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Product evaluation using uncertainty-based process capability index

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ABSTRACT

This paper introduces a new process capability index (PCI) under neutrosophic statistics. The proposed neutrosophic process capability index (NPCI) can be applied to evaluate the status of the industrial process in the presence of Neutrosophy. A new form of PCI is presented to deal with the uncertainty in the system. The performance of the proposed NPCI is checked using the specification limits of some real industrial datasets. By comparing the new NPCI with existing PCI, it is noted that the proposed NPCI provides the measure in indeterminacy interval which makes it more convenient to apply in an uncertain/indeterminate environment.

1. Introduction

The process capability index (PCI), say Cpk is an important tool that measures the ability either the industrial process is manufacturing the product according to the given specifications or not. The PCI is helpful to produce the product within the tolerance range for producer and consumer. The PCI tells the industrial engineers how close the process to the given target. Under the consistent performance, the PCI is a useful tool to estimate future process performance, see [1]. The higher the values of PCI mean the process is better. [2] introduced the PCI Cpk which considered both process location and process magnitude. For example, the value of Cpk greater than 1.33 indicates that the process is capable and the manufactured items are within the limits of the specifications. [3] provided the yield bounds of a process. According to [3], the value of Cpk = 1 indicates that an industrial personal can expect 2700 items out of one million outside the given customer tolerance range. The values of Cpk more than 1.33 and 1.67 mean one can expect 66 items and 0.554 items defective in one million, respectively. [4] presented a graphical approach to study PCI. On the other hand, [1] suggested that the negative values of Cpk indicate that the process is not capable and the product does not meet the given specification limits. [5] proposed the modified PCI. [6] introduced PCI using the Bayesian method. [7] noted the decrease in out of specification product when $1 < Cpk < 1.33$. [8] presented a detailed review of PCI and applications. [9] designed a control chart using PCI. More applications of PCI can be seen in [10], [11] and [12].

The traditional PCI under classical statistics is applied under the assumption that the specification limits or observations in data are determined. In the complex systems, it may not possible that the specification limits or observations are always precise. In this case, to see the capability of the process, the PCI using fuzzy logic is applied. [13] presented the PCI using the fuzzy logic. [14] introduced the inertial PCI using fuzzy logic. [15] worked on PCI using a type-II fuzzy approach. [16] worked on the asymmetrical intervals in PCI. More details on PCI based on fuzzy logic can be seen in [17], [18], [19], [20] and [21].

The neutrosophic logic proposed by [22] is an extension of fuzzy logic and interval-based approach. The earlier is used to get the three degrees of measures including truthiness, falseness, and indeterminacy. [23], [24], [25], [26], [27], [28], [29], [30], [31], [32] and [33] presented neutrosophic logic and its applications. [34] introduced the neutrosophic statistics (NS) as the extension of classical statistics (CS). The NS deals to analyze the neutrosophic data. [35] and [36] intruded the methods for analyzing the data having neutrosophic numbers. [37] introduced sampling plan using NPCI. More applications on NS can be seen in [38] and [39], [40] and [41].

The measure of indeterminacy is an important measure when the data is obtained from the complex process or data having Neu-

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trosophy. [37] proposed the sampling plan using NPCI but did not study the performance of NPCI. In addition, the existing PCI based on CS and fuzzy logic is cannot be applied when uncertainty/indeterminacy is presented in the data. By exploring the literature and the best of our knowledge, there is no work on PCI under NS. In this paper, the PCI under NS will be introduced originally. The neutrosophic process capability index (NPCI) will be applied to see the capability of the industrial process in uncertainty. The comparison of the proposed NPCI will be given with the PCI under CS. Three real examples from the industrial processes are selected to implement the proposed NPCI. It is expected that the proposed NPCI will be efficient than PCI in terms of measures of indeterminacy, flexibility and information. The paper is organized as follows: the preliminaries are introduced in section 2. The proposed NPCI is introduced in Section 3. The evaluation methods are introduced in Section 4. The simulation and application of the proposed methods are given in Section 5 and Section 6. The comparative study is given in Section 7 and some concluding remarks are given in Section 8.

2. Preliminaries

Defination-1: *Suppose that a neutrosophic random variable* $X_N = X_L + X_U I_N; I_{NX} \epsilon [I_{UX}, I_{UX}]$ *follows the neutrosophic normal distribution with neutrosophic mean* $\mu_N = \mu_L + \mu_U I_{N\mu}; I_{N\mu} \epsilon [I_{U\mu}, I_{U\mu}]$ *and neutrosophic standard deviation* $\sigma_N = \sigma_L + \sigma_U I_{N\sigma}; I_{N\sigma} \epsilon [I_{U\sigma}, I_{U\sigma}]$. *Note here that* $I_{NX} \epsilon [I_{UX}, I_{UX}]$, $I_{N\mu} \epsilon [I_{U\mu}, I_{U\mu}]$ *and* $I_{N\sigma} \epsilon [I_{U\sigma}, I_{U\sigma}]$ *are indeterminacy interval. Also note, that* $X_U I_N$, $\mu_U I_{N\mu}$ *and* $\sigma_U I_{N\sigma}$ *are indeterminate parts of the neutrosophic random variable* $X_N \epsilon [X_L, X_U]$, *neutrosophic mean* $\mu_N \epsilon [\mu_L, \mu_U]$ *and neutrosophic standard deviation* $\sigma_N \epsilon [\sigma_L, \sigma_U]$. *These measures reduce to measures under classical statistics when no indeterminacy is found in the observations. In practice,* $\mu_N \epsilon [\mu_L, \mu_U]$ *and* $\sigma_N \epsilon [\sigma_L, \sigma_U]$ *are unknown and can be estimated using the neutrosophic sample information. Suppose that* $x_{N1} = x_{L1} + x_{U1} I_{N1}$; $I_{Nx1} \epsilon [I_{Ux1}, I_{Ux1}]$, $\ldots, x_{Nn} = x_{Ln} + x_{Un} I_{Nn}; I_{Nxn} \epsilon [I_{Uxn}, I_{Uxn}]$ *be sample random variable consisting of the neutrosophic numbers.*

By following [36] and [36], the neutrosophic sample mean $\bar{x}_N \epsilon [\bar{x}_L, \bar{x}_U]$ based on sample size $n_N \epsilon [n_L, n_U]$ from population size $N_N \epsilon [N_L, N_U]$ is given by.

$\bar{x}_N = \bar{x}_L + \bar{x}_U I_N; \quad I_N \epsilon [I_{L\bar{x}}, I_{U\bar{x}}]$ (1).where $\bar{x}_L = \frac{1}{n_N}\sum_{i=1}^{n_N} x_{Li}$ and.$\bar{x}_U = \frac{1}{n_N}\sum_{i=1}^{n_N} x_{Ui}$.

The neutrosophic sum of squares of the values from $\bar{x}_N \epsilon [\bar{x}_L, \bar{x}_U]$ is given by.

$$\sum_{i=1}^{n_N}\left(x_{Ni}-\bar{x}_{iN}\right)^2=\sum_{i=1}^{n_N}\begin{bmatrix}\min\begin{pmatrix}(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_U\right)\\(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_U\right)\end{pmatrix}\\\max\begin{pmatrix}(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_U\right)\\\left(a_i+b_iI^U\right)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_U\right)\end{pmatrix}\end{bmatrix},I_N\epsilon[I_L,I_U] \quad (2)$$

Note here that $x_{Li} = a_i$ and $x_{Ui} = b_i$.

The neutrosophic variance $S_N^2 \epsilon [S_L^2, S_U^2]$ and neutrosophic standard deviation $S_N \epsilon [S_L, S_U]$ are given by.

$$S_N^2 = \frac{\sum_{i=1}^{n_N}\left(x_{Ni}-\bar{x}_{iN}\right)^2}{n_N}; \; n_N \epsilon [n_L, n_U]; \; S_N^2 \epsilon \left[S_L^2, S_U^2\right] \quad (3)$$

and.

$$S_N = \sqrt{\frac{\sum_{i=1}^{n_N}\left(x_{Ni}-\bar{x}_{iN}\right)^2}{n_N}}; \; n_N \epsilon [n_L, n_U]; \; S_N \epsilon [S_L, S_U] \quad (4)$$

3. Neutrosophic process capability index

In this section, the index C_{pkN} will be introduced under the NS.

Definition 2. *Suppose that* $USL_N \epsilon [USL_L, USL_U]$ *and* $LSL_N \epsilon [LSL_L, LSL_U]$ *are neutrosophic upper specification limit (NUSL) and neutrosophic lower specification limit (NLSL), respectively. The proposed index* C_{pkN} *based on* $\mu_N \epsilon [\mu_L, \mu_U]$ *and* $\sigma_N \epsilon [\sigma_L, \sigma_U]$ *is defined by.*

$$C_{pkN} = min\left\{\frac{USL_N - \mu_N}{3\sigma_N}, \frac{\mu_N - LSL_N}{3\sigma_N}\right\} \quad (5)$$

The population neutrosophic mean and NSD are defined as.

$$\mu_N = \mu_L + \mu_U I_N; \; I_N \epsilon [I_{L\mu}, I_{U\mu}] \quad (6)$$

and.

$$\sigma_N = \sqrt{\frac{1}{N_N}\sum_{i=1}^{N_N}\left(x_{Ni}-\mu_{iN}\right)^2} \quad (7)$$

where.

$$\sum_{i=1}^{N_N}\left(x_{Ni}-\mu_{iN}\right)^2=\sum_{i=1}^{N_N}\begin{bmatrix}min\begin{pmatrix}(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_U\right)\\(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_U\right)\end{pmatrix}\\max\begin{pmatrix}(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_U\right)\\\left(a_i+b_iI^U\right)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_U\right)\end{pmatrix}\end{bmatrix},I_N\epsilon[I_L,I_U]$$

where $N_N \epsilon [N_L, N_U]$ presents the neutrosophic population size. The index C_{pkN} based on neutrosophic data can be written as.

$$C_{pkN} = \min\left\{\frac{USL_N - (\mu_L + \mu_U I_N)}{3\sqrt{\frac{1}{N}\sum_{i=1}^{N_N}\left(x_{Ni}-\mu_{iN}\right)^2}}, \frac{(\mu_L + \mu_U I_N) - LSL_N}{3\sqrt{\frac{1}{N}\sum_{i=1}^{N_N}\left(x_{Ni}-\mu_{iN}\right)^2}}\right\} \quad (8)$$

As mentioned earlier, the population parameters $\mu_N \epsilon [\mu_L, \mu_U]$ and $\sigma_N \epsilon [\sigma_L, \sigma_U]$ are unknown in practice. The neutrosophic sample mean and NSD are the best estimates of the population parameters. The index $\hat{C}_{pkN} \epsilon \left[\hat{C}_{pkL}, \hat{C}_{pkU}\right]$ based on the sample estimators are defined by.

$$\hat{C}_{pkN} = \min\left\{\frac{USL_N - \bar{x}_N}{3S_N}, \frac{\bar{x}_N - LSL_N}{3S_N}\right\} \quad (9)$$

The neutrosophic sample average is given by.

$$\bar{x}_N = \bar{a}_{1N} + \bar{b}_{1N} I_N; \; I_N \epsilon [I_{LX^-}, I_{UX^-}]$$

The NSD is given by.

$$S_N = \sqrt{\frac{1}{n_N}\sum_{i=1}^{n_N}\left(x_{iN}-\bar{x}_{iN}\right)^2} \quad (10)$$

where.

$$\sum_{i=1}^{n_N}\left(x_{iN}-\bar{x}_{iN}\right)^2=\sum_{i=1}^{n_N}\begin{bmatrix}\min\begin{pmatrix}(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_U\right)\\(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_U\right)\end{pmatrix}\\\max\begin{pmatrix}(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_L)\left(\bar{a}+\bar{b}I_U\right)\\(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_L\right),(a_i+b_iI_U)\left(\bar{a}+\bar{b}I_U\right)\end{pmatrix}\end{bmatrix},I_N\epsilon[I_L,I_U]$$

The index $\hat{C}_{pkN}$ is defined by.

$$\hat{C}_{pkN} = \min\left\{\frac{USL_N - \left(\bar{a}_{1N} + \bar{b}_{1N} I_N\right)}{3\sqrt{\frac{1}{n}\sum_{i=1}^{n_N}\left(x_{iN}-x^-{}_{iN}\right)^2}}, \frac{\left(\bar{a}_{1N} + \bar{b}_{1N} I_N\right) - LSL_N}{3\sqrt{\frac{1}{n}\sum_{i=1}^{n_N}\left(x_{iN}-x^-{}_{iN}\right)^2}}\right\} \quad (11)$$

Table 1
The values of index by method-I.

Indeterminacy	$LSL = 9.2$, USL=10.4		$LSL = 5.5$, USL=6.4		$LSL = 40$, USL=90	
	C_{pkN}		C_{pkN}		C_{pkN}	
	C_{pkL}	C_{pkU}	C_{pkL}	C_{pkU}	C_{pkL}	C_{pkU}
0	0.134396	0.063022	0.467485	0.28383	0.866742	1.007874
0.10	0.107973	0.089446	−0.04341	0.540971	0.605437	1.269179
0.20	0.081549	0.115869	−0.5543	0.540971	0.344132	1.530484
0.30	0.055125	0.126621	−1.0652	0.540971	0.082827	1.791789
0.40	0.028702	0.126621	−1.57609	0.540971	−0.17848	2.018018
0.50	0.002278	0.126621	−2.08699	0.540971	−0.43978	2.018018
0.60	−0.02415	0.126621	−2.59788	0.540971	−0.70109	2.018018
0.70	−0.05057	0.126621	−3.10877	0.540971	−0.96239	2.018018
0.80	−0.07699	0.126621	−3.61967	0.540971	−1.2237	2.018018
0.90	−0.10342	0.126621	−4.13056	0.540971	−1.485	2.018018
1.00	−0.12984	0.126621	−4.64146	0.540971	−1.74631	2.018018

Note here that the proposed index $\widehat{C}_{pkN}$ is the extension of PCI under CS. The proposed index reduces to existing PCI when no uncertainty is found in specification limits or observations. According to the proposed index $\widehat{C}_{pkN}$, the minimum values from the neutrosophic values are selected to see the capability of the industrial process. The application of the index $\widehat{C}_{pkN}$ will be explained in the next section.

4. Evaluation methods

In this section, we will discuss two evaluation methods based on the proposed index $\widehat{C}_{pkN}$. Suppose that $\widehat{C}_{LN} = USL_N - \left(\bar{a}_{1N} + \bar{b}_{1N}I_N\right)/3\sqrt{\frac{1}{n}\sum_{i=1}^{n_N}\left(x_{iN} - \bar{x}_{iN}\right)^2}; \widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right]$ and $\widehat{C}_{UN} = \left(\bar{a}_{1N} + \bar{b}_{1N}I_N\right) - LSL_N/3\sqrt{\frac{1}{n}\sum_{i=1}^{n_N}\left(x_{iN} - \bar{x}_{iN}\right)^2}; \widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right]$. We discuss the following two methods to find the minimum values of the index $\widehat{C}_{pkN}$.

4.1. Method-I

As mentioned earlier that the proposed index $\widehat{C}_{pkN}$ will be calculated based on the data having the neutrosophic numbers. Therefore, the resulting output of the index $\widehat{C}_{pkN}$ will be in indeterminacy interval. Based on the calculations of neutrosophic data, $\widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right]$ and $\widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right]$ be indeterminacy interval having lower and upper values. According to the general theory of the PCI, the smaller values of $\widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right]$ and $\widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right]$ will be chosen. In this case, the proposed index $\widehat{C}_{pkN}$ can be modified as follows.

$$\widehat{C}_{pkN} = \min\left\{\min \widehat{C}_{LN}, \min \widehat{C}_{UN}\right\};\ \widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right],\ \widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right] \quad (12).$$

The minimum neutrosophic values of $\widehat{C}_{pkN}\epsilon\left[\widehat{C}_{pkL}, \widehat{C}_{pkU}\right]$ will be used to see the capability of the process.

4.2. Method-II

[34] discussed the method to calculate the sample average based on the neutrosophic data. The second method is proposed based on the idea of [34]. According to this method, the values of $\widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right]$ and $\widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right]$ are calculated using the data having the neutrosophic numbers. The averages of index $\widehat{C}_{LN}$ and $\widehat{C}_{UN}$ are calculated. The operational procedure of this method is explained as follows.

Step-1: calculate the values of $\widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right]$ and $\widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right]$ using the sample data.
Step-2: Find the neutrosophic averages of indeterminacy intervals $\widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right]$ and $\widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right]$.
Step-3: Select the minimum neutrosophic values of the index $\widehat{C}_{pkN}\epsilon\left[\widehat{C}_{pkL}, \widehat{C}_{pkU}\right]$ using the following modified index $\widehat{C}_{pkN}\epsilon\left[\widehat{C}_{pkL}, \widehat{C}_{pkU}\right]$

$$\widehat{C}_{pkN} = \min\left\{\min\left(\text{averageof}\widehat{C}_{LN}\right), \min\left(\text{averageof}\widehat{C}_{UN}\right)\right\};$$
$$\widehat{C}_{LN}\epsilon\left[\widehat{C}_{LL}, \widehat{C}_{LU}\right],\ \widehat{C}_{UN}\epsilon\left[\widehat{C}_{UL}, \widehat{C}_{UU}\right] (13).$$

The values of index $\widehat{C}_{pkN}\epsilon\left[\widehat{C}_{pkL}, \widehat{C}_{pkU}\right]$ is used to see the capability of the process. The higher values of this index mean the process is better.

5. Simulation study

In this section, we will discuss the application of the proposed index using method-I and method-II. To see the performance of the proposed index $\widehat{C}_{pkN}$, a simulation study using Excel is conducted for various combinations of LSL_N and USL_N. The values for the proposed index for various values of $I_N\epsilon[I_L, I_U]$ using method-I are presented in Table 1. In Table 1, the values of index in first row represent when no uncertainty is found in the data. From Table 1, it is quite clear that the proposed index seems sensitive

Table 2
The values of index by method-II.

Indeterminacy	$LSL = 9.2$, USL=10.4	$LSL = 5.5$, USL=6.4	$LSL = 40$, USL=90
	C_{pkN}	C_{pkN}	C_{pkN}
0	0.094821	0.4124	1.301089
0.10	0.108033	0.423801	1.170436
0.20	0.121245	0.168353	1.039784
0.30	0.134457	−0.08709	0.909131
0.40	0.147669	−0.34254	0.778479
0.50	0.13615	−0.59799	0.647826
0.60	0.122939	−0.85343	0.517174
0.70	0.109727	−1.10888	0.386522
0.80	0.096515	−1.36433	0.255869
0.90	0.083303	−1.61978	0.125217
1.00	0.070091	−1.87522	−0.00544

Table 3
The values of index for real examples.

Data	Specification limits	Mean	S.D	C_{pkN}	
				Method-I	Method-II
I	$LSL = 9.2$, USL=10.4	[6.86,6.96]	[4.37,8.78]	**[**0.134396, 0.063022]	0.094821
II	$LSL = 5.5$, USL=6.4	[5.84,6.12]	[0.2095,0.3993]	**[**0.467485, 0.28383]	0.4124
III	$LSL = 40$, USL=90	[66.88,69.69]	[4.44,8.89]	**[**0.866742, 1.007874]	1.301089

for the higher degree of indeterminacy in the process. As the values of I_N increases, the resulting values of the index are negative. As suggested by [1], the negative values of the index show that the process is lying outside the specification limits. For example, when $LSL = 9.2$ andUSL = 10.4, the values of $\widehat{C}_{pkL}$ become negative. For these specification limits, the process is not capable when $I_U > 0.5$. Table 2 is presented using the method-II. From Table 2, it is clear that the proposed index $\widehat{C}_{pkN}$ based on the single value rather than in interval as in Table 1. From Table 2, it quite clear that index values for $LSL = 9.2$, USL=10.4 and $LSL = 5.5$, USL=6.4 are less than 1.33 which clearly indicates that the process is not capable. For $LSL = 40$ and USL = 90, it can see that the values $\widehat{C}_{pkN}$ decreases as the values of I_U increases. From this study, it can be concluded that the evaluation of the index $\widehat{C}_{pkN}$ by method-I provides the results in indeterminacy interval which is required in uncertainty. On the other hand, a single value of index $\widehat{C}_{pkN}$ is obtained by method-II. In addition, it can be concluded that the degree of indeterminacy affects the values of the index $\widehat{C}_{pkN}$ significantly. The higher values of I_U leads to the process beyond the given specification limits. Therefore, industrial engineers should consider the effect of indeterminacy while making the decision about the industrial process based on process capability index.

6. Applications of the index $\widehat{C}_{pkN}$

To discuss the application on the proposed index $\widehat{C}_{pkN}$, three real examples from the industries are selected. The first data is selected from the automobile industry. The data of diameters of gears when $LSL = 9.2$, USL=10.4 is selected from [15]. The industrial engineers are neutrosophic in measuring the diameter of the gears. The second factory data with $LSL = 5.5$, USL=6.4 having the neutrosophic observations is selected from [13]. The third data about light-emitting diodes (LEDs) with $LSL = 40$ and USL = 90 is selected from [14]. According to [14] "an LED-based lighting fixture (LED-LF) is investigated as an example; the LED-LF is manufactured in Tainan Industrial Park, Taiwan. Due to high demands on the LED-LFs, the company does not have enough production capacity to supply-one type of LED components used in the LED-LFs. Therefore, the decision-makers decide to purchase the LED components from some possible suppliers. The luminous intensity of LED sources is a critical characteristic for this type of LEDs. Thus far, all light measurements and rating systems depend on the perception of the human eye or imprecise terminology and calibration standards. This implies that the randomness is not the only aspect of uncertainty for data collected on the luminous intensity of LED sources". From the three industrial examples, it can be seen that industrial engineers are uncertain about the measurement of the quality of interest. In this case, the presence of indeterminacy can affect the performance of the process. Therefore, the uses of the existing PCI or the PCI based on the fuzzy approach do not provide the measure of indeterminacy. Therefore, the use of the proposed index is quite suitable and effective to be applied for the analysis of these industry data. The values of the proposed index $\widehat{C}_{pkN}$ from both methods are placed in Table 3. Note that in Table 3, the first values 0.134396, 0.467485 and 0.866742 present the traditional values of PCI. Note here that the limitation of the traditional PCI is that it cannot be applied when the data at hand is imprecise and in intervals. From Table 3, it is quite clear that the values of index $\widehat{C}_{pkN}$ by method-I are indeterminacy intervals. On the other hand, the second method provides the determined values of the proposed index. For example, under uncertainty, the gears engineers can expect the index $\widehat{C}_{pkN}$ from 0.134396 to 0.063022; i.e. $\widehat{C}_{pkN}\epsilon$**[**0.134396, 0.063022]. The index is less than 1, it means, the industrial process is not capable to produce the gears product according to the given specification limits. From method-II, the index value is 0.09. Both methods show that the process is not capable to produce the gears product according to the specification limits.

Table 4
The measure of indeterminacy.

Data	C_{pkN}	
	Method-I	Existing PCI
I	$0.1343 - 0.0630I_N; I_N\epsilon[0, 1.1317]$	0.1343
II	$0.4674 - 0.2838I_N; I_N\epsilon[0, 0.6469]$	0.4674
III	$0.8667 + 1.007I_N; I_N\epsilon[0, 0.1393]$	0.8667

7. Comparative studies

In this section, the advantages of the index $\widehat{C}_{pkN}\epsilon\left[\widehat{C}_{pkL}, \widehat{C}_{pkU}\right]$ be discussed over the existing PCI under NS. The proposed index $\widehat{C}_{pkN}\epsilon\left[\widehat{C}_{pkL}, \widehat{C}_{pkU}\right]$ into the neutrosophic form can be written as: $\widehat{C}_{pkN} = \widehat{C}_{pkL} + \widehat{C}_{pkU}I_N; I_N\epsilon[I_L, I_U]$. The proposed index is consisted of two parts. The first part $\widehat{C}_{pkL}$ is known as the determined part and presents the value of the index under CS. The second part $\widehat{C}_{pkU}I_N; I_N\epsilon[I_L, I_U]$ is known as the indeterminate part. From the neutrosophic form, the proposed index reduces to index under CS, when $I_L = 0$. From Table 4, it can be seen that the proposed method provides the index values in indeterminacy interval with the measure of indeterminacy. For example, for the 3rd data set, the neutrosophic form of the index is $\widehat{C}_{pkN} = 0.8667 + 1.007I_N$; $I_N\epsilon[0, 0.1393]$. It means that, under uncertainty, the values of $\widehat{C}_{pkN}$ can be expected from 0.8667 to 1.007 with degree of indeterminacy 0.1393. From this study, it is concluded that the proposed method provides the values of the index $\widehat{C}_{pkN}$ which is required in the indeterminate environment. In addition, the proposed methods give the information about the measure of indeterminacy. In nutshell, the proposed index $\widehat{C}_{pkN}$ is quite effective, informative, and reasonable to apply to monitor the capability of the complex systems process.

8. Concluding remarks

A new PCI index under NS was introduced. The two evaluation methods under neutrosophic statistics were introduced. From the simulation study, real example, and comparisons, it is concluded that the proposed NPCI is the generalization of the classical index under CS. In addition, the proposed NPCI found to be effective, flexible and informative to be applied when the data or specification limits are neutrosophic. The proposed NPCI can be used to evaluate the industrial process in uncertainty environment. The proposed index can be developed for big data as future research. The present study using Pythagorean fuzzy uncertain environments can be extended as future research, see [42].

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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