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MATHEMATICAL METHODS IN DATA SCIENCE

Neutrosophic statistical analysis of split-plot designs

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Abstract

The classic split-plot designs are unable to analyze indeterminate and uncertain data resulting from circumstances beyond our control. To this end, proposing a generalized approach to be applied to split-plot designs in uncertain environments is desired. In this study, a new approach is proposed using neutrosophic statistics to analyze split-plot and split-block designs. By such an approach neutrosophic hypothesis is formulated, a decision rule is suggested, and neutrosophic ANOVA Tables, including the F_N-test, are derived. Furthermore, a numerical example and a simulation study are established to evaluate the effectiveness of the proposed designs. The results confirm that the neutrosophic logic of the proposed designs is more efficient and flexible than the classic designs in the event of facing uncertain data.

Keywords Split-block designs · Neutrosophic statistics · Neutrosophic ANOVA · Uncertainty

Abbreviations

ANOVA	Analysis of variance
MC	Monte Carlo
NSPD	Neutrosophic split-plot design
NSBD	Neutrosophic split-block design
NMS	Neutrosophic mean square
NND	Neutrosophic normal distribution
NRV	Neutrosophic random variable
NSS	Neutrosophic sum of square

Symbols

X_N	The neutrosophic random variable
n_N	The neutrosophic random sample selected from a population
μ_N	The neutrosophic population mean
σ_N^2	The neutrosophic population variance
I_N	The indeterminacy interval
$\overline{X}_N$	The neutrosophic sample mean
s_N^2	The neutrosophic sample variance
y_{Nhijk}	The neutrosophic response of the $Nhij$thsplit-plot experimental unit
ρ_{Nh}	The neutrosophic effect of the hth block or replicate
τ_{Ni}	The neutrosophic effect of the $i$thwhole-plot
η_{Nhi}	The neutrosophic whole-plot error
β_{Nj}	The neutrosophic effect of the jth split-plot
δ_{Nhj}	The neutrosophic split-plot error
$(\tau\beta)_{Nij}$	The neutrosophic interaction effect of the ith whole plot with the jth split-plot
ε_{Nhij}	The neutrosophic interaction error or neutrosophic split-plot error
SS_{NR}	The neutrosophic replicate or block sum of squares
SS_{NA}	The neutrosophic whole-plot sum of squares
$SS_{NE(A)}$	The neutrosophic error sum of squares for whole-plot
SS_{NB}	The neutrosophic split-plot sum of squares
$SS_{NE(B)}$	The neutrosophic error sum of squares for split-plot
SS_{NAB}	The neutrosophic interaction sum of squares
$SS_{NE(AB)}$	The neutrosophic error sum of squares for interaction
SS_{NT}	The neutrosophic total sum of squares

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MS_{NR}	The neutrosophic replicate or block mean squares
MS_{NA}	The neutrosophic whole-plot mean squares
$MS_{NE(A)}$	The neutrosophic error mean squares for whole-plot
MS_{NB}	The neutrosophic split-plot mean squares
$MS_{NE(B)}$	The neutrosophic error mean squares for split-plot
MS_{NAB}	The neutrosophic interaction mean squares
$MS_{NE(AB)}$	The neutrosophic error mean squares for interaction
F_{NA}	The neutrosophic whole-plot $f-$ distribution
F_{NB}	The neutrosophic split-plot $f-$ distribution
F_{NAB}	The neutrosophic interaction $f-$ distribution
$p_N - value$	The neutrosophic p value
α	Level of significance
F_N	The neutrosophic $f-$ distribution

1 Introduction

Split-plot designs are very in demand in agricultural experiments, which allow the testing of two factors in plots with different sizes. Split-plot designs were first introduced in the 20 s of the last centuries by Fisher (1936). This design is commonly used when the experiment is a factorial treatment structure. Moreover, it can be used when one of two factors might be more expensive, more time-consuming, or needs more material than the other factors. To overcome such a challenge, two levels of randomization are applied to assign experimental units to treatments. The first level of randomization is adapted to the whole-plot (main plot), and then, it is used to appoint experimental units to levels of the treatment factor-A. Afterward, the whole-plot is partitioned into sub-plots (split-plots), and the second level of randomization is applied to appoint the sub-plot experimental units to the levels of treatment factor-B. Since the split-plot design has two experimental units with different sizes, the portions, namely the whole-plot and sub-plot portions, have been separated by different experimental errors. In this context, (Federer and King 2007) highlighted the construction and analysis of split-plot and augmented split-plot designs. The uses of the split-plot design are discussed by Wooding (1973). The efficiency of SBDs versus SPDs for hypothesis testing is discussed by Wang and Hering (2005). An overview of some of the most important procedures for analyzing SPDs has been presented and discussed by Næs et al. (2007). An exchange algorithm was developed by Goos and Vandebroek (2001) for the construction of D-optimal SPDs, and the results of these designs are analyzed. A new class of second-order response surface SPDs has been proposed by Parker et al. (2007) in which ordinary least squares estimates of the proposed model are equivalent to generalized least squares estimates. Formulas for the optimality criteria for SPDs and SBDs have been updated (Arnouts and Goos 2010). The split-plot fractional factorial designs have been studied by Bingham and Sitter (2001) in the impact of randomization restrictions. Other research works on split-plot designs include (Hoshmand 2018; Jones and Nachtsheim 2009; Khargonkar 1948; Montgomery 2017; Nair 1944; Ott and Longnecker 2015). All of the above researchers have considered the determinant datasets; however, there are situations where the experimenters face indeterminate data under uncertain circumstances.

Neutrosophic statistics (NS) derived from neutrosophic logic is a generalization of fuzzy logic and is seen as a measure of indeterminacy. In other words, NS is an extension of classical statistics that deals with the analysis of observations in an uncertain environment. In this context, (Smarandache 2010) is the first to claim neutrosophic logic is more efficient than fuzzy logic. Smarandache (2014) introduced neutrosophic statistics (NS), which are extensions of classical statistics (CS) that are applied to samples collected from populations with uncertain observations. Chen et al. (2017a, b) presented neutrosophic numbers for rock engineering issues. Moreover, Aslam (2018) conducted the first study on neutrosophic statistical quality control (NSQC). Aslam (2019c) provided a table illustrating the differences between fuzzy statistics, the NS, and classical statistics. Aslam (2019b) has highlighted neutrosophic ANOVA. In a newer paper, AlAita and Aslam (2022) discussed neutrosophic analysis of covariance with respect to completely randomized designs, randomized complete block designs, and split-plot designs. Aslam and Albassam (2020) have proposed the use of post hoc multiple comparison tests based on neutrosophic statistics. A number of neutrosophic statistical tests have been discussed (Aslam 2019a, 2020a,, 2021; Aslam and Aldosari 2020; Sherwani et al. 2021a, b; Sherwani et al. 2021a, b). An innovative new distance measures have been proposed for single-valued neutrosophic sets by Huang (2016). In a more recent study, (Chutia et al. 2021) proposed ordering single-valued neutrosophic numbers based on flexible parameters and their reasonable properties. Moreover, an optimized score function in multi-attribute group decision-making based on fuzzy neutrosophic sets was suggested by Nafei et al. (2021). A discussion has been presented by Fahmi et al. (2017) regarding aggregation operators on triangular cubic fuzzy numbers and their application to multi-criteria decision-making problems. Likewise, (Amin et al. 2018) presented triangular cubic linguistic fuzzy aggregation operators and their applications in group decision-making. Fahmi et al. (2019a, b) defined the trapezoidal cubic fuzzy number Einstein hybrid weighted averaging operators and its application to decision-making. The weighted average rating (war) method for solving group decision-making problems using a triangular cubic fuzzy hybrid aggregation operator was presented by Fahmi et al. (2018c). Fahmi et al. (2018a, b) proposed the cubic fuzzy Einstein aggregation operators and their application to decision-making. Fahmi et al. (2018a, b) studied some geometric operators with the triangular cubic linguistic hesitant fuzzy number and their application in group decision-making. Fahmi et al. (2019a, b) used a new cubic fuzzy multi-attribute group decision-making model for selection in sol–gel synthesis of titanium carbide nanopowders. Garai and Garg (2021) discuss possible single-valued multi-attribute decision-making for water resource management problems under a single-valued bipolar neutrosophic environment. (Pamucar et al. 2020) established a novel fuzzy hybrid neutrosophic decision-making approach for the resilient supplier selection problem. Detailed information about the neutrosophic theory can be found in the following articles and books (Aslam 2020b, c; Nagarajan et al. 2021; Sumathi et al. 2019).

Based on a review of the literature and to the best of our knowledge, there has been no work on split-plot and split-block designs in an indeterminate environment. We will introduce the split-plot and split-block designs under the neutrosophic statistics, called NSPD and NSBD. The proposed designs are a generalized form of the existing SPDs and SBDs under classical statistics, as they can deal with both exact and vague data sets. We will examine the efficiency of NSPDs and NSBDs with the SPDs and SBDs with the help of a numerical example and MC simulation study to show that using the developed proposed tests will be beneficial when the observations are coming from an uncertain source, fuzzy, indeterminate, interval-valued, or neutrosophic form.

The next section introduces some basic concepts under neutrosophic statistics. In Sect. 3, we present an analysis of variance test for NSPD. Section 4 reproduces the results from Sect. 3 for NSBD. A discussion of hypotheses and decision rules is presented in Sect. 5. The performance of the proposed designs is evaluated in Sect. 6 through the use of a numerical example and a simulation study. The comparison analysis is presented in Sect. 7. The discussion and conclusions are presented in Sect. 8.

2 Preliminaries

Nowadays, due to the development of information science and the possession of a large amount of data in addition to the great advances in technology, statistical inference has become a very important option for analyzing these data in various fields. In classic statistics, observations are precisely determined; that is, the set of observations is deterministic. However, there are some situations where one is uncertain about the value of the observation which leads to a level of indeterminacy in the data set. In our current world, we have more indeterminate data than determinate data that has some degree of indeterminacy and could cause a major problem in statistical analysis because it could lead to unsatisfactory results of research projects.

Smarandache introduced the neutrosophic set as being a set where each element of the universe has a degree of truth, indeterminacy, and falsity, respectively. This is a theory that is known as neutrosophic logic and sets which is more general than fuzzy sets. So, to account for the measure of indeterminacy of the observations it can be used an interval to state an observation. The set of such intervals is called the neutrosophic data (Ndata). Smarandache (2014) introduced neutrosophic statistics using the Ndata to analyze the data. Afterward, researchers have been focused on the problem of data with some degree of indeterminacy using NS. Following is a brief discussion of some basic concepts in neutrosophic statistics.

Suppose that a neutrosophic random variable (NRV) $X_N \in [X_L, X_U]$ has neutrosophic normal distribution (NND) with a neutrosophic population mean $\mu_N \in [\mu_L, \mu_U]$ and a neutrosophic population variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$, where X_L and X_U are smaller and larger values of indeterminacy interval. Let $X_N = X_L + X_U I_N$ is the neutrosophic form of NRV having determinate part X_L and indeterminate part $X_U I_N$; $I_N \in [I_L, I_U]$, where $I_N \in [I_L, I_U]$ is indeterminate interval.

Suppose $n_N \in [n_L, n_U]$ is a neutrosophic random sample selected from a population of size N_N having indeterminate observations. The neutrosophic population mean and also $\mu_N \in [\ldots] = [\mu_L, \mu_U]$ and $\sigma_N^2 \in [\ldots] = [\sigma_L^2, \sigma_U^2]$ are expressed as follows:

$$\mu_N \in \left[\frac{\sum_{i=1}^{N_L} X_{Li}}{N_L}, \frac{\sum_{i=1}^{N_U} X_{Ui}}{N_U}\right]; \mu_N \in [\mu_L, \mu_U] \text{ and } \sigma_N^2 \in \left[\frac{\sum_{i=1}^{N_L} (X_{Li} - \mu_L)^2}{N_L}, \frac{\sum_{i=1}^{N_U} (X_{Ui} - \mu_U)^2}{N_U}\right]; \sigma_N^2 \in \left[\sigma_L^2, \sigma_U^2\right].$$

But, in the numerical examples, μ_N and σ_N^2 are unknown and can be estimated using the sample observations. The neutrosophic sample mean $\overline{X}_N$ and the variance s_N^2 are expressed by:

$\overline{X}_N \in \left[\frac{\sum_{i=1}^{n_L} X_{Li}}{n_L}, \frac{\sum_{i=1}^{n_U} X_{Ui}}{n_U}\right]; \overline{X}_N \in \left[\overline{X}_L, \overline{X}_U\right]$ and

$s_N^2 \in \left[\frac{\sum_{i=1}^{n_L}\left(X_{Li}-\overline{X}_L\right)^2}{n_L-1}, \frac{\sum_{i=1}^{n_U}\left(X_{Ui}-\overline{X}_U\right)^2}{n_U-1}\right];$

$s_N^2 \in \left[s_L^2, s_U^2\right].$

3 Neutrosophic split-plot design (NSPD)

The linear model for the neutrosophic split-plot design can be expressed as follows:

$$y_{Nhij} = \mu_N + \rho_{Nh} + \tau_{Ni} + \eta_{Nhi} + \beta_{Nj} + (\tau\beta)_{Nij} + \varepsilon_{Nhij}. \quad (1)$$

The neutrosophic form of y_{Nhij} can be expressed as

$$y_{Nhij} = y_{Lhij} + y_{Uhij} I_N; I_N \in [I_L, I_U], \quad (2)$$

where $h = 1, 2, \ldots, r_N$, $i = 1, 2, \ldots, a_N$ and $j = 1, 2, \ldots, b_N$, y_{Njij} is the neutrosophic response for the hij th experimental unit, μ_N is a neutrosophic general mean effect, ρ_{Nh}, τ_{Ni}, and η_{Nhi} represent neutrosophic replications or blocks, whole-plot (main factor$A$) and whole-plot error (replications $\times$ A), respectively. β_{Nj}, $(\tau\beta)_{Nij}$ and ε_{Nhij} represent the neutrosophic split-plot treatment (main factor$B$), AB interaction and the split-plot error**.** Table 1 presents the ANOVA for NSPD.

Neutrosophic sum of squares for NSPD are given as follows:

$$SS_{NT} = \sum_{h=1}^{r_N}\sum_{i=1}^{a_N}\sum_{j=1}^{b_N} y_{Nhij}^2 - CF_N; SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NR} = \frac{\sum_{h=1}^{r_N} y_{Nh.}^2}{a_N b_N} - CF_N; SS_{NR} \in [SS_{LR}, SS_{UR}]$$

$$SS_{NA} = \frac{\sum_{i=1}^{a_N} y_{N.i.}^2}{r_N b_N} - CF_N; SS_{NA} \in [SS_{LA}, SS_{UA}],$$

$$SS_{NE(A)} = \frac{\sum_{h=1}^{r_N}\sum_{i=1}^{a_N} y_{Nhi.}^2}{b_N} - CF_N - SS_{NA} - SS_{NR};$$

$$SS_{NE(A)} \in \left[SS_{LE(A)}, SS_{UE(A)}\right],$$

$$SS_{NB} = \frac{\sum_{j=1}^{b_N} y_{N..j}^2}{r_N a_N} - CF_N;\ SS_{NB} \in [SS_{LB}, SS_{UB}],$$

$$SS_{NAB} = \frac{\sum_{i=1}^{a_N}\sum_{j=1}^{b_N} y_{N.ij}^2}{r_N} - CF_N - SS_{NA} - SS_{NB};\ SS_{NAB} \in [SS_{LAB}, SS_{UAB}],$$

$$SS_{NE(B)} = SS_{NT} - SS_{NR} - SS_{NA} - SS_{NE(A)} - SS_{NB} - SS_{NAB};$$

$$SS_{NE(B)} \in \left[SS_{LE(B)}, SS_{UE(B)}\right],$$

where $CF_N = \frac{y_{N...}^2}{r_N a_N b_N}$.

Neutrosophic mean squares for NSPD are given as follows:

$$MS_{NR} = \frac{SS_{NR}}{r_N - 1}; MS_{NR} \in [MS_{LR}, MS_{UR}],$$

$$MS_{NA} = \frac{SS_{NA}}{a_N - 1};\ MS_{NA} \in [MS_{LA}, MS_{UA}],$$

$$MS_{NE(A)} = \frac{SS_{NE(A)}}{(r_N - 1)(a_N - 1)}; MS_{NE(A)} \in \left[MS_{LE(A)}, MS_{UE(A)}\right],$$

$$MS_{NB} = \frac{SS_{NB}}{b_N - 1};\quad MS_{NB} \in [MS_{LB}, MS_{UB}],$$

$$MS_{NAB} = \frac{SS_{NAB}}{(a_N - 1)(b_N - 1)};\quad MS_{NAB} \in [MS_{LAB}, MS_{UAB}],$$

$$MS_{NE(B)} = \frac{SS_{NE(B)}}{a_N(r_N - 1)(b_N - 1)}; MS_{NE(B)} \in \left[MS_{LE(B)}, MS_{UE(B)}\right],$$

The neutrosophic statistic F_N tests become

$$F_{NA} = \frac{MS_{NA}}{MS_{NE(A)}}; F_{NA} \in [F_{LA}, F_{UA}],$$

$$F_{NB} = \frac{MS_{NB}}{MS_{NE(B)}};\ F_{NB} \in [F_{LB}, F_{UB}],$$

$$F_{NAB} = \frac{MS_{NAB}}{MS_{NE(B)}};\ F_{NAB} \in [F_{LAB}, F_{UAB}],$$

Table 1 ANOVA for the NSPD

Source of variation	*NSS*	*ndf*	*NMS*	F_N
Replicate	SS_{NR}	$r_N - 1$	$MS_{NR} = \frac{SS_{NR}}{r_N-1}$	
A	SS_{NA}	$a_N - 1$	$MS_{NA} = \frac{SS_{NA}}{a_N-1}$	$\frac{MS_{NA}}{MS_{NE(A)}}$
Error A	$SS_{NE(A)}$	$(r_N - 1)(a_N - 1)$	$MS_{NE(A)} = \frac{SS_{NE(A)}}{(r_N-1)(a_N-1)}$	
B	SS_{NB}	$(b_N - 1)$	$MS_{NB} = \frac{SS_{NB}}{b_N-1}$	$\frac{MS_{NB}}{MS_{NE(B)}}$
$A \times B$	SS_{NAB}	$(a_N - 1)(b_N - 1)$	$MS_{NAB} = \frac{SS_{NAB}}{(a_N-1)(b_N-1)}$	$\frac{MS_{NAB}}{MS_{NE(B)}}$
Error B	$SS_{NE(B)}$	$a_N(r_N - 1)(b_N - 1)$	$MS_{NE(B)} = \frac{SS_{NE(B)}}{a_N(r_N-1)(b_N-1)}$	
Total	SS_{NT}	$r_N a_N b_N - 1$		

The neutrosophic form of the proposed tests F_N can be expressed as:

$$F_N = F_L + F_U I_{F_N}; I_{F_N} \in [I_{F_L}, I_{F_U}],$$

where F_L and $F_U I_{F_N}$ are determinate and indeterminate parts of each proposed test. These tests reduce to tests under classical statistic if I_{F_N} = 0.

4 Neutrosophic split-block design (NSBD)

The linear model for the neutrosophic split-block design can be expressed as follows:

$$y_{Nhij} = \mu_N + \rho_{Nh} + \tau_{Ni} + \eta_{Nhi} + \beta_{Nj} + \delta_{Nhj} + (\tau\beta)_{Nij} + \varepsilon_{Nhij}, \tag{3}$$

The neutrosophic form of y_{Nhij} is the same as in (2), where $h = 1, 2, ..., r_N$, $i = 1, 2, ..., a_N$ and $j = 1, 2, ..., b_N$, y_{Njij} is the neutrosophic response for the hij th experimental unit, μ_N is a neutrosophic general mean effect, ρ_{Nh}, τ_{Ni}, and η_{Nhi} represent neutrosophic replications or blocks, whole-plot (main factor A) and whole-plot error (replications $\times$ A), respectively.β_{Nj}, δ_{Nhj}, $(\tau\beta)_{Nij}$ and ε_{Nhij} represent the neutrosophic split-plot treatment (main factor B), the split-plot error (replications $\times$ B), AB interaction and the interaction error (replications $\times$ AB). Table 2 presents ANOVA for NSBD.

Neutrosophic sum of squares for NSBD are given as follows:

$$SS_{NT} = \sum_{h=1}^{r_N}\sum_{i=1}^{a_N}\sum_{j=1}^{b_N} y_{Nhij}^2 - CF_N; SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NR} = \frac{\sum_{h=1}^{r_N} y_{Nh..}^2}{a_N b_N} - CF_N; SS_{NR} \in [SS_{LR}, SS_{UR}],$$

$$SS_{NA} = \frac{\sum_{i=1}^{a_N} y_{N.i.}^2}{r_N b_N} - CF_N; SS_{NA} \in [SS_{LA}, SS_{UA}],$$

$$SS_{NE(A)} = \frac{\sum_{h=1}^{r_N}\sum_{i=1}^{a_N} y_{Nhi.}^2}{b_N} - CF_N - SS_{NA} - SS_{NR}; SS_{NE(A)} \in \left[SS_{LE(A)}, SS_{UE(A)}\right],$$

$$SS_{NB} = \frac{\sum_{j=1}^{b_N} y_{N..j}^2}{r_N a_N} - CF_N; SS_{NB} \in [SS_{LB}, SS_{UB}],$$

$$SS_{NE(B)} = \frac{\sum_{h=1}^{r_N}\sum_{j=1}^{b_N} y_{Nh.j}^2}{a_N} - CF_N - SS_{NB} - SS_{NR}; SS_{NE(B)} \in \left[SS_{LE(B)}, SS_{UE(B)}\right],$$

$$SS_{NAB} = \frac{\sum_{i=1}^{a_N}\sum_{j=1}^{b_N} y_{N.ij}^2}{r_N} - CF_N - SS_{NA} - SS_{NB}; SS_{NAB} \in [SS_{LAB}, SS_{UAB}],$$

$$SS_{NE(AB)} = SS_{NT} - SS_{NR} - SS_{NA} - SS_{NE(A)} - SS_{NB} - SS_{NE(B)} - SS_{NAB};$$

$$SS_{NE(AB)} \in \left[SS_{LE(AB)}, SS_{UE(AB)}\right],$$

where $CF_N = \frac{y_{N...}^2}{r_N a_N b_N}$.

Neutrosophic mean squares for NSBD are given as follows:

$$MS_{NR} = \frac{SS_{NR}}{r_N - 1}; MS_{NR} \in [MS_{LR}, MS_{UR}],$$

$$MS_{NA} = \frac{SS_{NA}}{a_N - 1}; MS_{NA} \in [MS_{LA}, MS_{UA}],$$

$$MS_{NE(A)} = \frac{SS_{NE(A)}}{(r_N - 1)(a_N - 1)};$$

$$MS_{NE(A)} \in \left[MS_{LE(A)}, MS_{UE(A)}\right],$$

$$MS_{NB} = \frac{SS_{NB}}{b_N - 1}; MS_{NB} \in [MS_{LB}, MS_{UB}],$$

Table 2 ANOVA for the NSBD

Source of variation	SS	*df*	MS	F_0
Block	SS_{NR}	$r_N - 1$	$MS_{NR} = \frac{SS_{NR}}{r_N-1}$	
A	SS_{NA}	$a_N - 1$	$MS_{NA} = \frac{SS_{NA}}{a_N-1}$	$\frac{MS_{NA}}{MS_{NE(A)}}$
Error A	$SS_{NE(A)}$	$(r_N - 1)(a_N - 1)$	$MS_{NE(A)} = \frac{SS_{NE(A)}}{(r_N-1)(a_N-1)}$	
B	SS_{NB}	$(b_N - 1)$	$MS_{NB} = \frac{SS_{NB}}{b_N-1}$	$\frac{MS_{NB}}{MS_{NE(B)}}$
Error B	$SS_{NE(B)}$	$(r_N - 1)(b_N - 1)$	$MS_{NE(B)} = \frac{SS_{NE(B)}}{(r_N-1)(b_N-1)}$	
$A \times B$	SS_{NAB}	$(a_N - 1)(b_N - 1)$	$MS_{NAB} = \frac{SS_{NAB}}{(a_N-1)(b_N-1)}$	$\frac{MS_{NAB}}{MS_{NE(AB)}}$
Error AB	$SS_{NE(AB)}$	$(r_N - 1)(a_N - 1)(b_N - 1)$	$MS_{NE(AB)} = \frac{SS_{NE(AB)}}{(r_N-1)(a_N-1)(b_N-1)}$	
Total	SS_{NT}	$r_N a_N b_N - 1$		

$$MS_{NE(B)} = \frac{SS_{NE(B)}}{(r_N - 1)(b_N - 1)};$$
$$MS_{NE(B)} \in \left[MS_{LE(B)}, MS_{UE(B)}\right],$$
$$MS_{NAB} = \frac{SS_{NAB}}{(a_N - 1)(b_N - 1)};$$
$$MS_{NAB} \in [MS_{LAB}, MS_{UAB}],$$
$$MS_{NE(AB)} = \frac{SS_{NE(AB)}}{(r_N - 1)(a_N - 1)(b_N - 1)};$$
$$MS_{NE(AB)} \in \left[MS_{LE(AB)}, MS_{UE(AB)}\right].$$

The neutrosophic statistic F_N tests become

$$F_{NA} = \frac{MS_{NA}}{MS_{NE(A)}}; F_{NA} \in [F_{LA}, F_{UA}],$$
$$F_{NB} = \frac{MS_{NB}}{MS_{NE(B)}}; F_{NB} \in [F_{LB}, F_{UB}],$$
$$F_{NAB} = \frac{MS_{NAB}}{MS_{NE(AB)}}; F_{NAB} \in [F_{LAB}, F_{UAB}].$$

The neutrosophic form of the proposed tests F_N can be expressed as:

$$F_N = F_L + F_U I_{F_N}; I_{F_N} \in [I_{F_L}, I_{F_U}],$$

where F_L and $F_U I_{F_N}$ are determinate and indeterminate parts of each proposed test under NS. These tests reduce to tests under CS if $I_{F_N} = 0$.

5 Neutrosophic hypotheses and decision rule

The neutrosophic null and alternative hypotheses are formulated as follows;

$$H_{N0}: \tau_{N1} = \tau_{N2} = \cdots = \tau_{Na} = 0 \text{ vs } H_{N1}: \text{at least one } \tau_{Ni} \neq 0,$$

$$H_{N0}: \beta_{N1} = \beta_{N2} = \cdots = \beta_{Nb} = 0 \text{ vs } H_{N1}: \text{at least one } \beta_{Nj} \neq 0,$$

$$H_{N0}: (\tau\beta)_{N11} = (\tau\beta)_{N12} = \cdots = (\tau\beta)_{Nab} = 0 \text{ vs}$$
$$H_{N1}: \text{at least one } (\tau\beta)_{Nij} \neq 0.$$

Smarandache (2014) outlined that the neutrosophic decision rule can be summarized as follows:

1. If $\min\{p_N - \text{value}\} > \alpha$, then we accept the null hypothesis H_{N0} at the level α.
2. If $\max\{p_N - \text{value}\} \leq \alpha$, then we reject the null hypothesis H_{N0} at the level α.
3. If $\min\{p_N - \text{value}\} < \alpha < \max\{p_N - \text{value}\}$, then there is indeterminacy. Thus $\frac{\alpha - \min\{p_N - \text{value}\}}{\max\{p_N - \text{value}\} - \min\{p_N - \text{value}\}}$ represents the chance to reject H_{N0} at the level α, and $\frac{\max\{p_N - \text{value}\} - \alpha}{\max\{p_N - \text{value}\} - \min\{p_N - \text{value}\}}$ represents the chance to accept H_{N0} at the level α.

6 Numerical example and simulation

In this section, we will verify the efficiency and methodology of the proposed tests in analysis through both a numerical example as well as a simulation study.

6.1 Numerical example

To illustrate this purpose, we have generated hypothetical data. Consider $A = (A1, A2, A3)$, $B = (B1, B2, B3, B4)$, and $R = 3$ replications (blocks). By using these data, we will discuss the following two cases:

- NSPD
- NSBD

These data are distributed as shown in Table 3.

In order to conduct the proposed F_N-tests for NSPD and NSBD, the following steps will need to be taken:

Step 1: Assigning the neutrosophic null hypothesis and alternative hypotheses to test the effect of factors.

$$H_{N0}: \tau_{Ni} = 0 \text{ vs } \quad H_{N1}: \text{at least one } \tau_{Ni} \neq 0,$$

$$H_{N0}: \beta_{Nj} = 0 \text{ vs } \quad H_{N1}: \text{at least one } \beta_{Nj} \neq 0,$$

$H_{N0}: (\tau\beta)_{Nij} = 0$ vs $\quad H_{N1}$: at least one $(\tau\beta)_{Nij} \neq 0$, $i = 1, 2, \ldots, a$, and $j = 1, 2, \ldots, b$.

Step 2: Preparation of the NANOVA Tables 4 and 5 for NSPD and NSBD, respectively.

Step 3: Calculation of the measure of indeterminacy for each factor using the neutrosophic form of the F_N-test under NS, as illustrated in Tables 4 and 5 where the F_N-test is reduced to existing F-test under CS if $I_N = 0$.

Step 4: Calculation of the $p_N - value$ at the level $\alpha = 0.05$.

In Tables 4 and 5: $p_{NA} - value = [0.5192,\ 0.1327] > 0.05$., i.e., there is no difference in the mean levels of factor A.

6.2 Simulation study

In the simulation study, the proposed tests $F_N \in [F_L, F_U]$ for NSPD and NSBD were compared with respect to empirical Type I error rate and the test power $(1 - \beta)$ with the existing test under classical statistics in terms of a measure of uncertainty. The considered distribution is neutrosophic standard normal distribution. The MC simulations are repeated 10,000 times. Further, it is assumed that homogeneity of neutrosophic variances is satisfied and neutrosophic balanced designs. Moreover, different values of the level of significance α are deemed $(0.05, 0.01)$. The numbers of the neutrosophic observations are selected based on various previously existing examples for the proposed tests (e.g., Montgomery 2017).

Table 3 Data for the NSPD and NSBD

	Block 1			Block 2			Block 3		
	A1	A2	A3	A1	A2	A3	A1	A2	A3
B1	[31, 35]	[35, 40]	[30, 40]	[31, 38]	[30, 38]	[30, 40]	[30, 38]	[32, 40]	[32, 40]
B2	[32, 38]	[33, 40]	[31, 39]	[30, 38]	[30, 39]	[30, 39]	[34, 39]	[31, 40]	[30, 40]
B3	[30, 35]	[31, 40]	[32, 40]	[33, 37]	[30, 39]	[30, 35]	[30, 38]	[32, 40]	[31, 39]
B4	[32, 38]	[30, 39]	[30, 39]	[34, 39]	[32, 40]	[33, 39]	[30, 40]	[32, 38]	[31, 40]

Table 4 ANOVA table for the NSPD

Source of variation	*NSS*	*ndf*	*NMS*	F_N	Neutrosophic form F_N	p_N − value
Replicate	[0.6667, 5.7222]	[2, 2]	[0.3333, 2.8611]			
A	[3.1667, 19.3889]	[2, 2]	[1.5833, 9.6944]	[0.7755, 3.4900]	$0.7755 + 3.4900I_N; I_N \in [0, 0.7778]$	[0.5192, 0.1327]
Error A	[8.1667, 11.1111]	[4, 4]	[2.0417, 2.7778]			
B	[1.4167, 6.0000]	[3, 3]	[0.4722, 2.0000]	[0.1855, 2.0187]	$0.1855 + 2.0187I_N; I_N \in [0, 0.9081]$	[0.9049, 0.1473]
AB	[7.5000, 12.1667]	[6, 6]	[1.2500, 2.0278]	[0.4909, 2.0467]	$0.4909 + 2.0467I_N; I_N \in [0, 0.7601]$	[0.8067, 0.1117]
Error B	[45.8333, 17.833]	[18, 18]	[2.5463, 0.9907]			
Total	[66.7500, 72.2222]	[35, 35]				

Table 5 ANOVA table for the NSBD

Source of variation	*NSS*	*ndf*	*NMS*	F_N	Neutrosophic form F_N	p_N − value
Block	[0.6667, 5.7222]	[2, 2]	[0.3333, 2.8611]			
A	[3.1667, 19.3889]	[2, 2]	[1.5833, 9.6944]	[0.7755, 3.4900]	$0.7755 + 3.4900I_N; I_N \in [0, 0.7778]$	[0.5192, 0.1327]
Error A	[8.1667, 11.1111]	[4, 4]	[2.0417, 2.7778]			
B	[1.4167, 6.0000]	[3, 3]	[0.4722, 2.0000]	[0.1417, 2.6667]	$0.1417 + 2.6667I_N; I_N \in [0, 0.9469]$	[0.9313, 0.1416]
Error B	[20.0000, 4.5000]	[6, 6]	[3.3333, 0.7500]			
AB	[7.5000, 12.1667]	[6, 6]	[1.2500, 2.0278]	[0.5806, 1.8250]	$0.5806 + 1.8250I_N; I_N \in [0, 0.6819]$	[0.7395, 0.1766]
Error AB	[25.8333, 13.3333]	[12, 12]	[2.1528, 1.1111]			
Total	[66.7500, 72.2222]	[35, 35]				

Let us review the following simulation:

An MC experiment for calculating the empirical Type I error rate under NS consists of the following steps:

- For each replicate, $i = 1, 2, \ldots, a_N$:
- Generate the ith neutrosophic random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, \ldots, x_{Nn}^{(i)}$ under H_{N0}.
- Calculate the test statistic F_{Ni} from the ith sample.
- Record results of the test $I_{Ni} = 1$ if H_{N0} is rejected at the significance level α and accepted $I_{Ni} = 0$.
- Calculate the ratio of significant tests $\frac{1}{a_N}\sum_{i=1}^{a_N} I_{Ni}$ and take it as $\alpha_{\text{Empirical}}$ under NS.

An MC experiment for calculating the estimate of the power of a test under NS consists of the following steps:

- Select a particular value of the parameters. For instance, $(\mu_{N1}, \mu_{N2}, \mu_{N3}) = (0, 1, 1)$ for factor A.
- For each replicate, $i = 1, 2, \ldots, a_N$:
- Generate the i th random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, \ldots, x_{Nn}^{(i)}$ under H_{N1}.
- Calculate the test statistic F_{Ni} from the i th sample.
- Record results of the test $I_{Ni} = 1$ if H_{N0} is rejected at the significance level α and accepted $I_{Ni} = 0$.
- Calculate the ratio of significant tests $\widehat{\pi}(\mu_{Ni}) = \frac{1}{a_N}\sum_{i=1}^{a_N} I_{Ni}$ and take it as test $\text{Power}_{\text{Empirical}}$ under NS.

7 Comparison analysis

An assessment of the legitimacy and viability of the proposed methodology is conducted by comparing neutrosophic designs with existing classic designs.

Table 6 The simulation results for NSPD and NSBD (Block = 3, factor A = 3, factor B = 4)

Test	Factor	α	Mean empirical type I error			Mean empirical power		
NSPD	A	0.05	[0.0500, 0.0519]	[0.0514, 0.0524]	[0.1217, 0.1321]	[0.3544, 0.3932]	[0.6527, 0.7088]	[0.8791, 0.9139]
		0.01	[0.0104, 0.0110]	[0.0098, 0.0103]	[0.0284, 0.0311]	[0.1053, 0.1231]	[0.2619, 0.2988]	[0.4803, 0.5381]
	B	0.05	[0.0501, 0.0504]	[0.0496, 0.0515]	[0.1325, 0.1456]	[0.4236, 0.4809]	[0.7934, 0.8493]	[0.9667, 0.9807]
		0.01	[0.0100, 0.0101]	[0.0083, 0.0088]	[0.0338, 0.0379]	[0.1833, 0.2221]	[0.5265, 0.6033]	[0.8476, 0.9025]
NSBD	A	0.05	[0.0500, 0.0519]	[0.0514, 0.0524]	[0.1217, 0.1321]	[0.3544, 0.3932]	[0.6527, 0.7088]	[0.8791, 0.9139]
		0.01	[0.0104, 0.0110]	[0.0098, 0.0103]	[0.0284, 0.0311]	[0.1053, 0.1231]	[0.2619, 0.2988]	[0.4803, 0.5381]
	B	0.05	[0.0483, 0.0515]	[0.0502, 0.0514]	[0.1014, 0.1075]	[0.2875, 0.3272]	[0.5922, 0.6501]	[0.8360, 0.8831]
		0.01	[0.0098, 0.0113]	[0.0092, 0.0097]	[0.0230, 0.0250]	[0.0859, 0.1049]	[0.2397, 0.2839]	[0.4828, 0.5471]

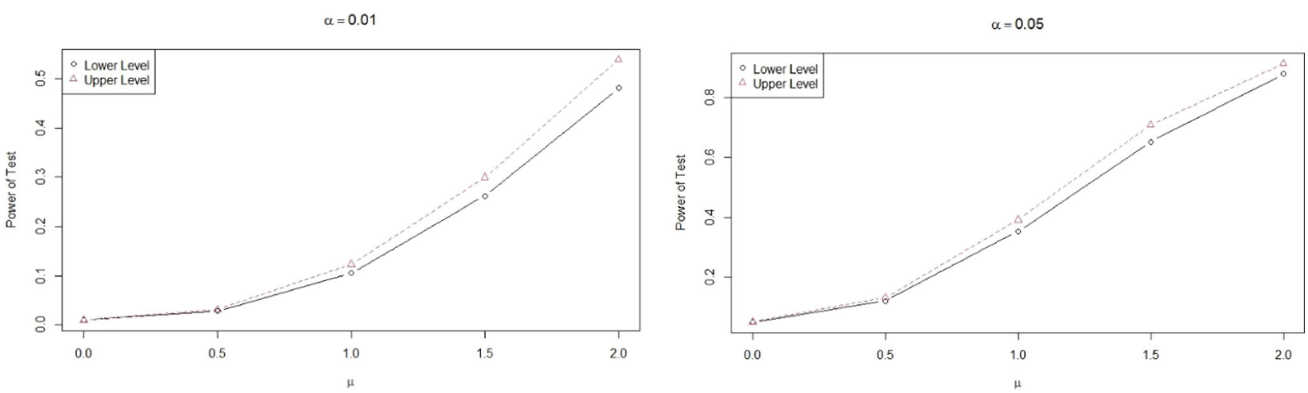

Fig. 1 Power curves for NSPD and NSBD (Factor A)

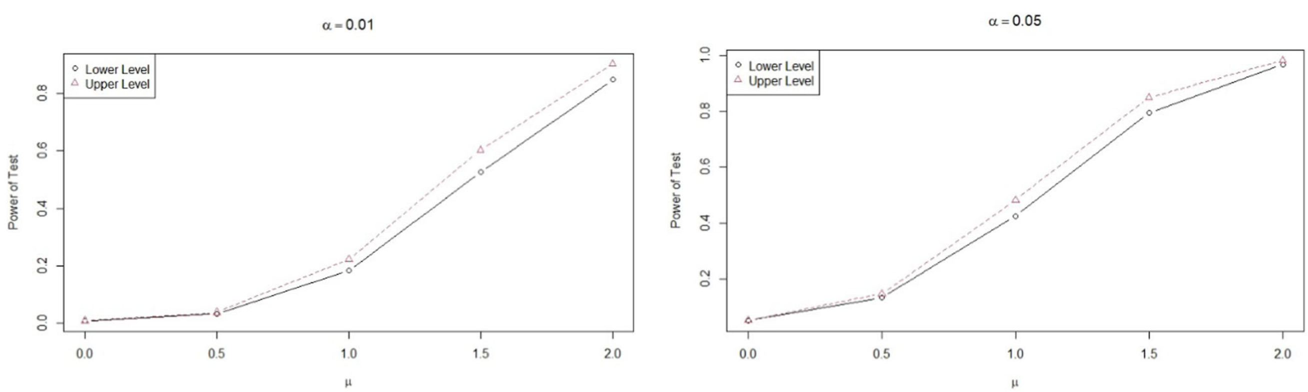

Fig. 2 Power curves for NSPD (Factor B)

7.1 Comparison using the neutrosophic form of F_N-test

The neutrosophic form of F_N-test is $F_N = F_L + F_U I_N; I_N \in [I_L, I_U]$, where the first part F_L is known as the determined part and presents the value of F-test under CS. The second part $F_U I_N; I_N \in [I_L, I_U]$ is known as the indeterminate part. Note that the F_N-test reduces to F-test under classical statistics if $I_{F_N} = 0$. This means that the NS approach provides the F_N-test values in an interval with the measure of indeterminacy which is more general and includes the determinate part of the CS. As an example, the neutrosophic form of F_N-test for factor A in Tables 4, 5 is $0.7755 + 3.4900 I_N; I_N \in [0, 0.7778]$. It means that the proposed F_N-test for NSPD and NSBD indicates that F_N ranges between 0.7755 and 3.4900 with the degree of indeterminacy 0.7778. Therefore, in light of the comparisons, it is concluded that the proposed designs under NS are more informative than the existing designs under CS.

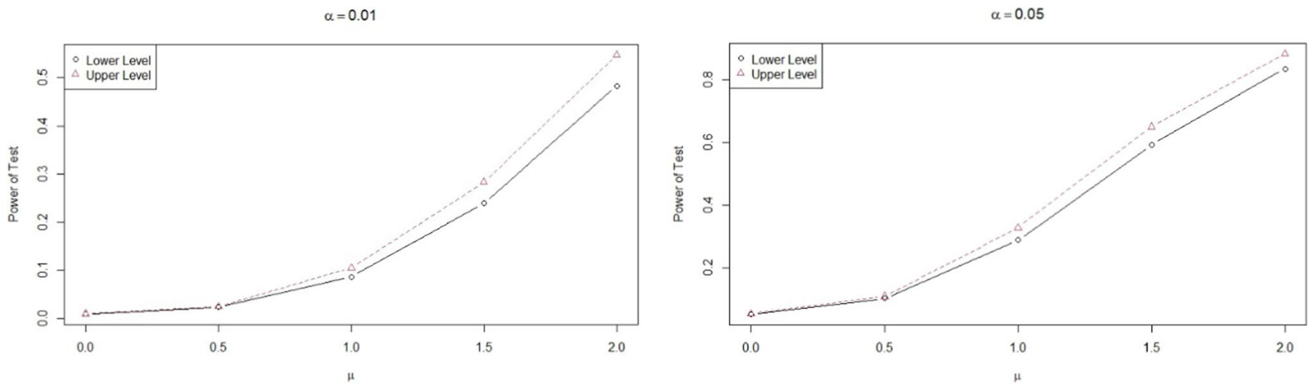

Fig. 3 Power curves for NSBD (Factor B)

7.2 Performance comparison based on empirical α and test power

In this section, we illustrate the relative performance of the proposed neutrosophic approach to analyze SPD and SBD in the event of the presence of indeterminate data based on the empirical type I error rate ($\alpha_{\text{Empirical}}$) and power of the test ($\text{Power}_{\text{Empirical}}$). The results in Table 6 of the numerical section indicates that the neutrosophic empirical type I error and power are more general, precise and flexible than the classic ones. For example, $\alpha_{\text{Empirical}}$ at the nominal level $\alpha = 0.05$ for factor A is in interval [0.0500, 0.0519] which is very close to the nominal level $\alpha = 0.05$ and is more general by including the classic $\alpha_{\text{Empirical}}$ as the lower bound; it also leads to a higher power of the test as demonstrated in Fig. 1. In light of the rest of the experimental results, it is evident that the empirical type I error rates between the determinate and indeterminate parts are relatively close to nominal 0.05 and 0.01 under NS. The differences between the empirical power values of the determinate part and the indeterminate part reveal not only the superiority but also the flexibility and generalization of the proposed approach. Accordingly, in Figs. 1, 2, 3, the power values in Table 6 were displayed. These figures demonstrate that the empirical test power of factors A and B for the indeterminate part is uniformly higher than that of the determinate which corresponds to the classic designs. This discloses the neutrosophic approach is more efficient than the existing classic statistic in analyzing the SP and SB designs.

8 Discussion and conclusions

In the case of indeterminate data coming from uncertain experimental measurements, using classic statistics may lead to biased inferences about the parameter models. Under such a situation, we considered the SPD and SBD with indeterminate observations and suggested using a neutrosophic statistical approach in analyzing the data. To this end, we formulated the corresponding neutrosophic ANOVA Tables of these designs based on the neutrosophic estimates of the model parameters. Numerical examples assure the performance of the proposed approach. To assess the efficiency of the neutrosophic approach, a simulation study was conducted to calculate the type I error and the power of the treatment tests. This study revealed the superiority of the proposed approach in terms of accuracy, flexibility and generality of analysis under the uncertain circumstances. The proposed approach can potentially be extended to the various classic designs under uncertainty in measured data.

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