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RESEARCH ARTICLE

Comparing the Performance of Navigation Systems Under Complex Environment

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Abstract

Mardia–Watson–Wheeler is applied for testing whether the circulation observations differ significantly or not. The existing Mardia–Watson–Wheeler test is applied when all circulation observations are determined, precise, and certain. In this paper, Mardia–Watson–Wheeler is introduced under neutrosophic statistics. The proposed test applies when the circular data are not exact, in the interval, imprecise and indeterminate. The test statistic of Mardia–Watson–Wheeler is extended for neutrosophic statistics. The decision criteria for testing the null hypothesis are explained with the help of circular observations obtained from two boat navigation systems. The real example and comparative studies show that the proposed test outperforms the existing Mardia–Watson–Wheeler test under classical statistics in terms of flexibility and information. From the application and comparative studies, it can be concluded that the proposed test can be applied effectively in an uncertain environment.

Keywords Circular observations · Indeterminate · Mardia–Watson–Wheeler test · Navigation systems · Neutrosophic

Abbreviation

MWM MardiaWatsonWheeler

1 Introduction

Usually, statistical tests are developed to analyze linear data. The traditional tests cannot be applied when the observations are recorded in angles or for circulation observations. Mardia–Watson–Wheeler test is widely applied for testing whether the mean or variance of circulation observations is significantly differing or not. This test is applied under the assumption that circulation observations (the data that are measured on a circle in degree) are obtained from continuous circular observations. The aim of the Mardia–Watson–Wheeler test is to test the null hypothesis that circulation observations obtained from two navigation systems (a set of equipment designed to assist in ship and aircraft) are the same or differ significantly. [20] "Circular data need special treatment in data analysis: consider that an angle of 355° is much nearer to an angle of 5° than it is to an angle of 330°, and so simple arithmetic mean for example can be quite misleading". [24, 29–31, 33] and [, 20–22, 26, 37] and [20] proposed the various tests for circular data and applied in the areas like education, medical sciences, space science, ecology, wind, and ocean.

When uncertainty is found in the circulation observations, the Mardia–Watson–Wheeler test under classical statistics cannot be applied. The fuzzy-based tests can be applied in case of indeterminacy in the data. [11, 16, 19, 23, 25, 38] and [25] discussed various statistical tests for circular observations under fuzzy logic. More information on the applications of the fuzzy-based analysis can be seen in [1, 4] and [5].

[34] introduced the neutrosophic (a study between true and false) logic that is found to be more efficient than fuzzy logic. Later on, [35] provided the efficiency of the neutrosophic logic over the results obtained from fuzzy and interval analyses. [2, 12, 13, 17] and [3] presented several applications of neutrosophic logic. Classical statistics cannot be applied in the presence of neutrosophy. To overcome this issue, [36] introduced neutrosophic statistics to deal with imprecise and indeterminate data. Later on, [6–8, 10, 14, 14, 15, 15, 27] and [28] proved the efficiency of neutrosophic statistics over classical statistics in terms of flexibility and information. Recently, [9] proposed the test for circular angles data under neutrosophic statistics. More details on neutrosophic statistics can be read on https://archive.org/details/neutrosophic-statistics?tab=about, https://archive.

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org/details/neutrosophic-statistics?tab=collection, http://fs.unm.edu/NS/NeutrosophicStatistics.htm, and [32].

Mardia–Watson–Wheeler test in the literature can be applied only when all circular observations are determined, precise, and certain. In case of uncertainty in circular observations, the use of the existing Mardia–Watson–Wheeler (MWW) test cannot apply or may mislead the decision-makers. According to the best of our knowledge, there is no work on the MWM test under neutrosophic statistics. In this paper, the modification of the MWM test under neutrosophic statistics is given. The neutrosophic statistic of the MWM test is introduced and applied using the circular observations recorded from two boat navigation systems. It is expected that the proposed MWM test will be more effective than the existing test in terms of information, flexibility, and adequacy.

1.1 Preliminaries

Suppose that $n_N = n_L + n_U I_{nN}; I_{nN} \epsilon \left[I_{nL}, I_{nU} \right]$ be the first neutrosophic random sample having neutrosophic circular observations $\Phi_N = \Phi_L + \Phi_U I_{\Phi N}; I_{\Phi N} \epsilon \left[I_{\Phi L}, I_{\Phi U} \right]$, where $I_{nN} \epsilon \left[I_{nL}, I_{nU} \right]$ and $I_{\Phi N} \epsilon \left[I_{\Phi L}, I_{\Phi U} \right]$ are the measure of uncertainty associated with the first neutrosophic sample size $n_N \epsilon \left[n_L, n_U \right]$ and neutrosophic circular observations $\Phi_N \epsilon \left[\Phi_L, \Phi_U \right]$. Suppose that $m_N = m_L + m_U I_{mN}; I_{mN} \epsilon \left[I_{mL}, I_{mU} \right]$ be the second neutrosophic random sample having neutrosophic circular observations $\psi_N = \psi_L + \psi_U I_{\psi N}; I_{\psi N} \epsilon \left[I_{\psi L}, I_{\psi U} \right]$, where $I_{mN} \epsilon \left[I_{mL}, I_{mU} \right]$ and $I_{\psi N} \epsilon \left[I_{\psi L}, I_{\psi U} \right]$ are the measure of uncertainty associated with the second neutrosophic sample size $m_N \epsilon \left[m_L, m_U \right]$ and neutrosophic circular observations $\psi_N \epsilon \left[\psi_L, \psi_U \right]$. Note that n_L, m_L,Φ_L and ψ_L denote the determinate (exact) parts and $n_U I_{nN}$, $\Phi_U I_{\Phi N}$,$m_U I_{mN}$ and $\psi_U I_{\psi N}$ are indeterminate parts of neutrosophic forms. Let $N_N = n_N + m_L; N_N \epsilon \left[N_L, N_U \right]$ be the total number of circular observations in both samples. The two samples are arranged in order and assigned the ranks on the basis of mid-value when the circular data are in intervals.

1.2 Neutrosophic Mardia–Watson–Wheeler test

The existing MWW test under classical statistics cannot be applied when the decision-makers are uncertain about the sample size or circular observations. In this section, the neutrosophic Mardia–Watson–Wheeler (NMWW) test will be introduced to test either circular observations that came from two independent samples are different in mean angle or variance. Therefore, the main aim of the proposed test is to test the null hypothesis H_0 : two samples have the same neutrosophic mean angle vs. the alternative hypothesis H_1: two samples have different neutrosophic mean angles. The proposed test will be implemented under the assumption that the data follow the neutrosophic continuous distribution and there is no tie between the two neutrosophic samples. Angular data having all certain observations are known as regular angles data. Angular data having imprecise, vague, and fuzzy observations are known as neutrosophic angular (angles) data. Let $r_1, r_2, \ldots, r_{n_N}$ be the rank of the first sample and $\beta_{iN} = r_{iN} \delta_N \left(i = 1, 2, 3, \ldots, n_N \right)$ be neutrosophic angles. The values of $\delta_N \epsilon \left[\delta_L, \delta_U \right]$ can be obtained as

$$\delta_N = \frac{360^0}{n_N + m_N}; n_N \epsilon \left[n_L, n_U \right], m_N \epsilon \left[m_L, m_U \right] \quad (1)$$

The first neutrosophic sample has the following components

$$C_{1N} = \sum Cos\left(\beta_{iN} \right) \quad (2)$$

$$S_{1N} = \sum Sin\left(\beta_{iN} \right) \quad (3)$$

The length of the resultant neutrosophic vector (a sum of two vectors when these are oriented in the same direction) is given by

$$R_{1N} = \left(C_{1N}^2 + S_{1N}^2 \right)^{\frac{1}{2}} \quad (4)$$

The neutrosophic test statistics $B_N \epsilon \left[B_L, B_U \right]$ is defined as:

$$B_N = R_{1N}^2; B_N \epsilon \left[B_L, B_U \right] \quad (5)$$

The neutrosophic form of the proposed test is given as follows:

$$B_N = B_L + B_U I_{BN}; I_{BN} \epsilon \left[I_{BL}, I_{BU} \right]; B_N \epsilon \left[B_L, B_U \right] \quad (6)$$

Note that the neutrosophic form of $B_N \epsilon \left[B_L, B_U \right]$ have two parts. The first part B_L is known as the determinate part and the second part $B_U I_{BN}$ is known as the indeterminate part, where $I_{BN} \epsilon \left[I_{BL}, I_{BU} \right]$ is the measure of indeterminate associated with the proposed test statistic $B_N \epsilon \left[B_L, B_U \right]$. The proposed statistic $B_N \epsilon \left[B_L, B_U \right]$ reduces to statistic under classical statistics if $B_U = 0$. The operational procedure of the proposed test is explained in the following steps.

Step-1: State H_0 : two samples have the same neutrosophic mean angle vs.H_1: two samples have different neutrosophic mean angles.

Step-2: Set the value of the level of significance α and choose the critical value $B_{N,m,\alpha}$ from [18].

Step-3: Do not reject H_0 if $B_N \epsilon \left[B_L, B_U \right] \geq B_{N,m,\alpha}$, otherwise reject it.

The operational process of the proposed test can be seen in Fig. 1.

The algorithm of the operational procedure of the proposed test is given as:

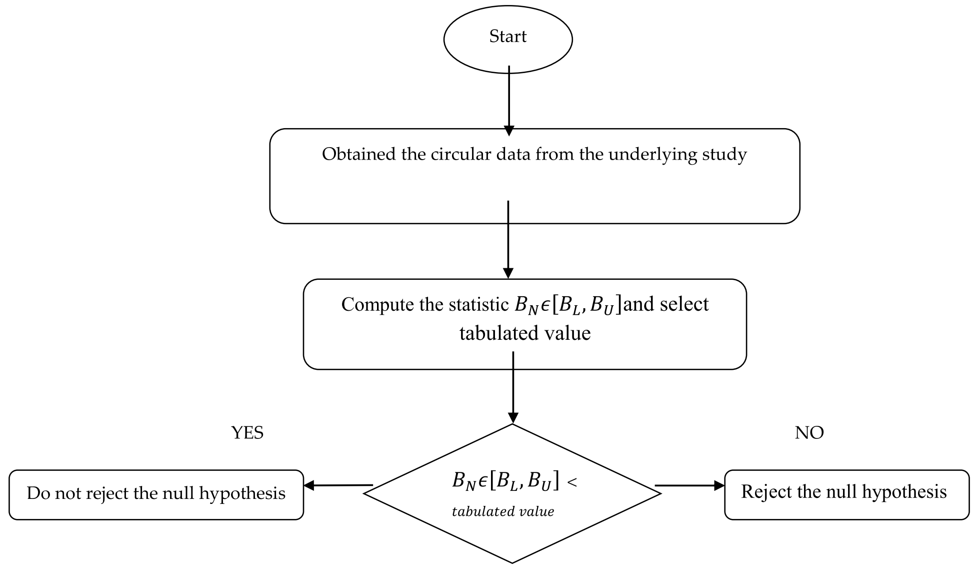

Fig. 1 The operational process of the proposed NMWW test

Step-1: Select the first neutrosophic sample size $n_N \epsilon \left[n_L, n_U\right]$ and the second neutrosophic sample size $m_N \epsilon \left[m_L, m_U\right]$

Step-2: Assign the rank to the first sample and the second sample.

Step-3: Compute $\delta_N \epsilon \left[\delta_L, \delta_U\right]$, $R_{1N} \epsilon \left[R_{1L}, R_{1U}\right]$ and $B_N \epsilon \left[B_L, B_U\right]$

Step-4: Accept H_0 when $B_N \epsilon \left[B_L, B_U\right] \leq B_{N,m,\alpha}$

1.3 Application of the Proposed Test

A sailor wants to compare a new boat navigation system with the old boat navigation system. The sailor is interested to see whether both boat navigation systems work similarly or not on the based-on observations taken from the two systems, see [18]. Suppose that the sailor is uncertain in selecting the suitable sample size for $n_N \epsilon \left[n_L, n_U\right]$ and $m_N \epsilon \left[m_L, m_U\right]$ with measures of indeterminacy of 0.25 and 0.30, respectively. Under uncertainty, the sailor decided to apply the proposed neutrosophic Mardia–Watson–Wheeler test to test whether a new boat navigation system works similarly to an old boat navigation system. For the testing, the neutrosophic forms of the first sample and the second sample are given as $n_N = 6 + 8I_{nN}; I_{nN} \epsilon [0, 0.25]$ and $m_N = 4 + 6I_{mN}; I_{mN} \epsilon [0, 0.3]$. Based on this information, the sailor decided to select $n_N \epsilon [6, 8]$ and $m_N \epsilon [4, 6]$ which leads to $N_N \epsilon [10, 14]$. The ranks of the first sample when N_L=10 are given as:

$r_1 = 2, r_2 = 5, r_3 = 7, r_4 = 8, r_5 = 9$ and r_6=10.

The ranks of the second sample when N_U=14 are given as:

$r_1 = 2$, $r_2 = 4, r_3 = 6$, $r_4 = 8$, $r_5 = 9$, $r_6 = 12, r_7 = 13$ and $r_8 = 14$

The values of δ_N when N_L=10 and N_U=14 are calculated as $\delta_N = \frac{360^0}{[10,14]}$;[36, 25.71]. The values of C_{1N} and S_{1N} when N_L=10 are given as:

$$C_{1N} = \sum Cos\left(\beta_{iN}\right) = 0.309, -1, -0.309, 0.309, 0.809, 1, 1.118 = 1.118.$$

$$S_{1N} = \sum Sin\left(\beta_{iN}\right) = 0.9511, 0, -0.9511, -0.9511, -0.5878, 0, -1.5389 = -1.5389.$$

The values of C_{1N} and S_{1N} when N_L=14 are given as:

$$C_{1N} = \sum Cos\left(\beta_{iN}\right) = 0.6234, -0.2225, -0.9009, -0.9009, -0.6234, 0.6235, 0.9009, 1 = 0.5001.$$

$S_{1N} = \sum Sin\left(\beta_{iN}\right)$=0.7818, 0.9749, 0.4338, −0.4338, −0.7833, −0.7818, −0.4338, 0 = −-0.2422. The values of $R_{1N} \epsilon \left[R_{1L}, R_{1U}\right]$ are calculated as R_{1N} = [1.90214, 0.555663]. The neutrosophic values of statistic $B_N \epsilon \left[B_L, B_U\right]$ are calculated as B_N =[3.61, 0.30]. The neutrosophic form of $B_N \epsilon \left[B_L, B_U\right]$ is given by $B_N = 3.61 - 0.30I_{BN}; I_{BN} \epsilon [0, 11.03]$. The proposed test for two boat navigation systems is explained as:

Step-1: H_0 : no difference between work of new and old boat navigation systems vs. H_1: there is difference between work of new and old boat navigation systems.

Step-2: Let $\alpha = 0.05$ and the critical value $B_{N,m,\alpha}$ = [9.47, 12.21] from [18].

Step-3: Do not reject H_0 as $B_N \epsilon \left[B_L, B_U\right]$ < [9.47, 12.21].

As $B_N \epsilon \left[B_L, B_U\right]$ < [9.47, 12.21], therefore, H_0 will be accepted. From the study, it is concluded that a new boat navigation system works similarly to old boat navigation systems. Based on the analysis, it is concluded that there is a difference between the two boat navigation systems.

1.4 Simulation Study

In this section, the simulation study is given to see the effect of the measure of indeterminacy on $n_N\epsilon\left[n_L, n_U\right]$, $m_N\epsilon\left[m_L, m_U\right]$ and $B_N\epsilon\left[B_L, B_U\right]$. For the simulation study, we consider $n_N\epsilon[6, 8]$, B_N =[3.61, 0.30], $m_N\epsilon[4, 6]$ and values of measure of indeterminacy from 0 to 1.0. To see the effect of indeterminacy, several values of $I_{BU} = I_{nU} = I_{mU}$ are considered. The values of $n_N\epsilon\left[n_L, n_U\right]$, $m_N\epsilon\left[m_L, m_U\right]$ and $B_N\epsilon\left[B_L, B_U\right]$ are generated using the neutrosophic forms for various values of measure of indeterminacy. The results are shown in Table 1.

The determinate part that presents the classical statistics is greater than the indeterminate part in the neutrosophic form of B_N. Therefore, from Table 1, it is clear that the values of B_N decrease as the measure of indeterminacy increases from 0 to 1.0. For example, when I_{BU}=0.1, the value of B_N is 3.58 and the value of B_N is 3.31 when I_{BU} =1. The value of B_N at I_{BU}=0 shows the value of statistic under classical statistics. On the other hand, the values of $n_N\epsilon\left[n_L, n_U\right]$ and $m_N\epsilon\left[m_L, m_U\right]$ increase as the measure of indeterminacy increases. For example, when $I_{nU} = I_{mU}$ =0.1, the values of n_N and m_N are 6 and 4, respectively. When $I_{nU} = I_{mU}$=1, the values of n_N and m_N are 14 and 10, respectively. The values of $I_{nU} = I_{mU}$=0 present the values of n_N and m_N under classical statistics. From this study, it can be seen that the measure of indeterminacy affects the values of statistic B_N. On the other hand, for the larger values of measure of indeterminacy, the larger values of the first sample and the second sample are needed. From the study, it can be concluded that under uncertainty, a large number of samples are needed. From the study, it can be concluded that the decision-makers should be very careful in the selection of sample size in the presence of indeterminacy. Larger sample size is needed when the indeterminacy measure is high in the data.

Table 1 Effect of Indeterminacy on sample and statistic

$I_{BU} = I_{nU} = I_{mU}$	$B_N = 3.61 - 0.30I_{BN}$	$n_N = 6 + 8I_{nN}$	$m_N = 4 + 6I_{mN}$
0	3.61	6	4
0.1	3.58	6.8	4.6
0.2	3.55	7.6	5.2
0.3	3.52	8.4	5.8
0.4	3.49	9.2	6.4
0.5	3.46	10	7
0.6	3.43	10.8	7.6
0.7	3.4	11.6	8.2
0.8	3.37	12.4	8.8
0.9	3.34	13.2	9.4
1.0	3.31	14	10

1.5 Comparative Study

The proposed statistic $B_N\epsilon\left[B_L, B_U\right]$ is the extension of statistic under classical statistics. As mentioned earlier, the neutrosophic form $B_N = B_L + B_U I_{BN}; I_{BN}\epsilon\left[I_{BL}, I_{BU}\right]$ consists of two parts. The proposed statistic reduces to classical statistic when I_{BL}=0. The efficiency of the proposed test is given in terms of a measure of indeterminacy. For the real data, the neutrosophic form of $B_N\epsilon\left[B_L, B_U\right]$ is $B_N = 3.61 - 0.30I_{BN}; I_{BN}\epsilon[0, 11.03]$. In this neutrosophic form, the first value 3.61 presents the value of statistic under classical statistics. The second part $0.30I_{BN}$ presents the indeterminate part and $I_{BN}\epsilon[0, 11.03]$ is the measure of indeterminacy. Under uncertainty, the value of B_N can be reduced from 3.61 to 0.30. From the study, it can be seen that the proposed test gives the value of $B_N\epsilon\left[B_L, B_U\right]$ in indeterminate intervals rather than the exact value as in the classical statistics case. On the other hand, the proposed test gives more information about the testing of H_0. For example, when $\alpha = 0.05$, the confidence level will be 0.95 and the measure of indeterminacy is 11. This is the case of paraconsistent probability where the sum of probability can be larger than 1, see [36]. From the study, it can be seen that the proposed test illustrates that the probability of accepting H_0, the probability of committing a type-I error is 0.05, and the measure of indeterminacy is 11. Therefore, the proposed test gives more information about H_0 and flexibility in uncertain environments as compared to test under classical statistics. From the study, it can be concluded that the proposed test gives the value of statistic in a range rather than the exact value of the statistic as in classical statistics. In addition, the proposed test gives more information about the testing of the hypothesis of two boat navigation systems. Based on this study, it can be concluded that in the presence of uncertainty, the application of the proposed test will be more useful and informative than the existing MWM test.

1.6 Characteristics and Limitations of the Proposed Test

In this section, some characteristics and limitations of the proposed test will be discussed. The proposed NMWW test has the following characteristics:

1. The proposed test was found to be effective and informative when it is applied in uncertain environment.

2. The proposed NMWW test evaluates the effect of a measure of indeterminacy/uncertainty on the test statistics and sample size.
3. The proposed NMWW test is useful to test the independence of two samples from the circular data.

1.7 Some Limitations of the Proposed Test are Given as:

1. The proposed test can be applied only when uncertainty is presented in the data.
2. This test cannot be applied if there are ties between the samples and population [18].

1.8 Concluding Remarks

The existing MWW test was applied for testing whether the circulation observations differ significantly or not. The existing MWW test was applied when all circulation observations are determined, precise, and certain. In this paper, MWW was introduced under neutrosophic statistics. The proposed test is applied when the circular observations are not exact, in the interval, imprecise, and indeterminate. The proposed test is an extension of the existing MWW test. From the simulation study, it can be seen that indeterminacy affects the selection of sample size. From the application and comparative studies, it can be concluded that the proposed test can be applied effectively in an uncertain environment. The simulation and comparative studies showed that the proposed test is informative, flexible and adequate to be applied under uncertainty. The proposed test for big data can be studied for future research. The proposed test using other sampling schemes can be considered for future research. The various properties of the proposed test can be studied in future research. The development of software to perform the proposed test is another fruitful area for future research.

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Conflict of interest No
conflict of interest regarding this paper.

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Consent for Publication Not applicable.

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