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Research Article

Some discrete neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables

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Abstract

The study of random variables and their distributions have been a great area of interest for many researchers. Recently the study of neutrosophic random variables have been introduced, that is why, in this paper, we apply discrete random distribution such as the uniform discrete distribution, Bernoulli distribution, binomial distribution, geometric distribution, negative binomial distribution, hypergeometric distribution and Poisson distribution by using neutrosophic random variables. This study opens a new way for dealing with issues that follow the classical distributions which appear in classical random variables and at the same time contain data not specified accurately.

Mathematics Subject Classification (2020). 03B52

Keywords. neutrosophic logic, neutrosophic random variable, neutrosophic discrete distributions, neutrosophic probability

1. Introduction

The notion of neutrosophic probability measure as a function $\mathcal{NP} : Y \to [0,1]^3$ was introduced by F. Smarandache where U is a neutrosophic sample space, and defined the probability mapping to take the form $\mathcal{NP}(S) = (ch(S), ch(neutS), ch(antiS)) = (\alpha, \beta, \gamma)$ where $0 \leq \alpha, \beta, \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$ [30]. Besides, many researchers have investigated many neutrosophic probability distributions like Poisson, exponential, binomial, normal, uniform, Weibull, etc. (see [2, 4, 22]). Additionally, researchers have investigated the notion of neutrosophic queueing theory in [33, 34], this is one branch of neutrosophic stochastic modelling. Furthermore, researchers have also studied neutrosophic time series prediction and modelling in many cases like neutrosophic moving averages, neutrosophic logarithmic models, neutrosophic linear models and so on [3, 8].

Recently, researchers have started to study the notion of neutrosophic random variable (see Definition 2.5). Bisher and Hatip [6] presented the first notion of neutrosophic random variables in which they presented some basics notions. later on, Granados [14] showed new notions on neutrosophic random variables and then Granados and Sanabria [17] studied independence neutrosophic random variables.

On the other hand, neutrosophic logic is an extension of intuitionistic fuzzy logic by adding indeterminacy component (I) where $I^2 = I, ..., I^n = I, 0.I = 0; n \in \mathbb{N}$ and I^{-1} is

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undefined (see [30]). Neutrosophic logic has a huge brand of applications in many fields including decision making [18,21,26], machine learning [24,31], intelligent disease diagnosis [11,28], communication services [7], pattern recognition [23], social network analysis and e-learning systems [25], physics [32], sequences spaces [15] and so on. Neutrosophic logic has solved many decision-making problems efficiently like evaluating green credit rating, personnel selection, etc. [1,19,20]. For more notions related to neutrosophic theory, we refer the reader to [9,10,12,13,15,16].

In this paper, we highlight the use of neutrosophic random variable theory [6] with the classical probability distributions, particularly uniform discrete distribution, Bernoulli distribution, binomial distribution, geometric distribution, negative binomial distribution, hypergeometric distribution and Poisson distribution, which opens the way for dealing with issues that follow the classical distributions and at the same time contain data not specified accurately. In this paper, we discuss discrete random distributions such as the uniform discrete distribution, Bernoulli distribution, binomial distribution, geometric distribution, negative binomial distribution, hypergeometric distribution and Poisson distribution by using neutrosophic random variables. Further, we show some examples and properties of each distribution defined in this paper.

2. Preliminaries

In this section, we procure some well-known notions which will be useful for the development of this paper. Throughout this paper, the set of real number is denoted by $\mathbb{R}$, Ω denotes the set of sample space and ω denotes an event of the sample space, X_N and Y_N denote neutrosophic random variables.

Next, we show some well-known definitions and properties of neutrosophic logic and neutrosophic probability which are useful for the development of this paper.

Definition 2.1. (see [29]) Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $\{x, (\mu A(x), \delta A(x), \gamma A(x)) : x \in X\}$, where $\mu A(x), \delta A(x)$ and $\gamma A(x)$ represent the degree of membership, the degree of indeterminacy, and the degree of non-membership respectively of each element $x \in X$to the set A.

Definition 2.2. (see [5]) Let K be a field, the neutrosophic filed generated by K and I is denoted by $\langle K \cup I \rangle$ under the operations of K, where I is the neutrosophic element with the property $I^2 = I$.

Definition 2.3. (see [30]) Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such that $0.I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers n.

Definition 2.4. (see [30]) The neutrosophic probability of event A occurrence is
$NP(A) = (ch(A), ch(neutA), ch(antiA)) = (T, I, F)$ where T, I, F are standard or non-standard subsets of the non-standard unitary interval $]^{-}0, 1^{+}[$.

Recently, Bisher and Hatip [6] introduced and studied the notions of neutrosophic random variables by using the concepts presented by [30], these notions were defined as follows:

Definition 2.5. Consider the real valued crisp random variable X which is defined as follows:

$$X : \Omega \to \mathbb{R},$$

where Ω is the events space. Now, they defined a neutrosophic random variable X_N as follows:

$$X_N : \Omega \to \mathbb{R}(I)$$

and

$$X_N = X + I,$$

where I is indeterminacy.

Theorem 2.6. *Consider the neutrosophic random variable $X_N = X + I$ where cumulative distribution function of X is $F_X(x) = P(X \leq x)$. Then, the following statements hold:*

(1) $F_{X_N}(x) = F_X(x - I)$,
(2) $f_{X_N}(x) = f_X(x - I)$,

where F_{X_N} and f_{X_N} are cumulative distribution function and probability density function of X_N, respectively.

Theorem 2.7. *Consider the neutrosophic random variable $X_N = X + I$, expected value can be found as follows:*

$$E(X_N) = E(X) + I.$$

Proposition 2.8 (Properties of expected value of a neutrosophic random variable)**.** *Let X_N and Y_N be neutrosophic random variables, then the following properties holds:*

(1) $E(aX_N + b + cI) = aE(X_N) + b + cI; a, b, c \in \mathbb{R}$.
(2) *If X_N and Y_N are neutrosophic random variables, then $E(X_N \pm E_(Y_N) = E(X_N) \pm E(Y_N)$.*
(3) $E[(a + bI)X_N] = aE(X_N) + bIE(X_N); a, b \in \mathbb{R}$.
(4) $|E(X_N)| \leq E|X_N|$.

Theorem 2.9. *Consider the neutrosophic random variable $X_N = X + I$, variance of X_N is equal to variance of X, i.e. $V(X_N) = V(X)$.*

3. Main results

In this section, we define new neutrosophic discrete distribution by using the notion of neutrosophic random variable. Also, we present some examples and properties of each neutrosophic discrete distribution.

3.1. Neutrosophic uniform discrete distribution(NUDD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic uniform discrete distribution denoted by $X_N \sim unif\{x_1, ..., x_n\}$ or $X \sim unif\{x_1 + I, ..., x_n + I\}$ if the probability that X_N takes any value in constant i.e., it is $\frac{1}{n+I}$. Nuetorsophic probability function is given by

$$f_X(x - I) = \begin{cases} \dfrac{1}{n + I}, & if\ x = x_1 + I, x_2 + I, ..., x_n + I. \\ 0, & otherwise. \end{cases}$$

Proof.

$$\sum_{x=I}^{n+I} \frac{1}{n + I} = \frac{1}{n + I}(n + I) = 1.$$

□

It can be easily shown that expected and variance of neutrosophic uniform discrete distribution are given by

$$E(X_N) = \mu,$$

$$Var(X_N) = \frac{1}{m} \sum_{i=1}^{m} (x_i - \mu),$$

where $\mu = \frac{1}{m} \sum_{i=1}^{m} x_i$ and $m = n + I$.

Example 3.1. in the manufacture of a certain product occurs with failures, assuming that the number of failures follows a neutrosophic uniform discrete distribution,

$$f_X(x-I) = \begin{cases} \dfrac{1}{3+I}, & if\ x = 2+I, 4+I, ..., 5+I. \\ 0, & otherwise. \end{cases}$$

Determine the probability that more than three faults are found in a certain product in this has a grade of indeterminacy between [0,0.5].

Solution:

$$\begin{aligned} P(X_N > 3) &= f_{X_N}(4) + f_{X_N}(5) + f_{X_N}(6) + ... \\ &= \frac{1}{3+[0,0.5]} + \frac{1}{3+[0,0.5]} + 0 + 0 + 0 + ... \\ &= \frac{2}{3+[0,0.5]} \\ &= \frac{2}{[3,3.5]} \\ &= [0.57, 0.66]. \end{aligned}$$

Therefore, the probability that more than three faults are found in a certain product is [0.57,0.66]. If we make this exercise in classical way, we obtain that $P(X > 3) = 0.66 \in [0.57, 0.66] = P(X_N > 3)$.

By using computational software *R*, we can randomly obtain the neutrosophic distribution uniform discrete by using the following command:

```
# 15 randomly values of the distribution unif{2,3,4,5,...,9,10,11} where I=1 .
> sample(0:10,15)
[1] 1 7 3 4 1 1 3 2 1 6 5 3 8 2 9 1
```

3.2. Neutrosophic Bernoulli distribution (NBD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic Bernoulli distribution denoted by $X_N \sim Ber(p_N)$ where p_N is set with one or more elements (may p_N be an interval). Nuetorsophic probability function is given by

$$f_X(x-I) = \begin{cases} p_N^{x-I}(1-p_N)^{1-x+I}, & if\ x = I, 1+I. \\ 0, & otherwise. \end{cases}$$

Proof.

$$\begin{aligned} \sum_{x=I}^{1+I} p_N^{x-I}(1-p_N)^{1-x+I} &= [p_N^0(1-p_N)] + [p_N(1-p_N)^0] \\ &= 1 - p_N + p_N \\ &= 1. \end{aligned} \tag{3.1}$$

□

By Theorems (2.7) and (2.9), we can see that

$$E(X_N) = p_N + I,$$
$$Var(X_N) = p_N(1-p_N).$$

Example 3.2. A machine is known to produce [2%,4%] defective parts. We choose a piece at random to check if it has no defects with an indeterminacy of [0,0.2]. How is the neutrosophic random variableX_N distributed, which is $1 + I$ if the part is not defective and I if is it defective? What is its expected?

Solution:

X_N has a neutrosophic Bernoulli distribution wit parameters $p_N = [0.96, 0.98]$ and its expected is gived by $E(X_N) = [0.96, 0.98] + [0.0.2] = [0.96, 1]$.

If we make this exercise in classical way with parameter $p = 0.97 \in p_N$, then $E(X) \in E(X_N)$ and produce defective parts will be followed by $3\% \in [2\%, 4\%]$.

By using computational software R, we can randomly obtain the neutrosophic Bernoulli distribution by using the following command:

```
# rbinom(j,1,p) takes j randomly values for the neutrosophic distribution Ber(p)
> rbinom(11,1,0.3)
[1] 1+I I 1+I I I I I 1+I I 1+I 1+I
```

Theorem 3.3. *Let X_N be a neutrosophic random variable with neutrosophic Bernoulli distribution and let $a, b \in \mathbb{R}$ with $a \neq 0$. Taking $Y_N = aX_N + b$, then*

(1) *The neutrosophic density function of Y_N is given by*

$$f_Y(y - I) = \begin{cases} p_N^{(y-b-I)/a}(1 - p_N)^{1-[(y-b-I)/a]}, & if\ y = b + I, a + b + I. \\ 0, & otherwise. \end{cases}$$

(2) $E(Y_N) = ap + b + I$.

Proof.

(1) It follows from Equation (3.1).

(2) Since $E(Y) = ap + b + I$, then by Theorem 2.7, $E(Y_N) = ap + b + I$.

□

Theorem 3.4. *Let $X_{N_1}, X_{N_2}, ..., X_{N_n}$ be neutrosophic random variables independence and identically distributed with neutrosophic Bernoulli distribution $X_{N_n} \sim Ber(p_N)$. Then, the neutrosophic Bernoulli distribution of $X_{N_1}X_{N_2}...X_{N_n}$ is given by $X_{N_1}X_{N_2}...X_{N_n} \sim Ber(p_N^n)$*

Proof. The neutrosophic random variable $X_{N_1}X_{N_2}...X_{N_n}$ takes values in 1 and 0. Thus, its probabilities are $P(X_{N_1}X_{N_2}...X_{N_n}) = 1 = P(X_{N_1} = 1)...P(X_{N_n}) = 1 = P(X_1 = 1 - I)...P(X_n) = 1 - I = p_N^n$, and for its complement, $P(X_{N_1}X_{N_2}...X_{N_n}) = 0 = P(X_{N_1} = 0)...P(X_{N_n}) = 0 = 1 - p_N^n$. Therefore, $X_{N_1}X_{N_2}...X_{N_n} \sim Ber(p_N^n)$. □

3.3. Neutrosophic binomial distribution (NbD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic binomial distribution denoted by $X_N \sim bin(n, p_N)$ where p_N is set with one or more elements (may p_N be an interval). Nuetorsophic probability function is given by

$$f_X(x - I) = \begin{cases} \binom{n}{x-I} p_N^{x-I}(1 - p_N)^{n-x+I}, & if\ x = I, 1 + I, 2 + I, ..., n + I. \\ 0, & otherwise. \end{cases}$$

Proof.

$$\sum_{x=I}^{\infty} \binom{n}{x - I} p_N^{x-I}(1 - p_N)^{n-x+I} = (p_N + (1 - p_N))^{n+I} = 1.$$

□

By Theorems (2.7) and (2.9), we can see that

$$E(X_N) = np_N + I,$$
$$Var(X_N) = np_N(1 - p_N).$$

Example 3.5. If X denotes the number of questions answered correctly, then X has distribution $bin(n, p_N)$ with $n = 10$ and $p_N = 1/3$. Assuming that the minimum passing grade is 6, then what is the probability of passing the exam if there is an indeterminacy of 0.3?

Solution:

$$P(X_N \geq 6) = P(X \geq 5.7) = \sum_{x=5.7}^{10} \binom{10}{x} (1/3)^x (2/3)^{10-x}$$
$$= 0.06789.$$

This probability is surprisingly small and hence the strategy followed by the student to answer the exam does not seem to be the best.

Example 3.6. A seed producer knows from experience that [10%,20%] of a large batch of seeds does not germinate. The producer sells his seeds in packages of 20 seeds, guaranteeing that at least 18 of them will germinate. He calculates the percentage of packages that will not meet the guarantee if there is a indeterminacy of 40%.

Solution:

Let X_N be the number of seeds that will end up germinating in a packages of 20 seeds. Then, X_N has a neutrosophic binomial distribution $bin(n, p_N)$ with $n = 20$ and $p_N = [0.8, 0.9]$. Then,

$$P(X_N \leq 17) = P(X \leq 16.6) = 1 - P(X \geq 17.6)$$
$$= 1 - \sum_{x=17.6}^{20} \binom{20}{x} ([0.8, 0.9])^x ([0.1, 0.2])^{20-x} = [0.15, 0.37].$$

Therefore, [15%,37%] of packages will not meet the guarantee. If we make this exercise in a classical way, taking $p_N = 0.9$, we obtain $P(X \leq 17) = 32\% \in P(X_N \leq 17) = [0.15, 0.37]$.

By using computational software R, we can randomly obtain the neutrosophic binomial distribution by using the following command:

```
# rbinom(j,n,p) takes j randomly values for the neutrosophic binomial distribution
# bin(n,p)
> rbinom(11,7,0.7)
[1] 2 1 8 5 8 9 3 1 1 4 3
```

Theorem 3.7. *Let X_N and Y_N be two independence neutrosophic random variables with neutrosophic distribution $bin(n, p_N)$ and $bin(m, p_N)$, respectively. Then, $X_N + Y_N \sim bin(n + m, p_N)$.*

Proof. Let u be a value from the set $\{0, 1, ..., m+n\}$. Then, $0 \leq x \leq n+I$, $0 \leq y \leq m+I$ and $x+y = u+2I$. By hypothesis of independence (for more notion related to independence of neutrosophic random variables, we refer the reader to [17]),

$$\begin{aligned}
P(X+Y=u+2I) &= \sum_{x-I,y-I} P(X=x-I)P(Y=y-I) \\
&= \sum_{x-I,y-I} \binom{n}{x-I} p_N^{x-I}(1-p)^{n-x+I} \binom{m}{y-I} p_N^{y-I}(1-p)^{m-y+I} \\
&= p^{u-2I}(1-p)^{n+m-u+2I} \sum_{x-I,y-I} \binom{n}{x-I}\binom{m}{y-I} \\
&= \binom{n+m}{u-2I} p^{u-2I}(1-p)^{n+m-u+2I}.
\end{aligned}$$

□

Theorem 3.8. *Let X_N be a neutrosophic random variable with $X_N \sim bin(n, p_N)$, then $n - X_N \sim bin(n, 1-p_N)$.*

Proof. For any $x = I, 1+I, ..., n+I$,

$$\begin{aligned}
P(n-X=x-I) = P(X=n-x+I) &= \binom{n}{n-x+I} p_N^{n-x+I}(1-p_N)^{x-I} \\
&= \binom{n}{x-I}(1-p_N)^{x-I} p^{n-x+I}.
\end{aligned}$$

□

Theorem 3.9. *Let's consider that we have any random experiment and that B is an event with strictly positive probability. Suppose that n independent trials of the random experiment are performed and that X_{N_n} denotes the number of times event B is observed to occur in these n trials with an indeterminacy $I \in [0,1]$. Then, for any fixed value $j \geq 1$,*

$$\lim_{n\to\infty} P(X_n > j - I_n) = 1.$$

Proof.

$$\begin{aligned}
\lim_{n\to\infty} P(X_n > j - I_n) &= 1 - \lim_{n\to\infty} P(X_n \leq j - I_n) \\
&= 1 - \lim_{n\to\infty} \sum_{x=I}^{j+I} \binom{n}{x-I} p_N^{x-I}(1-p_N)^{n-x+I} \\
&= 1 - \sum_{x=I}^{j+I} \frac{1}{(x-I)!} p_N^{x-I}(1-p_N)^{-(x-I)} \lim_{n\to\infty} \frac{n!}{(n-x+I)!}(1-p_N)^n.
\end{aligned}$$

Now,

$$\begin{aligned}
\lim_{n\to\infty} \frac{n!}{(n-x+I)!}(1-p_N)^n &= (n-x+I+1)...n(1-p_N)^n \\
&= n^{x-I}(1-p_N)^n \\
&= e^{(x-I)ln(n)+nln(1-p_N)} \\
&= e^{x-I} n e^{nln(1-p)} \\
&\to 0,
\end{aligned}$$

since $ln(1-p) < 0$. Therefore, $\lim_{n\to\infty} P(X_n > j - I_n) = 1$. □

3.4. Neutrosophic geometric distribution (NGD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic geometric distribution denoted by $X_N \sim geo(p_N)$ where p_N is set with one or more elements (may p_N be an interval). Nuetorsophic probability function is given by

$$f_X(x-I) = \begin{cases} p_N(1-p_N)^{x-I}, & if\ x = I, 1+I, 2+I, ..., n+I. \\ 0, & otherwise. \end{cases}$$

Proof.

$$\sum_{x=I}^{\infty} p_N(1-p_N)^{x-I} = p_N \sum_{x=I}^{\infty}(1-p_N)^{x-I}$$
$$= 1.$$

□

By Theorems (2.7) and (2.9), we can see that

$$E(X_N) = \frac{1-p_N}{p_N} + I,$$
$$Var(X_N) = \frac{1-p_N}{p_N^2}.$$

By using computational software *R*, we can obtain the neutrosophic geometric probability by using the following command:

```
# dgeom(x−I,p) evaluates f(x−I) in the neutrosophic distribution geo(p) where I=0.3
    and x=5.3
> dgeom(5,0.4)
[1] 0.03
```

Example 3.10. One person participates each week with a ticket in a lottery game, where the probability of winning the first prize is $p_N = [10^{-8}, 10^{-6}]$ with an indeterminacy of $[0.2, 0.4]$. How many years on average does this person play the game until you get the first prize?

Solution:

$$E(X_N+1) = \frac{1-p_N}{p_N} + 1 + I$$
$$= \frac{1}{p_N} + I$$
$$= \frac{1}{[10^{-8}, 10^{-6}]} + [0.2, 0.4]$$
$$= [10^6, 10^8] + [0.2, 0.4]$$
$$= [1000000.2, 100000000.4].$$

This is the average number of weeks a person must play to get the top prize and is approximately equivalent to [19164.96,1916495.61] years.

Theorem 3.11. *Let $X_{N_0}, X_{N_1}, ...$ be a sequence of independence neutrosophic random variables with neutrosophic distribution $Ber(p_N)$. If $X_N = \min\{n \geq 0 : X_n = 1 - I\}$. Then, X_N has neutrosophic geometric distribution.*

Proof. For each $n \geq 0$, the event $(X = n - I)$ is equal to the event $(X_0 = -I, X_1 = -I, ..., X_{n-1} = -I, X_n = 1 - I)$. Therefore,

$$\begin{aligned} P(X = n - I) &= P(X_0 = -I, X_1 = -I, ..., X_{n-1} = -I, X_n = 1 - I) \\ &= P(X_0 = -I)P(X_1 = -I)...P(X_{n-1} = -I)P(X_n = 1 - I) \\ &= (1 - p_N)^{n-1-I} p_N. \end{aligned}$$

□

Theorem 3.12. *Let X_N and Y_N be two independence neutrosophic random variable which have geo(p_N) distribution. Then,*

$$P(X_N + Y_N = j - 2I) = \binom{j + 1 - 2I}{j - 2I} (1 - p_N)^{j-2I} p_N^2,$$

for $j = I, 1 + I, 2 + I, ...$

Proof.

$$\begin{aligned} P(X_N + Y_N = j - 2I) &= \sum_{x=I}^{j+I} P(X = x - I, Y = j - x - I) \\ &= \sum_{x=I}^{j+I} P(X = x - I)P(Y = j - x - I) \\ &= \sum_{x=I}^{j+I} (1 - p_N)^{x-I} p_N (1 - p_N)^{j-x-I} p_N \\ &= (j + 1 - 2I)(1 - p_N)^{j-2I} p_N^2. \end{aligned}$$

□

Theorem 3.13. *Let $Y_N = 1 + X_N$ be a neutrosophic random variable where X_N has neutrosophic geometric distribution. Then,*

(1) *The neutrosophic density function of Y_N is given by*

$$f_Y(y - I) = \begin{cases} p_N(1 - p_N)^{y-I-1}, & if\ y = 1 + I, 2 + I, ..., n + I. \\ 0, & otherwise. \end{cases}$$

(2) $E(Y_N) = \dfrac{1}{p_N} + I.$

(3) $Var(Y_N) = \dfrac{1 - p_N}{p_N^2}.$

Proof.

(1) The neutrosophic random variable $Y_N = 1 + X_N$ takes values in $1 + I, 2 + I, ...$ with probabilities
$f_Y(y - I) = P(Y_N = y) = P(1 + X_N = y) = P(X_N = y - 1) = (1 - p_N)^{y-1-I} p_N.$

(2) $E(Y_N) = E(1 + X_N) = 1 + E(X_N) = 1 + (1 - p_N)/p_N + I = 1/p_N + I.$

(3) $Var(Y_N) = Var(1 + X_N) = Var(X_N) = (1 - p_N)/p_N^2.$

□

3.5. Neutrosophic negative binomial distribution(NNBD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic negative binomial distribution denoted by $X_N \sim binneg(r_N, p_N)$ where r_N and p_N are sets with one or more elements (may r_N and p_N be an intervals). Nuetorsophic probability function is given by

$$f_X(x-I) = \begin{cases} \binom{r_N+x-I-1}{x-I} p_N^{r_N}(1-p_N)^{x-I}, & if\ x = I, 1+I, 2+I, ..., n+I. \\ \\ 0, & otherwise. \end{cases}$$

Proof.

$$\begin{aligned}\sum_{x=I}^{\infty}\binom{r_N+x-I-1}{x-I}p_N^{r_N}(1-p_N)^{x-I} &= p_N^{r_N}\sum_{x=I}^{\infty}(-1)^{x-I}\binom{-r_N}{x-I}(1-p_N)^{x-I} \\ &= p_N^{r_N}\sum_{x=I}^{\infty}\binom{-r_N}{x-I}(p_N-1)^{x-I} \\ &= p_N^{r_N}(1+p_N-1)^{-r_N} \\ &= 1.\end{aligned}$$

□

By Theorems (2.7) and (2.9), we can see that

$$E(X_N) = r_N\frac{1-p_N}{p_N} + I,$$
$$Var(X_N) = r_N\frac{1-p_N}{p_N^2}.$$

By using computational software R, we can obtain the neutrosophic negative binomial distribution by using the following command:

```
# dnbinom(x−I,r,p) evaluates f(x−I) in the neutrosophic distribution binneg(r,p)
    where I=0.4 and x=3.4
> dnbinom(3,5,0.5)
[1] 0.13
```

Example 3.14. A balanced coin is tossed repeatedly whose two results are heads and tails. What is the probability of getting the third tail on the fifth toss if there is an indeterminacy of [0.1,0.5]?

Solution:

$$P(X = 2-I) = P(X = [1.5, 1.9]) = \binom{[5.5, 5.9]}{[1.5, 1.9]}(1/2)^5 = [0.16, 0.20].$$

Therefore, the probability of getting the third tail on the fifth toss is [0.16,0.20]. If we make this exercise in classical way, we obtain $P(X = 2) = 0.18 \in P(X_N = 2) = [0.16, 0.20]$.

Theorem 3.15. *Let $X_{n_1}, X_{N_2}, ...$ be a sequence of independence neutrosophic random variables with distribution $Ber(p_N)$ and let $r_N \geq 1$ such that*

$$X_N = \min\{n \geq r_N : \sum_{k=1+I}^{n+I} X_{N_k} = r_N\} - r_N.$$

Then, X_N has neutrosophic distribution binneq(r_N, p_N).

Proof. X_N can take values in $I, 1+I, ...$ For any of these values,

$$\begin{aligned}P(X_N = x) &= P(\min\{n \geq r_N : \sum_{k=1+I}^{n+I} X_{N_k} = r_N\} = r_N + x - I \\ &= P(X_{N_{r_N+x-I}} = 1)\ and\ in\ (X_{N_1}, X_{N_2}, ..., X_{N_{r_N+x-I-1}})\ there\ is\ r_N - 1 \\ &= \binom{r_N+x-I-1}{r_N-1}(1-p_N)^{x-I}p_N^{r_N}.\end{aligned}$$

□

Theorem 3.16. *Let $Y_N = r_N + X_N$ be a neutrosophic random variable such that X_N has distribution $binneq(r_N, p_N)$. Then,*

(1) *The neutrosophic density function of Y_N is given by*

$$f_Y(y-I) = \begin{cases} \binom{y-1-I}{y-I-r_N} p_N^{r_N}(1-p_N)^{y-I-r_N}, & if\ y = r_N + I, r_N + 1 + I, ... \\ 0, & otherwise. \end{cases}$$

(2) $E(Y_N) = \frac{r_N}{p_N} + I$.

(3) $Var(Y_N) = r_N \frac{1-p_N}{p_N^2}$.

Proof.

(1) The neutrosophic random variable Y_N takes values in $y = r_N + I, r_N + 1 + I, ...$ with probabilities

$$\begin{aligned} f_Y(y-I) = P(Y = y - I) &= P(r_N + X_N = y) \\ &= P(X = y - I - r_N) \\ &= \binom{y-I-1}{y-I-r_N}(1-p_N)^{y-I-r_N} p_N^{r_N}. \end{aligned}$$

(2) $E(Y_N) = E(r_N + X_N) = r_N + E(X_N) = r_N + r_N(1-p_N)/p_N + I = r_N/p_N + I$.

(3) $Var(Y_N) = Var(r_N + Y_N) = Var(X) = r_N \frac{1-p_N}{p_N^2}$.

□

3.6. Neutrosophic hypergeometric distribution (NHGD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic hypergeometric distribution denoted by $X_N \sim hypergeo(N_N, K_N, n)$ where N_N and KN are sets with one or more elements (may N_N and K_N be an intervals). Nuetorsophic probability function is given by

$$f_X(x-I) = \begin{cases} \frac{\binom{K_N}{x-I}\binom{N_N-K_N}{n-x+I}}{\binom{N_N}{n}}, & if\ x = I, 1+I, 2+I, ..., n+I. \\ 0, & otherwise. \end{cases}$$

Proof. It follows form the fact $(a+b)^{N_N} = (a+b)^{K_N}(a+b)^{N_N-K_N}$. □

By Theorems (2.7) and (2.9), we can see that

$$\begin{aligned} E(X_N) &= n\frac{K_N}{N_N} + I, \\ Var(X_N) &= n\frac{K_N}{N_N}\frac{N_N-K_N}{N_N}\frac{N_N-n}{N_N-1}. \end{aligned}$$

By using computational software R, we can obtain the neutrosophic hypergeometric distribution by using the following command:

```
# hyper(x-I,K,K-N,n) evaluates f(x-I) in the neutrosophic distribution where I=0.2
    and x=3.2
> dnbinom(3,7,13,5)
[1] 0.17
```

Example 3.17. In a fish tank there are 10 fish of which 6 are male and 4 are female. 5 pieces are drawn at random and without replacement. What is the probability that there are 3 males and 2 females if there is an indeterminacy of [10%,30%]?

Solution:

$$P(X_N = 3) = P(X = [2.7, 2.9]) = \frac{\binom{6}{[2.7,2.9]}\binom{4}{[2.1,2.3]}}{\binom{10}{5}} = [0.40, 0.49].$$

If we make this exercise in classical way, we obtain $P(X = 3) = 0.47 \in P(X_N = 3) = [0.40, 0.49]$.

Remark 3.18. As $N_N, K_N \to \infty$ with $K_N/N_N = \gamma \in \mathbb{R}$, then

$$P(X_N = x) \to \binom{n}{x-I}\gamma^{x-I}(1-\gamma)^{n-x+I},$$

so the neutrosophic distribution tends to a neutrosophic Binomial distribution.

The following theorem shows a general description of Remark 3.18.

Theorem 3.19. *Let X_N be a neutrosophic random variable with neutrosophic hypergeoemtric distribution. Then, neutrosophic density probability function of X_N converges to the neutrosophic density function $bin(n, p_N)$ when $N_N \to \infty$ when $K_N/N_N \to p_N \in \mathbb{R}$.*

Proof. We begin to discompose the neutrosophic density probability function as can been shown as follows. In particular, $N_N! = (N_N - x + I)!N_N(N_N - 1)...(N_N - x + I + 1)$. Then,

$$\begin{aligned} f_X(x-I) &= \frac{\binom{N_N}{x-I}\binom{N_N-K_N}{n-x+I}}{\binom{N_N}{n}} \\ &= \frac{K_N!(N_N-K_N)!n!(N_N-n)!}{(x-I)!(K_N-x+I)!(n-x+I)!(N_N-K_N-n+x-I)!N_N!} \\ &= \binom{n}{x-I}\frac{K_N!(N_N-K_N)!}{N_N!}\frac{(N_N-n)!}{(K_N-x+I)!(N_N-K_N-n+x-I)!} \\ &= \binom{n}{x-I}\frac{K_N(K_N-1)...(K_N-x+I+1)}{N_N(N_N-1)...(N_N-x+I+1)} \\ &\frac{(N_N-K_N)(N_N-K_N-1)...(N_N-K_N-n+x-I+1)}{(N_N-x+I)(N_N-x+I-1)...(N_N-n+1)}. \end{aligned}$$

Thus,

$$\frac{K_N(K_N-1)...(K_N-x+I+1)}{N_N(N_N-1)...(N_N-x+I+1)}, \tag{3.2}$$

tends to p_N^{x-I}, and

$$\frac{(N_N-K_N)(N_N-K_N-1)...(N_N-K_N-n+x-I+1)}{(N_N-x+I)(N_N-x+I-1)...(N_N-n+1)}, \tag{3.3}$$

tends to $(1-p_N)^{n-x+I}$. Hence, by Equations (3.2) and (3.3) we conclude the proof. □

3.7. Neutrosophic Poisson distribution (NPD)

Let X_N be a neutrosophic random variable. We say that X_N has neutrosophic Poisson distribution denoted by $X_N \sim Poisson(\lambda_N)$ where λ_N is set with one or more elements (may λ_N be an interval). Nuetorsophic probability function is given by

$$f_X(x-I) = \begin{cases} e^{-\lambda_N} \dfrac{\lambda_N^{x-I}}{(x-I)!}, & if\ x = I, 1+I, 2+I, ..., n+I. \\ \\ 0, & otherwise. \end{cases}$$

Proof. It follows from the fact $e^{x-I} = \sum_{j=I}^{\infty} \dfrac{(x-I)^j}{j!}$. □

By Theorems (2.7) and (2.9), we can see that

$$E(X_N) = \lambda_N + I,$$
$$Var(X_N) = \lambda_N.$$

By using computational software R, we can obtain the neutrosophic Poisson distribution by using the following command:

```
# dpois(x−I,\lambda) evaluates f(x−I) in the neutrosophic Poisson distribution
    where I=0.6 and x=3.6
> dnbinom(3,2)
[1] 0.18
```

Example 3.20. On average [2,4] requests to access a page are received web for any given minute with an indeterminacy of 20%. What is the probability that in a given minute:

(1) No one requests access to the page.
(2) More than two requests are received.

Solution:
Let X_N be the number of requests per minute with an indeterminacy of 20%. We will suppose that X_N has neutrosophic distribution $Poisson(\lambda_N)$ with $\lambda_N = [2, 4]$. Then,

(1) $P(X = -I) = e^{-[2,4]} \dfrac{[2,4]^{-0.2}}{-0.2!} = [0.15, 0.23].$
(2)

$$\begin{aligned} P(X > 2-I) &= 1 - P(X \leq 2-I) \\ &= 1 - (P(X = -I) + P(X = 1-I) + P(X = 2-I)) \\ &= 1 - (P(X = -0.2) + P(X = 0.8) + P(X = 1.8)) \\ &= 1 - e^{-[2,4]}(\frac{[2,4]^{-0.2}}{-0,2!} + \frac{[2,4]^{0.8}}{0.8!} + \frac{[2,4]^{1.8}}{1,8!}) \\ &= [0.24, 0.37]. \end{aligned}$$

Theorem 3.21. *Let X_N and Y_N be two independence neutrosophic random variables with neutrosophic distribution $Poisson(\lambda_{N_1})$ and $Poisson(\lambda_{N_2})$, respectively. Then, $X_N + Y_N \sim Pisson(\lambda_{N_1} + \lambda_{N_2})$.*

Proof. For any $u \geq 0$,

$$\begin{aligned}
P(X_N + Y_N = u - 2I) &= \sum_{x=I}^{u+I} P(X_N = x, Y_N = u - x) \\
&= \sum_{x=I}^{u+I} P(X_N = x) P(Y_N = u - x) \\
&= \sum_{x=I}^{u+I} e^{\lambda_{N_1}} \frac{\lambda_{N_1}^{x-I}}{(x-I)!} e^{\lambda_{N_2}} \frac{\lambda_{N_1}^{u-x+I}}{(u-x+I)!} \\
&= e^{-(\lambda_{N_1}+\lambda_{N_2}} \frac{1}{(u-2I)!} \sum_{x=I}^{u+I} \binom{u-2I}{x-I} \lambda_{N_1}^{x-I} \lambda_{N_2}^{u-x+I} \\
&= e^{-(\lambda_{N_1}+\lambda_{N_2}} \frac{(\lambda_{N_1} + \lambda_{N_2})^{u-2I}}{(u-2I)!}.
\end{aligned}$$

□

3.8. Relationship between neutrosophic discrete distributions in random variables

As well as happens in classical probability, the distributions presented in this paper can be related to each other, as can be seen in the following diagram:

$$\begin{array}{ccccc}
NHGD & \leftarrow & NBD & \rightarrow & NGD \\
 & \searrow & \downarrow & & \downarrow \\
 & & NbD & & NNBD \\
 & & \downarrow & \swarrow & \\
 & & NPD & &
\end{array}$$

Meanings:

(1) Neutrosophic Bernoulli distribution: NBD.
(2) Neutrosophic binomial distribution: NbD.
(3) Neutrosophic geometric distribution: NGD.
(4) Neutrosophic negative binomial distribution:NNBD.
(5) Neutrosophic hypergeometric distribution: NHGD
(6) Neutrosophic Poisson distribution: NPD.

It is easy to verify these relations, since they are proved in a similar way to classical probability, only the degree indeterminacy must be taken into account.

4. Conclusion

The neutrosophic probability distributions only deal with the specified undetermined values. In this paper, we contributed to the study of classical distributions and classical neutrosophic probability distribution and applied them in neutrosophic random variable an we define its discrete distribution. We called these distributions neutrosophic discrete distributions in neutrosophic random variables. On the other hand, We conclude from this paper that the neutrosophic discrete distributions in neutrosophic random variables gives us a more general and clarity study of the studied issue. In this paper, we presented several solved for the problems that classic logic and classical neutrosophic probability. We look forward in the future to apply these distrbutions in decision making, engineer and social science environment.

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