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Process Quality Evaluation Model with Taguchi Cost Loss Index

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Abstract: Process Capability Indices (PCIs) are not only a good communication tools between sales departments and customers but also convenient tools for internal engineers to evaluate and analyze process capabilities of products. Many statisticians and process engineers are dedicated to research on process capability indices, among which the Taguchi cost loss index can reflect both the process yield and process cost loss at the same time. Therefore, in this study the Taguchi cost loss index was used to propose a novel process quality evaluation model. After the process was stabilized, a process capability evaluation was carried out. This study used Boole's inequality and DeMorgan's theorem to derive the $(1 - \alpha) \times 100\%$ confidence region of (δ, γ^2) based on control chart data. The study adopted the mathematical programming method to find the $(1 - \alpha) \times 100\%$ confidence interval of the Taguchi cost loss index then employed a $(1 - \alpha) \times 100\%$ confidence interval to perform statistical testing and to determine whether the process needed improvement.

Keywords: process capability indices; Taguchi cost loss index; confidence interval; mathematical programming; control chart data

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1. Introduction

Many studies have pointed out that Process Capability Indices (PCIs) are not only good communication tools between sales departments and customers but also convenient tools for internal engineers to evaluate and analyze the process capabilities of products [1–10]. In fact, Process Capability Indices have been widely used in the machine tool industry, semiconductor manufacturing process, and IC packaging industry, electronics industry. [11–14]. Let random variable X represent the relevant quality characteristic of a manufacturing process followed by a normal distribution with process mean μ and process standard deviation δ. Then, the probability density function of X can be expressed as follows:

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}exp\left\{\frac{(x-\mu)^2}{2\sigma^2}\right\},x\in R \quad (1)$$

Kane [15] proposed a process yield based capability index C_{PK} in the following:

$$C_{PK}=\frac{d-|\mu-T|}{3\sigma} \quad (2)$$

where $d=(USL-LSL)/2$, $T=(USL+LSL)/2$, and USL and LSL are, respectively, the upper specification limit and the lower specification limit. The relationship between the process yield and capability index C_{PK} is demonstrated as follows:

$$Yield\% \geq 2\,\Phi(3C_{pk})-1 \quad (3)$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal distribution. For example, when $C_{PK}=1$, we can guarantee the process yield is equal to at least $2\,\Phi(3)-1=99.73\%$. Chan et al. [16] proposed the Taguchi cost loss index C_{PM} as follows:

$$C_{PM}=\frac{d}{3\sqrt{(\mu-T)^2+\sigma^2}} \quad (4)$$

where $(\mu-T)^2+\sigma^2$ is the expected value of the Taguchi loss function. As noted by Chen et al. [17] and Chen and Chang [18], the relationship between the process yield and Taguchi cost loss index C_{PM} is described as follows:

$$Yield\% \geq 2\,\Phi(3C_{PM})-1,\ C_{PM}>1/3 \quad (5)$$

Obviously, the process-yield-based capability index C_{PK} can reflect the process yield, while the Taguchi cost loss index can reflect not only the process yield but also the process loss, since its denominator is the expected value of the Taguchi cost loss function.

Zimmer et al. [19] and Parchami et al. [20] derived the confidence intervals of the Taguchi cost loss index and determined the sample size for this index. Perakis et al. [21] developed a new method for constructing confidence intervals for the Taguchi cost loss index. Perakis et al. [22] proposed some new methods for comparing the capabilities of two processes using the Taguchi cost loss index. In addition, many other scholars have done a lot of research on the Taguchi cost loss index C_{PM}.

Since the statistical distribution of the index estimation formula is relatively complicated, it is also relatively difficult to make statistical inferences. Thus, Smarandache developed neutrosophic statistics that are more flexible than traditional statistics [23]. In addition, Chen et al. [17] used a mathematical programming method to derive the confidence intervals of PCIs, discussed the results, and found accuracy as high as 95% or more. Subsequently, many articles also applied the mathematical programming method to derive the confidence intervals of various indicators [24,25]. Chen et al. [17] adopted the model to derive the confidence interval of the indices, and the confident interval was also extended [18] by a one-sided Fuzzy test to a two-sided Fuzzy test. Therefore, this study proposes a novel evaluation model for process quality with the Taguchi cost loss index, the advantages being: (1) to help quality management engineers in the industry understand better and apply more conveniently in practice; (2) to conform to an enterprise's quick response demand, and (3) to help the enterprise to meet the goal of intelligent manufacturing management.

Numerous studies have pointed out that industries usually uses a statistical process control chart to monitor the quality of processes [26–29] as well as evaluate process capability when the process is stabilized [30–34]. Thus, this study employs control chart data to estimate the Taguchi cost loss index. First, we apply the mathematical programming method to find the confidence interval of this index, then we adopt this confidence interval to evaluate the process quality and determine whether to improve the process. We also use a case to explain how to apply the evaluation method proposed by this study. Finally, we make conclusions, state research limitations and propose future research.

2. Find Confidence Intervals Using the Mathematical Programming Method

Let random variable $Y = (X - T)/d$. Then, Y is distributed as a normal distribution with mean δ and standard deviation γ, where $\delta = (\mu - T)/d$ and $\gamma = \sigma/d$. Thus, the Taguchi cost loss index C_{PM} can be rewritten as follows:

$$C_{PM} = \frac{1}{3\sqrt{\delta^2 + \gamma^2}}, \tag{6}$$

where $\delta^2 + \gamma^2$ is the expected value of Taguchi cost loss.

As noted above, random variable Y is distributed as a normal distribution. In the statistical control, the subsamples are obtained over a period of time. Each subsample contains n observations related to the quality characteristics, and m subsamples are available. Consequently, for each subsample, let $\overline{Y}_j$ and S_j^2 respectively, represent the sample mean and sample variance of the *i*th subsample, and let $N = m \times n$ represent the total number of observations as follows:

$$\overline{Y}_i = \frac{1}{n}\sum_{j=1}^{n} Y_{ij} \tag{7}$$

and

$$S_j^2 = \frac{1}{n}\sum_{j=1}^{n}\left(Y_{ij} - \overline{Y}_i\right)^2 \tag{8}$$

The overall sample mean and the pooled sample variance, unbiased estimators of δ and γ^2, are respectively expressed as follows:

$$\hat{\delta} = \overline{\overline{Y}} = \frac{1}{m}\sum_{i=1}^{m}\overline{Y}_i \tag{9}$$

and

$$\hat{\gamma}^2 = \frac{1}{m}\sum_{i=1}^{m} S_i^2 \tag{10}$$

Obviously, the estimator $\hat{\delta}$ is distributed as a normal distribution with mean δ and variance γ^2/N, while the estimator $\hat{\gamma}^2$ is distributed as $\gamma^2\chi^2_{N-m}/(N-m)$. Furthermore, we let

$$T_N = \frac{\sqrt{N}\left(\hat{\delta} - \delta\right)}{\hat{\gamma}} \tag{11}$$

and

$$K_N = \frac{(N-m)\hat{\gamma}^2}{\gamma^2}. \tag{12}$$

Then, T_N and K_N are distributed as t_{N-m} and χ^2_{N-m} respectively. t_{N-m} is t distribution with $N - m$ degree of freedom and χ^2_{N-m} is chi-squire distribution with $N - m$ degree of freedom. Thus, we have

$$1-\frac{\alpha}{2}=\left\{-t_{\alpha/4;N-m}\le T_N\le t_{\alpha/4;N-m}\right\}$$
$$=\left\{-t_{\alpha/4;N-m}\le\frac{\sqrt{N}\left(\hat{\delta}-\delta\right)}{\hat{\gamma}}\le t_{\alpha/4;N-m}\right\} \tag{13}$$
$$=\left\{\hat{\delta}-t_{\alpha/4;N-m}\left(\frac{\hat{\gamma}}{\sqrt{N}}\right)\le\delta\le\hat{\delta}+t_{\alpha/4;N-m}\left(\frac{\hat{\gamma}}{\sqrt{N}}\right)\right\}$$

and

$$1-\frac{\alpha}{2}=\left\{\chi^2_{\alpha/4;N-m}\le K_N\le\chi^2_{1-\alpha/4;N-m}\right\}$$
$$=\left\{\chi^2_{\alpha/4;N-m}\le\frac{(N-m)\hat{\gamma}^2}{\gamma^2}\le\chi^2_{1-\alpha/4;N-m}\right\} \tag{14}$$
$$=\left\{\frac{(N-m)\hat{\gamma}^2}{\chi^2_{1-\alpha/4;N-m}}\le\gamma^2\le\frac{(N-m)\hat{\gamma}^2}{\chi^2_{\alpha/4;N-m}}\right\}$$

To derive the $(1-\alpha)\times 100\%$ confidence interval of the Taguchi cost loss index, this study defines the event E_T and event E_K as follows:

$$E_T=\left\{\hat{\delta}-t_{\alpha/4;N-m}\left(\frac{\hat{\gamma}}{\sqrt{N}}\right)\le\delta\le\hat{\delta}+t_{\alpha/4;N-m}\left(\frac{\hat{\gamma}}{\sqrt{N}}\right)\right\} \tag{15}$$

and

$$E_T=\left\{\frac{(N-m)\hat{\gamma}^2}{\chi^2_{1-\alpha/4;N-m}}\le\gamma^2\le\frac{(N-m)\hat{\gamma}^2}{\chi^2_{\alpha/4;N-m}}\right\}, \tag{16}$$

where $t_{\alpha/4;N-m}$ is the upper $\alpha/4$ quantile of t_{N-m}, and $\chi^2_{a;N-m}$ is the upper a quantile of χ^2_{N-m}. Based on Boole's inequality and DeMorgan's theorem, we have

$$P(E_T\cap E_K)\ge 1-P\left(E_T^C\right)-P\left(E_K^C\right), \tag{17}$$

where $p(E_T)=p(E_K)=1-\alpha/2$ and $p\left(E_T^C\right)=p\left(E_K^C\right)=\alpha/2$. Then,

$$p\left\{\hat{\delta}-t_{\alpha/4;N-m}\times\sqrt{\frac{\hat{\gamma}^2}{N}}\le\delta\le\hat{\delta}+t_{\alpha/4;N-m}\times\sqrt{\frac{\hat{\gamma}^2}{N}},\frac{(N-m)\hat{\gamma}^2}{\chi^2_{1-\alpha/4;N-m}}\le\gamma^2\le\frac{(N-m)\hat{\gamma}^2}{\chi^2_{\alpha/4;N-m}}\right\}=1-\alpha\,. \tag{18}$$

Let $(y_{i1},y_{i2},\cdots,y_{in})$ represent the observed value of $(Y_{i1},Y_{i2},\cdots,Y_{in})$. $\hat{\delta}_0$ and $\hat{\gamma}_0^2$ are the observed values of $\hat{\delta}$ and $\hat{\gamma}^2$ respectively as follows:

$$\hat{\delta}_0=\overline{\overline{y}}=\frac{1}{m}\sum_{i=1}^{m}\overline{y}_i\,, \tag{19}$$

and

$$\hat{\gamma}_0^2=\frac{1}{m}\sum_{i=1}^{m}s_i^2 \tag{20}$$

Therefore, the confidence region can be displayed as:

$$CR=\left\{\hat{\delta}_0-t_{\alpha/4;N-m}\times\sqrt{\frac{\hat{\gamma}^2}{N}}\le\delta\le\hat{\delta}_0+t_{\alpha/4;N-m}\times\sqrt{\frac{\hat{\gamma}_0^2}{N}},\frac{(N-m)\hat{\gamma}^2}{\chi^2_{1-\alpha/4;N-m}}\le\gamma^2\le\frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{\alpha/4;N-m}}\right\} \tag{21}$$

According to Chen et al. [13], the mathematical program model can be presented as follows:

$$\begin{cases} LC_{PM} = Min \dfrac{1}{3\sqrt{\delta^2+\gamma^2}} \\ subject\ to \\ \delta_L \le \delta \le \delta_U \\ \gamma_L^2 \le \gamma^2 \le \gamma_U^2 \end{cases} \text{and} \begin{cases} UC_{PM} = Max \dfrac{1}{3\sqrt{\delta^2+\gamma^2}} \\ subject\ to \\ \delta_L \le \delta \le \delta_U \\ \gamma_L^2 \le \gamma^2 \le \gamma_U^2 \end{cases},$$

where

$$\delta_L = \hat{\delta}_0 - t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}},\ \delta_U = \hat{\delta}_0 + t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}$$

$$\gamma_L^2 = \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{1-\alpha/4;N-m}}, \tag{22}$$

$$\gamma_U^2 = \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{\alpha/4;N-m}}. \tag{23}$$

Let $\Delta(\delta,\gamma) = \sqrt{\delta^2+\gamma^2}$, where $\Delta(\delta,\gamma)$ is the function of (δ,γ) and represents the distance from the punctuation point (δ,γ) to the origin coordinate (0, 0). Obviously, the closer the punctuation point (δ,γ) to the origin is, the greater the index value, whereas the farther the punctuation point (δ,γ) from the origin, the smaller the index value. According to this concept, this study solves the values of lower confidence limit LC_{PM} and upper confidence limit UC_{PM} in three situations as follows:

Situation 1: $\delta_U < 0$

In this situation, we can conclude that $\delta < 0$ and for any $(\delta,\gamma) \in CR$, $\Delta(\delta_U,\gamma_L) \le \Delta(\delta,\gamma) \le \Delta(\delta_L,\gamma_U)$. Therefore,

$$LC_{PM} = \frac{1}{3\sqrt{\delta_L^2+\gamma_U^2}} = 1 \Big/ 3\sqrt{\left(\hat{\delta}_0 - t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{\alpha/4;N-m}}} \tag{24}$$

and

$$UC_{PM} = \frac{1}{3\sqrt{\delta_U^2+\gamma_L^2}} = 1 \Big/ 3\sqrt{\left(\hat{\delta}_0 + t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{1-\alpha/4;N-m}}}. \tag{25}$$

Situation 2: $\delta_L > 0$

In this situation, we can conclude that $\delta > 0$ and for any $(\delta,\gamma) \in CR$, $\Delta(\delta_L,\gamma_L) \le \Delta(\delta,\gamma) \le \Delta(\delta_U,\gamma_U)$. Therefore, the lower confidence limit LC_{PM} and upper confidence limit UC_{PM} are expressed as follows:

$$LC_{PM} = \frac{1}{3\sqrt{\delta_U^2+\gamma_U^2}} = 1 \Big/ 3\sqrt{\left(\hat{\delta}_0 + t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{\alpha/4;N-m}}} \tag{26}$$

and

$$UC_{PM} = \frac{1}{3\sqrt{\delta_L^2+\gamma_L^2}} = 1 \Big/ 3\sqrt{\left(\hat{\delta}_0 - t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{1-\alpha/4;N-m}}}. \tag{27}$$

Situation 3: $\delta_L \le 0 \le \delta_U$

In this situation, we can conclude that $\delta = 0$ and for any $(\delta, \gamma) \in CR$, then $\Delta(0, \gamma_L) \leq \Delta(0, \gamma) \leq \Delta(0, \gamma_U)$. Therefore, the lower confidence limit LC_{PM} and upper confidence limit UC_{PM} are denoted as follows:

$$LC_{PM} = \sqrt{\frac{\chi^2_{\alpha/4;N-m}}{(N-m)}\left(\frac{1}{3\hat{\gamma}_0}\right)} \tag{28}$$

and

$$UC_{PM} = \sqrt{\frac{\chi^2_{1-\alpha/4;N-m}}{(N-m)}\left(\frac{1}{3\hat{\gamma}_0}\right)}. \tag{29}$$

Obviously, using the mathematical programming method to derive the confidence intervals of PCIs is much easier than using the probability density function of the index estimation formula, and its accuracy can reach 95% or above. It is convenient for readers or engineers in the industry to apply, and can master process quality evaluation and improve timeliness.

3. Process Quality Evaluation by Statistical Testing

In this section, we use a statistical testing method to evaluate process quality by means of the confidence interval of the Taguchi cost loss index. As described in Section 2, interval (LC_{PM}, UC_{PM}) is the $100(1-\alpha)\%$ confidence interval of the Taguchi cost loss index, rewritten as follows:

1. When $\delta_U < 0$, then the lower confidence limit LC_{PM} is as follows:

$$LC_{PM} = 1\Bigg/3\sqrt{\left(\hat{\delta}_0 - t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{\alpha/4;N-m}}}, \tag{30}$$

and the upper confidence limit UC_{PM} is as follows:

$$UC_{PM} = 1\Bigg/3\sqrt{\left(\hat{\delta}_0 + t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{1-\alpha/4;N-m}}}. \tag{31}$$

2. When $\delta_L > 0$, then the lower confidence limit LC_{PM} is as follows:

$$LC_{PM} = 1\Bigg/3\sqrt{\left(\hat{\delta}_0 + t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{\alpha/4;N-m}}}, \tag{32}$$

and the upper confidence limit UC_{PM} is as follows:

$$UC_{PM} = 1\Bigg/3\sqrt{\left(\hat{\delta}_0 - t_{\alpha/4;N-m} \times \sqrt{\frac{\hat{\gamma}_0^2}{N}}\right)^2 + \frac{(N-m)\hat{\gamma}_0^2}{\chi^2_{1-\alpha/4;N-m}}}. \tag{33}$$

3. When $\delta_L \leq 0 \leq \delta_U$, then the lower confidence limit LC_{PM} is as follows:

$$LC_{PM} = \sqrt{\frac{\chi^2_{\alpha/4;N-m}}{(N-m)}\left(\frac{1}{3\hat{\gamma}_0}\right)}, \tag{34}$$

and the upper confidence limit UC_{PM} is as follows:

$$UC_{PM} = \sqrt{\frac{\chi^2_{1-\alpha/4;N-m}}{(N-m)}\left(\frac{1}{3\hat{\gamma}_0}\right)}. \tag{35}$$

As noted by Lo et al. [35], the process capability level is "Capable" for $C_{PM} \geq 1.00$. To determine whether the process capability level is capable or not, we adopted the following null hypothesis $H_0: C_{PM} = 1.00$ versus the alternative hypothesis $H_1: C_{PM} \neq 1.00$. Let α be the significance level of the test, then the three statistical testing rules can be described as follows:

(1) If $LC_{PM} > 1.00$, then reject H_0 and conclude that $C_{PM} > 1.00$ (consider cutting costs).
(2) If $UC_{PM} < 1.00$, then reject H_0 and conclude that $C_{PM} < 1.00$. Then, the process needs to improve.
(3) If $LC_{PM} \leq 1.00 \leq UC_{PM}$, then do not reject H_0 and conclude that $C''_{pk} = c$ (maintain the status quo).

Next, these three statistical testing rules can be adopted to evaluate the process quality and decide whether the process needs to improve, maintain its status quo, or cut its costs.

4. A Practical Application

As mentioned earlier, the process quality evaluation model developed by the Taguchi loss index in this paper is applicable to the machine tool industry, semiconductor manufacturing processes, the IC packaging industry, and the electronics industry, among others. Central Taiwan is a place of strategic importance for the machine tool industry, which is formed due to the division of labor. The improvement of process quality enhances the competitiveness of the industrial chain. The shaft is an important component of many mechanical products. The outer diameter of the shaft is an important quality characteristic. The outer diameter tolerance of the shaft of a fan motor is 1.1 ± 0.05. Therefore, the lower specification limit is $LSL = 1.05$, while the upper specification limit is $USL = 1.15$. Then, the second-generation capability index can be exhibited as follows:

$$C_{PM} = \frac{0.05}{3\sqrt{(\mu - 1.1)^2 + \sigma^2}}$$

Let random variable $Y = (X - 1.1)/0.05$. Then, the new tolerance and the Taguchi cost loss index can be rewritten as follows:

$$C_{PM} = \frac{1}{3\sqrt{\delta^2 + \gamma^2}}$$

To test whether the Taguchi cost loss index value is equal to 1.00 with $\alpha = 0.01$, the null hypothesis and alternative hypothesis can be defined as follows:

null hypothesis $H_0: C_{PM} = 1.00$
and
alternative hypothesis $H_1: C_{PM} \neq 1.00$.

We collected 20 subsamples ($m = 20$) of a control chart data with sample size $n = 11$, where the dates $(y_{i1}, \ldots, y_{ij}, \ldots, y_{i11})$ are the ith sub-sample ($i = 1, 2, \ldots, 20$). Then,

$$\hat{\delta}_0 = \frac{1}{20}\sum_{i=1}^{20} \bar{y}_i = 0.16$$

and

$$\hat{\gamma}_0^2 = \frac{1}{20}\sum_{i=1}^{20} s_i^2 = 0.11.$$

Therefore, we have

$$\delta_L = \hat{\delta}_0 - t_{0.025;200} \times \sqrt{\frac{\hat{\gamma}_0^2}{220}} = 0.16 - 1.972 \times \sqrt{\frac{0.11}{220}} = 0.16 - 0.001 = 0.159,$$

$$\delta_U = \hat{\delta}_0 + t_{0.025;200} \times \sqrt{\frac{\hat{\gamma}_0^2}{220}} = 0.25 + 1.972 \times \sqrt{\frac{0.11}{220}} = 0.16 + 0.001 = 0.161,$$

$$\gamma_L^2 = \frac{200 \times \hat{\gamma}_0^2}{\chi^2_{0.975;200}} = \frac{200 \times 0.11}{241.06} = 0.091,$$

and

$$\gamma_U^2 = \frac{200 \times \hat{\gamma}_0^2}{\chi^2_{0.025;200}} = \frac{200 \times 0.11}{162.73} = 0.135.$$

Obviously, when $\delta_L = 0.159 > 0$, we can conclude that $\delta > 0$,

$$LC_{PM} = 1\Big/3\sqrt{\left(\hat{\delta}_0 + t_{0.025;200} \times \sqrt{\frac{\hat{\gamma}_0^2}{220}}\right)^2 + \frac{200\hat{\gamma}_0^2}{\chi^2_{0.025;200}}} = 1\Big/3\sqrt{(0.161)^2 + 0.135} = 0.83,$$

and

$$UC_{PM} = 1\Big/3\sqrt{\left(\hat{\delta}_0 - t_{0.025;200} \times \sqrt{\frac{\hat{\gamma}_0^2}{220}}\right)^2 + \frac{200\hat{\gamma}_0^2}{\chi^2_{0.975;200}}} = 1\Big/3\sqrt{(0.159)^2 + 0.091} = 0.98.$$

Based on the statistical testing rule (2), if $UC_{PM} = 0.98 < 1.00$, then reject H_0 and conclude that $C_{PM} < 1.00$, which means the process needs to improve. According to the previous research, the assessments of process capability indices were performed under the statistical process control condition [32], and the data of statistical process control used to estimate process capability indices resulting in some advantages:

(1) Because of large volume of data collected in long term of statistical process control condition, the estimated confidence interval of indices would be smaller and result in greater accuracy of estimation [33].
(2) The confidence interval of indices derived from large volume of data used as assessment tools suggest the risk of misjudgments by sampling error would decline [36–38].
(3) Assessment of process capability and monitoring the process with a control chart helps with parameter tuning and keeps process capability stable [39].

5. Conclusions

The Taguchi cost loss index can simultaneously reflect the process yield and process cost loss. Therefore, this study proposed a novel process quality evaluation model with the Taguchi cost loss index. The study employed a control chart data to estimate the Taguchi cost loss index. Based on Boole's inequality and DeMorgan's theorem, this study derived the $(1-\alpha)\times 100\%$ confidence region of (δ, γ^2). Subsequently, the study used the Taguchi cost loss index as the objective function and a confidence region of (δ, γ^2) as the feasible solution area, and also employed the mathematical programming method to find the confidence interval of the Taguchi cost loss index. At the same time, a $(1-\alpha)\times 100\%$ confidence interval was adopted to perform statistical testing to evaluate the process quality to see whether it could reach the level required by customers, and thereby decide whether to improve the process.

The projected Taguchi cost loss index reflected the process yield and process cost loss at the same time, and we used a large amount of statistical process control data to reduce the risk of misjudgment caused by sampling errors and enhance accuracy of evaluation. Furthermore, the control chart was implemented for process monitoring, and was helpful in adjusting the machine parameters to maintain process capability stability. The evaluation model proposed in this article can be widely used in various industries.

6. Research Limitations and Future Research

The study assumed that the data were normally distributed and had symmetric tolerances. When e data are abnormally distributed or have asymmetric tolerances, the error may be relatively large when this model is used. Therefore, abnormally distributed data and asymmetric tolerances must be considered in future work.

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