Advances and Applications in Statistics
© 2024 Pushpa Publishing House, Prayagraj, India
https://pphmjopenaccess.com
http://dx.doi.org/10.17654/0972361724023
Volume 91, Number 4, 2024, Pages 439-450 **P-ISSN: 0972-3617**

VAGUE DATA ANALYSIS USING SEQUENTIAL TEST

Muhammad Aslam and Muhammad Saleem

Department of Statistics
King Abdulaziz University
Jeddah 21551, Saudi Arabia
e-mail: aslam_ravian@hotmail.com

Department of Industrial Engineering
Faculty of Engineering-Rabigh
King Abdulaziz University
Jeddah 21589, Saudi Arabia
e-mail: msaleim1@kau.edu.sa

Abstract

Objective. The existing sequential test using Bernoulli distribution can only be applied when no uncertainty/indeterminacy is found in testing the hypothesis. This paper introduces neutrosophic Bernoulli distribution and sequential test using the distribution.

Method. The operational procedure of the proposed test will be introduced and applied for testing the hypothesis in the presence of uncertainty.

Received: October 29, 2023; Accepted: December 16, 2023

2020 Mathematics Subject Classification: 97K80.

Keywords and phrases: sequential test, neutrosophy, classical test, industry, Bernoulli distribution.

How to cite this article: Muhammad Aslam and Muhammad Saleem, Vague data analysis using sequential test, Advances and Applications in Statistics 91(4) (2024), 439-450.
http://dx.doi.org/10.17654/0972361724023

Published Online: February 12, 2024

Results. The advantages of the proposed test will be discussed using manufacturing data. From the comparison and simulation studies, it is found that the proposed test is efficient than the existing test under classical statistics.

Conclusions. The proposed test may be more economic and time-saving as the decision may make on the basis of the first sample. Therefore, the proposed test is efficient, economical and adequate than the existing test.

1. Introduction

Usually, the statistical tests give decisions about the acceptance or rejection of the null hypothesis using the information obtained from a single sample. In practice, the decision-makers sometimes are unable to reach the decision using single sample of information. In case, when decision-makers are in-decision on the basis of the first sample, a second sample is taken and the process is continued until a decision is reached. The sequential test using the Bernoulli distribution is also applied when no decision about the null hypothesis is made on the first sample. Causey [13] provided the exact calculation of the test. This test is applied for testing the parameter of the Bernoulli distribution. Kanji [26] discussed the sequential test for Bernoulli distribution. Bartroff and Song [10] studied the power of the sequential tests. Pereira et al. [31] proposed the sequential test using the Bayesian approach. Pramanik et al. [32] presented the modification of the sequential test. More details can be read in Gardonyi and Samu [19], and Pramanik et al. [32] presented the modified forms of SPRT. More applications of SPRT can be seen in Wijsman [44], Thomas [41], Bacanli and Icen [9], and Pan et al. [28].

The fuzzy-based statistical tests attract the decision-makers when uncertainty/indeterminacy is found in the observations or parameters. The fuzzy-based tests are found to be effective when testing of hypothesis is done in uncertainty. Kacprzyk et al. [24] and Taheri and Hesamian [39] pointed out the fields where imprecise observations can be recorded. Torabi and Behboodian [42], Torabi and Mirhosseini [43], Parchami [29], Talukdar and Baruah [40], İçen et al. [23], Kahraman et al. [25], Dubois and Prade [18], Grzegorzewski [20], Denoeux et al. [17], Grzegorzewski [21], Taheri and Arefi [38], Taheri and Hesamian [37], Kacprzyk et al. [24], Taheri and Hesamian [39], Shafiq et al. [33], Grzegorzewski and Śpiewak [22] and, Chukhrova and Johannssen [16] applied statistical tests on imprecise data.

Smarandache [35] introduced neutrosophic logic and showed the efficiency over the fuzzy-based logic and interval-based analysis. The neutrosophic logic gives information about the measure of indeterminacy in addition to the fuzzy logic. Broumi and Smarandache [12], Abdel-Basset et al. [1], Peng and Dai [30], Shahin et al. [34], Broumi et al. [11], Abdel-Basset et al. [2, 3] and Nabeeh et al. [27] showed the efficiency of the neutrosophic logic over the existing logic. Smarandache [36] used neutrosophic logic to develop neutrosophic statistics to analyze imprecise data. Chen et al. [14, 15], and Aslam [4-7] showed that neutrosophic statistics is efficient than classical statistics.

The sequential test using Bernoulli distribution can be applied only when the parameter of the distribution is certain, precise and exact. The existing sequential test using Bernoulli distribution under classical statistics cannot be applied when the observation/parameter is indeterminate. By exploring the literature, no work is found on sequential test using Bernoulli distribution under neutrosophic statistics. In this paper, we introduce neutrosophic Bernoulli distribution first. We present the design of a sequential test using Bernoulli distribution under neutrosophic statistics. The application of the proposed test will be given in the manufacturing industry. From the study, it is expected that the proposed will be adequate than the existing sequential test using the Bernoulli distribution.

2. Method

Suppose that $X_{1N}, X_{2N}, X_{3N}, \ldots, X_{nN}$ is independently and identically neutrosophic random variable which follows the neutrosophic Bernoulli distribution with the neutrosophic probability density function (npdf), say $f_{0N}^{x_N}$ is given by

$$f_{0N}^{x_N} = f_0^{x_N} + f_0^{x_N} I_{Nf0}; \quad I_{Nf0} \in [I_{Lf0}, I_{Uf0}]. \tag{1}$$

Note that $f_0^{x_N}$ represents the Bernoulli distribution under classical statistics, $f_0^{x_N} I_{Nf0}$ presents the indeterminate part and $I_{Nf0} \in [I_{Lf0}, I_{Uf0}]$ is the measure of indeterminacy. The current neutrosophic form reduces to Bernoulli distribution (BD) under classical statistics when $I_{Lf0} = 0$, see Aslam [5]. Using the neutrosophic theory information, the npdf of neutrosophic Bernoulli distribution (NBD) is given by

$$f_{0N}^{x_N} = \theta_N^{x_N}(1 - \theta_N^{x_N})^{1-x_N} + \theta_N^{x_N}(1 - \theta_N^{x_N})^{1-x_N} I_{Nf0};$$

$$I_{Nf0} \in [I_{Lf0}, I_{Uf0}], \tag{2}$$

where $[0, 0] \le \theta_N^{x_N} \le [1, 1]$. The proposed NBD reduces to BD when $I_{Lf0} = 0$. The proposed NBD can be expressed as follows:

$$f_{0N}^{x_N} = \theta_N^{x_N}(1 - \theta_N^{x_N})^{1-x_N}(1 + I_{Nf0}); \quad I_{Nf0} \in [I_{Lf0}, I_{Uf0}]. \tag{3}$$

Suppose that the decision-makers are interested to apply the proposed test when uncertain about the specified value of the unknown parameter. Let $\theta_{0N} = \theta_{0N} + \theta_{0N} I_{\theta_{0N}}$ and $I_{\theta_{0N}} \in [I_{\theta_{0L}}, I_{\theta_{0U}}]$ be the neutrosophic form against the null hypothesis H_0 of the unknown parameter, where θ_{0N} is the exact value, $\theta_N I_{\theta_{0N}}$ is the indeterminate part and $I_{\theta_{0N}} \in [I_{\theta_{0L}}, I_{\theta_{0U}}]$ is an indeterminate interval. Let $\theta_{1N} = \theta_{1N} + \theta_{1N} I_{\theta_{1N}}$ and $I_{\theta_{1N}} \in [I_{\theta_{1L}}, I_{\theta_{1U}}]$ be the neutrosophic form against the alternative hypothesis H_1 of the unknown parameter, where θ_{1N} is the exact value, $\theta_{1N} I_{\theta_{0N}}$ is the indeterminate part and $I_{\theta_{1N}} \in [I_{\theta_{1L}}, I_{\theta_{1U}}]$ is an indeterminate interval. Let $S_{mN} = \sum_{i=1}^{mN} X_{iN}$ be the sum of neutrosophic random variables, α_N be a type-I and β_N be a type-II error. The neutrosophic lower limit, say $a_{mN} \in [a_{mL}, a_{mU}]$ of the proposed test is given by

$$a_{mN} = \frac{\log\left(\frac{\beta_N}{1-\alpha_N}\right)}{\log\left(\frac{\theta_{1N}(1+I_{\theta_{1N}})}{\theta_{0N}(1+I_{\theta_{0N}})}\right) - \log\left(\frac{1-\{\theta_{1N}(1+I_{\theta_{1N}})\}}{1-\{\theta_{0N}(1+I_{\theta_{0N}})\}}\right)} + \frac{m_N \log\left(\frac{1-\{\theta_{0N}(1+I_{\theta_{0N}})\}}{1-\{\theta_{1N}(1+I_{\theta_{1N}})\}}\right)}{\log\left(\frac{\theta_{1N}(1+I_{\theta_{1N}})}{\theta_{0N}(1+I_{\theta_{0N}})}\right) - \log\left(\frac{1-\{\theta_{1N}(1+I_{\theta_{1N}})\}}{1-\{\theta_{0N}(1+I_{\theta_{0N}})\}}\right)}. \quad (4)$$

The neutrosophic upper limit, say $r_{mN} \in [r_{mL}, r_{mU}]$ of the proposed test is given by

$$r_{mN} = \frac{\log\left(\frac{1-\beta_N}{\alpha_N}\right)}{\log\left(\frac{\theta_{1N}(1+I_{\theta_{1N}})}{\theta_{0N}(1+I_{\theta_{0N}})}\right) - \log\left(\frac{1-\{\theta_{1N}(1+I_{\theta_{1N}})\}}{1-\{\theta_{0N}(1+I_{\theta_{0N}})\}}\right)} + \frac{m_N \log\left(\frac{1-\{\theta_{0N}(1+I_{\theta_{0N}})\}}{1-\{\theta_{1N}(1+I_{\theta_{1N}})\}}\right)}{\log\left(\frac{\theta_{1N}(1+I_{\theta_{1N}})}{\theta_{0N}(1+I_{\theta_{0N}})}\right) - \log\left(\frac{1-\{\theta_{1N}(1+I_{\theta_{1N}})\}}{1-\{\theta_{0N}(1+I_{\theta_{0N}})\}}\right)}. \quad (5)$$

The proposed limits are the generalization of the limits under classical statistics. The proposed two limits under neutrosophic statistics reduce to limits under classical statistics when $I_{\theta_{0N}} = 0$ and $I_{\theta_{1N}} = 0$.

The testing process of the proposed test is stated as follows:

Step 1. State $H_0 : \theta_{0N} = \theta_{0N}(1 + I_{\theta_{0N}});\ I_{\theta_{0N}} \in [I_{\theta_{0L}}, I_{\theta_{0U}}]$ vs. the alternative hypothesis $\theta_{1N} = \theta_{1N}(1 + I_{\theta_{1N}});\ I_{\theta_{1N}} \in [I_{\theta_{1L}}, I_{\theta_{1U}}]$.

Step 2. Specify α_N and β_N and compute the values of $a_{mN} \in [a_{mL}, a_{mU}]$ and $r_{mN} \in [r_{mL}, r_{mU}]$.

Step 3. The decision-makers failed to reject H_0 if $S_{mN} < a_{mN}$ and reject H_0 if $S_{mN} \geq r_{mN}$. The decision-makers should continue to take samples if $a_{mN} < S_{mN} < r_{mN}$.

3. Application

The application of the proposed test will be given using the information obtained from the manufacturing industry. Suppose that θ_N is the proportion of defective items during the manufacturing process, $1 - \theta_N$ is the non-defective product and the manufacturer is uncertain about the defective and non-defective items with I_{θ_N}. Based on the information, the manufacturer may willing to adopt the new manufacturing process if $\theta_N \leq \theta_{0N}(1 + I_{\theta_{0N}})$; $I_{\theta_{0N}} \in [I_{\theta_{0L}}, I_{\theta_{0U}}]$ and reject $\theta_N \geq \theta_{1N}(1 + I_{\theta_{1N}})$; $I_{\theta_{1N}} \in [I_{\theta_{1L}}, I_{\theta_{1U}}]$ and he may in-decision when $\theta_{0N}(1 + I_{\theta_{0N}})$; $I_{\theta_{0N}} \in [I_{\theta_{0L}}, I_{\theta_{0U}}] < \theta_N < \theta_{1N}(1 + I_{\theta_{1N}})$; $I_{\theta_{1N}} \in [I_{\theta_{1L}}, I_{\theta_{1U}}]$. Suppose that $\theta_{0N} = 0.05$, $\theta_{1N} = 0.30$, $I_{\theta_{0U}} = 0.05$, $I_{\theta_{1N}} = 0.30$, $\alpha_N = 0.05$ and $\beta_N = 0.10$. For the given data, $a_{mN} \in [a_{mL}, a_{mU}]$ and $r_{mN} \in [r_{mL}, r_{mU}]$ are calculated by

$$a_{mN} = -0.9205 + 0.1800\, m_N,$$

$$r_{mN} = 1.1818 + 0.1800\, m_N.$$

The testing process of the proposed test is stated as follows:

Step 1. State $H_0 : \theta_{0N} = 0.0525$ vs. the alternative hypothesis $\theta_{1N} = 0.39$.

Step 2. Specify $\alpha_N = 0.05$ and $\beta_N = 0.10$ and compute the values of $a_{mN} = -0.9205 + 0.1800 m_N$ and $r_{mN} = 1.1818 + 0.1800 m_N$.

Step 3. The decision-makers failed to reject H_0 if $S_{mN} < -0.9205 + 0.1800 m_N$ and reject H_0 if $S_{mN} \geq 1.1818 + 0.1800 m_N$. The decision-makers should continue to take samples if $-0.9205 + 0.1800m_N < S_{mN} < 1.1818 + 0.1800m_N$.

The operational process of the proposed test for the real example is shown in Figure 1.

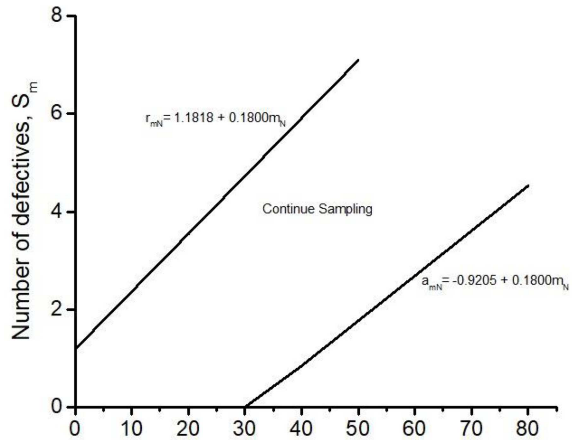

Figure 1. The operational process for the example.

4. Simulation

In this section, a simulation study is performed to check the effect of the measure of uncertainty/indeterminacy on the lower and upper limits of the proposed test. As mentioned earlier, two measures of uncertainty $I_{\theta_{0N}} \in [I_{\theta_{0L}}, I_{\theta_{0U}}]$ and $I_{\theta_{1N}} \in [I_{\theta_{1L}}, I_{\theta_{1U}}]$ are associated with null hypothesis and alternative hypothesis. In the simulation study, various values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ are considered. The values of a_{mN} and r_{mN} are reported in Table 1. From Table 1, it can be seen that the intercept of a_{mN} decreases as the values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ increase. On the other hand, the values of rate of change (slope) increase as the values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ increase. We also note the decreasing trends in intercept of r_{mN} as the values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ increase. The increasing trend is also noted in the slope of r_{mN} as the values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ increase. From the simulation study, it can be concluded that as $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ increase, the two limits become narrow and the change of continuing sampling is reduced.

Table 1. Effect of measure of uncertainty on limits

$I_{\theta_{0N}}$	$I_{\theta_{1N}}$	a_{mN}	r_{mN}
0	0	$-1.0735 + 0.1456\, m_N$	$1.3782 + 0.1456\, m_N$
0.01	0.02	$-1.0644 + 0.1482\, m_N$	$1.3665 + 0.1482\, m_N$
0.02	0.03	$-1.0625 + 0.1497\, m_N$	$1.3641 + 0.1497\, m_N$
0.03	0.04	$-1.0606 + 0.1512\, m_N$	$1.3617 + 0.1512\, m_N$
0.04	0.05	$-1.0588 + 0.1528\, m_N$	$1.3593 + 0.1528\, m_N$
0.05	0.06	$-1.0569 + 0.1543\, m_N$	$1.3569 + 0.1543\, m_N$
0.06	0.07	$-1.0550 + 0.1559\, m_N$	$1.3545 + 0.1559\, m_N$
0.07	0.08	$-1.0512 + 0.1590\, m_N$	$1.3497 + 0.1590\, m_N$
0.09	0.10	$-1.0493 + 0.1605\, m_N$	$1.3472 + 0.1605\, m_N$

5. Comparative Study

The proposed sequential test for the Bernoulli distribution is the extension of the existing test under classical statistics. The proposed test reduces to the classical test when $I_{\theta_{0N}} = 0$ and $I_{\theta_{1N}} = 0$. In this section, the comparison of the proposed test is given with the existing test in terms of a measure of uncertainty. The limits of the classical test when $I_{\theta_{0N}} = 0$ and $I_{\theta_{1N}} = 0$ are shown in Table 1. The limits for various values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ are also shown in Table 1. From Table 1, it can be seen that the gap between two limits reduces as the values of $I_{\theta_{0N}}$ and $I_{\theta_{1N}}$ increase. On the other hand, a big gap can be noted in limits when $I_{\theta_{0N}} = 0$ and $I_{\theta_{1N}} = 0$. From the study, it can be seen that as the gap between two limits decreases, it also decreases the chance of continued sampling. In addition, the comparison shows that there is a high chance of resampling in the case of the existing test. In nutshell, the proposed test may be more economic and time-saving as the decision may make on the basis of the first sample. Therefore, the proposed test is efficient, economical and adequate than the existing test.

6. Concluding Remarks

We introduced neutrosophic Bernoulli distribution and used it in designing the sequential test. The proposed test was found to be in more generalized form than the existing sequential test. The operational procedure of the proposed test was explained with the help of a diagram. The application of the proposed test was given using industrial data. From the comparison and simulation studies, the proposed test was found more efficient than the existing sequential test. The proposed test has the limitations that it can be applied only when the data is obtained from neutrosophic Bernoulli distribution. The proposed test using a cost model can be studied in future.

References

[1] M. Abdel-Basset, A. Atef and F. Smarandache, A hybrid neutrosophic multiple criteria group decision making approach for project selection, Cognitive Systems Research 57 (2019), 216-227.

[2] M. Abdel-Basset, M. Mohamed, M. Elhoseny, F. Chiclana and A. E.-N. H. Zaied, Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases, Artificial Intelligence in Medicine 101 (2019), 101735.

[3] M. Abdel-Basset, N. A. Nabeeh, H. A. El-Ghareeb and A. Aboelfetouh, Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises, Enterprise Information Systems 14 (2020), 1304-1324.

[4] M. Aslam, Introducing Kolmogorov-Smirnov Tests under Uncertainty: An Application to Radioactive Data, ACS Omega, 2019a.

[5] M. Aslam, Neutrosophic analysis of variance: application to university students, Complex and Intelligent Systems 5(4) (2019b), 403-407.

[6] M. Aslam, Design of the Bartlett and Hartley tests for homogeneity of variances under indeterminacy environment, Journal of Taibah University for Science 14(1) (2020), 6-10.

[7] M. Aslam, Radar data analysis in the presence of uncertainty, European Journal of Remote Sensing 54(1) (2021a), 140-144.

[8] M. Aslam, Testing average wind speed using sampling plan for Weibull distribution under indeterminacy, Scientific Reports 11(1) (2021b), 1-9.

[9] S. Bacanli and D. Icen, Sequential probability ratio test of correlation coefficient using fuzzy hypothesis testing, Open Journal of Statistics 3 (2013), 195-199.

[10] J. Bartroff and J. Song, Sequential tests of multiple hypotheses controlling type I and II familywise error rates, J. Statist. Plann. Inference 153 (2014), 100-114.

[11] S. Broumi, M. Talea, A. Bakali, F. Smarandache and K. Ullah, Bipolar neutrosophic minimum spanning tree, Smart Application and Data Analysis for Smart Cities (SADASC'18), 2018. Available at SSRN: https://ssrn.com/abstract=3127519.

[12] S. Broumi and F. Smarandache, Correlation coefficient of interval neutrosophic set, Applied Mechanics and Materials 436 (2013), 511-517.

[13] B. D. Causey, Exact calculations for sequential tests based on Bernoulli trials, Comm. Statist. Simulation Comput. 14(2) (1985), 491-495.

[14] J. Chen, J. Ye and S. Du, Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics, Symmetry 9(10) (2017), 208.

[15] J. Chen, J. Ye, S. Du and R. Yong, Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers, Symmetry 9(7) (2017), 123.

[16] N. Chukhrova and A. Johannssen, Nonparametric fuzzy hypothesis testing for quantiles applied to clinical characteristics of COVID-19, International Journal of Intelligent Systems 36(6) (2021), 2922-2963.

[17] T. Denoeux, M.-H. Masson and P.-A. Hébert, Nonparametric rank-based statistics and significance tests for fuzzy data, Fuzzy Sets and Systems 153(1) (2005), 1-28.

[18] D. Dubois and H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Inform. Sci. 30(3) (1983), 183-224.

[19] G. Gardonyi and K. Samu, An enhanced evaluation method of sequential probability ratio test, Math. Probl. Eng. Volume 2019, Article ID 4724507. https://doi.org/10.1155/2019/4724507.

[20] P. Grzegorzewski, Statistical inference about the median from vague data, Control Cybernet. 27 (1998), 447-464.

[21] P. Grzegorzewski, *k*-sample median test for vague data, International Journal of Intelligent Systems 24(5) (2009), 529-539.

[22] P. Grzegorzewski and M. Śpiewak, The sign test and the signed-rank test for interval-valued data, International Journal of Intelligent Systems 34(9) (2019), 2122-2150.

[23] D. İçen, S. Bacanlı and S. Günay, Fuzzy approach for group sequential test, Advances in Fuzzy Systems Volume 2014, Article ID 896150. https://doi.org/10.1155/2014/896150.

[24] J. Kacprzyk, E. Szmidt, S. Zadrożny, K. T. Atanassov and M. Krawczak, Advances in fuzzy logic and technology 2017, Proceedings of: EUSFLAT-2017-The 10th Conference of the European Society for Fuzzy Logic and Technology, September 11-15, 2017, Warsaw, Poland IWIFSGN'2017-The Sixteenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets, September 13-15, 2017, Warsaw, Poland, Springer, Vol. 2, 2017.

[25] C. Kahraman, C. E. Bozdag, D. Ruan and A. F. Özok, Fuzzy sets approaches to statistical parametric and nonparametric tests, International Journal of Intelligent Systems 19(11) (2004), 1069-1087.

[26] G. K. Kanji, 100 Statistical Tests, Sage Publishing, 2006.

[27] N. A. Nabeeh, F. Smarandache, M. Abdel-Basset, H. A. El-Ghareeb and A. Aboelfetouh, An integrated neutrosophic-TOPSIS approach and its application to personnel selection: a new trend in brain processing and analysis, IEEE Access 7 (2019), 29734-29744.

[28] J. Pan, Y. Li and V. Y. Tan, Asymptotics of sequential composite hypothesis testing under probabilistic constraints, 2021. arXiv preprint arXiv:2106.00896.

[29] A. Parchami, Fuzzy decision in testing hypotheses by fuzzy data: two case studies, Iran. J. Fuzzy Syst. 17(5) (2020), 127-136.

[30] X. Peng and J. Dai, Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function, Neural Computing and Applications 29(10) (2018), 939-954.

[31] C. A. D. B. Pereira, E. Y. Nakano, V. Fossaluza, L. G. Esteves, M. A. Gannon and A. Polpo, Hypothesis tests for Bernoulli experiments: ordering the sample space by Bayes factors and using adaptive significance levels for decisions, Entropy 19(12) (2017), 696.

[32] S. Pramanik, V. E. Johnson and A. Bhattacharya, A modified sequential probability ratio test, J. Math. Psych. 101 (2021), 102505.

[33] M. Shafiq, M. Atif and R. Viertl, Generalized likelihood ratio test and Cox's *F*-test based on fuzzy lifetime data, International Journal of Intelligent Systems 32(1) (2017), 3-16.

[34] A. Shahin, Y. Guo, K. Amin and A. A. Sharawi, A novel white blood cells segmentation algorithm based on adaptive neutrosophic similarity score, Health Information Science and Systems 6(1) (2018), 1.

[35] F. Smarandache, Neutrosophy Neutrosophic Probability, Set, and Logic, ProQuest Information and Learning, Ann. Arbor, Michigan, USA, Vol. 105, 1998, pp. 118-123.

[36] F. Smarandache, Introduction to Neutrosophic Statistics, Sitech and Education Publisher, Romania-Educational Publisher, Craiova, Columbus, Ohio, USA, 2014.

[37] S. Taheri and G. Hesamian, A generalization of the Wilcoxon signed-rank test and its applications, Statist. Papers 54(2) (2013), 457-470.

[38] S. M. Taheri and M. Arefi, Testing fuzzy hypotheses based on fuzzy test statistic, Soft Computing 13(6) (2009), 617-625.

[39] S. M. Taheri and G. Hesamian, Non-parametric statistical tests for fuzzy observations: fuzzy test statistic approach, International Journal of Fuzzy Logic and Intelligent Systems 17(3) (2017), 145-153.

[40] R. Talukdar and H. K. Baruah, A two sample sequential *t*-test with fuzzy observations, Appl. Math. Sci. 4(68) (2010), 3361-3374.

[41] E. A. Thomas, A note on the sequential probability ratio test, Psychometrika 40(1) (1975), 107-111.

[42] H. Torabi and J. Behboodian, Sequential probability ratio test for fuzzy hypotheses testing with vague data, Austrian Journal of Statistics 34(1) (2005), 25-38.

[43] H. Torabi and S. Mirhosseini, Sequential probability ratio tests for fuzzy hypotheses testing, Appl. Math. Sci. 3(33) (2009), 1609-1618.

[44] R. A. Wijsman, Examples of exponentially bounded stopping time of invariant sequential probability ratio tests when the model may be false, Vol. 1: Theory of Statistics, University of California Press, 1972, pp. 109-128.