $\mathcal{A}^c$ is NMRCo.*

Proof. It follows from Lemma 3. □

Remark 2. *It is obvious that every NMROS (NMRCoS) is an NMOS (NMCoS). The converse need not be true.*

Example 5. *Let $X = \{a, b\}$ and*

$$\mathcal{A} = \left\{ \begin{array}{l} < a, 0.8, 0.1, 0.2 >, < a, 0.7, 0.1, 0.3 >, < a, 0.6, 0.2, 0.4 >, \\ < b, 0.7, 0.2, 0.3 >, < b, 0.6, 0.3, 0.4 >, < b, 0.4, 0.2, 0.5 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{l} < a, 0.9, 0.1, 0.1 >, < a, 0.8, 0.1, 0.2 >, < a, 0.7, 0.2, 0.3 >, \\ < b, 0.8, 0.2, 0.2 >, < b, 0.7, 0.2, 0.3 >, < b, 0.5, 0.2, 0.4 > \end{array} \right\}.$$

Then $\tau = \{0_X, 1_X, \mathcal{B}\}$ is a neutrosophic multi topological space.
Then $Cl(\mathcal{B}) = 1_X,\ Int(Cl(\mathcal{B})) = 1_X$, which is not NMROS.

Remark 3. *The union (intersection) of any two NMROSs (NMRCoS) need not be an NMROS (NMRCoS).*

Example 6. *Let* $X = \{a, b\}$ *and*
$\tau = \{0_X, 1_X, \mathcal{A}, \mathcal{B}, \mathcal{A} \longrightarrow \mathcal{B}\}$ is a neutrosophic multi topological space, where

$$\mathcal{A} = \left\{ \begin{array}{c} < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.7 >, < a, 0.2, 0.6, 0.8 >, \\ < b, 0.7, 0.5, 0.3 >, < b, 0.6, 0.5, 0.4 >, < b, 0.4, 0.5, 0.6 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{c} < a, 0.6, 0.5, 0.4 >, < a, 0.7, 0.5, 0.3 >, < a, 0.8, 0.4, 0.2 >, \\ < b, 0.3, 0.5, 0.7 >, < b, 0.4, 0.5, 0.6 >, < b, 0.6, 0.5, 0.4 > \end{array} \right\};$$

$$\mathcal{A} \bigcup \mathcal{B} = \left\{ \begin{array}{c} < a, 0.6, 0.5, 0.4 >, < a, 0.7, 0.5, 0.3 >, < a, 0.8, 0.4, 0.2 >, \\ < b, 0.7, 0.5, 0.3 >, < b, 0.6, 0.5, 0.4 >, < b, 0.4, 0.5, 0.6 > \end{array} \right\}.$$

Here, $Cl(\mathcal{A}) = \mathcal{B}^C$, $Int(Cl(\mathcal{A})) = \mathcal{A}$, and $Cl(\mathcal{B}) = \mathcal{A}^C$, $Int(Cl(\mathcal{B})) = \mathcal{B}$.
Then $Cl(\mathcal{A} \bigcup \mathcal{B}) = 1_X$.
Thus, $Int(Cl(\mathcal{A} \bigcup \mathcal{B})) = 1_X$.
Hence, $\mathcal{A}$ and $\mathcal{B}$ is NROS, but $\mathcal{A} \bigcup \mathcal{B}$ is not NROS.

Theorem 5. *(i) The intersection of any two NMROSs is an NMROS;*
(ii) The union of any two NMRCoSs is an NMRCoS.

Proof. (i) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two NMROSs of an NMTS (X, τ_X). Since $\mathcal{A}_1 \Cap \mathcal{A}_2$ is NMOS (from Remark 3), we have $\mathcal{A}_1 \Cap \mathcal{A}_2 \preccurlyeq NM \backsim Int(NM \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2))$. Now, $NM \backsim Int(NM \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2)) \preccurlyeq NM \backsim Int(NM \backsim Cl(\mathcal{A}_1)) = \mathcal{A}_1$ and $NM \backsim Int(NM \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2)) \preccurlyeq NM \backsim Int(NM \backsim Cl(\mathcal{A}_2)) = \mathcal{A}_2$ implies that $NM \backsim Int(NM \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2)) \preccurlyeq \mathcal{A}_1 \Cap \mathcal{A}_2$, hence the theorem;

(ii) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two NMROSs of an NMTS (X, τ_X). Since $\mathcal{A}_1 \Cup \mathcal{A}_2$ is NMOS (from Remark 3), we have $\mathcal{A}_1 \Cup \mathcal{A}_2 \succcurlyeq NM \backsim Cl(NM \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2))$. Now, $NM \backsim Cl(NM \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2)) \succcurlyeq NM \backsim Cl(NM \backsim Int(\mathcal{A}_1)) = \mathcal{A}_1$ and $NM \backsim Cl(NM \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2)) \succcurlyeq NM \backsim Cl(NM \backsim Int(\mathcal{A}_2)) = \mathcal{A}_2$ implies that $\mathcal{A}_1 \Cup \mathcal{A}_2 \preccurlyeq NM \backsim Cl(NM \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2))$, hence the theorem. □

Theorem 6. *(i) The closure of an NMOS is an NMRCoS;*
(ii) The interior of an NMCoS is an NMROS.

Proof. (i) Let $\mathcal{A}$ be an NMOS of an NMTS (X, τ_X), clearly, $NM \backsim Int(NM \backsim Cl(\mathcal{A})) \preccurlyeq NM \backsim Cl(\mathcal{A}) \Rightarrow NM \backsim Cl(NM \backsim Int(NM \backsim Cl(\mathcal{A}))) \preccurlyeq NM \backsim Cl(\mathcal{A})$. Now, $\mathcal{A}$ is NMOS implies that $\mathcal{A} \preccurlyeq NM \backsim Int(NM \backsim Cl(\mathcal{A}))$, and hence, $NM \backsim Cl(\mathcal{A}) \preccurlyeq NM \backsim Cl(NM \backsim Int(NM \backsim Cl(\mathcal{A})))$. Thus, $NM \backsim Cl(\mathcal{A})$ is NMRCoS;

(ii) Let $\mathcal{A}$ be an NMCoS of an NMTS (X, τ_X), clearly, $NM \backsim Cl(NM \backsim Int(\mathcal{A})) \succcurlyeq NM \backsim Int(\mathcal{A}) \Rightarrow NM \backsim Int(NM \backsim Cl(NM \backsim Int(\mathcal{A}))) \succcurlyeq NM \backsim Int(\mathcal{A})$. Now, $\mathcal{A}$ is NMCoS implies that $\mathcal{A} \succcurlyeq NM \backsim Cl(NM \backsim Int(\mathcal{A}))$, and hence, $NM \backsim Int(\mathcal{A}) \succcurlyeq NM \backsim Int(NM \backsim Cl(NM \backsim Int(\mathcal{A})))$. Thus, $NM \backsim Int(\mathcal{A})$ is NMROS. □

Definition 17. *Let* $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ *be a mapping from an NMTS* (X, τ_X) *to another NMTS* (Y, τ_Y)*, then* ϕ *is known as a neutrosophic multi continuous mapping (NMCM), if* $\phi^{-1}(\mathcal{A}) \in \tau_X$ *for each* $\mathcal{A} \in \tau_Y$*, or equivalently* $\phi^{-1}(\mathcal{B})$ *is an NMCoS of* X *for each CoNMS* $\mathcal{B}$ *of* Y*.*

Example 7. *Let* $X = Y = \{a, b, c\}$ *and*

$$\mathcal{A} = \left\{ \begin{array}{c} < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.7 >, < a, 0.2, 0.6, 0.8 >, \\ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{c} < a, 0.6, 0.1, 0.2 >, < a, 0.5, 0.1, 0.3 >, < a, 0.4, 0.2, 0.4 >, \\ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\}.$$

Then $\tau_X = \{0_X, 1_X, \mathcal{A}\}$ and $\tau_Y = \{0_Y, 1_Y, \mathcal{B}\}$ are neutrosophic multi topological spaces.
Now, define a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ by $f(a) = f(c) = c$ and $f(b) = b$.
Thus, f is NMCM.

Definition 18. *Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS (X, τ_X) to another NMTS (Y, τ_Y), then ϕ is called a neutrosophic multi open mapping (NMOM) if $\phi(\mathcal{A}) \in \tau_Y$ for each $\mathcal{A} \in \tau_X$.*

Definition 19. *Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS (X, τ_X) to another NMTS (Y, τ_Y), then ϕ is said to be a neutrosophic multi-closed mapping (NMCoM) if $\phi(\mathcal{B})$ is an NMCoS of Y for each NMCoS $\mathcal{B}$ of X.*

Definition 20. *Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS (X, τ_X) to another NMTS (Y, τ_Y), then ϕ is called a neutrosophic multi semi-continuous mapping (NMSCM), if $\phi^{-1}(\mathcal{A})$ is the NMSOS of X, for each $\mathcal{A} \in \tau_Y$.*

Definition 21. *Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS (X, τ_X) to another NMTS (Y, τ_Y), then ϕ is called a neutrosophic multi semi-open mapping (NMSOM) if $\phi(\mathcal{A})$ is a SONMS for each $\mathcal{A} \in \tau_X$.*

Example 8. *Let $X = Y = \{a, b, c\}$ and*

$$\mathcal{A} = \left\{ \begin{array}{c} < a, 0.6, 0.1, 0.2 >, < a, 0.5, 0.1, 0.3 >, < a, 0.4, 0.2, 0.4 >, \\ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{c} < a, 0.3, 0.5, 0.4 >, < a, 0.2, 0.5, 0.6 >, < a, 0.1, 0.5, 0.7 >, \\ < b, 0.6, 0.1, 0.2 >, < b, 0.5, 0.1, 0.3 >, < b, 0.4, 0.2, 0.4 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\}.$$

Then $\tau_X = \{0_X, 1_X, \mathcal{A}\}$ and $\tau_Y = \{0_Y, 1_Y, \mathcal{B}\}$ are neutrosophic multi topological spaces.
Clearly, $\mathcal{A}$ is a semi-open set.
Then a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ defined by $f(a) = b, f(b) = a$ and $f(c) = c$.
Hence, f is NMSOM.

Definition 22. *Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS (X, τ_X) to another NMTS (Y, τ_Y), then ϕ is called a neutrosophic multi semi-closed mapping (NMSCoM) if $\phi(\mathcal{B})$ is an NMSCoS for each NMCoS $\mathcal{B}$ of X.*

Remark 4. *From Remark 1, an NMCM (NMOM, NMCoM) is also an NMSCM (NMSOM, NMSCoM).*

Example 9. *Let $X = Y = \{a, b, c\}$ and*

$$\mathcal{A} = \left\{ \begin{array}{c} < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.7 >, < a, 0.2, 0.6, 0.8 >, \\ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{c} < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.7 >, < a, 0.2, 0.6, 0.8 >, \\ < b, 0.4, 0.6, 0.4 >, < b, 0.3, 0.5, 0.5 >, < b, 0.2, 0.5, 0.6 >, \\ < c, 0.6, 0.5, 0.5 >, < c, 0.4, 0.5, 0.6 >, < c, 0.2, 0.6, 0.9 > \end{array} \right\}.$$

Then $\tau_X = \{0_X, 1_X, \mathcal{A}\}$ and $\tau_Y = \{0_Y, 1_Y, \mathcal{B}\}$ are neutrosophic multi topological spaces.
Let us define a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ by $f(a) = f(c) = c$ and $f(b) = b$.
Thus, f is NMSCM, which is not an NMCM.

Theorem 7. *Let X_1, X_2, Y_1 and Y_2 be NMTSs such that X_1 is product related to X_2. Then, the product $\phi_1 \times \phi_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ of NMSCMs $\phi_1 : X_1 \longrightarrow Y_1$ and $\phi_2 : X_2 \longrightarrow Y_2$ is NMSCM.*

Proof. Let $\mathcal{A} \equiv \Cup(\mathcal{A}_\alpha \times \mathcal{B}_\beta)$, where $\mathcal{A}_\alpha$'s and $\mathcal{B}_\beta$'s are NMOSs of Y_1 and Y_2, respectively, be an NMOS of $Y_1 \times Y_2$. By using Lemma 1(i) and Lemma 3, we have

$$(\phi_1 \times \phi_2)^{-1}(\mathcal{A}) = \Cup\left[{\phi_1}^{-1}(\mathcal{A}_\alpha) \times {\phi_2}^{-1}(\mathcal{A}_\beta)\right]$$

where $(\phi_1 \times \phi_2)^{-1}(\mathcal{A})$ is an NMSOS follows from Theorem 3 and Theorem 2 (i). □

Theorem 8. *Let X, X_1 and X_2 be NMTSs and $p_i : X_1 \times X_2 \longrightarrow X_i$ $(i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then, if $\phi : X \longrightarrow X_1 \times X_2$ is an NMSCM, $p_i\phi$ is also NMSCM.*

Proof. For an NMOS $\mathcal{A}$ of X_i, we have $(p_i\phi)^{-1}(\mathcal{A}) = \phi^{-1}\left({p_i}^{-1}(\mathcal{A})\right)$. p_i is an NMCM and ϕ is an NMSCM, which implies that $(p_i\phi)^{-1}(\mathcal{A})$ is an NMSOS of X. □

Theorem 9. *Let $\phi : X \longrightarrow Y$ be a mapping from an NMTS X to another NMTS Y. Then if the graph $\psi : X \longrightarrow X \times Y$ of ϕ is NMSCM, ϕ is also NMSCM.*

Proof. From Lemma 4, $\phi^{-1}(\mathcal{A}) = 1_{\mathrm{NM}} \Cap \phi^{-1}(\mathcal{A}) = \psi^{-1}(1_{\mathrm{NM}} \times \mathcal{A})$, for each NMOS $\mathcal{A}$ of Y. Since ψ is an NMSCM and $1_{\mathrm{NM}} \times \mathcal{A}$ is an NMOS $X \times Y$, $\phi^{-1}(\mathcal{A})$ is an NMSOS of X and hence ϕ is an NMSCM. □

Remark 5. *The converse of Theorem 9 is not true.*

Definition 23. *A mapping $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ from an NMTS X to another NMTS Y is known as a neutrosophic multi almost continuous mapping (NMACM), if $\phi^{-1}(\mathcal{A}) \in \tau_X$ for each NMROS $\mathcal{A}$ of Y.*

Example 10. *Let $X = Y = \{a, b\}$ and*

$$\mathcal{A} = \left\{ \begin{array}{l} < a, 0.4, 0.5, 0.5 >, < a, 0.3, 0.5, 0.6 >, < a, 0.2, 0.6, 0.7 >, \\ < b, 0.5, 0.7, 0.6 >, < b, 0.4, 0.5, 0.7 >, < b, 0.3, 0.5, 0.8 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{l} < a, 0.5, 0.7, 0.6 >, < a, 0.4, 0.5, 0.7 >, < a, 0.3, 0.5, 0.8 >, \\ < b, 0.4, 0.5, 0.5 >, < b, 0.3, 0.5, 0.6 >, < b, 0.2, 0.6, 0.7 > \end{array} \right\}.$$

Then $\tau_X = \{0_X, 1_X, \mathcal{A}\}$ and $\tau_Y = \{0_Y, 1_Y, \mathcal{B}\}$ are neutrosophic multi topological spaces.
Clearly, $Cl(\mathcal{B}) = \mathcal{B}^C$, $Int(Cl(\mathcal{B})) = \mathcal{B}$.
Hence, $\mathcal{B}$ is NMROS.
Now, let us define a mapping $f : (X, \tau_X) \to (Y, \tau_Y)$ by $f(a) = b, f(b) = a$.
Thus, f is NMACM.

Theorem 10. *Let $\phi : (X, \tau_X) \to (Y, \tau_Y)$ be a mapping. Then the below statements are equivalent:*

(a) *ϕ is an NMACM;*
(b) *$\phi^{-1}(\mathcal{F})$ is an NMCoS, for each NMRCoS $\mathcal{F}$ of Y;*
(c) *$\phi^{-1}(\mathcal{A}) \preccurlyeq NM \backsim Int(\phi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A}))))$, for each NMOS $\mathcal{A}$ of Y;*
(d) *$NM \backsim Cl\left(\phi^{-1}(NM \backsim Cl(NM \backsim Int(\mathcal{F})))\right) \preccurlyeq \phi^{-1}(\mathcal{F})$, for each NMCoS $\mathcal{F}$ of Y.*

Proof. Consider that $\phi^{-1}(\mathcal{A}^c) = \left(\phi^{-1}(A)\right)^c$, for any NMS $\mathcal{A}$ of Y, (a) $\Leftrightarrow$ (b) follows from Theorem 4.

(a) $\Rightarrow$ (c). Since $\mathcal{A}$ is an NMOS of Y, $\mathcal{A} \preccurlyeq NM \backsim Int(Cl(\mathcal{A}))$, hence, $\phi^{-1}(\mathcal{A}) \preccurlyeq \phi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A})))$. From Theorem 6 (ii), $NM \backsim Int(NM \backsim Cl(\mathcal{A}))$ is an NMROS of Y, hence $\phi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A})))$ is an NMOS of X. Thus, $\phi^{-1}(\mathcal{A}) \preccurlyeq \phi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A}))) = NM \backsim Int(\phi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A}))).$

(c) $\Rightarrow$ (a). Let $\mathcal{A}$ be an NMROS of Y, then we have $\phi^{-1}(\mathcal{A}) \preccurlyeq NM \backsim Int\left(\phi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A})))\right) = NM \backsim Int\left(\phi^{-1}(\mathcal{A})\right)$. Thus, $\phi^{-1}(\mathcal{A}) = NM \backsim Int\left(\phi^{-1}(\mathcal{A})\right)$. This shows that $\phi^{-1}(\mathcal{A})$ is an NMOS of X.

(b) $\Leftrightarrow$ (d) similarly can be proved. □

Remark 6. *Clearly, an NMCM is an NMACM. The converse need not be true.*

Example 11. *Let* $X = Y = \{a, b\}$ *and*

$$\mathcal{A} = \left\{ \begin{array}{l} < a, 0.4, 0.5, 0.5 >, < a, 0.3, 0.5, 0.6 >, < a, 0.2, 0.6, 0.7 >, \\ < b, 0.5, 0.7, 0.6 >, < b, 0.4, 0.5, 0.7 >, < b, 0.3, 0.5, 0.8 > \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{l} < a, 0.5, 0.5, 0.6 >, < a, 0.6, 0.5, 0.7 >, < a, 0.2, 0.6, 0.9 >, \\ < b, 0.4, 0.4, 0.7 >, < b, 0.3, 0.5, 0.5 >, < b, 0.4, 0.5, 0.6 > \end{array} \right\}.$$

Then, $\tau_X = \{0_X, 1_X, \mathcal{A}\}$ and $\tau_Y = \{0_Y, 1_Y, \mathcal{B}\}$ are neutrosophic multi topological spaces.
Clearly, $Cl(\mathcal{B}) = \mathcal{B}^C$, $Int(Cl(\mathcal{B})) = \mathcal{B}$.
Hence, $\mathcal{B}$ is NMROS in τ_Y.
Now, a mapping $f : (X, \tau_X) \to (Y, \tau_Y)$ defined by $f(a) = a, f(b) = b$.
Then clearly, f is NMACM but not NMCM.

Theorem 11. *Neutrosophic multi semi-continuity and neutrosophic multi almost continuity are independent notions.*

Definition 24. *AN NMTS* (X, τ_X) *is called a neutrosophic multi semi-regularly space (NMSRS) if and only if the collection of all NMROSs of* X *forms a base for NMT* τ_X.

Theorem 12. *Let* $\phi : (X, \tau_X) \to (Y, \tau_Y)$ *be a mapping from an NMTS* X *to an NMSRS* Y*. Then* ϕ *is NMACM iff* ϕ *is NMCM.*

Proof. From Remark 6, it suffices to prove that if ϕ is NMACM, then it is NMCM. Let $\mathcal{A} \in \tau_Y$, then $\mathcal{A} = \Cup \mathcal{A}_\alpha$, where $\mathcal{A}_\alpha$'s are NMROSs of Y. Now, from Lemma 1(i), 5, and Theorem 10 (c), we obtain

$$\phi^{-1}(\mathcal{A}) = \Cup \phi^{-1}(\mathcal{A}_\alpha) \preccurlyeq \Cup NM \backsim Int\Big(\phi^{-1}(NM \backsim Cl(\mathcal{A}_\alpha))\Big) = \Cup NM \backsim Int\Big(\phi^{-1}(\mathcal{A}_\alpha)\Big).$$

$$\preccurlyeq NM \backsim Int \Cup \Big(\phi^{-1}(\mathcal{A}_\alpha)\Big) = NM \backsim Int\Big(\phi^{-1}(\mathcal{A}_\alpha)\Big).$$

which shows that $\phi^{-1}(\mathcal{A}_\alpha) \in \tau_X$. □

Theorem 13. *Let* X_1, X_2, Y_1 *and* Y_2 *be the NMTSs, such that* Y_1 *is product related to* Y_2*. Then the product* $\phi_1 \times \phi_2 : X_1 \times X_2 \to Y_1 \times Y_2$ *of NMACMs* $\phi_1 : X_1 \to Y_1$ *and* $\phi_2 : X_2 \to Y_2$ *is NMACM.*

Proof. Let $\mathcal{A} = \Cup \left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)$, where $\mathcal{A}_\alpha$'s and $\mathcal{B}_\beta$'s are NMOSs of Y_1 and Y_2, respectively, be an NMOS of $Y_1 \times Y_2$. From Lemma 1(i), 3, 5, and *Theorems* 6, and 10 (c), we have

$$(\phi_1 \times \phi_2)^{-1}(\mathcal{A}) = \Cup \left\{ {\phi_1}^{-1}(\mathcal{A}_\alpha) \times {\phi_2}^{-1}(\mathcal{B}_\beta) \right\}$$

$$\preccurlyeq \Cup \begin{bmatrix} NM \backsim Int\left(\phi_1{}^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A}_\alpha)))\right) \\ \times NM \backsim Int\left(\phi_2{}^{-1}\left(NM \backsim Int\left(NM \backsim Cl\left(\mathcal{B}_\beta\right)\right)\right)\right) \end{bmatrix}$$

$$\preccurlyeq \Cup \left[NM \backsim Int\left\{\phi_1{}^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A}_\alpha))) \times \phi_2{}^{-1}\left(NM \backsim Int\left(NM \backsim Cl\left(\mathcal{B}_\beta\right)\right)\right)\right\}\right]$$

$$\preccurlyeq NM \backsim Int\left[\Cup \ (\phi_1 \times \phi_2)^{-1}\left\{NM \backsim Int(NM \backsim Cl(\mathcal{A}_\alpha)) \times NM \backsim Int\left(NM \backsim Cl\left(\mathcal{B}_\beta\right)\right)\right\}\right]$$

$$= NM \backsim Int\left[\Cup \ (\phi_1 \times \phi_2)^{-1}\left\{NM \backsim Int\left(NM \backsim Cl\left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)\right)\right\}\right]$$

$$\preccurlyeq NM \backsim Int\left[(\phi_1 \times \phi_2)^{-1}\left\{NM \backsim Int\left(NM \backsim Cl\left(\Cup \ \left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)\right)\right)\right\}\right]$$

$$= NM \backsim Int\left[(\phi_1 \times \phi_2)^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A})))\right]$$

Thus, by Theorem 10 (c), $\phi_1 \times \phi_2$ is NMACM. □

Theorem 14. *Let X, X_1 and X_2 be an NMTSs and $p_i : X_1 \times X_2 \to X_i (i = 1,\ 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then if $\phi : X \to X_1 \times X_2$ is an NMACM, $p_i\phi$ is also an NMACM.*

Proof. Since p_i is NMCM Definition 16, for any NMS $\mathcal{A}$ of X_i, we have (i) $NM \backsim Cl\left(p_i{}^{-1}(\mathcal{A})\right) \preccurlyeq p_i{}^{-1}(\mathrm{NM} \backsim \mathrm{Cl}(\mathcal{A}))$ and (ii) $NM \backsim Int\left(p_i{}^{-1}(\mathcal{A})\right) \succcurlyeq p_i{}^{-1}(\mathrm{NM} \backsim Int(\mathcal{A}))$. Again, since (i) each p_i is an NMOS, and (ii) for any NMS $\mathcal{A}$ of X_i (a) $\mathcal{A} \preccurlyeq p_i{}^{-1}p_i(\mathcal{A})$ and (b) $p_i{}^{-1}p_i(\mathcal{A}) \preccurlyeq \mathcal{A}$, we have $p_i\left(NM \backsim Int\left(p_i{}^{-1}(\mathcal{A})\right)\right) \preccurlyeq p_i p_i{}^{-1}(\mathcal{A}) \preccurlyeq \mathcal{A}$, and hence, $p_i\left(NM \backsim Int\left(p_i{}^{-1}(\mathcal{A})\right)\right) \preccurlyeq NM \backsim Int(\mathcal{A})$. □

Thus, $NM \backsim Int\left(p_i{}^{-1}(\mathcal{A})\right) \preccurlyeq p_i{}^{-1}p_i\left(NM \backsim Int\left(p_i{}^{-1}(\mathcal{A})\right)\right) \preccurlyeq \left(p_i{}^{-1}(NM \backsim Int(\mathcal{A})\right)$ establishes that $NM \backsim Int\left(p_i{}^{-1}(\mathcal{A})\right) \preccurlyeq p_i{}^{-1}(NM \backsim Int(\mathcal{A}))$. Now, for any NMOS $\mathcal{A}$ of X_i,

$$\begin{aligned} (p_i\phi)^{-1}(\mathcal{A}) = \ & \phi^{-1}\left(p_i{}^{-1}(\mathcal{A})\right) \\ & \preccurlyeq NM \backsim Int\left\{\phi^{-1}\left(NM \backsim Int\left(NM \backsim Cl\left(p_i{}^{-1}(\mathcal{A})\right)\right)\right)\right\} \\ & \preccurlyeq NM \backsim Int\left\{\phi^{-1}\left(NM \backsim Int\left(p_i{}^{-1}(NM \backsim Cl(\mathcal{A}))\right)\right)\right\} \\ & = NM \backsim Int\left\{\phi^{-1}\left(p_i{}^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A})))\right)\right\} \\ & = NM \backsim Int(p_i\phi)^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A}))) \end{aligned}$$

Theorem 15. *Let X and Y be NMTSs such that X is product related to Y and let $\phi : X \to Y$ be a mapping. Then, the graph $\psi : X \to X \times Y$ of ϕ is NMACM if ϕ is NMACM.*

Proof. Consider that ψ is an NMACM and $\mathcal{A}$ is an NMOS of Y. Then, using Lemma 4 and Theorems 10 (c), we have

$$\begin{aligned} \phi^{-1}(\mathcal{A}) \ & = 1_{NM} \Cap \phi^{-1}(\mathcal{A}) \\ & = \psi^{-1}(1_{NM} \times \mathcal{A}) \preccurlyeq NM \backsim Int\left(\psi^{-1}(NM \backsim Int(NM \backsim Cl(1_{NM} \times \mathcal{A})))\right) \\ & = NM \backsim Int\left(\psi^{-1}(1_{NM} \times NM \backsim Int(NM \backsim Cl(\mathcal{A})))\right) \\ & = NM \backsim Int\left(\psi^{-1}(NM \backsim Int(1_{NM} \times NM \backsim Cl(\mathcal{A})))\right) \\ & = NM \backsim Int\left(\psi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{A})))\right) \end{aligned}$$

Thus, by Theorem 10 (c), ϕ is NMACM.

Conversely, let ϕ be an NMACM and $\mathcal{B} = \Cup \left(\mathcal{B}_\alpha \times \mathcal{A}_\beta\right)$, where $\mathcal{B}_\alpha$'s and $\mathcal{A}_\beta$'s are NMOSs of X and Y, respectively, be an NMOS of $X \times Y$.

Since $\mathcal{B}_\alpha \Cap NM \backsim Int\left(\phi^{-1}\left(NM \backsim Int\left(NM \backsim Cl\left(\mathcal{A}_\beta\right)\right)\right)\right)$ is an NMOSs of X contained in

$$NM \backsim Int(NM \backsim Cl(\mathcal{B}_\alpha)) \Cap \ \phi^{-1}\left(NM \backsim Int\left(NM \backsim Cl\left(\mathcal{A}_\beta\right)\right)\right),$$

$$\mathcal{B}_\alpha \Cap NM \backsim Int\left(\phi^{-1}\left(NM \backsim Int\left(NM \backsim Cl\left(\mathcal{A}_\beta\right)\right)\right)\right)$$

$$\preccurlyeq NM \backsim Int\Big[NM \backsim Int(NM \backsim Cl(\mathcal{B}_\alpha)) \Cap \phi^{-1}\big(NM \backsim Int\big(NM \backsim Cl\big(\mathcal{A}_\beta\big)\big)\big)\Big]$$

and hence, using Lemmas 1(i), 4 and 5, and Theorems 10 (c), we have

$$\begin{aligned}
\phi^{-1}(\mathcal{B}) &= \phi^{-1}\big(\Cup\,\big(\mathcal{B}_\alpha \times \mathcal{A}_\beta\big)\big) \\
&= \Cup\,\big[\mathcal{B}_\alpha \Cap \phi^{-1}\big(\mathcal{A}_\beta\big)\big] \\
&\preccurlyeq \Cup\,\big[\mathcal{B}_\alpha \Cap NM \backsim Int\big(\phi^{-1}\big(NM \backsim Int\big(NM \backsim Cl\big(\mathcal{A}_\beta\big)\big)\big)\big)\big] \\
&\preccurlyeq \Cup\,\big[NM \backsim Int(NM \backsim Int(NM \backsim Cl(\mathcal{B}_\alpha))) \Cap \phi^{-1}\big(NM \backsim Int\big(NM \backsim Cl\big(\mathcal{A}_\beta\big)\big)\big)\big] \\
&\preccurlyeq NM \backsim Int\big[\Cup\;\psi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{B}_\alpha))) \times NM \backsim Int\big(NM \backsim Cl\big(\mathcal{A}_\beta\big)\big)\big] \\
&= NM \backsim Int\big[\psi^{-1}\big(\Cup\,\big(NM \backsim Int\big(NM \backsim Cl\big(\mathcal{B}_\alpha \times \mathcal{A}_\beta\big)\big)\big)\big)\big] \\
&\preccurlyeq NM \backsim Int\big[\psi^{-1}\big(NM \backsim Int\big(NM \backsim Cl\big(\Cup\,\big(\mathcal{B}_\alpha \times \mathcal{A}_\beta\big)\big)\big)\big)\big] \\
&= NM \backsim Int\big[\psi^{-1}(NM \backsim Int(NM \backsim Cl(\mathcal{B})))\big]
\end{aligned}$$

Thus, by Theorem 10(c), ψ is NMACM. □

Definition 25. *Let $\mathbb{G}$ be an NMG on a group X. Now, if τ_X is an NMT on $\mathbb{G}$, then $(\mathbb{G}\,, \tau_X)$ is said to be a neutrosophic multi almost topological group (NMATG) if the given conditions are satisfied:*

(i) $\alpha : (\mathbb{G}\,, \tau_X) \times (\mathbb{G}, \tau_X) \;\to (\mathbb{G}\,, \tau_X) : \alpha(m, n) = mn$ *is NMACM;*

(ii) $\beta : (\mathbb{G}\,, \tau_X) \;\to (\mathbb{G}\,, \tau_X)\; : \beta(m) = m^{-1}$ *is NMACM.*

Then $(\mathbb{G}\,, \tau_X)$ *is known as an NMATG.*

Remark 7. $(\mathbb{G}\,, \tau_X)$ *is an NMATG if the below conditions hold good:*

(i) *For $g_1, g_2 \in \mathbb{G}$ and every NMROS $\mathcal{P}$ containing g_1g_2 in $\mathbb{G}$, $\exists$ open neighborhoods $\mathcal{R}$ and $\mathcal{S}$ of g_1 and g_2 in $\mathbb{G}$ such that $\mathcal{R} * \mathcal{S} \preccurlyeq \mathcal{P}$;*

(ii) *For $g \in \mathbb{G}$ and every N in $\mathbb{G}$ containing g^{-1}, $\exists$ open neighborhood $\mathcal{R}$ of g in $\mathbb{G}$ so that $\mathcal{R}^{-1} \preccurlyeq \mathcal{S}$.*

Remark 8. *For any $\mathcal{P}, \mathcal{Q} \preccurlyeq \mathbb{G}$, we denote $\mathcal{P} * \mathcal{Q}$ by $\mathcal{P}\mathcal{Q}$ and defined as $\mathcal{P}\mathcal{Q} = \{gh : g \in \mathcal{P},\ h \in \mathcal{Q}\}$ and $\mathcal{P}^{-1} = \big\{g^{-1} : g \in \mathcal{P}\big\}$. If $\mathcal{P} = \{a\}$ for each $a \in \mathbb{G}$, we denote $\mathcal{P} * \mathcal{Q}$ by $a\mathcal{Q}$ and $\mathcal{Q} * \mathcal{P}$ by $\mathcal{P}a$.*

Example 12. *Let, $\mathbb{G} = (\mathbb{Z}_3, +)$ be a classical group and*

$$\mathcal{A} = \left\{ \begin{array}{c} <0,\ 0.4, 0.5, 0.6>, <0,\ 0.3, 0.5, 0.7>, <0,\ 0.2, 0.6, 0.8>, \\ <1,\ 0.3, 0.5, 0.4>, <1,\ 0.2, 0.5, 0.6>, <1,\ 0.1, 0.5, 0.7>, \\ <2,\ 0.4, 0.5, 0.6>, <2,\ 0.3, 0.5, 0.7>, <2,\ 0.2, 0.6, 0.8> \end{array} \right\}$$

Then $\tau_{\mathbb{G}} = \{0_G, 1_G, \mathcal{A}\}$ is NTS and the mapping $\alpha : (\mathbb{G}\,, \tau_{\mathbb{G}}) \times (\mathbb{G}, \tau_{\mathbb{G}}) \;\to (\mathbb{G}, \tau_{\mathbb{G}}) : \alpha(m, n) = mn$ and $\beta : (\mathbb{G}\,, \tau_{\mathbb{G}}) \;\to (\mathbb{G}\,, \tau_{\mathbb{G}}) : \beta(m) = m^{-1}$ are NMACM. Hence, $(\mathbb{G}\,, \tau_{\mathbb{G}})$ is NMATG.

Theorem 16. *Let $(\mathbb{G}\,, \tau_X)$ be an NMATG and let a be any element of $\mathbb{G}$. Then*

(a) $\mu_a : (\mathbb{G}\,, \tau_X) \to (\mathbb{G}\,, \tau_X) : \mu_a(x) = ax\,, \forall x \in \mathbb{G}$, *is NMACM;*

(b) $\lambda_a : (\mathbb{G}\,, \tau_X) \to (\mathbb{G}\,, \tau_X) : \lambda_a(x) = xa, \forall x \in \mathbb{G}$, *is NMACM.*

Proof. (a) Let $p \in \mathbb{G}$ and let $\mathcal{R}$ be an NMROS containing ap in $\mathbb{G}$. By Definition 25, $\exists$ open neighborhoods $\mathcal{P}, \mathcal{Q}$ of a, p in $\mathbb{G}$ such that $\mathcal{P}\mathcal{Q} \preccurlyeq \mathcal{R}$. Especially, $a\mathcal{Q} \preccurlyeq \mathcal{R}$, i.e., $\mu_a(\mathcal{Q}) \preccurlyeq \mathcal{R}$. This proves that μ_a is NMACM at p, and hence, μ_a is NMACM.

(b) Suppose $p \in \mathbb{G}$ and $\mathcal{R} \in NMRO(\mathbb{G})$ contain pa. Then $\exists$ open sets $p \in \mathcal{P}$ and $a \in \mathcal{Q}$ in $\mathbb{G}$ such that $\mathcal{P}\mathcal{Q} \preccurlyeq \mathcal{R}$. This proves $\mathcal{P}a \preccurlyeq \mathcal{R}$. This shows that λ_a is NMACM at p. Since arbitrary element p is in $\mathbb{G}$, hence, λ_a is NMACM. □

Theorem 17. *Let $\mathcal{U}$ be NMROS in a NMATG $(\mathbb{G}\,, \tau_X)$. The below conditions hold good:*

(a) $m\mathcal{U} \in NMROS(\mathbb{G}), \forall m \in \mathbb{G}$;

(b) $\mathcal{U}m \in NMROS(\mathbb{G}), \forall m \in \mathbb{G}$;

(c) $\mathcal{U}^{-1} \in NMROS(\mathbb{G})$.

Proof. (a) We first show that $m\mathcal{U} \in \tau_X$. Let $p \in m\mathcal{U}$. Then by Definition 25 of NMATGs, $\exists$ NMOSs $m^{-1} \in W_1$ and $p \in W_2$ in $\mathbb{G}$ such that $W_1W_2 \preccurlyeq \mathcal{U}$. Especially, $m^{-1}W_2 \preccurlyeq \mathcal{U}$. That is, equivalently, $W_2 \preccurlyeq m\mathcal{U}$. This indicates that $p \in NM \backsim Int(m\mathcal{U})$ and thus, $NM \backsim Int(m\mathcal{U}) = m\mathcal{U}$. That is $m\mathcal{U} \in \tau_X$. Consequently, $m\mathcal{U} \preccurlyeq NM \backsim Int(NM \backsim Cl(m\mathcal{U}))$.

Now, we have to prove that $NM \backsim Int(NM \backsim Cl(m\mathcal{U})) \preccurlyeq m\mathcal{U}$. As $\mathcal{U}$ is NMOS, $NM \backsim Cl(\mathcal{U}) \in NMRCS(\mathbb{G})$. By Theorem 16, $\mu_{m^{-1}} : (\mathbb{G}, \tau_X) \to (\mathbb{G}, \tau_X)$ is NMACM, and therefore, $mNM \backsim Cl(\mathcal{U})$ is NMCoS. Thus, $NM \backsim Int(NM \backsim Cl(m\mathcal{U})) \preccurlyeq NM \backsim Cl(m\mathcal{U}) \preccurlyeq mNM \backsim Cl(\mathcal{U})$, i.e., $m^{-1}NM \backsim Int(NM \backsim Cl(m\mathcal{U})) \preccurlyeq NM \backsim Cl(\mathcal{U})$. Since $NM \backsim Int(NM \backsim Cl(m\mathcal{U}))$ is NMROS, it follows that $m^{-1}NM \backsim Int(NM \backsim Cl(m\mathcal{U})) \preccurlyeq NM \backsim Int(NM \backsim Cl(\mathcal{U})) = \mathcal{U}$, i.e., $NM \backsim Int(NM \backsim Cl(m\mathcal{U})) \preccurlyeq m\mathcal{U}$. Thus $m\mathcal{U} = NM \backsim Int(NM \backsim Cl(m\mathcal{U}))$. This proves that $m\mathcal{U} \in NMROS(\mathbb{G})$.

(b) Following the same steps as in part (1) above, we can prove that $\mathcal{U}m \in NMROS(\mathbb{G}), \forall\, m \in \mathbb{G}$.

(c) Let $p \in \mathcal{U}^{-1}$, then $\exists$ open set $p \in W$ in $\mathbb{G}$ such that $W^{-1} \preccurlyeq \mathcal{U} \Rightarrow W \preccurlyeq \mathcal{U}^{-1}$. Thus, $\mathcal{U}^{-1}$ has interior point p. Thus, $\mathcal{U}^{-1}$ is NMOS. That is, $\mathcal{U}^{-1} \preccurlyeq NM \backsim Int\big(NM \backsim Cl\big(\mathcal{U}^{-1}\big)\big)$. Now we have to prove that $NM \backsim Int\big(NM \backsim Cl\big(\mathcal{U}^{-1}\big)\big) \preccurlyeq \mathcal{U}^{-1}$. Since $\mathcal{U}$ is NMOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS and thus $NM \backsim Cl(\mathcal{U})^{-1}$ is CoNMS in $\mathbb{G}$. Thus, $NM \backsim Int\big(NM \backsim Cl\big(\mathcal{U}^{-1}\big)\big) \preccurlyeq NM \backsim Cl\big(\mathcal{U}^{-1}\big) \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1} \Rightarrow NM \backsim Int\big(NM \backsim Cl\big(\mathcal{U}^{-1}\big)\big) \preccurlyeq (NM \backsim Cl(\mathcal{U}))^{-1} \preccurlyeq \mathcal{U}^{-1}$. Thus, $\mathcal{U}^{-1} = NM \backsim Int\big(NM \backsim Cl\big(\mathcal{U}^{-1}\big)\big)$. This proves that $\mathcal{U}^{-1} \in NMROS(\mathbb{G})$. □

Corollary 1. *Let $\mathcal{Q}$ be any NMRCoS in an NMATG in $\mathbb{G}$. Then*

(a) $m\mathcal{Q} \in NMRCS(\mathbb{G})$*, for each* $m \in \mathbb{G}$*;*

(b) $\mathcal{Q}^{-1} \in NMRCS(\mathbb{G})$.

Theorem 18. *Let $\mathcal{U}$ be any NMROS in an NMATG $\mathbb{G}$. Then*

(a) $NM \backsim Cl(\mathcal{U}m) = NM \backsim Cl(\mathcal{U})m$*, for each* $m \in \mathbb{G}$*;*

(b) $NM \backsim Cl(m\mathcal{U}) = mNM \backsim Cl(\mathcal{U}),\ for\ each\ m \in \mathbb{G};$

(c) $NM \backsim Cl\big(\mathcal{U}^{-1}\big) = NM \backsim Cl(\mathcal{U})^{-1}$.

Proof. (a) Assume $p \in NM \backsim Cl(\mathcal{U}m)$ and consider $q = pm^{-1}$. Let $q \in W$ be NMOS in $\mathbb{G}$. Then $\exists$ NMOSs $m^{-1} \in V_1$ and $p \in V_2$ in $\mathbb{G}$, such that $V_1V_2 \preccurlyeq NM \backsim Int(NM \backsim Cl(W))$. By hypothesis, there is $g \in \mathcal{U}m \Cap V_2 \Rightarrow gm^{-1} \in \mathcal{U} \Cap V_1V_2 \preccurlyeq \mathcal{U} \Cap NM \backsim Int(NM \backsim Cl(W)) \Rightarrow \mathcal{U} \Cap NM \backsim Int(NM \backsim Cl(W)) \neq 0_{\mathrm{NM}} \Rightarrow \mathcal{U} \Cap (NM \backsim Cl(W)) \neq 0_{\mathrm{NM}}$. Since $\mathcal{U}$ is NMOS, $\mathcal{U} \Cap W \neq 0_{\mathrm{NM}}$. That is, $m \in NM \backsim Cl(\mathcal{U})m$.

Conversely, let $q \in NM \backsim Cl(\mathcal{U})m$. Then $q = pg$ for some $p \in NM \backsim Cl(\mathcal{U})$.

To prove $NM \backsim Cl(\mathcal{U})m \preccurlyeq NM \backsim Cl(\mathcal{U}m)$.

Let $pg \in W$ be an NMOS in $\mathbb{G}$. Then $\exists$ NMOSs $m \in V_1$ in $\mathcal{G}$ and $p \in V_2$ in $\mathbb{G}$ so that $V_1V_2 \preccurlyeq NM \backsim Int(NM \backsim Cl(W))$. Since $p \in NM \backsim Cl(\mathcal{U})$, $\mathcal{U} \Cap V_2 \neq 0_{\mathrm{NM}}$. There is $g \in \mathcal{U} \Cap V_2$. This implies $gm \in (\mathcal{U}m) \Cap NM \backsim Int(NM \backsim Cl(W)) \Rightarrow (\mathcal{U}m) \Cap (NM \backsim Cl(W)) \neq 0_{\mathrm{NM}}$. From Theorem 17, $\mathcal{U}m$ is NMOS and thus $(\mathcal{U}m) \Cap W \neq 0_{\mathrm{NM}}$, therefore $q \in NM \backsim Cl(\mathcal{U}m)$. Therefore $NM \backsim Cl(\mathcal{U}m) = NM \backsim Cl(\mathcal{U})m$.

(b) Following the same steps as in part (1) above, we can prove that $NM \backsim Cl(m\mathcal{U}) = mNM \backsim Cl(\mathcal{U})$.

(c) Since $NM \backsim Cl(\mathcal{U})$ is NMRCoS, $NM \backsim Cl(\mathcal{U})^{-1}$ is NMCoS in $\mathbb{G}$. Therefore, $\mathcal{U}^{-1} \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1}$ this gives $NM \backsim Cl\big(\mathcal{U}^{-1}\big) \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1}$. Next, let $q \in NM \backsim Cl(\mathcal{U})^{-1}$. Then $q = p^{-1}$, for some $p \in NM \backsim Cl(\mathcal{U})$. Let $q \in V$ be any NMOS in $\mathbb{G}$. Then $\exists$ open set $\mathcal{U}$ in $\mathbb{G}$ such that $p \in \mathcal{U}$ with $\mathcal{U}^{-1} \preccurlyeq NM \backsim Int(NM \backsim Cl(V))$. Moreover, there is $m \in \mathcal{A} \Cap \mathcal{U}$ which implies $m^{-1} \in \mathcal{U}^{-1} \Cap NM \backsim Int(NM \backsim Cl(V))$. That is, $\mathcal{U}^{-1} \Cap NM \backsim$

$Int(NM \backsim Cl(V)) \neq 0_{\mathrm{NM}} \Rightarrow \mathcal{U}^{-1} \Cap NM \backsim Cl(V) \neq 0_{\mathrm{NM}} \Rightarrow \mathcal{U}^{-1} \Cap V \neq 0_{\mathrm{NM}}$, since $\mathcal{U}^{-1}$ is NMOS. Therefore, $q \in NM \backsim Cl(\mathcal{U})^{-1}$. Hence, $NM \backsim Cl(\mathcal{U}^{-1}) \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1}$. □

Theorem 19. *Let $\mathcal{Q}$ be NMRCo subset in an NMATG $\mathbb{G}$. Then the below assertions are true:*

(a) $NM \backsim Int(m\mathcal{Q}) = aNM \backsim Int(\mathcal{Q}), \forall m \in \mathbb{G}$;
(b) $NM \backsim Int(\mathcal{Q}m) = NM \backsim Int(\mathcal{Q})a, \forall m \in \mathbb{G}$;
(c) $NM \backsim Int(\mathcal{Q}^{-1}) = NM \backsim Int(\mathcal{Q})^{-1}$.

Proof. (a) Since $\mathcal{Q}$ is NMRCoS, $NM \backsim Int(\mathcal{Q})$ is NMROS in $\mathbb{G}$. Consequently, $mNM \backsim Int(\mathcal{Q}) \preccurlyeq NM \backsim Int(m\mathcal{Q})$. Conversely, let $q \in NM \backsim Int(m\mathcal{Q})$ be an arbitrary element. Suppose $q = mp$, for some $p \in \mathcal{Q}$. By hypothesis, this proves $m\mathcal{Q}$ is NMCoS, and that is $NM \backsim Int(m\mathcal{Q})$ is NMROS in $\mathbb{G}$. Assume that $m \in U$ and $p \in V$ be NMOSs in $\mathbb{G}$, such that $UV \preccurlyeq NM \backsim Int(m\mathcal{Q})$. Then $mV \preccurlyeq m\mathcal{Q}$, which means that $mV \preccurlyeq mNM \backsim Int(\mathcal{Q})$. Thus, $NM \backsim Int(m\mathcal{Q}) \preccurlyeq mNM \backsim Int(\mathcal{Q})$.

(b) Following the same steps as in part (1) above, we can prove that $NM \backsim Int(\mathcal{Q}m) \preccurlyeq NM \backsim Int(\mathcal{Q})m$.

(c) Since $NM \backsim Int(\mathcal{Q})$ is NMROS, $NM \backsim Int(\mathcal{Q})^{-1}$ is NMOS in $\mathbb{G}$. Therefore, $\mathcal{Q}^{-1} \preccurlyeq NM \backsim Int(\mathcal{Q})^{-1}$ implies that $NM \backsim Int(\mathcal{Q}^{-1}) \preccurlyeq NM \backsim Int(\mathcal{Q})^{-1}$. Next, let q be an arbitrary element of $NM \backsim Int(\mathcal{Q})^{-1}$. Then $q = p^{-1}$, for some $p \in NM \backsim Int(\mathcal{Q})$. Let $q \in V$ be NMOS in $\mathbb{G}$. Then $\exists$ NMOS U is in $\mathbb{G}$, such that $p \in U$ with $U^{-1} \preccurlyeq NM \backsim Cl(NM \backsim Int(V))$. Moreover, there is $g \in \mathcal{Q} \Cap U$, which implies $g^{-1} \in \mathcal{Q}^{-1} \Cap NM \backsim Cl(NM \backsim Int(V))$. That is $\mathcal{Q}^{-1} \Cap NM \backsim Cl(NM \backsim Int(V)) \neq 0_{\mathrm{NM}} \Rightarrow \mathcal{Q}^{-1} \Cap NM \backsim Int(V) \neq 0_{\mathrm{NM}} \Rightarrow \mathcal{Q}^{-1} \Cap V \neq 0_{\mathrm{NM}}$, since $\mathcal{Q}^{-1}$ is NMCoS. Hence, $NM \backsim Int(\mathcal{Q}^{-1}) = NM \backsim Int(\mathcal{Q})^{-1}$. □

Theorem 20. *Let $\mathcal{U}$ be any NMSOS in an NMATG $\mathbb{G}$. Then*

(a) $NM \backsim Cl(m\mathcal{U}) \preccurlyeq mNM \backsim Cl(\mathcal{U}), \forall\, m \in \mathbb{G}$;
(b) $NM \backsim Cl(\mathcal{U}m) \preccurlyeq NM \backsim Cl(\mathcal{U})m, \forall\, m \in \mathbb{G}$;
(c) $NM \backsim Cl(\mathcal{U}^{-1}) \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1}$.

Proof. (a) As $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS. From Theorem 16, $\mu_{m^{-1}} : (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ is NMACM. Thus, $mNM \backsim Cl(\mathcal{U})$ is NMCoS. Hence, $NM \backsim Cl(m\mathcal{U}) \preccurlyeq mNM \backsim Cl(\mathcal{U})$.

(b) As $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS. From Theorem 16, $\lambda_{m^{-1}} : (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ is NMACM. Thus, $NM \backsim Cl(\mathcal{U})m$ is NMCoS. Therefore, $NM \backsim Cl(\mathcal{U}m) \preccurlyeq NM \backsim Cl(\mathcal{U})m$.

(c) Since $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS, and hence, $NM \backsim Cl(\mathcal{U})^{-1}$ is NMCoS. Consequently, $NM \backsim Cl(\mathcal{U}) \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1}$. □

Theorem 21. *Let $\mathcal{U}$ be both NMSO and NMSCo subset of an NMATG $\mathbb{G}$. Then the below statements hold:*

(a) $NM \backsim Cl(m\mathcal{U}) = mNM \backsim Cl(\mathcal{U})$, *for each* $m \in \mathbb{G}$;
(b) $NM \backsim Cl(\mathcal{U}m) = NM \backsim Cl(\mathcal{U})m,\ for\ each\ m \in \mathbb{G}$;
(c) $NM \backsim Cl(\mathcal{U}^{-1}) = NM \backsim Cl(\mathcal{U})^{-1}$.

Proof. (a) Since $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS, from which it implies that $NM \backsim Cl(m\mathcal{U}) \preccurlyeq mNM \backsim Cl(\mathcal{U})$. Further, neutrosophic multi semi-openness of $\mathcal{U}$ gives $NM \backsim Cl(\mathcal{U}) = NM \backsim Cl(NM \backsim Int(\mathcal{U})) \Rightarrow mNM \backsim Cl(\mathcal{U}) = mNM \backsim Cl(NM \backsim Int(\mathcal{U})$. As $\mathcal{U}$ is NMSCoS, $NM \backsim Int(\mathcal{U})$ is NMROS in $\mathbb{G}$. From Theorem 20, $mNM \backsim Cl(\mathcal{U}) = mNM \backsim Cl(NM \backsim Int(\mathcal{U})) = NM \backsim Cl(mNM \backsim Int(\mathcal{U})) \preccurlyeq NM \backsim Cl(m\mathcal{U})$. Hence, NM $\backsim Cl(m\mathcal{U}) = mNM \backsim Cl(\mathcal{U})$.

(b) Following the same steps as in part (1) above, we can prove that $NM \backsim Cl(\mathcal{U}m) = NM \backsim Cl(\mathcal{U})m$.

(c) By hypothesis, this proves $NM \backsim Cl(\mathcal{U})$ is NMRCoS and therefore $NM \backsim Cl(\mathcal{U})^{-1}$ is NMCoS. Consequently, $NM \backsim Cl(\mathcal{U}^{-1}) \preccurlyeq NM \backsim Cl(\mathcal{U})^{-1}$. Next, since $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U}) = NM \backsim Cl(NM \backsim Int(\mathcal{U})) \Rightarrow NM \backsim Cl(\mathcal{U})^{-1} = NM \backsim Cl(NM \backsim Int(\mathcal{U})$. Moreover, as $\mathcal{U}$ is NMSCoS, $NM \backsim Int(\mathcal{U})$ is NMROS. From Theorem 18, $NM \backsim Cl(\mathcal{U})^{-1} = NM \backsim Cl\left(NM \backsim Int(\mathcal{U})^{-1}\right) \preccurlyeq NM \backsim Cl(\mathcal{U}^{-1})$. This shows that $NM \backsim Cl(\mathcal{U}^{-1}) = NM \backsim Cl(\mathcal{U})^{-1}$. □

Theorem 22. *From Theorem 21, the following statements hold:*

(a) $NM \backsim Int(m\mathcal{U}) = mNM \backsim Int(\mathcal{U})$, *for each* $m \in \mathbb{G}$*;*
(b) $NM \backsim Int(\mathcal{U}m) = NM \backsim Int(\mathcal{U})m,\ for\ each\ m \in \mathbb{G}$*;*
(c) $NM \backsim Int(\mathcal{U}^{-1}) = NM \backsim Int(\mathcal{U})^{-1}$.

Proof. (a) As $\mathcal{U}$ is NMSCoS, $NM \backsim Int(\mathcal{U})$ is NMROS. From Theorem 16, $\mu_{m^{-1}} : (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ is NMACM. Therefore, ${\mu^{-1}}_{m^{-1}}(NM \backsim Int(\mathcal{U})) = mNM \backsim Int(\mathcal{U})$ is NMOS. Thus, $mNM \backsim Int(\mathcal{U}) \preccurlyeq NM \backsim Int(m\mathcal{U})$. Next, by assumption, it implies that $NM \backsim Int(\mathcal{U}) = NM \backsim Int(NM \backsim Cl(\mathcal{U})) \Rightarrow mNM \backsim Int(\mathcal{U}) = mNM \backsim Int(NM \backsim Cl(\mathcal{U}))$. As $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS. From Theorem 19, $mNM \backsim Int(NM \backsim Cl(\mathcal{U})) = NM \backsim Int(mNM \backsim Cl(\mathcal{U})) \succcurlyeq NM \backsim Int(m\mathcal{U})$. That is, $NM \backsim Int(m\mathcal{U}) \preccurlyeq mNM \backsim Int(\mathcal{U})$. Therefore, we have, $NM \backsim Int(m\mathcal{U}) = mNM \backsim Int(\mathcal{U})$. Hence, it was proved.

(b) As $\mathcal{U}$ is NMSCoS, $NM \backsim Int(\mathcal{U})$ is NMROS. From Theorem 16, $\mu_{m^{-1}} : (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ is NMACM. Thus, ${\lambda^{-1}}_{m^{-1}}(NM \backsim Int(\mathcal{U})) = mNM \backsim Int(\mathcal{U})$ is NMOS. Therefore, $NM \backsim Int(\mathcal{U})m \preccurlyeq NM \backsim Int(\mathcal{U}m)$. Next, by assumption, this proves that $NM \backsim Int(\mathcal{U}) = NM \backsim Int(NM \backsim Cl(\mathcal{U})) \Rightarrow NM \backsim Int(\mathcal{U})m = NM \backsim Int(NM \backsim Cl(\mathcal{U}))m$. As $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS. From Theorem 19, $NM \backsim Int(NM \backsim Cl(\mathcal{U}))m = NM \backsim Int(NM \backsim Cl(\mathcal{U})m) \succcurlyeq NM \backsim Int(\mathcal{U}m)$. That is, $NM \backsim Int(\mathcal{U}m) \preccurlyeq NM \backsim Int(\mathcal{U})m$. Therefore, $NM \backsim Int(\mathcal{U}m) = NM \backsim Int(\mathcal{U})m$. Hence, it was proved.

(c) From assumption, this proves that $NM \backsim Int(\mathcal{U})$ is NMROS and therefore $NM \backsim Int(\mathcal{U})^{-1}$ is NMOS. Consequently, $NM \backsim Int(\mathcal{U}^{-1}) \preccurlyeq NM \backsim Int(\mathcal{U})^{-1}$. Next, as $\mathcal{U}$ is NMSCoS, $NM \backsim Int(\mathcal{U}) = NM \backsim Int(NM \backsim Cl(\mathcal{U})) \Rightarrow NM \backsim Int(\mathcal{U})^{-1} = NM \backsim Int(NM \backsim Cl(\mathcal{U}))^{-1}$. Moreover, as $\mathcal{U}$ is NMSOS, $NM \backsim Cl(\mathcal{U})$ is NMRCoS. From Theorem 19, $NM \backsim Int(\mathcal{U})^{-1} = NM \backsim Int(NM \backsim Cl(\mathcal{U})^{-1}) \preccurlyeq NM \backsim Int(\mathcal{U}^{-1})$. This proves that $NM \backsim Int(\mathcal{U}^{-1}) = NM \backsim Int(\mathcal{U})^{-1}$. □

Theorem 23. *Let* $\mathcal{A}$ *be NMOS in an NMATG* $\mathbb{G}$*. Then* $a\mathcal{A} \preccurlyeq NM \backsim Int(aNM \backsim Int(NM \backsim Cl(\mathcal{A})))$ *for* $a \in \mathbb{G}$*.*

Proof. Since $\mathcal{A}$ is NMOS, so $\mathcal{A} \preccurlyeq NM \backsim Int(NM \backsim Cl(\mathcal{A})) \Rightarrow a\mathcal{A} \preccurlyeq aNM \backsim Int(NM \backsim Cl(\mathcal{A}))$. From Theorem 17, $aNM \backsim Int(NM \backsim Cl(\mathcal{A}))$ is NMOS (in fact, NMROS). Hence, $a\mathcal{A} \preccurlyeq NM \backsim Int(aNM \backsim Int(NM \backsim Cl(\mathcal{A})))$. □

Theorem 24. *Let* $\mathcal{Q}$ *be any neutrosophic multi-closed subset in an NMATG* $\mathbb{G}$*. Then* $NM \backsim Cl(aNM \backsim Cl(NM \backsim Int(\mathcal{A}))) \preccurlyeq a\mathcal{Q}$ *for each* $a \in \mathbb{G}$*.*

Proof. Since $\mathcal{Q}$ is NMCoS, so $\mathcal{Q} \succcurlyeq NM \backsim Cl(NM \backsim Int(\mathcal{Q})) \Rightarrow a\mathcal{Q} \succcurlyeq aNM \backsim Cl(NM \backsim Int(\mathcal{Q}))$. From Theorem 17, $aNM \backsim Cl(NM \backsim Int(\mathcal{Q}))$ is NMCoS (in fact, NMRCoS). Therefore, $a\mathcal{Q} \succcurlyeq NM \backsim Cl(aNM \backsim Cl(NM \backsim Int(\mathcal{A})))$. Hence, $NM \backsim Cl(aNM \backsim Cl(NM \backsim Int(\mathcal{A}))) \preccurlyeq a\mathcal{Q}$. □

4. Conclusions

To deal with uncertainty, the NS uses the truth membership function, indeterminacy membership function, and falsity membership function. By discovering this concept, we were able to generalise the idea of an almost topological group to an NMATG. First, we developed the definitions of NMSOS, NMSCoS, NMROS, NMRCoS, NMCM, NMOM, NMCoM, NMSCM, NMSOM, NMSCoM to propose the definition of NMATG. Some properties of NMACM were demonstrated. Finally, we defined NMATG and demonstrated some of their properties using the definition of NMACM. In this study, an NMATG is conceptualised for the environments of the NS along with some of their elementary properties and theoretic operations. Novel numerical examples are given for definitions and remarks to study NMATG. We expect that our study may spark some new ideas for the construction of the NMATG. Future work may include the extension of this work for:

(1) The development of the NMATG of the neutrosophic multi-vector spaces, etc.;
(2) Dealing NMATG with multi-criteria decision-making techniques.

Author Contributions: Conceptualization: B.B., N.W., D.D.M., A.K.B., J.M., U.R.B. and J.B. All authors have contributed equally to this paper in all aspects. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: We would like to thank the anonymous reviewers for their very careful reading and valuable suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Chang, C.L. Fuzzy Topological Space. *J. Math. Anal. Appl.* **1968**, *24*, 182–190. [CrossRef]
3. Coker, D. An introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets Syst.* **1997**, *88*, 81–89. [CrossRef]
4. Lupianez, F.G. On neutrosophic topology. *Int. J. Syst. Cybern.* **2008**, *37*, 797–800. [CrossRef]
5. Lupianez, F.G. Interval neutrosophic sets and topology. *Int. J. Syst. Cybern.* **2009**, *38*, 621–624. [CrossRef]
6. Lupianez, F.G. On various neutrosophic topologies. *Int. J. Syst. Cybern.* **2009**, *38*, 1009–1013.
7. Lupianez, F.G. On neutrosophic paraconsistent topology. *Int. J. Syst. Cybern.* **2010**, *39*, 598–601. [CrossRef]
8. Salama, A.A.; Smarandache, F.; Kroumov, V. Closed sets and Neutrosophic Continuous Functions. *Neutrosophic Sets Syst.* **2014**, *4*, 4–8.
9. Kelly, J.C. Bitopological spaces. *Proc. Lond. Math. Soc.* **1963**, *3*, 71–89. [CrossRef]
10. Kandil, A.; Nouth, A.A.; El-Sheikh, S.A. On fuzzy bitopological spaces. *Fuzzy Sets Syst.* **1995**, *74*, 353–363. [CrossRef]
11. Lee, S.J.; Kim, J.T. Some Properties of Intuitionistic Fuzzy Bitopological Spaces. In Proceedings of the 6th International Conference on Soft Computing and Intelligent Systems, and The 13th IEEE International Symposium on Advanced Intelligence Systems, Kobe, Japan, 20–24 November 2012; pp. 20–24. [CrossRef]
12. Garg, H. A new improved score function of an interval-valued Pythagorean fuzzy set based TOPSIS method. *Int. J. Uncertain. Quantif.* **2017**, *7*, 463–474. [CrossRef]
13. Peng, X.; Yang, Y. Some results for Pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2015**, *30*, 1133–1160. [CrossRef]
14. Peng, X.; Selvachandran, G. Pythagorean fuzzy set: State of the art and future directions. *Artif. Intell. Rev.* **2017**, *52*, 1873–1927. [CrossRef]
15. Beliakov, G.; James, S. Averaging aggregation functions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs. In Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Beijing, China, 6–11 July 2014; pp. 298–305.
16. Dick, S.; Yager, R.R.; Yazdanbakhsh, O. On Pythagorean and complex fuzzy set operations. *IEEE Trans. Fuzzy Syst.* **2016**, *24*, 1009–1021. [CrossRef]
17. Gou, X.J.; Xu, Z.S.; Ren, P.J. The properties of continuous Pythagorean fuzzy Information. *Int. J. Intell. Syst.* **2016**, *31*, 401–424. [CrossRef]
18. He, X.; Du, Y.; Liu, W. Pythagorean fuzzy power average operators. *Fuzzy Syst. Math.* **2016**, *30*, 116–124.

19. Ejegwa, P.A. Distance and similarity measures of Pythagorean fuzzy sets. *Granul. Comput.* **2018**, *5*, 225–238. [CrossRef]
20. Yager, R.R. Generalized orthopair fuzzy sets. *IEEE Trans. Fuzzy Syst.* **2017**, *25*, 1222–1230. [CrossRef]
21. Yager, R.R.; Alajlan, N. Approximate reasoning with generalized orthopair fuzzy sets. *Inf. Fusion* **2017**, *38*, 65–73. [CrossRef]
22. Peng, X.; Liu, L. Information measures for q-rung orthopair fuzzy sets. *Int. J. Intell. Syst.* **2019**, *34*, 1795–1834. [CrossRef]
23. Pinar, A.; Boran, F.E. A q-rung orthopair fuzzy multi-criteria group decision making method for supplier selection based on a novel distance measure. *Int. J. Mach. Learn. Cybern.* **2020**, *11*, 1749–1780. [CrossRef]
24. Cuong, B.C.; Kreinovich, V. Picture Fuzzy Sets—A new concept for computational intelligence problems. In Proceedings of the 2013 Third World Congress on Information and Communication Technologies (WICT 2013), Hanoi, Vietnam, 15–18 December 2013; pp. 1–6. [CrossRef]
25. Cuong, B.C. Picture fuzzy sets. *J. Comput. Sci. Cybern.* **2014**, *30*, 409–420.
26. Phong, P.H.; Hieu, D.T.; Ngan, R.T.H.; Them, P.T. Some compositions of picture fuzzy relations. In Proceedings of the 7th National Conferenceon Fundamental and Applied Information Technology Research, FAIR'7, Thai Nguyen, Vietnam, 19–20 June 2014; pp. 19–20.
27. Cuong, B.C.; Hai, P.V. Some fuzzy logic operators for picture fuzzy sets. In Proceedings of the Seventh International Conference on Knowledge and Systems Engineering, Ho Chi Minh City, Vietnam, 8–10 October 2015; pp. 132–137.
28. Cuong, B.C.; Ngan, R.T.; Hai, B.D. An involutive picture fuzzy negator on picture fuzzy sets and some De Morgan triples. In Proceedings of the Seventh International Conference on Knowledge and Systems Engineering, Ho Chi Minh City, Vietnam, 8–10 October 2015; pp. 126–131.
29. Viet, P.V.; Chau, H.T.M.; Hai, P.V. Some extensions of membership graphs for picture inference systems. In Proceedings of the 2015 Seventh International Conference on Knowledge and Systems Engineering, (KSE), Ho Chi Minh City, Vietnam, 8–10 October 2015; IEEE: Piscataway, NJ, USA, 2015; pp. 192–197.
30. Singh, P. Correlation coefficients for picture fuzzy sets. *J. Intell. Fuzzy Syst.* **2015**, *28*, 591–604. [CrossRef]
31. Garg, H. Some picture fuzzy aggregation operators and their applications to multicriteria decision-making. *Arab. J. Sci. Eng.* **2017**, *42*, 5275–5290. [CrossRef]
32. Quek, S.G.; Selvachandran, G.; Munir, M.; Mahmood, T.; Ullah, K.; Son, L.H.; Pham, T.H.; Kumar, R.; Priyadarshini, I. Multi-attribute multi-perception decision-making based on generalized T-spherical fuzzy weighted aggregation operators on neutrosophic sets. *Mathematics* **2019**, *7*, 780. [CrossRef]
33. Garg, H.; Munir, M.; Ullah, K.; Mahmood, T.; Jan, N. Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators. *Symmetry* **2018**, *10*, 670. [CrossRef]
34. Zeng, S.; Garg, H.; Munir, M.; Mahmood, T.; Hussain, A. A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of T-spherical fuzzy sets and its application in the selection of solar cells. *Energies* **2019**, *12*, 4436. [CrossRef]
35. Munir, M.; Kalsoom, H.; Ullah, K.; Mahmood, T.; Chu, Y.M. T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems. *Symmetry* **2020**, *12*, 365. [CrossRef]
36. Mahmood, T.; Ullah, K.; Khan, Q.; Jan, N. An approach towards decision making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Comput. Appl.* **2018**, *31*, 7041–7053. [CrossRef]
37. Warren, R.H. Boundary of a fuzzy set. *Indiana Univ. Math. J.* **1977**, *26*, 191–197. [CrossRef]
38. Tang, X. Spatial Object Modeling in Fuzzy Topological Spaces with Applications to Land Cover Change in China. Ph.D. Thesis, University of Twente, Enschede, The Netherlands, 2004.
39. Kharal, A. A Study of Frontier and Semifrontier in Intuitionistic Fuzzy Topological Spaces. *Sci. World J.* **2014**, *2014*, 674171. [CrossRef]
40. Salama, A.A.; Alblowi, S. Generalized neutrosophic set and generalized neutrosophic topological spaces. *Comp. Sci. Eng.* **2012**, *2*, 129–132. [CrossRef]
41. Azad, K.K. On Fuzzy Semi-continuity, Fuzzy Almost Continuity and Fuzzy Weakly Continuity. *J. Math. Anal. Appl.* **1981**, *82*, 14–32. [CrossRef]
42. Smarandache, F. *Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics*; University of New Mexico: Gallup, NM, USA, 2002.
43. Smarandache, F. Neutrosophic set—A generalization of the intuitionistic fuzzy set. *Int. J. Pure Appl. Math.* **2005**, *24*, 287–297.
44. Mwchahary, D.D.; Basumatary, B. A note on Neutrosophic Bitopological Space. *Neutrosophic Sets Syst.* **2020**, *33*, 134–144.
45. Blizard, W. Multiset theory. *Notre Dame J. Form. Logic* **1989**, *30*, 36–66. [CrossRef]
46. Yager, R.R. On the theory of bags. *Int. J. Gen Syst.* **1986**, *13*, 23–37. [CrossRef]
47. Miyamoto, S. Fuzzy Multisets and Their Generalizations. In *Multiset Processing*; Springer: Berlin, Germany, 2001; pp. 225–235.
48. Onasanya, B.O.; Hoskova-Mayerova, S. Some Topological and Algebraic Properties of alpha-level Subsets Topology of a Fuzzy Subset. *An. St. Univ. Ovidius Constanta* **2018**, *26*, 213–227.
49. Onasanya, B.O.; Hoskova-Mayerova, S. Multi-fuzzy group induced by multisets. *Ital. J. Pure Appl. Math.* **2019**, *41*, 597–604.
50. Al Tahan, M.; Hoskova-Mayerova, S.; Davvaz, B. Fuzzy multi-polygroups. *J. Intell. Fuzzy Syst.* **2020**, *38*, 2337–2345. [CrossRef]
51. Al Tahan, M.; Hoskova-Mayerova, S.; Davvaz, B. Some results on (generalized) fuzzy multi-Hv-ideals of Hv-rings. *Symmetry* **2019**, *11*, 1376. [CrossRef]
52. Bakbak, D.; Uluçay, V.; Sahin, M. Neutrosophic Multigroups and Applications. *Mathemaics* **2019**, *7*, 95. [CrossRef]

53. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy sets: An extension of fuzzy sets. *Fuzzy Inf. Eng.* **2011**, *1*, 35–43. [CrossRef]
54. Dey, A.; Pal, M. Multi-fuzzy complex numbers and multi-fuzzy complex sets. *Int. J. Fuzzy Syst. Appl.* **2014**, *4*, 15–27. [CrossRef]
55. Yong, Y.; Xia, T.; Congcong, M. The multi-fuzzy soft set and its application in decision making. *Appl. Math. Model* **2013**, *37*, 4915–4923. [CrossRef]
56. Sebastian, S.; Ramakrishnan, T.T. Multi-Fuzzy Sets. *Int. Math. Forum* **2010**, *5*, 2471–2476.
57. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
58. Shinoj, T.K.; John, S.S. Intuitionistic fuzzy multisets and its application in medical diagnosis. *World Acad. Sci. Eng. Technol.* **2012**, *6*, 1418–1421.
59. Abdullah, S.; Naeem, M. A new type of interval valued fuzzy normal subgroups of groups. *New Trends Math. Sci.* **2015**, *3*, 62–77.
60. Mordeson, J.N.; Bhutani, K.R.; Rosenfeld, A. *Fuzzy Group Theory*; Springer: Berlin/Heidelberg, Germany, 2005.
61. Liu, Y.L. Quotient groups induced by fuzzy subgroups. *Quasigroups Related Syst.* **2004**, *11*, 71–78.
62. Baby, A.; Shinoj, T.K.; John, S.J. On Abelian Fuzzy Multi Groups and Orders of Fuzzy Multi Groups. *J. New Theory* **2015**, *5*, 80–93.
63. Muthuraj, R.; Balamurugan, S. Multi-Fuzzy Group and its Level Subgroups. *Gen. Math. Notes* **2013**, *17*, 74–81.
64. Tella, Y. On Algebraic Properties of Fuzzy Membership Sequenced Multisets. *Br. J. Math. Comput. Sci.* **2015**, *6*, 146–164. [CrossRef]
65. Fathi, M.; Salleh, A.R. Intuitionistic fuzzy groups. *Asian J. Algebra* **2009**, *2*, 1–10. [CrossRef]
66. Li, X.P.; Wang, G.J. (l, a)-Homomorphisms of Intuitionistic Fuzzy Groups. *Hacettepe J. Math. Stat.* **2011**, *40*, 663–672.
67. Palaniappan, N.; Naganathan, S.; Arjunan, K. A study on Intuitionistic L-fuzzy Subgroups. *Appl. Math. Sci.* **2009**, *3*, 2619–2624.
68. Sharma, P.K. Homomorphism of Intuitionistic fuzzy groups. *Int. Math. Forum* **2011**, *6*, 3169–3178.
69. Sharma, P.K. On the direct product of Intuitionistic fuzzy subgroups. *Int. Math. Forum* **2012**, *7*, 523–530.
70. Sharma, P.K. On intuitionistic fuzzy abelian subgroups. Adv. *Fuzzy Sets Syst.* **2012**, *12*, 1–16.
71. Xu, C.Y. Homomorphism of Intuitionistic Fuzzy Groups. In Proceedings of the 2007 International Conference on Machine Learning and Cybernetics, Hong Kong, China, 19–22 August 2007; pp. 178–183. [CrossRef]
72. Yuan, X.H.; Li, H.X.; Lee, E.S. On the definition of the intuitionistic fuzzy subgroups. *Comput. Math. Appl.* **2010**, *59*, 3117–3129. [CrossRef]
73. Shinoj, T.K.; John, S.S. Intuitionistic fuzzy multigroups. *Ann. Pure Appl. Math.* **2015**, *9*, 131–143.
74. Smarandache, F. *A Unifying Field in Logics*; Infinite Study: Conshohocken, PA, USA, 1998.
75. Deli, I.; Broumi, S.; Smarandache, F. On neutrosophic multisets and its application in medical diagnosis. *J. New Theory* **2015**, *6*, 88–98.
76. Ye, S.; Ye, J. Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis. *Neutrosophic Sets Syst.* **2014**, *6*, 49–54.
77. Ye, S.; Fu, J.; Ye, J. Medical Diagnosis Using Distance-Based Similarity Measures of Single Valued Neutrosophic Multisets. *Neutrosophic Sets Syst.* **2015**, *7*, 47–52.
78. Smarandache, F. *Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications*; Pons Publishing House Brussels: Brussels, Belgium, 2017; p. 323.