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Simulating imprecise data: sine–cosine and convolution methods with neutrosophic normal distribution

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Abstract

Objective: The primary aim of this research paper is to introduce and demonstrate the application of the sine–cosine method and the convolution method for simulating data by utilizing the neutrosophic normal distribution.

Method: The methodological framework presented in this paper elaborates on the incorporation of both the sine–cosine method and the convolution method into the realm of neutrosophic statistics. It also introduces algorithms engineered to produce random variables adhering to the neutrosophic normal distribution.

Results: Moreover, the study furnishes practical tables that encompass neutrosophic random normal variables generated via the sine–cosine method, as well as tables exhibiting neutrosophic random standard normal variables generated using the convolution method.

Conclusion: The analysis undertaken in this study conclusively establishes that the proposed sine–cosine and convolution simulation methods yield outcomes presented in the form of intervals. Furthermore, the study's conclusion emphasizes that the extent of indeterminacy significantly influences the characteristics of the random variates.

Keywords: Classical method of simulation, Uncertainty, Neutrosophy, Normal distribution, Simulation

Introduction

Random numbers hold significant importance across various domains, including experimental design, statistical analysis, cryptography, and diverse disciplines, as highlighted by [26]. In earlier times, random numbers were derived through methods such as coin flipping and other simulation techniques. However, with the advent of high-speed computing, contemporary approaches employ computer-based generators to produce random variations efficiently. These generators are tailored to the specific problems at hand and involve algorithmic designs aimed at generating random variables for a range of applications. Among various simulation methodologies, the sine–cosine simulation method and the convolution simulation method have gained substantial traction, particularly in the generation of standard normal variates. These techniques involve the creation of continuous uniform numbers

sourced from the continuous uniform distribution. Subsequently, these numbers are leveraged to generate standard normal variates. The sine–cosine method was introduced by [8]. Applied [13] the sine–cosine algorithm in solving the issues in machine scheduling problems [11]. used sine–cosine transformation in developing the probability distribution [18]. Discussed the algorithm to generate random variate from the normal distribution [7]. Applied both the sine–cosine and convolution methods in a neural network application [19]. Implemented the sine–cosine algorithms within a wastewater treatment system. More information on sine–cosine and convolution transformation can be seen in [12, 22] and [16]. Various studies on generating random numbers can be seen in [4, 14, 21] and [27].

The neutrosophic statistics (NS) introduced by [25] and applied when the observations in the data are in an interval or have imprecise observations. The NS was found to be more efficient than classical statistics (CS) in terms of flexibility and information [9, 10]. Presented a methodology to deal with neutrosophic numbers [20]. Introduced the t-test for the AR (1) process [2]. Outlined the experimental design within the framework of neutrosophic statistics [17]. Engaged with the probabilistic methodology employing neutrosophic statistics [1]. Investigated a discrete distribution utilizing neutrosophic statistics [5]. Expanded the theory of survey sampling through the application of neutrosophic statistics [6]. conducted the research on the neutrosophic statistical test for sequential contingency. The various applications of NS can be seen in [3, 15] and [23].

The current sine–cosine and convolution simulation techniques within the framework of classical statistics face limitations in simulating standard normal variates in the context of uncertainty. A thorough literature review, conducted to the best of the authors' knowledge, reveals a lack of research on sine–cosine and convolution simulation methods within the framework of neutrosophic statistics. This paper will introduce the routines and algorithms for both simulation methods. Neutrosophic standard normal tables for both the sine–cosine and convolution simulation methods will be presented and analyzed in detail. The significance of this work lies in its pioneering approach to addressing a gap in simulation methods within the realm of uncertainty. By introducing and exploring sine–cosine and convolution simulation methods under the framework of neutrosophic statistics, this research contributes to expanding the toolkit available for handling complex and uncertain data scenarios. As existing methods struggle to simulate standard normal variates accurately in the presence of uncertainty, this work offers a novel solution that has not been explored before in the literature. The presentation of routines, algorithms, and tables for both simulation methods not only fills this research gap but also opens new avenues for statistical analysis in uncertain contexts. Sect. "Preliminaries" provides an introduction to neutrosophic random variables. In Sect. "The Proposed Sine–Cosine Simulation Method", the Sine–Cosine Simulation Method is elaborated, followed by the presentation of the Convolution Method of Simulation in Sect. "The Proposed Convolution Method". Sect. "Simulation Studies" encompasses the simulation studies, while the final section offers concluding remarks.

Preliminaries

Suppose that $X_N \in [X_L, X_U]$ is a neutrosophic random variable follows the neutrosophic normal distribution having uncertain in the mean or in standard deviation or in mean and standard deviation. Let $\mu_N \in [\mu_L, \mu_U]$ be mean and $\sigma_N \in [\sigma_L, \sigma_U]$ be the

standard deviation of the neutrosophic normal distribution. The neutrosophic forms of $X_N \in [X_L, X_U]$ is expressed as

$$X_N = X_L + X_U I_N; I_N \in [I_L, I_U] \quad (1)$$

The neutrosophic forms of $\mu_N \in [\mu_L, \mu_U]$ is expressed as

$$\mu_N = \mu_L + \mu_U I_N; I_N \in [I_L, I_U] \quad (2)$$

The neutrosophic forms of $\sigma_N \in [\sigma_L, \sigma_U]$ is expressed as

$$\sigma_N = \sigma_L + \sigma_U I_N; I_N \in [I_L, I_U] \quad (3)$$

Note that the first values in the above neutrosophic forms denote the values under classical statistics and the second values denote the determinate part and I_N is the measure of uncertainty. Note that the neutrosophic forms reduce to determinate values when $I_L = 0$.

The neutrosophic probability density function (npdf) of the neutrosophic normal distribution is given by

$$f(X_N) = \frac{1}{\sigma_N \sqrt{2\pi}} \exp\left(-\frac{(X - \mu_N)^2}{2\sigma_N^2}\right) \quad (4)$$

The neutrosophic standard normal distribution is expressed by

$$f(Z_N) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z)^2}{2}\right) \quad (5)$$

More details about the neutrosophic normal distribution and its shape can be seen in [24].

The proposed sine–cosine simulation method

Suppose that $U_{1N} \in [U_{1L}, U_{1U}]$ and $U_{2N} \in [U_{2L}, U_{2U}]$ are the uniform continuous random numbers for the same neutrosophic continuous uniform distribution. Note that U_{1N} and U_{2N} are uniform continuous random numbers ranging from 0 to 1. By following [8], two neutrosophic standard normal random variates z_{1N} and z_{2N} can be generated as follows

$$z_{1N} \in \left[\left\{ \sqrt{-2\ln U_{1L}} \cos(2\pi (U_{2L})) \right\}, \left\{ \sqrt{-2\ln U_{1U}} \cos(2\pi (U_{2U})) \right\} \right] \quad (6)$$

$$z_{2N} \in \left[\left\{ \sqrt{-2\ln U_{1L}} \sin(2\pi (U_{2L})) \right\}, \left\{ \sqrt{-2\ln U_{1U}} \sin(2\pi (U_{2U})) \right\} \right] \quad (7)$$

The joint neutrosophic density $X_{1N} \in [X_{1L}, X_{1U}]$ and $X_{2N} \in [X_{2L}, X_{2U}]$ can be given by

$$f(z_{1N}, z_{2N}) = \left[\left\{ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{1L})^2}{2}\right), \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{1U})^2}{2}\right) \right\}, \left\{ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{2L})^2}{2}\right), \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{2U})^2}{2}\right) \right\} \right] = f(z_{1N})f(z_{2N}) \quad (8)$$

By following [8], it is concluded that $z_{1N} \in [z_{1L}, z_{1U}]$ and $z_{2N} \in [z_{2L}, z_{2U}]$ are independent random variate from the same neutrosophic standard normal distribution in Eq. (5). The process for generating neutrosophic standard normal and neutrosophic normal random variates is elucidated as follows:

Step-1: Generate two neutrosophic continuous uniform random numbers $U_{1N} \in [U_{1L}, U_{1U}]$ and $U_{2N} \in [U_{2L}, U_{2U}]$ from 0 to 1.

Step-2: Generate the neutrosophic standard random variable $z_{1N} \in [z_{1L}, z_{1U}]$ using $z_{1N} = [\{\sqrt{-2\ln U_{1L}} \cos(2\pi(U_{2L}))\}, \{\sqrt{-2\ln U_{1U}} \cos(2\pi(U_{2U}))\}]$.

Step-3: Generate the neutrosophic standard random variable $z_{2N} \in [z_{2L}, z_{2U}]$ using $z_{2N} = [\{\sqrt{-2\ln U_{1L}} \sin(2\pi(U_{2L}))\}, \{\sqrt{-2\ln U_{1U}} \sin(2\pi(U_{2U}))\}]$

Step-4: Return $z_{1N} \in [z_{1L}, z_{1U}]$ and $z_{2N} \in [z_{2L}, z_{2U}]$

Step-5: $x_{1N} = \mu_N + z_{1N} \times \sigma_N$ and $x_{2N} = \mu_N + z_{2N} \times \sigma_N$

Step-6: Return x_{1N} and x_{2N}

The operational procedure of the proposed simulation method is also shown in Fig. 1.

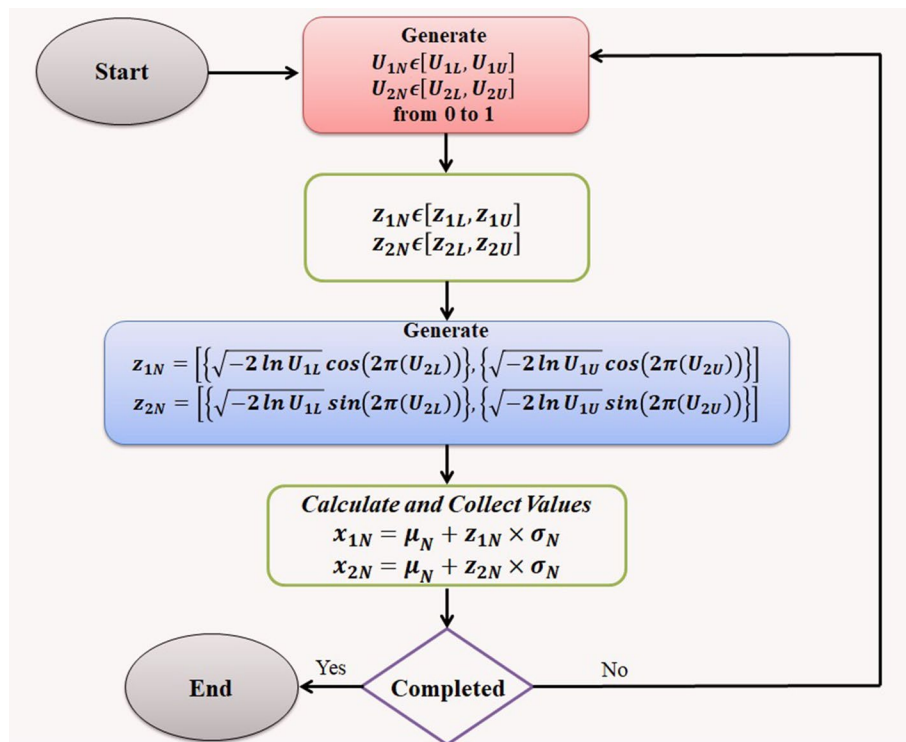


Fig. 1 Algorithm of sine–cosine method

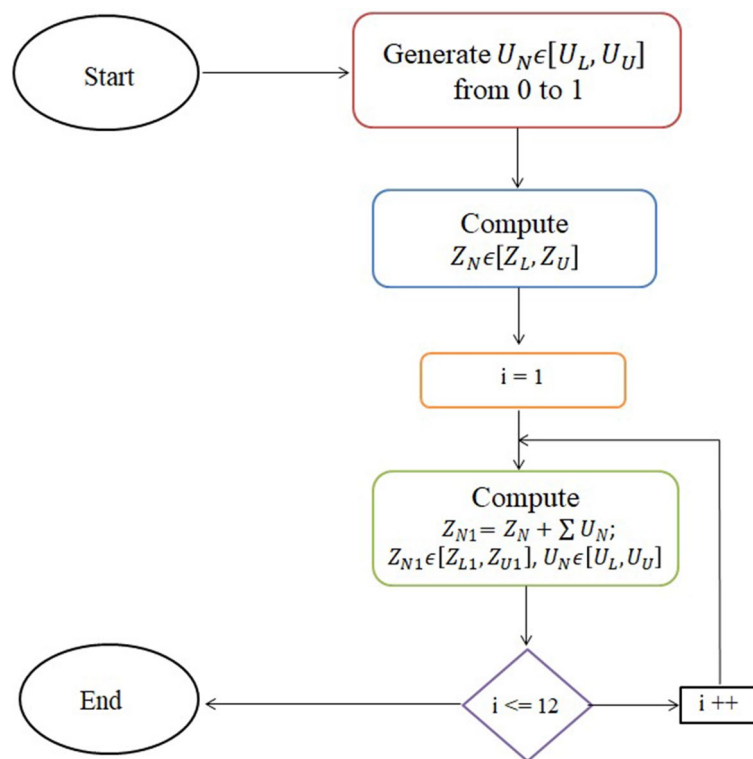


Fig. 2 Algorithm of the convolution method

The proposed convolution method

This section delves into the convolution method within the framework of neutrosophic statistics, drawing inspiration from [26]. The presented simulation technique operates as follows: utilizing a set (Fig. 2) of twelve neutrosophic continuous uniform random numbers to produce neutrosophic standard normal variates. The routine for implementing the proposed convolution method under the domain of neutrosophic statistics is outlined as follows:

Step-1: Generate continuous uniform random numbers $U_N \in [U_L, U_U]$ from 0 to 1.

Step-2: Set neutrosophic standard normal variable $Z_N \in [Z_L, Z_U]$.

Step-3: For $i=1$ to 12, calculate the sum of 12 continuous uniform random variates and compute $Z_{N1} = Z_N + \sum U_N$; $Z_{N1} \in [Z_{L1}, Z_{U1}], U_N \in [U_L, U_U]$

Step-4: return $Z_{N1} \in [Z_{L1}, Z_{U1}]$

Simulation studies

Within this segment, we will generate random normal variates using the sine–cosine method across a range of specified parameter values. Simultaneously, we will employ the convolution method to generate standard normal variates. Our approach involves adhering to the prescribed algorithms to generate these random standard normal variates.

Sine–cosine simulation

Tables 1, 2, 3, 4 exhibit the results obtained using the proposed sine–cosine method, showcasing two random normal variates, denoted as x_{1N} and x_{2N} . These

Table 1 Random normal variate when $\mu = [10, 12]$ and $\sigma = [1, 1.5]$

Sr#	$I_N=0$		$I_N=0.01$		$I_N=0.02$		$I_N=0.03$		$I_N=0.05$		$I_N=0.10$	
	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}
1	[9.2,10.8]	[11.6,14.4]	[9.4,11]	[11.2,13.7]	[10.9,13.5]	[8.9,10.5]	[10.2,12.8]	[7.8,8.8]	[10.2,12.4]	[9.5,11.54]	[9.5,11.54]	[10.1,11.77]
2	[10, 12]	[11.3,14]	[10.5,12.8]	[8.2,9.4]	[10.6,13.1]	[8.4,9.7]	[9.6,11.7]	[8.7,10.1]	[8.9,10]	[12.2,15.06]	[12.2,15.06]	[9.1,10.73]
3	[8.2,9.3]	[9.6,11.4]	[10,12.1]	[9.5,11.3]	[10.9,13.5]	[8.6,10.1]	[11.6,14.7]	[8.6,10.4]	[11.2,13.6]	[12.4,15.61]	[12.4,15.61]	[9.2,11.9]
4	[9.9,11.9]	[11.2,13.8]	[9.2,10.8]	[9.3,10.9]	[10.6,12.8]	[10.5,12.7]	[9.1,10.8]	[9.8,11.6]	[10.4,13]	[8.9,10.59]	[8.9,10.59]	[9.3,11.51]
5	[10.3,12.5]	[10.6,12.9]	[12.7,16.1]	[10.7,13.1]	[9.9,11.8]	[11.5,14.3]	[9.9,12.1]	[8.6,10]	[11,13.4]	[10.7,13.05]	[10.7,13.05]	[10.1,12.23]
6	[10.6,12.8]	[6.8,7.2]	[9.1,10.6]	[10.6,12.9]	[10.4,12.5]	[11.8,14.8]	[10,12]	[10.6,12.8]	[9.5,11.4]	[9.5,11.24]	[9.5,11.24]	[9.5,11.7]
7	[9.9,11.9]	[11,13.6]	[11,13.5]	[10.3,12.5]	[10.4,12.5]	[10.6,12.9]	[9.6,11.4]	[10.4,12.5]	[9,10.8]	[9.2,10.57]	[9.2,10.57]	[9.5,11.06]
8	[9,10.5]	[10.5,12.7]	[9.4,11.2]	[9.7,11.5]	[11.1,13.6]	[11.3,14]	[10.3,12.4]	[9.5,11.4]	[10.9,13.2]	[12.1,15.13]	[12.1,15.13]	[9.9,11.74]
9	[10.4,12.6]	[11.8,14.7]	[11.3,14]	[9,10.6]	[8.9,10.5]	[9.3,10.8]	[10.3,12.3]	[11.4,14.1]	[11,13.5]	[10,12.05]	[10,12.05]	[9.6,10.71]
10	[10.9,13.3]	[10.7,13.1]	[10.6,12.9]	[10.2,12.3]	[10.5,13]	[8.4,9.8]	[9.1,10.5]	[11.3,13.8]	[10.9,13.3]	[10.2,12.36]	[10.2,12.36]	[9.6,11]
11	[9.3,11]	[10.4,12.6]	[11.3,13.9]	[11,13.6]	[9.5,11.3]	[9.6,11.3]	[9.7,11.8]	[8.8,10.2]	[10,12.2]	[9.4,11.17]	[9.4,11.17]	[9.8,11.59]
12	[9.3,10.9]	[10.3,12.4]	[10,12.1]	[8.1,9.2]	[8.9,10.4]	[10.5,12.6]	[8.7,10.1]	[10.1,11.9]	[9.3,11]	[10.4,12.4]	[10.4,12.4]	[9.8,12.5]
13	[10.5,12.8]	[11.4,14.1]	[11.5,14.2]	[10.2,12.3]	[8.7,10.1]	[10,11.9]	[9,10.7]	[9.3,10.9]	[8.6,9.8]	[11.1,13.38]	[11.1,13.38]	[7.1,8.09]
14	[9.5,11.2]	[9.4,11.1]	[9.1,10.7]	[10,11.9]	[8.4,9.7]	[9.4,11]	[9.8,11.6]	[10.8,13.1]	[9.4,11.2]	[9.6,11.32]	[9.6,11.32]	[9.8,12.41]
15	[10.4,12.5]	[10.9,13.3]	[10.3,12.6]	[9.2,10.8]	[10.2,12.2]	[10.8,13.1]	[9.5,11.1]	[11,13.4]	[8.5,10]	[9.3,10.61]	[9.3,10.61]	[11.6,14.23]
16	[9.5,11.3]	[9.9,11.9]	[9.3,10.9]	[10.8,13.2]	[10.1,12.1]	[11.4,14.1]	[10.3,12.4]	[10.4,12.5]	[8.6,10.1]	[9.3,10.55]	[9.3,10.55]	[10,11.67]
17	[11.5,14.3]	[10.2,12.3]	[9,10.5]	[9.7,11.5]	[10,11.9]	[11.5,14.2]	[12.6,15.9]	[11.2,13.8]	[10.4,12.5]	[10.6,12.83]	[10.6,12.83]	[8.2,8.88]
18	[10.6,12.9]	[10.3,12.4]	[8.6,9.9]	[10.8,13.1]	[11.1,13.7]	[10.7,13.1]	[10,12.3]	[8.8,10.2]	[8.3,9.7]	[9.5,10.84]	[9.5,10.84]	[10.9,13.1]
19	[10.9,13.4]	[10,12]	[9.3,11]	[9.6,11.4]	[9.2,10.7]	[10.6,12.8]	[9.7,11.4]	[11,13.5]	[9.8,11.6]	[10.7,12.99]	[10.7,12.99]	[9.9,11.55]
20	[8.1,9.2]	[7.8,8.6]	[11,13.5]	[10.1,12.2]	[9.2,10.9]	[9,10.5]	[10.3,12.4]	[10.3,12.4]	[10.7,13.7]	[8.1,9.57]	[8.1,9.57]	[10.8,13.41]
21	[9.8,11.7]	[11.3,14]	[9.3,10.9]	[11.2,13.7]	[10.5,12.7]	[9.6,11.4]	[10.1,12.3]	[8.9,10.5]	[10.4,12.5]	[10.7,12.93]	[10.7,12.93]	[10.6,13.46]
22	[9.5,11.2]	[10.7,13]	[11.9,14.9]	[11,13.5]	[11.1,13.7]	[10.5,12.7]	[9.8,11.6]	[11,13.4]	[9.5,11.2]	[10.6,12.66]	[10.6,12.66]	[10.3,12.3]
23	[9.2,10.8]	[10.5,12.8]	[9.7,11.6]	[11,13.5]	[8.3,9.7]	[8.8,9]	[7.4,8.2]	[9.4,10.7]	[9.4,11.1]	[10.5,12.52]	[10.5,12.52]	[10.6,13.06]
24	[7.9,8.9]	[10.5,12.7]	[8.2,9.3]	[12.1,15.2]	[10.2,12.2]	[11.5,14.3]	[9.6,11.4]	[9.9,11.8]	[8.9,10.4]	[10.1,11.86]	[10.1,11.86]	[11.3,13.71]
25	[10,12]	[11.2,13.9]	[9.8,11.7]	[10.9,13.3]	[9.6,11.5]	[8.8,10.2]	[9,10.6]	[9.9,11.7]	[9.7,11.4]	[11,13.41]	[11,13.41]	[10.4,12.32]

Table 2 Random normal variate when $\mu = [10, 12]$ and $\sigma = [3, 5]$

Sr#	$I_N=0$		$I_N=0.01$		$I_N=0.02$		$I_N=0.03$		$I_N=0.05$		$I_N=0.10$	
	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}	x_{1W}	x_{2N}
1	[9.2,10.7]	[15.8,21.6]	[7.8,8.3]	[12.7,16.3]	[13,17]	[10.5,12.8]	[12.2,15.4]	[11.7,14.7]	[13.8,18.1]	[12.8,16.85]	[12.8,16.85]	[9.8,10.28]
2	[7.9,8.5]	[9.9,11.9]	[7.3,7.7]	[6.8,6.5]	[13.1,17.1]	[11.2,13.9]	[4.9,4.1]	[7.1,6.3]	[8.8,7]	[11.1,13.27]	[11.1,13.27]	[8.4,10.56]
3	[9.4,11]	[7.7,8.2]	[9.1,10.9]	[5.3,4.1]	[13.4,17.6]	[10.1,12.2]	[5.9,4.8]	[13.8,17.9]	[9.8,12.8]	[6.9,7.07]	[6.9,7.07]	[9.1,6.5]
4	[5.7,4.8]	[10.4,12.6]	[11.3,14]	[16.4,22.6]	[5.9,5.1]	[10.9,13.1]	[14.5,19.3]	[12.4,16]	[13.8,18.1]	[10.9,13.46]	[10.9,13.46]	[10.5,12.38]
5	[8.3,9.2]	[6.8,6.7]	[6.8,6.7]	[9.9,11.6]	[11.6,14.5]	[13.6,18]	[8.3,9]	[12.2,15.3]	[11.3,13.8]	[12.8,16.64]	[12.8,16.64]	[7.4,9.19]
6	[11.7,14.8]	[12,15.3]	[12.5,16.5]	[6.3,6]	[7.3,7.8]	[8.6,9.4]	[7.4,8]	[9.7,11.1]	[11,16.4]	[3.1,1.38]	[3.1,1.38]	[10,17.86]
7	[9.2,10.7]	[13.1,17.1]	[9.7,11.7]	[7.4,7]	[12.3,15.6]	[14,18.7]	[15.2,1.1]	[6.4,7.5]	[10.3,12.1]	[13.1,16.95]	[13.1,16.95]	[13,16.57]
8	[6.3,5.8]	[5.5,4.5]	[12.6,16.3]	[12.3,15.8]	[5.6,4.3]	[15,20]	[4.3,3.2]	[7.1,6.2]	[8.9,12.7]	[3.6,1.24]	[3.6,1.24]	[10.3,14.35]
9	[11.9,15.2]	[13.7,18.1]	[11.6,15.1]	[5.9,5.3]	[6.1,5.7]	[8.6,9.2]	[10.5,13.2]	[8.2,9.4]	[7.2,10.2]	[2.3,-1.47]	[2.3,-1.47]	[13.4,17.1]
10	[10.9,13.4]	[3.1,0.5]	[6.8,6.7]	[10.5,12.6]	[11.1,13.7]	[12.1,15.4]	[5.6,4.6]	[11.7,14.3]	[5.2,3.2]	[10.5,11.52]	[10.5,11.52]	[9.9,15.38]
11	[7.7]	[8.5,9.5]	[8.6,9.8]	[11,13.6]	[8.6,9.9]	[9.6,11.3]	[9,10.3]	[11.5,14.2]	[4.7,3.3]	[12,14.95]	[12,14.95]	[12.5,20.03]
12	[10.9,13.6]	[13.4,17.7]	[11.7,14.8]	[10.9,13.4]	[4.3,3.1]	[6.2,5]	[8.1,9.1]	[10.2,12.1]	[10,11.7]	[12,16]	[12,16]	[10.6,15.41]
13	[9.7,11.6]	[15.6,21.4]	[9.3,10.9]	[11.5,14.4]	[8.2,9.3]	[9.6,11.3]	[8.8,10.6]	[7.6,7.9]	[8.8,9.7]	[12.7,16]	[12.7,16]	[8.6,11.23]
14	[5.3,7]	[12.9,16.9]	[10.5,12.7]	[12.7,16.5]	[8.8,10.5]	[6.8,6.6]	[11.1,13.7]	[12.1,15.4]	[13,16.6]	[14.5,19.66]	[14.5,19.66]	[12.4,15.2]
15	[5.9,5.2]	[14.6,19.6]	[16.9,23.5]	[11.2,14]	[14,18.4]	[15,20.4]	[12.2,15.5]	[13,17.1]	[10.4,14.2]	[6.5,9.4]	[6.5,9.4]	[8,12.08]
16	[5.8,4.9]	[5.1,3.8]	[5.5,4.2]	[15.6,21.2]	[11,13.5]	[11.6,14.5]	[17.23,6]	[10.8,13.3]	[2.7,-0.7]	[13.4,15.87]	[13.4,15.87]	[5.5,5.23]
17	[10.8,13.3]	[8.9,10.1]	[7.8,8.2]	[16.5,22.8]	[5.8,4.9]	[12.1,15.2]	[7.4,7.6]	[11.5,14.1]	[10.8,13.7]	[8.4,10.17]	[8.4,10.17]	[10,15.36]
18	[12.4,16.1]	[12,15.4]	[10.5,12.8]	[11.4,14.3]	[5.3,9]	[9.1,9.9]	[8.9,1]	[8.7,9.6]	[10.4,12.4]	[11.5,14.03]	[11.5,14.03]	[7.7,6.4]
19	[14.2,18.9]	[12.5,16.2]	[12.4,15.9]	[10.9,13.5]	[12,15.6]	[8.1,9.3]	[10.4,12.5]	[11.4,14]	[10.6,12.3]	[15.4,20.9]	[15.4,20.9]	[13.5,19.18]
20	[9.5,11.1]	[5.7,4.9]	[8.4,9.3]	[11.3,14.1]	[8.1,9]	[9.8,11.5]	[7.7,8]	[12.2,15.2]	[7.1,7.2]	[10.7,12.39]	[10.7,12.39]	[3.8,3.34]
21	[17.5,24.5]	[11.6,14.7]	[8.6,9.8]	[9.5,11.1]	[9.1,10.6]	[11.1,13.7]	[5.5,5.2]	[6.8,5.9]	[13.3,17.3]	[11.1,13.93]	[11.1,13.93]	[10.5,12.32]
22	[12,15.3]	[14.8,20]	[16.3,22.4]	[12,15.4]	[15.4,20.9]	[14.7,19.9]	[14.6,19.4]	[13.2,17.4]	[6.3,5.9]	[10.7,12.21]	[10.7,12.21]	[7.3,7.48]
23	[11.6,14.6]	[12.1,15.5]	[9.8,11.5]	[14.3,19.2]	[11.4,15.3]	[4.3,2.9]	[12.5,16]	[13.1,17.2]	[5.8,4.5]	[12.7,15.47]	[12.7,15.47]	[12.4,16.76]
24	[9.4,11.1]	[8.5,9.4]	[11,13.8]	[6.8,6.9]	[9.3,10.8]	[11.6,14.5]	[7.3,7.3]	[12.3,15.4]	[15,20.2]	[10.3,12.58]	[10.3,12.58]	[7.6,8.29]
25	[12.2,15.6]	[10.8,13.3]	[8.7,10.1]	[6.9,6.8]	[6,5]	[16.2,7]	[9.2,10.6]	[11.6,14.3]	[7.2,8.7]	[6.3,5.17]	[6.3,5.17]	[6.5,7.06]

Table 3 Random normal variate when $\mu = [20, 22]$ and $\sigma = [1, 1.5]$

S#	$I_N=0$		$I_N=0.01$		$I_N=0.02$		$I_N=0.03$		$I_N=0.05$		$I_N=0.10$	
	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}
1	[19,621.5]	[20,823.2]	[20,422.6]	[21,624.4]	[21,223.8]	[21,724.6]	[18,619.8]	[20,923.1]	[18,920.8]	[18,619.66]	[18,619.66]	[21,523.64]
2	[21,624.4]	[18,319.5]	[20,422.6]	[19,721.6]	[21,924.8]	[20,222.3]	[20,923.2]	[20,923.4]	[18,920.2]	[20,923.11]	[20,923.11]	[21,23.34]
3	[21,324]	[22,24.9]	[19,320.9]	[20,322.5]	[19,120.8]	[18,820.1]	[20,222.2]	[22,25]	[19,821.7]	[20,622.71]	[20,622.71]	[20,422.19]
4	[20,322.5]	[20,723]	[20,622.8]	[20,422.6]	[18,419.7]	[19,921.8]	[18,19.1]	[19,721.2]	[20,823.1]	[20,122.21]	[20,122.21]	[19,621.24]
5	[19,220.8]	[21,123.7]	[20,923.4]	[20,823.2]	[19,821.9]	[18,319.5]	[19,822]	[18,820.2]	[19,220.7]	[20,923.18]	[20,923.18]	[20,321.95]
6	[20,122.2]	[20,923.4]	[19,521.2]	[21,23.5]	[17,618.4]	[20,722.9]	[18,820.2]	[19,921.7]	[19,621.4]	[20,522.65]	[20,522.65]	[20,723.26]
7	[20,22]	[21,824.8]	[19,521.4]	[18,119.1]	[19,321.1]	[19,220.7]	[19,621.5]	[19,721.6]	[20,122.5]	[19,20.67]	[19,20.67]	[21,224.25]
8	[18,119.2]	[20,422.6]	[20,122.2]	[20,622.8]	[20,823.3]	[19,20.6]	[21,324]	[20,322.5]	[20,823.1]	[21,223.74]	[21,223.74]	[20,822.86]
9	[18,319.4]	[19,521.2]	[18,19]	[19,220.7]	[18,419.6]	[20,823]	[20,622.9]	[20,622.9]	[20,21.9]	[21,123.55]	[21,123.55]	[18,820.49]
10	[19,621.4]	[18,920.3]	[20,723]	[20,322.5]	[18,619.9]	[20,522.6]	[20,622.9]	[21,724.6]	[20,923.5]	[19,221.29]	[19,221.29]	[19,21.38]
11	[19,821.8]	[20,723]	[20,322.4]	[20,622.9]	[20,222.4]	[19,521.3]	[19,822.1]	[18,419.7]	[20,923.9]	[18,319.87]	[18,319.87]	[20,21.98]
12	[20,723]	[19,220.8]	[19,721.5]	[22,425.6]	[19,921.8]	[20,522.7]	[20,522.7]	[20,222.2]	[20,322.8]	[18,920.52]	[18,920.52]	[20,322.87]
13	[20,522.7]	[19,120.6]	[20,723.2]	[18,920.5]	[19,421.2]	[18,920.3]	[21,223.7]	[22,525.8]	[19,721.5]	[20,522.54]	[20,522.54]	[18,420.06]
14	[20,923.4]	[20,823.2]	[20,22.1]	[18,720]	[20,723]	[20,122.1]	[19,120.5]	[20,823.1]	[20,322.6]	[19,421.37]	[19,421.37]	[19,322.54]
15	[18,419.6]	[19,621.4]	[20,422.7]	[18,720]	[21,724.8]	[18,620.2]	[19,721.9]	[18,820.2]	[21,23.4]	[21,123.66]	[21,123.66]	[18,920.86]
16	[19,120.7]	[19,421.1]	[20,522.8]	[19,120.8]	[21,223.9]	[20,222.3]	[18,119.2]	[19,821.5]	[18,419.7]	[19,821.3]	[19,821.3]	[21,424.01]
17	[19,821.7]	[20,723]	[19,921.9]	[20,522.7]	[18,319.6]	[19,320.8]	[21,524.1]	[21,624.4]	[20,322.3]	[20,923.3]	[20,923.3]	[17,417.72]
18	[21,223.8]	[20,522.7]	[21,23.4]	[21,524.3]	[20,22]	[20,723]	[22,926.4]	[20,623]	[20,323.2]	[17,919.08]	[17,919.08]	[19,320.33]
19	[19,621.4]	[19,821.6]	[20,22.1]	[18,519.8]	[20,322.5]	[20,422.5]	[18,419.7]	[19,721.3]	[19,421.3]	[19,721.51]	[19,721.51]	[21,824.55]
20	[20,522.7]	[20,122.2]	[20,622.9]	[20,322.5]	[20,122.1]	[20,622.8]	[18,319.9]	[17,518]	[18,19.1]	[20,322.07]	[20,322.07]	[20,924.49]
21	[18,720]	[19,621.4]	[19,821.8]	[19,120.7]	[19,220.7]	[21,824.6]	[21,724.5]	[21,424.1]	[18,720.4]	[19,220.5]	[19,220.5]	[20,522.57]
22	[20,222.3]	[19,20.5]	[20,22]	[20,923.4]	[20,322.7]	[18,720.1]	[18,820.2]	[20,622.8]	[19,220.4]	[22,525.57]	[22,525.57]	[20,121.85]
23	[20,923.4]	[20,422.6]	[20,122.1]	[21,223.8]	[19,120.7]	[20,122]	[19,421.6]	[17,618.4]	[21,424.1]	[21,324.05]	[21,324.05]	[20,824.23]
24	[18,319.5]	[19,921.8]	[21,924.9]	[18,319.6]	[22,425.6]	[20,22]	[19,821.7]	[20,522.6]	[21,223.8]	[20,723.12]	[20,723.12]	[19,922.55]
25	[21,524.2]	[20,122.1]	[20,422.6]	[19,421.1]	[19,220.9]	[20,222.2]	[18,419.5]	[20,522.5]	[20,523]	[19,120.98]	[19,120.98]	[21,23.38]

Table 4 Random normal variate when $\mu = [20, 22]$ and $\sigma = [3, 5]$

Sr#	$I_N=0$		$I_N=0.01$		$I_N=0.02$		$I_N=0.03$		$I_N=0.05$		$I_N=0.10$	
	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}	x_{1N}	x_{2N}
1	[22.5,26.1]	[23.5,27.9]	[21.1,23.8]	[24.5,29.5]	[18.4,19.7]	[17.2,17.2]	[14.8,14.1]	[16.7,23]	[17.7,18.6]	[16.9,17.86]	[16.9,17.86]	[19.5,19.77]
2	[20.7,23.2]	[12.9,10.2]	[14.8,13.7]	[15.9,14.9]	[18.4,19.2]	[22.25,2]	[23.1,27]	[21.8,28.1]	[15.2,15.5]	[18.9,19.81]	[18.9,19.81]	[22.3,24.89]
3	[18.5,19.4]	[21.7,24.9]	[16.7,16.7]	[17.3,17.4]	[19.7,21.5]	[22.2,25.6]	[22.3,25.7]	[21.3,25.1]	[23.1,27.1]	[25.32,19]	[25.32,19]	[16.2,21.06]
4	[17.9,18.5]	[22.8,26.7]	[18,18.8]	[20.2,19]	[18,19]	[17.8,18.1]	[22.7,27.5]	[16,23.4]	[14.5,13.1]	[18.8,19.29]	[18.8,19.29]	[23.1,26.32]
5	[16.7,16.5]	[23.1,27.2]	[18.2,19.2]	[17.9,18.4]	[21.5,24.3]	[26.8,33.4]	[17.8,18.7]	[19.4,27.1]	[25.8,31.9]	[13.9,12.16]	[13.9,12.16]	[20.9,20.47]
6	[17.7,18.2]	[17,17]	[17,17]	[21.6,24.5]	[20.5,23.1]	[17.9,18.7]	[18.5,19.4]	[22.9,18.5]	[17.5,17.3]	[20.7,24.79]	[20.7,24.79]	[17.3,18.98]
7	[23.9,28.5]	[22.2,25.6]	[23.7,28.1]	[21.3,24.1]	[17.6,17.8]	[22.7,26.2]	[21,23.4]	[22.1,12.3]	[17.2,15.5]	[23.1,26.05]	[23.1,26.05]	[25.7,31.82]
8	[18.4,19.3]	[20.6,23]	[21.1,23.7]	[25.5,31.2]	[16.3,15.5]	[26.9,33.2]	[17.4,17.3]	[23.7,25]	[17.3,18.3]	[19.9,20.66]	[19.9,20.66]	[24.9,29.7]
9	[23.26,9]	[22.6,26.3]	[22.4,25.9]	[20.3,22.5]	[24.3,29.1]	[23.9,28.6]	[24,28.5]	[20.3,15.7]	[16.3,14.6]	[22.5,25.58]	[22.5,25.58]	[20.8,23.28]
10	[16.3,15.8]	[19.3,20.8]	[13.8,11.8]	[19.3,20.5]	[19.6,21.2]	[22.7,26.4]	[22.9,26.6]	[23.9,24.6]	[18.4,20.2]	[22.9,26.31]	[22.9,26.31]	[21.6,24.68]
11	[21.4,24.3]	[12.8,10]	[20.4,22.8]	[18.5,19.7]	[22.2,25.6]	[20.22]	[25.7,32.4]	[15.9,23.5]	[17.7,18.7]	[24.2,28.46]	[24.2,28.46]	[22.6,26.53]
12	[24.3,29.1]	[21.6,24.7]	[23.2,27.4]	[20.5,22.8]	[23.7,28.1]	[20.3,22.6]	[17.5,17.7]	[21.6,32.4]	[15.9,17.9]	[24.30,16]	[24.30,16]	[16.9,21.15]
13	[15.7,14.9]	[26.4,32.7]	[21.4,24.7]	[15.4,14.4]	[19.8,21.6]	[21.4,24.1]	[16.8,17.1]	[18.6,20.1]	[20.7,22.7]	[22.1,24.93]	[22.1,24.93]	[21.2,23.86]
14	[20.9,23.5]	[17.6,18]	[21.4,24.7]	[16.6,16.5]	[15.7,14.7]	[22.1,25.2]	[12.9,7]	[14.6,22.4]	[25.8,31.5]	[21.1,22.98]	[21.1,22.98]	[19.2,1.82]
15	[21.23,7]	[21.7,24.9]	[22.3,26.3]	[15.3,14.5]	[18,18.9]	[18.7,19.7]	[21,24.2]	[17.7,13.3]	[19.8,20.2]	[24.32,19]	[24.32,19]	[14.6,17.72]
16	[21.2,24]	[16.8,16.7]	[20.1,22.1]	[22.7,26.4]	[15.5,14.7]	[19.8,21.3]	[20.2,22.9]	[17.6,20.4]	[22.8,26.3]	[19.2,21.9]	[19.2,21.9]	[18.5,20.13]
17	[16.8,16.7]	[18.2,18.9]	[21.2,24.3]	[17.2,17.5]	[16,15.5]	[19,19.9]	[19,20.8]	[18.5,30.1]	[20.2,22.3]	[18.6,21.32]	[18.6,21.32]	[19.8,21.65]
18	[23.26,9]	[17.7,18.2]	[19.1,20.8]	[17,17]	[19.8,21.3]	[26.6,32.9]	[20.5,22.6]	[22.3,25.8]	[21.3,24.1]	[22.4,25.41]	[22.4,25.41]	[20.3,22.42]
19	[24.7,29.8]	[21.6,24.7]	[16.3,15.9]	[18.4,19.1]	[19.5,21.6]	[17.4,17.8]	[21.3,25.3]	[15.3,26]	[18.3,20.4]	[17.1,20.37]	[17.1,20.37]	[15.8,14]
20	[18.1,18.8]	[16.7,16.6]	[17.7,18.3]	[19.9,21.6]	[22.4,25.8]	[22.6,26.4]	[20.3,22.2]	[23.2,24.2]	[15.5,15]	[18.9,21.23]	[18.9,21.23]	[20.8,22.33]
21	[23.4,27.7]	[23.6,27.9]	[22.4,25.9]	[21.3,24.2]	[23.8,28.2]	[21.8,25]	[21.1,23.4]	[24.7,25.7]	[23.7,28.2]	[20.5,23.2]	[20.5,23.2]	[18.5,20.9]
22	[12.9,10.2]	[19.8,21.7]	[23.6,27.9]	[20.4,22.7]	[17.1,17.1]	[22.25,1]	[19,121]	[18.25,5]	[20.8,23.3]	[20.1,21.69]	[20.1,21.69]	[22.3,25.08]
23	[19.8,21.7]	[22.25,3]	[10.4,6.2]	[17.8,17.8]	[21.3,24]	[22.9,26.9]	[22.1,25.2]	[20.9,13.8]	[25.4,30]	[21.7,25.87]	[21.7,25.87]	[17.8,20.47]
24	[16.6,16.3]	[16.6,16.3]	[22.5,26.1]	[21.9,25.1]	[12.9,10.1]	[22.6,25.7]	[17,16.9]	[21.4,18.2]	[26.3,1.5]	[18.1,18.65]	[18.1,18.65]	[21.9,24.06]
25	[20.4,22.7]	[23.27]	[17.6,18.1]	[17.6,17.8]	[18.7,19.7]	[22.4,25.8]	[14.4,13.2]	[17.3,18]	[17.9,17.9]	[21.7,27.83]	[21.7,27.83]	[15.6,17.09]

tables provide insights across a range of specified parameter values, encompassing $\mu_N \in [\mu_L, \mu_U]$ and $\sigma_N \in [\sigma_L, \sigma_U]$. The investigation is extended to account for varying levels of indeterminacy, represented as I_U , which influences the generation of random normal variates. In particular, the values of I_U considered encompass 0, 0.01, 0.02, 0.05, and 0.10, generating pairs of random normal variates. To elaborate, Table 1 is tailored for $\mu_N \in [10, 12]$ and $\sigma_N \in [1, 1.5]$, whereas Table 2 corresponds to the parameter ranges $\mu_N \in [10, 12]$ and $\sigma_N \in [3, 5]$. Similarly, Table 3 and Table 4 correspond to distinct parameter combinations, namely $\mu_N \in [20, 22]$ and $\sigma_N \in [1, 1.5]$, and $\mu_N \in [20, 22]$ and $\sigma_N \in [3, 5]$, respectively. Upon scrutiny of Tables 1, 2, 3, 4, it becomes evident that no discernible trend exists within the neutrosophic random normal variates as the values of I_U fluctuate between 0 and 0.10. Additionally, the outcomes underscore that the proposed sine–cosine simulation method yields random normal variates represented as intervals, rather than determinate values. For instance, consider the case where $I_U = 0.01$, as seen in Table 1. Here, the random normal variate spans from 9.4 to 11 in the cosine simulation approach and from 11.2 to 13.7 using the sine method. It is important to note that the values in each interval represent the random normal variate for various degree of indeterminacy.

Table 5 Standard normal variate from convolution method

Sr#	$I_N=0$	$I_N=0.01$	$I_N=0.02$	$I_N=0.03$	$I_N=0.05$	$I_N=0.10$
1	[−1.34,−1.34]	[−0.86,−0.81]	[−0.93,−0.83]	[1.91,2.15]	[−1.04,−0.79]	[−0.31,0.26]
2	[0.65,0.65]	[−0.98,−0.93]	[−0.36,−0.24]	[1.02,1.23]	[0.24,0.55]	[1.1,1.81]
3	[−0.63,−0.63]	[−0.3,−0.24]	[−1.64,−1.56]	[1.49,1.72]	[1.04,1.39]	[−1.16,−0.68]
4	[0.18,0.18]	[−0.07,−0.01]	[−2.26,−2.18]	[1.04,1.25]	[−0.34,−0.06]	[−1.07,−0.58]
5	[1.18,1.18]	[0.56,0.63]	[−0.06,0.05]	[1.08,1.29]	[−2.28,−2.1]	[−0.43,0.12]
6	[−0.44,−0.44]	[−1.16,−1.11]	[0.81,0.95]	[−0.39,−0.22]	[−0.57,−0.29]	[−0.09,0.5]
7	[0.62,0.62]	[−0.7,−0.64]	[−0.48,−0.37]	[−0.05,0.13]	[0.15,0.46]	[0.4,1.04]
8	[−0.01,−0.01]	[−0.23,−0.18]	[−1.14,−1.04]	[−0.5,−0.34]	[−1.21,−0.97]	[−0.41,0.15]
9	[−0.8,−0.8]	[0.33,0.39]	[0.51,0.64]	[−0.69,−0.53]	[0.92,1.27]	[−0.82,−0.3]
10	[1.29,1.29]	[−0.74,−0.69]	[−0.05,0.07]	[−0.13,0.05]	[−0.73,−0.47]	[−0.2,0.38]
11	[−0.58,−0.58]	[0.81,0.88]	[3.03,3.21]	[−2.44,−2.33]	[−0.36,−0.08]	[−1.64,−1.2]
12	[0.86,0.86]	[−0.5,−0.44]	[0.14,0.26]	[−2.1,−1.98]	[−0.14,0.15]	[1.63,2.39]
13	[0.48,0.48]	[−0.34,−0.28]	[0.36,0.49]	[0.37,0.56]	[1.07,1.43]	[0.39,1.03]
14	[0.42,0.42]	[1.67,1.74]	[1.38,1.53]	[0.22,0.41]	[−0.85,−0.59]	[0.3,0.93]
15	[0.08,0.08]	[0.26,0.32]	[0.66,0.79]	[−0.42,−0.25]	[−0.26,0.03]	[−0.34,0.23]
16	[−1.48,−1.48]	[−2.13,−2.09]	[0.92,1.06]	[0.54,0.73]	[−1.74,−1.53]	[0.18,0.8]
17	[0.15,0.15]	[0.82,0.89]	[−1.31,−1.22]	[1.42,1.64]	[−0.37,−0.09]	[−1.57,−1.13]
18	[0.33,0.33]	[−0.18,−0.13]	[−0.58,−0.47]	[1.5,1.72]	[0.51,0.84]	[0.93,1.63]
19	[0.02,0.02]	[0.89,0.96]	[0.63,0.76]	[−0.99,−0.84]	[1,1.35]	[−1.94,−1.53]
20	[−0.82,−0.82]	[0.35,0.42]	[−0.06,0.06]	[−1.45,−1.31]	[0.28,0.59]	[−0.56,−0.02]
21	[0.87,0.87]	[−1,−0.95]	[0.3,0.43]	[−0.43,−0.26]	[0.03,0.33]	[0.72,1.39]
22	[0.92,0.92]	[−1.18,−1.14]	[0.99,1.13]	[−0.8,−0.65]	[0,0.4]	[−1.23,−0.75]
23	[−0.65,−0.65]	[0.77,0.83]	[1.05,1.19]	[−0.37,−0.2]	[−0.61,−0.34]	[−0.76,−0.23]
24	[−2.31,−2.31]	[−0.82,−0.77]	[0.96,1.1]	[0.61,0.8]	[1.01,1.36]	[−0.92,−0.41]
25	[0.75,0.75]	[−0.59,−0.54]	[−0.4,−0.29]	[−0.44,−0.27]	[−0.24,0.05]	[−0.02,0.58]

Convolution Simulation

The results of the convolution method yield random standard normal variates for varying levels of indeterminacy, with I_U values of 0, 0.01, 0.02, 0.05, and 0.10. The convolution method's algorithm is utilized to generate these random standard normal variates, as per the prescribed approach. The resulting values are compiled in Table 5. Analyzing Table 5 reveals that the convolution method produces standard normal variates represented as intervals characterized by indeterminacy. These intervals, rather than offering precise numerical values, capture the uncertainty inherent in the process. The proposed simulation approach follows this trend, providing random standard normal variates presented as intervals. For instance, consider the case where $I_U = 0.01$; the resulting random standard normal variate spans from -0.86 to -0.81.

Comparative study

This section aims to contrast the techniques for generating random variates based on neutrosophic statistics and classical statistics. In the proposed simulation methods, a concept of indeterminacy, denoted as I_U , is introduced within the parameters μ_N and σ_N . The neutrosophic expressions for μ_N and σ_N take the form $\mu_N = \mu_L + \mu_U I_N$; $I_N \in [I_L, I_U]$ and $\sigma_N = \sigma_L + \sigma_U I_N$; $I_N \in [I_L, I_U]$. It is worth noting that these neutrosophic expressions simplify to the mean and variance of the normal distribution when I_L equals zero. For instance, in the scenario where $\mu_N = [10, 12]$ and $\sigma_N = [1, 1.5]$, the corresponding neutrosophic forms of mean and standard deviation will be $\mu_N = 10 + 12I_N$ and $\sigma_N = 1 + 1.5I_N$, respectively. Setting I_L to zero in these expressions results in the classical statistics' mean and standard deviation values of 10 and 1, respectively. Tables 1, 2, 3, 4, 5 showcase results for various values of I_U . The initial column in these tables represents the random variable outcomes under classical statistics. To illustrate, considering the sine–cosine simulation method within the realm of classical statistics, Table 1 demonstrates that the random normal variate x_{1N} falls within the range of 9.2 to 10.8. However, when I_U equals 0.02, this range shifts to 10.9 to 13.5. From this comparative analysis, it becomes evident that the degree of indeterminacy significantly impacts the random variate. Consequently, in situations involving uncertainty, relying on simulation methods rooted in classical statistics might lead decision-makers astray.

Concluding remarks

The paper introduced the sine–cosine method for generating random normal variates and also presented the convolution method for generating random standard normal variates. Algorithms for both methods were provided as well. Various tables were presented, showcasing the impact of indeterminacy on mean and standard deviation. The outcomes of the comparative study underscored that the degree of indeterminacy influences random normal variates. The proposed methods and algorithms offer practical solutions for simulation in scenarios where precise data recording might be challenging. These simulation methods find applicability in diverse fields including reliability analysis, engineering and manufacturing, healthcare, and environmental modeling. As a potential direction for future research, there is room for an extensive

investigation into adapting the proposed sine–cosine simulation method and convolution method for neutrosophic multivariate normal distributions. This could enhance the applicability and scope of the method for more complex and multidimensional data scenarios.

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M.A wrote the paper.

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