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APPLICATION ARTICLE

Selection of waste water treatment plans alternative: a neutrosophy based MCDM approach

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Abstract

In the present context of environmental scenario, proper management of waste water from different sources is an emerging topic as the crisis of ground water is rudely seen from the last decade. This study addresses a multi-criteria decision making frame work on the selection of appropriate waste water treatment plans alternative in uncertain atmosphere. The approach merges Technique for Order Preference by Similarity to Ideal Solution method. To cultivate the indeterminacy and inconsistency on decision making with uncertainty more precisely, all the experimental data are designed in term of neutrosophic sets. Using a distance function proposed here and by practice of prospect theory, the degree of effectiveness of each treatment plan is evaluated. A user friendly algorithm is drawn to sketch that approach, and then it is demonstrated in practical field. The outcome is analysed and compared with the existing frame to validate the superiority of this work.

Keywords Neutrosophic opinion matrix · MCDM problem · TOPSIS method · Prospect theory · Efficiency of waste water treatment plan

1 Introduction

Utilization of water resources is necessary to keep the economical growth but consequently, the deterioration of environmental stability is not welcome. Waste water excreted from several domestic and industrial sources causes a rusty pollution of soil, usable water and green environment, even it may bring many severe water born diseases to human beings. Now if the authorities bring so many restrictions towards controlling the pollution, then the development of human society will not be in a well sequence. Hence, a best policy has to be adopted in order to make a

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balance in that concern. If there be a proper recycling process of waste water, then it will reward a huge renewable source and will be helpful for the improvement of regional economy as well as for the progress of mankind. Moreover, in developing countries, crisis of ground water is an emerging environmental issue as being seen from the last decade. Hence it is urgent to drive a proper management of waste water from different sources, so that it will be reused for the sake of benefit of human society. But the treatment of waste water in improper way may call a degradation of our natural environment rudely instead of being good. There are several parameters depending on which waste water plan selection is made. So, selection of a suitable treatment plan is a very crucial job and it needs a critical scientific analysis.

In today's uncertain scenario, experts suffer from a lot of hesitations while making opinion on several issues. Insufficiency in the available information regarding any fact, lack of attention and raising target of expert, time bounds on works, etc compel the experts to undergo to a hesitation. Moreover between favour and disfavour of an opinion, there may be a neutral zone as seen in the result of sports game, casting of elector in a poll, decision making, etc. The Intuitionistic fuzzy (IF) set theory innovated by Atanassov [2] brought a scope to cultivate the hesitancy of experts in making decision. Therein an opinion is characterised by two dependent angles namely degree of truth and degree of falsity measured individually in [0,1] so that their sum also lies in [0,1]. Then, the hesitancy of expert's opinion on a fact is calculated as: 1 - (degree of truth + degree of falsity). To draw the hesitancy of experts more precisely in setting of opinion on a fact, Smarandache [20] introduced the concept of Neutrosophic set (*neuts*) theory. By *neuts* theory, expert's opinion on a fact is enlighten in three independent directions: degree of truth, degree of hesitancy and degree of falsity. Then experts feel comfort by making their opinions in a more flexible way and it results a fruitful conclusion of decision making in uncertain climate.

To deal with uncertainty in real fields, researchers made their attentions in several directions over fuzzy and IF climate. Majumdar and Sasikumar [15] designed a fuzzy optimization model for seasonal water quality management of river systems based on the varying river flow in different seasons. Two levels of uncertainty associated with low water quality and low risk for violation of water quality were quantified and incorporated there in a fuzzy probabilistic frame. The model provided a treatment of each pollutant to be independent in nature. Combined effect for the treatment of multiple relevant pollutants was not thought about here, and hence this framework is dependent on a specific class of pollutants. Li and Nan [14] and Zhou et al. [26] solved multi-attribute group decision making problems under IF atmosphere using TOPSIS technique. Thus experts' hesitancy in decision making were not treated independently in experimental data as seen in *neuts* theory and so a question on fair outcome in decision making with uncertainty is being raised. During decision making, experts may have different priority to the alternatives, which should be aggregated. Li and Nan [14] allowed weight $w_k(w_k \in [0,1]$ with $\sum w_k = 1)$ to the alternatives by arbitrary choice of experts. But the present study calculates the weight to be acted on alternative from the initial data provided and thus burden of extra data handling is reduced here (see the Step 7 of proposed algorithm). By two examples, Pavic and Novoselac [18] shown that the best

TOPSIS was neither closest to the positive ideal solution nor the farthest from the negative ideal solution and it was against the general theme of TOPSIS method. An overview on the developments of TOPSIS technique practiced in decision making by several researchers from the years 2000 to 2015 was made by Zavadskas et al. [25]. Dursun [7] proposed a fuzzy MCDM approach for the assessment of waste water treatment plans (wwt-plans) alternative using fuzzy linguistic representation model, decision making trial and fuzzy TOPSIS method. Hence the uncertainty was not measured precisely there by nature of fuzzy set. Moreover practice of linguistic terms may mislead the decision because the fuzzy number representing a particular linguistic variable may not be suitably everywhere, it may be somewhere narrow or may be broad. So, instead of use a rating scale for alternatives, it is better to focus on practical data setting. Geometric programming gives better result than other nonlinear programming. With this truth, Ghosh et al. [9] designed a goal geometric programming over IF environment for removal of maximum five day biochemical oxygen demand at minimum cost to purify waste water from pulp and paper manufacturing industry. A new approach in multiple attribute decision making (MADM) field namely prospective MADM was claimed by Zolfani et al. [27] and it has a wide field in ranking of several MADM methods like as TOPSIS, VIKOR, COPRAS, ARAS, WASPAS, etc. Then Choudhury et al. [6] investigated a performance regarding applicability of different strategies of operation and effectiveness of various microbial fuel cells to achieve cost effective bioelectricity generation from waste water. Liu et al. [11] developed a type-2 fuzzy programming method within a two-stage stochastic programming with recourse framework to address water pollution control of basin systems where chemical oxygen demand, total nitrogen, total phosphorus and soil loss were taken as major indicators. Two MCDM methods based on hybrid SWARA-WASPAS were proposed by Khodadadi et al. [13] to deploy different advance oxidation processes: ozonation, fenton, electrochemical oxidation, UV/Photo-catalysis, UV/H_2O_2 as an effective tool of chemical treatment for degradation of toxic from water in industrial processing plant. For a case study in Iran's steel industry, Mahjouri et al. [16] framed a MCDM technique based on AHP and TOPSIS in combination with fuzzy logic for optimal selection of iron and steel waste water treatment technology. In a MCDM process, AHP methodology is usually applied to establish the importance of assigning weights for associated criteria in defining overall goal and it is done by comparing the relative importance of the criteria pairwise. Clearly the assigned weights are pairwise dependent in nature and hence the scope to choose the weight preference on relevant criteria by experts in a flexible way is lost. That is why, the current study is developed based on TOPSIS method only. An IF set theory based group MADM method for sustainable measurement of waste water treatment was addressed by Ren and Liang [19]. This assessment was done with respect to four key aspects addressing environment, economy, society-politic, and technology dimensions along with its sub-criteria in term of some linguistic variables and corresponding IF numbers. So, the study had a limitation in view of the generalized case of wwt-plans, the sustainable measurement of wwt-plans precisely in uncertain practical field and the number of criteria with respect to which this measurement was done. For decision making trial, Xing et al. [24] proposed a ranking method of IF values based on the

Euclidean distance notion with a principle: closer the IF value is as to the most favorable IF value, higher the ranking of IF value is. A critical path problem was designed by Mehlawat and Grover [17] as a multi-criteria group decision making in IF sense to evaluate the critical path. Therein, the quantitative and qualitative assessments of the decision makers were performed in light of both preferences and non-preferences simultaneously using cut set of triangular IF numbers. Because of having limitation in decision making over fuzzy and IF state, researchers took care it over *neuts* theory. The structure of matrix and its characteristics was extended by Bera and Mahapatra [3] in soft and neutrosophic sense. Using this matrix structure, a decision making algorithm was also drawn therein. In pandemic period, to determine the priority groups in society for allocating available COVID-19 vaccine, Hezam et al. [10] brought a neutrosophic MCDM approach using AHP and TOPSIS based on four main-criteria: age, health status, a woman's status, the type of job and fifteen sub-criteria. Therein AHP was used to evaluate the weights of both main and sub-criteria, and then the TOPSIS method found the rank of COVID-19 vaccine alternatives in the early stage. Bera and Mahapatra [4] later established the ranking of a number of coal based thermal power plants based on the emission rate of harmful air pollutants and this emission rate was assessed in neutrosophic sense to use the hesitancy of experts more precisely in decision making with uncertainty. More MCDM techniques were developed in neutrosophic platform by several researchers for instance Elhassouny and Smarandache [8], Karasan and Kahraman [12], Tian et al. [21], Abdel-Baset et al. [1]. But these developments had some limitations like use of linguistic terms for criteria and alternatives, single expert treated decision making i.e., not cross verified the alternatives by several experts etc.

The present study addresses a MCDM frame work on the selection of sustainable wwt-plans alternative based on TOPSIS with uncertainty modelling. It will bring a healthy competition among wwt-plans to do good for human beings. The chosen alternative here admits the shortest distance from the best solution and the longest distance from the worst solution. A group of experts initially makes their opinion on the treatment plans to be assessed with respect to a set of relevant parameters. The opinions are all put in term of *neuts* to promote the indeterminacy and inconsistency on decision making more precisely in uncertain climate. A neutrosophic weight preference is also imposed on each parameter by individual expert to put its importance on the entire performances. Applying TOPSIS technique and prospect theory, the degree of effectiveness of each treatment plan is finally evaluated. The approach is sketched by developing a user friendly algorithm and then it is demonstrated in practical field. The outcome is analysed and compared with the existing frame to validate the superiority of present study. The work is organised in the following manner.

After recalling some necessary existing results in Sect. 2, the study develops some definitions and operations in Sect. 3. The proposed model is then framed by three phases in Sect. 4 and it is also sketched by a suitable algorithm. In Sect. 5, the algorithm is demonstrated step wise in two case studies. An analysis of the outcome from these particular case studies and on the experimental data setting ensure the potentiality of proposed work. A brief note on this study and its managerial impact, limitation, future aspects are all sought in Sect. 6.

2 Preliminary result

Let us recall the following existing results to be helpful to build the present study.

2.1 Definition (Atanassov, [2])

An object x of the universe U is enlighten by two dependent characters i.e., degree of belonging ($\mu_{\widetilde{D}}$) and degree of non-belonging ($\nu_{\widetilde{D}}$) in the parlance of an intuitionistic fuzzy set $\widetilde{D}$. Thus $\widetilde{D}$ is put by the expression: $\widetilde{D} = \{x, (\mu_{\widetilde{D}}(x), \nu_{\widetilde{D}}(x)) : x \in U\}$ satisfying $\mu_{\widetilde{D}}(x) \in [0, 1], \nu_{\widetilde{D}}(x) \in [0, 1]$ and $0 \leq \mu_{\widetilde{D}}(x) + \nu_{\widetilde{D}}(x) \leq 1$. Then the quantity $1 - (\mu_{\widetilde{D}}(x) + \nu_{\widetilde{D}}(x))$ refers the degree of indeterminacy of object x in $\widetilde{D}$.

2.2 Definition (Tversky and Kahneman, [22])

Under prospect theory, decision makers compute a value (utility / score) based on the potential outcomes x_ks and their respective probabilities q_ks in the subsequent evaluation phase. The alternatives are then chosen having a higher utility. The expected utility of the outcomes to the decision made by individual is designed as:

$$V = \sum_{k=1}^{n} \zeta(q_k) S(x_k) \tag{1}$$

for $\zeta(q) = q^\gamma / \{q^\gamma + (1-q)^\gamma\}^{\frac{1}{\gamma}}$ being an increasing function of probability, is called the decision weight and $S(x)$ is the score function described as:

$$S(x) = \begin{cases} x^\eta & \text{if } x \geq 0 \\ -\xi(-x)^\tau & \text{if } x < 0. \end{cases}$$

γ, η, ξ, τ are the parameters chosen by experts according to subjective feelings.

2.3 Definition (Smarandache, [20])

An object $x \in U$ (U being universal set) is characterised by three independent components namely truth ($T_{\widetilde{Q}}$), hesitancy ($I_{\widetilde{Q}}$) and falsity ($F_{\widetilde{Q}}$) in a *neuts* $\widetilde{Q}$. These are real standard or non standard subset of $]^{-}0, 1^{+}[$.

Thus $\widetilde{Q}$ is designed as: $\widetilde{Q} = \{< x, (T_{\widetilde{Q}}(x), I_{\widetilde{Q}}(x), F_{\widetilde{Q}}(x)) >: x \in U\}$ with ${}^{-}0 \leq \sup T_{\widetilde{Q}}(x) + \sup I_{\widetilde{Q}}(x) + \sup F_{\widetilde{Q}}(x) \leq 3^{+}$. The concept was primarily designed in philosophical ground. To incorporate it in real life, the three components are restricted on [0, 1] only.

2.4 Definition (Wang et al., [23])

When three components of a *neuts* are standard subset of [0, 1] only, it is called single valued neutrosophic set (*SVneuts*). Thus a *SVneuts* $\widetilde{M}$ is presented as: $\widetilde{M} = \{< x, (T_{\widetilde{M}}(x), I_{\widetilde{M}}(x), F_{\widetilde{M}}(x)) >: x \in U\}$ and $T_{\widetilde{M}}(x), I_{\widetilde{M}}(x), F_{\widetilde{M}}(x) \in [0, 1]$ with $0 \leq \sup T_{\widetilde{M}}(x) + \sup I_{\widetilde{M}}(x) + \sup F_{\widetilde{M}}(x) \leq 3$.

2.5 Definition (Bera and Mahapatra, [3])

Let $\widetilde{M} = \{< \alpha, f_{\widetilde{M}}(\alpha) >: \alpha \in E\}$ be a neutrosophic soft set over (U, E) where $U = \{x_1, x_2, \ldots, x_m\}$ be the universal set of objects, $E = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be the set of parameters, and $f_{\widetilde{M}}(\alpha) = \{< x, (T_{f_{\widetilde{M}}(\alpha)}(x), I_{f_{\widetilde{M}}(\alpha)}(x), F_{f_{\widetilde{M}}(\alpha)}(x)) >: x \in U\}$. Clearly $f_{\widetilde{M}}(\alpha)$ corresponds a symmetric relation on $\{\alpha\} \times U$ for each $\alpha \in E$. The characteristic function of that relation is defined by a mapping $\{\alpha\} \times U \to [0, 1] \times [0, 1] \times [0, 1]$ and is given by $\chi_{f_{\widetilde{M}(\alpha)}}(x, \alpha) = (T_{f_{\widetilde{M}}(\alpha)}(x), I_{f_{\widetilde{M}}(\alpha)}(x), F_{f_{\widetilde{M}}(\alpha)}(x))$. Thus a matrix of order $m \times n$ can be drawn taking all its entries as characteristic function $\chi_{f_{\widetilde{M}(\alpha)}}(x_i, \alpha_j) = \widetilde{O}_{ij}$ over all objects and all parameters.

$$[\widetilde{O}_{ij}]_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \ldots & \alpha_n \end{array} \\ \begin{pmatrix} \widetilde{O}_{11} & \widetilde{O}_{12} & \ldots & \widetilde{O}_{1n} \\ \widetilde{O}_{21} & \widetilde{O}_{22} & \ldots & \widetilde{O}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{O}_{m1} & \widetilde{O}_{m2} & \ldots & \widetilde{O}_{mn} \end{pmatrix} \end{array}$$

In the present study, we shall use $[\widetilde{O}_{ij}]_{m\times n}$ as a neutrosophic opinion matrix.

3 Key results of present study

Here we develop some definitions in order to design the key line of present study.

3.1 Definition

Let $\widetilde{M} = \{x, (T_{\widetilde{M}}(x), I_{\widetilde{M}}(x), F_{\widetilde{M}}(x)) : x \in U\}$ and $\widetilde{P} = \{x, (T_{\widetilde{P}}(x), I_{\widetilde{P}}(x), F_{\widetilde{P}}(x)) : x \in U\}$ be two *SVneuts* over the universe U. Then their multiplication is denoted by $\widetilde{M} \otimes \widetilde{P}$ and is defined for all $x \in U$ as:

$$\widetilde{M} \otimes \widetilde{P} = \{x, (T_{\widetilde{M}}(x)T_{\widetilde{P}}(x), I_{\widetilde{M}}(x) + I_{\widetilde{P}}(x) - I_{\widetilde{M}}(x)I_{\widetilde{P}}(x), F_{\widetilde{M}}(x) + F_{\widetilde{P}}(x) - F_{\widetilde{M}}(x)F_{\widetilde{P}}(x))\}$$

3.2 Definition

Let a class of m objects be assessed by an expert with respect to n indicators. Based on the nature of indicators, expert provides an initial neutrosophic opinion matrix $OE = [\widetilde{O}_{ij}]_{m\times n} = [(T_{ij}, I_{ij}, F_{ij})]_{m\times n}$ to the objects and also a neutrosophic weight preference $\rho = \{\tilde{\rho}_1, \tilde{\rho}_2, \ldots, \tilde{\rho}_n\} = \{(u_1, v_1, w_1), (u_2, v_2, w_2), \ldots, (u_n, v_n, w_n)\}$ to be imposed on n indicators during assessment, for all $(u_n, v_n, w_n) \in [0,1] \times [0,1] \times [0,1]$ with $u_n + v_n + w_n \in [0, 3]$. Then the weighted neutrosophic opinion matrix of expert is defined as:

$$\begin{aligned} &\rho \otimes OE = [\tilde{\rho}_1 \otimes \widetilde{O}_{i1}, \tilde{\rho}_2 \otimes \widetilde{O}_{i2}, \ldots, \tilde{\rho}_n \otimes \widetilde{O}_{in}]_{m\times n} \text{ where} \\ &\tilde{\rho}_n \otimes \widetilde{O}_{in} = (\tilde{\rho}_n \otimes \widetilde{O}_{1n}, \tilde{\rho}_n \otimes \widetilde{O}_{2n}, \ldots, \tilde{\rho}_n \otimes \widetilde{O}_{mn})^t \text{ for all } n \text{ and} \\ &\tilde{\rho}_n \otimes \widetilde{O}_{mn} = (u_n, v_n, w_n) \otimes (T_{mn}, I_{mn}, F_{mn}) \text{ for all } m \\ &\qquad = (u_n T_{mn}, v_n + I_{mn} - v_n I_{mn}, w_n + F_{mn} - w_n F_{mn}) \end{aligned} \tag{2}$$

3.3 Definition

For a neutrosophic opinion matrix $OE = [\widetilde{O}_{ij}]_{m\times n} = [(T_{ij}, I_{ij}, F_{ij})]_{m\times n}$, its positive ideal solution is defined as:

$$\begin{aligned} &OE^* = \{(T_1^*, I_1^*, F_1^*), (T_2^*, I_2^*, F_2^*), \ldots, (T_n^*, I_n^*, F_n^*)\} \text{ where} \\ &T_n^* = \max_{1\le i\le m}\{T_{in}\}, I_n^* = \min_{1\le i\le m}\{I_{in}\}, F_n^* = \min_{1\le i\le m}\{F_{in}\} \text{ for all } n \end{aligned} \tag{3}$$

The negative ideal solution for opinion matrix OE is designed as:

$$\begin{aligned} &OE^\diamond = \{(T_1^\diamond, I_1^\diamond, F_1^\diamond), (T_2^\diamond, I_2^\diamond, F_2^\diamond), \ldots, (T_n^\diamond, I_n^\diamond, F_n^\diamond)\} \text{ where} \\ &T_n^\diamond = \min_{1\le i\le m}\{T_{in}\}, I_n^\diamond = \max_{1\le i\le m}\{I_{in}\}, F_n^\diamond = \max_{1\le i\le m}\{F_{in}\} \text{ for all } n \end{aligned} \tag{4}$$

3.4 Definition

Let $\widetilde{M} = \{x, (T_{\widetilde{M}}(x), I_{\widetilde{M}}(x), F_{\widetilde{M}}(x)) : x \in U\}$ and $\widetilde{P} = \{x, (T_{\widetilde{P}}(x), I_{\widetilde{P}}(x), F_{\widetilde{P}}(x)) : x \in U\}$ be two *SVneuts* over the universe U. Then the Euclidean distance between $\widetilde{M}$ and $\widetilde{P}$ is drawn as:

$$d(\widetilde{M}, \widetilde{P}) =$$

$$\sqrt{\langle \frac{1}{3n} \sum_{k=1}^{n} [\{T_{\widetilde{M}}(x_k) - T_{\widetilde{P}}(x_k)\}^2 + \{I_{\widetilde{M}}(x_k) - I_{\widetilde{P}}(x_k)\}^2 + \{F_{\widetilde{M}}(x_k) - F_{\widetilde{P}}(x_k)\}^2] \rangle}$$

3.5 Definition

Let $OE = [(T_{ij}, I_{ij}, F_{ij})]_{m\times n}$ be a neutrosophic opinion matrix formed after assessment of m objects in the parlance of n indicators by an expert. Suppose $OE^* = \{(T_1^*, I_1^*, F_1^*), (T_2^*, I_2^*, F_2^*), \ldots, (T_n^*, I_n^*, F_n^*)\}$ and $OE^\diamond = \{(T_1^\diamond, I_1^\diamond, F_1^\diamond), (T_2^\diamond, I_2^\diamond, F_2^\diamond), \ldots, (T_n^\diamond, I_n^\diamond, F_n^\diamond)\}$ be the respective positive and negative ideal solution of OE. Then the distance of m-th object (i.e., the assessment for m-th object in opinion matrix OE) to OE^* is designed as:

$$d_m^* = \sqrt{\left\{ \frac{1}{3} \sum_{j=1}^{n} [(T_{mj} - T_j^*)^2 + (I_{mj} - I_j^*)^2 + (F_{mj} - F_j^*)^2] \right\}} \tag{5}$$

The distance of m-th object to $OE^\diamond$ is directed as:

$$d_m^\diamond = \sqrt{\left\{ \frac{1}{3} \sum_{j=1}^{n} [(T_{mj} - T_j^\diamond)^2 + (I_{mj} - I_j^\diamond)^2 + (F_{mj} - F_j^\diamond)^2] \right\}} \tag{6}$$

3.6 Definition

The distance between positive ideal solution OE^* and negative ideal solution $OE^\diamond$ for a neutrosophic opinion matrix OE is represented by:

$$D(OE^*, OE^\diamond) = \sqrt{\left\{ \frac{1}{3} \sum_{j=1}^{n} [(T_j^* - T_j^\diamond)^2 + (I_j^* - I_j^\diamond)^2 + (F_j^* - F_j^\diamond)^2] \right\}} \tag{7}$$

It tells, if D is high (small) then the deviation between positive and negative ideal solution is high (low). Suppose there be $k\,(> 1)$ number of experts appointed to assess the objects. Then the weight W_{E_l} of l-th expert $(1 \leq l \leq k)$ is calculated using the respective distance function D_l and it is expressed as follows.

$$W_{E_l} = \left(1 - \frac{D_l}{D_1 + D_2 + \ldots + D_l + \ldots + D_k}\right) / (k-1),\ (k > 1) \tag{8}$$

Obviously $W_{E_l} \in [0,1]$ with $\sum W_{E_l} = 1$. Thus smaller (larger) the deviation refers higher (lower) the expert's weight.

3.7 Definition

A MCDM problem in neutrosophic frame is represented by a matrix as:

$$[\widetilde{O}_{ij}]_{m\times n} = \begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array} \begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \dots & \alpha_n \end{array} \\ \begin{pmatrix} \widetilde{O}_{11} & \widetilde{O}_{12} & \dots & \widetilde{O}_{1n} \\ \widetilde{O}_{21} & \widetilde{O}_{22} & \dots & \widetilde{O}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{O}_{m1} & \widetilde{O}_{m2} & \dots & \widetilde{O}_{mn} \end{pmatrix} \end{array}$$

where $P_1, P_2, \dots, P_m$ are alternatives and $\alpha_1, \alpha_2, \dots, \alpha_n$ are predefined indicators. $\widetilde{O}_{ij}, 1 \le i \le m, 1 \le j \le n$ are the opinions (described by *SVneuts*) on alternatives made by an expert based on the indicators. In a MCDM problem, the key motive of expert is to assess the overall importance of the alternatives depending on some permissible norms. An initial evaluation of alternatives are generally done with respect to each indicator to draw some class of indicators specific priority scores which, in aggregation, provide the overall performances.

In the present study, a MCDM framework is designed to select the best wwt-plans alternative depending on some relevant criterions and each entry of the matrix is taken as *SVneuts* to include the hesitancy of expert precisely in making opinion.

4 Framework of proposed model

The present study innovates a group decision making model for the selection of best wwt-plan from a set of alternatives in neutrosophic atmosphere. The model is furnished based on TOPSIS method and it includes following phases.

Phase 1: To chose a number of wwt-plans to be evaluated its efficiency and appoint a panel of experts from different management fields related to this job. By consultation, experts will fix some relevant indicators depending on which the state of efficiency of plans will be evaluated.

Phase 2: To construct the opinion matrix on wwt-plans in neutrosophic sense and to set neutrosophic weight preferences which are imposed on each indicator by individual expert, to emphasise the importance of indicator on entire assessment.

Phase 3: To measure the efficiency of wwt-plans by use of TOPSIS method in uncertain climate. Using TOPSIS method, the level of closeness of each treatment plan to its positive and negative ideal solution for individual expert is calculated first in neutrosophic arena. Then the distance between positive and negative ideal solution is evaluated corresponding to each expert using a proposed distance function (7) and with the help of obtained distances, the importance of each expert for decision making is worked out. Thus a weighted matrix on group decision making (i.e., focusing the evaluation of all treatment plans by appointed experts at a glance) is

Table 1 Tabular form of opinion matrix OE_l

OE_l	α_1	α_2	$\cdots$	α_n
$\rho^l \rightarrow$	$\tilde{\rho}^l_{\alpha_1}$	$\tilde{\rho}^l_{\alpha_2}$	$\cdots$	$\tilde{\rho}^l_{\alpha_n}$
P_1	$(T^l_{11}, I^l_{11}, F^l_{11})$	$(T^l_{12}, I^l_{12}, F^l_{12})$	$\cdots$	$(T^l_{1n}, I^l_{1n}, F^l_{1n})$
P_2	$(T^l_{21}, I^l_{21}, F^l_{21})$	$(T^l_{22}, I^l_{22}, F^l_{22})$	$\cdots$	$(T^l_{2n}, I^l_{2n}, F^l_{2n})$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
P_m	$(T^l_{m1}, I^l_{m1}, F^l_{m1})$	$(T^l_{m2}, I^l_{m2}, F^l_{m2})$	$\cdots$	$(T^l_{mn}, I^l_{mn}, F^l_{mn})$

now drawn. Finally, the efficiency of each waste water treatment plan is evaluated by applying TOPSIS method and prospect theory on weighted matrix of group decision making. Now highest efficiency refers the best wwt-plan and least efficiency refers the worst wwt-plan.

4.1 Proposed algorithm

Suppose a set of $k\,(>1)$ number of experts $\{E_1, E_2, \dots, E_k\}$ are going to evaluate the benefit of a set of m number of wwt-plans $\{P_1, P_2, \dots, P_m\}$ over a set of n number of indicators $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$. Based on the nature of indicators, experts put their opinions (given by *neuts*) to each plan. There be a set of k opinion matrices $\{OE_1, OE_2, \dots, OE_k\}$ given by k number of experts where $OE_l = [(T^l_{ij}, I^l_{ij}, F^l_{ij})]_{m\times n}$ for $1 \le i \le m$, $1 \le j \le n, 1 \le l \le k$. Also there be a set of k number of neutrosophic weight preferences $\{\rho^1, \rho^2, \dots, \rho^k\}$ imposed on the indicators by k number of experts where $\rho^l = \{\tilde{\rho}^l_{\alpha_1}, \tilde{\rho}^l_{\alpha_2}, \dots, \tilde{\rho}^l_{\alpha_n}\}$ be assigned by expert $E_l, 1 \le l \le k$ and $\tilde{\rho}^l_{\alpha_j} = (u^l_{\alpha_j}, v^l_{\alpha_j}, w^l_{\alpha_j})$ for all $1 \le j \le n$. The tabular form of opinion matrix $OE_l, 1 \le l \le k$ is displayed below (Table 1).

Following steps we will consider here.

Step 1 Calculate weighted opinion matrix $\overline{OE_l} = \rho^l \otimes OE_l$ for all l.

Step 2 Extract positive and negative ideal solution $\overline{OE}^*_l, \overline{OE}^\diamond_l$ from weighted opinion matrix $\overline{OE_l}$ for all l based on parameters.

Step 3 Find the distances d^{l*}_i and $d^{l\diamond}_i$ of $\overline{OE_l}$ to $\overline{OE}^*_l, \overline{OE}^\diamond_l$ respectively for all l and for all i. Arrange these in the following tabular form (Table 2):

Table 2 Distance of $\overline{OE_l}$ to $\overline{OE}^*_l, \overline{OE}^\diamond_l$

	E_1	E_2	$\cdots$	E_k
P_1	$d^{1*}_1, d^{1\diamond}_1$	$d^{2*}_1, d^{2\diamond}_1$	$\cdots$	$d^{k*}_1, d^{k\diamond}_1$
P_2	$d^{1*}_2, d^{1\diamond}_2$	$d^{2*}_2, d^{2\diamond}_2$	$\cdots$	$d^{k*}_2, d^{k\diamond}_2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
P_m	$d^{1*}_m, d^{1\diamond}_m$	$d^{2*}_m, d^{2\diamond}_m$	$\cdots$	$d^{k*}_m, d^{k\diamond}_m$

Step 4 Calculate the level of closeness p_{il} of the plan P_i to $\overline{OE}_l^*, \overline{OE}_l^\diamond$ for the expert E_l by: $p_{il} = \frac{d_i^{l\diamond}}{d_i^{l*}+d_i^{l\diamond}}$ for $1 \le i \le m$ and $1 \le l \le k$.
Step 5 Construct a matrix OE_d of order $m \times k$ whose entries are p_{il} s for $1 \le i \le m$ and $1 \le l \le k$.

$$OE_d = [p_{il}]_{m\times k} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mk} \end{pmatrix}$$

Step 6 Find the distance D_l between positive and negative ideal solution $\overline{OE}_l^*, \overline{OE}_l^\diamond$ obtained from weighted opinion matrix $\overline{OE}_l$ for all l.
Step 7 Calculate the weight W_{E_l} of each expert using D_l obtained in Step 6 for all l and the expression (8).
Step 8 Multiply the weight W_{E_l} with the level of closeness p_{il} in the matrix OE_d for $1 \le l \le k$ and name the new matrix as $\overline{OE}_d$. Thus, for $\overline{p}_{il} = W_{E_l} \times p_{il}$,

$$\overline{OE}_d = [\overline{p}_{il}]_{m\times k} = \begin{pmatrix} \overline{p}_{11} & \overline{p}_{12} & \cdots & \overline{p}_{1k} \\ \overline{p}_{21} & \overline{p}_{22} & \cdots & \overline{p}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{p}_{m1} & \overline{p}_{m2} & \cdots & \overline{p}_{mk} \end{pmatrix}$$

Step 9 Extract positive and negative ideal solution $\overline{OE}_d^*, \overline{OE}_d^\diamond$ from $\overline{OE}_d$ as follows:

$$\overline{OE}_d^* = \{\max_l\{\overline{p}_{1l}\}, \max_l\{\overline{p}_{2l}\}, \dots, \max_l\{\overline{p}_{ml}\}\}$$
$$\overline{OE}_d^\diamond = \{\min_l\{\overline{p}_{1l}\}, \min_l\{\overline{p}_{2l}\}, \dots, \min_l\{\overline{p}_{ml}\}\}$$

Step 10 Calculate the distance λ_i^* and $\lambda_i^\diamond$ of $\overline{OE}_d$ to its respective positive and negative ideal solution $\overline{OE}_d^*, \overline{OE}_d^\diamond$ obtained in Step 9 as follows:

$$\lambda_i^* = \sqrt{\sum_{l=1}^{k}(\overline{p}_{il} - \max_l\{\overline{p}_{il}\})^2}, \quad 1 \le i \le m$$
$$\lambda_i^\diamond = \sqrt{\sum_{l=1}^{k}(\overline{p}_{il} - \min_l\{\overline{p}_{il}\})^2}, \quad 1 \le i \le m$$

Step 11 Evaluate the score functions by following expressions.

$$S^\diamond(\lambda_i^*) = -\xi(\lambda_i^*)^\tau, \quad S^*(\lambda_i^\diamond) = (\lambda_i^\diamond)^\eta$$

Step 12 Determine the effectiveness $B(P_i)$ of wwt-plan $P_i, 1 \le i \le m$:

$$B(P_i) = |S^*(\lambda_i^\diamond)|/|S^\diamond(\lambda_i^*)|$$

4.2 Overview of methodology

Select the wwt-plans to be evaluated its efficiencies, and then appoint a number of experts to drive that job.

⇓

Fix some relevant indicators with respect to which the evaluation will be performed.

⇓

Construct neutrosophic opinion matrix to the plans and also set neutrosophic weight preferences on the assumed indicators for individual expert.

⇓

Calculate weighted opinion matrices, and then find the level of closeness of each plan to its positive and negative ideal solution for individual expert using TOPSIS method.

⇓

Determine the distance between positive and negative ideal solution for each weighted opinion matrix. Using this, work out importance of experts for decision making process and then construct a weighted matrix on group decision making.

⇓

Apply TOPSIS approach first and then prospect theory to assess the efficiency of plans.

4.3 Impact of present study

The decision making approach for wwt-plan selection drawn in this study and the entire outcome are effective, realistic in the present consequences. This design enlightens the following reasonable features.

(1) The decision making framework and its parameter design are all timeliness for real situation.
(2) The experimental data of all parameters are put in term of *SVneuts*. This brings a scope to the experts to exercise their hesitancy independently in setting of experimental data, and thus experts' opinions for the assessment of plans can be made in a more flexible way.
(3) A weight preference of parameters is considered by each expert to execute the degree of importance of parameters on the decision making and these preferences are further taken as *SVneuts* instead of conventional way (i.e., as normalised weight) to emphasize the hesitancy of expert in uncertain climate.
(4) In decision making process, different priority given by experts to the alternatives is generally aggregated. Instead of choice the priority on the alternatives arbitrarily by the experts, the present study finds the weight to be acted on alternative from the initial data provided and thus burden of extra data handling is reduced here (see the Step 7 of proposed algorithm).
(5) Moreover the practice of prospect theory provides an opportunity to the experts to drive the strategy in the demand of situation arisen.

5 Demonstration of proposed model

The proposed model of decision making on selection of wwt-plans alternative is here demonstrated in practical ground. For that, two case studies are brought. The first one shows that experts have a scope of flexibility for making opinion on wwt-plans under neutrosophic environment. Second one brings the comparison of outcomes between neutrosophic and IF sense. Finally, by analyzing the state of experimental data and the outcomes, a proper validation on the necessity to perform the decision making in neutrosophic platform are drawn to have a realistic and fruitful result.

5.1 Case study 1

Let us consider three wwt-plans $\{P_1, P_2, P_3\}$ and there are three experts $\{E_1, E_2, E_3\}$ deployed to choice the best plan depending on three parameters $\{\alpha_1, \alpha_2, \alpha_3\}$. The experts come from the area of researcher in waste water treatment plan, environment protection, financial investment respectively. The parameters are described as financial benefit (α_1: the benefit from initial investment in comparison to others, various operational and maintenance cost like the cost for energy consumption, staff salary or labour charges, recovery cost for machinery system disturbances, etc), availability of management (α_2: the availability of land to initiate the plant, energy supply and resource of labour to drive and monitoring

Table 3 Opinion given by E_1 on wwt-plans

OE_1	α_1	α_2	α_3
P_1	(0.68, 0.28, 0.32)	(0.56, 0.51, 0.38)	(0.62, 0.44, 0.27)
P_2	(0.81, 0.37, 0.31)	(0.75, 0.52, 0.47)	(0.84, 0.54, 0.49)
P_3	(0.74, 0.63, 0.34)	(0.80, 0.55, 0.31)	(0.76, 0.43, 0.38)

Table 4 Opinion given by E_2 on wwt-plans

OE_2	α_1	α_2	α_3
P_1	(0.86, 0.31, 0.60)	(0.79, 0.61, 0.53)	(0.69, 0.60, 0.28)
P_2	(0.90, 0.19, 0.29)	(0.88, 0.40, 0.18)	(0.83, 0.38, 0.30)
P_3	(0.71, 0.29, 0.62)	(0.77, 0.40, 0.43)	(0.66, 0.46, 0.24)

Table 5 Opinion given by E_3 on wwt-plans

OE_3	α_1	α_2	α_3
P_1	(0.62, 0.52, 0.40)	(0.59, 0.61, 0.49)	(0.67, 0.50, 0.28)
P_2	(0.73, 0.54, 0.37)	(0.76, 0.27, 0.53)	(0.74, 0.42, 0.59)
P_3	(0.92, 0.26, 0.51)	(0.86, 0.40, 0.38)	(0.90, 0.33, 0.19)

Table 6 $\overline{OE_1} = \rho^1 \otimes OE_1$

$\overline{OE_1}$	α_1	α_2	α_3
P_1	(.3400, .4888, .4560)	(.2240, .6570, .4730)	(.3534, .6640, .4525)
P_2	(.4050, .5527, .4480)	(.3000, .6640, .5495)	(.4788, .7240, .6175)
P_3	(.3700, .7373, .4720)	(.3200, .6850, .4135)	(.4332, .6580, .5350)

Table 7 $\overline{OE_2} = \rho^2 \otimes OE_2$

$\overline{OE_2}$	α_1	α_2	α_3
P_1	(.3010, .4618, .7000)	(.3555, .7192, .6146)	(.2760, .6600, .4240)
P_2	(.3150, .3682, .4675)	(.3960, .5680, .3276)	(.3320, .4730, .4400)
P_3	(.2485, .4462, .7150)	(.3465, .5680, .5326)	(.2640, .5410, .3920)

Table 8 $\overline{OE_3} = \rho^3 \otimes OE_3$

$\overline{OE_3}$	α_1	α_2	α_3
P_1	(.2356, .6160, .4900)	(.1770, .7114, .5920)	(.1876, .6200, .4168)
P_2	(.2774, .6320, .4645)	(.2280, .4598, .6240)	(.2072, .5592, .6679)
P_3	(.3496, .4080, .5835)	(.2580, .5560, .5040)	(.2520, .4908, .3439)

the entire plant, scope of earlier recovery for any system disturbance or technical problem, transport communications, flexibility of plant, etc), scope of local sustainable development (α_3: the environmental refreshment of locality by less emission of waste or harmful gases, accessibility of usable water after treatment, degree of improvement of water quality, scope of employment in locality by appointing technical or non-technical persons and hence the improvement of socioeconomic status and life style of residents, development of local infrastructure facility by Govt. support like enhancement of road condition, transport communication, continuous power supply in locality, etc). Followings are the opinion of experts given by Tables 3, 4 and 5 on wwt-plans.

The neutrosophic weight preference on parameters by three experts are given as:

$$\rho^1 = \{\alpha_1(0.50, 0.29, 0.20),\ \alpha_2(0.40, 0.30, 0.15),\ \alpha_3(0.57, 0.40, 0.25)\}$$
$$\rho^2 = \{\alpha_1(0.35, 0.22, 0.25),\ \alpha_2(0.45, 0.28, 0.18),\ \alpha_3(0.40, 0.15, 0.20)\}$$
$$\rho^3 = \{\alpha_1(0.38, 0.20, 0.15),\ \alpha_2(0.30, 0.26, 0.20),\ \alpha_3(0.28, 0.24, 0.19)\}$$

The proposed algorithm is now applied stepwise below.

Step 1 The weighted opinion of three experts are displayed in Tables 6, 7 and 8 respectively.

For convenience of Table 6, the calculation of weighted opinion for plan P_1 is provided.

Table 9 $d_i^{l*}, d_i^{l\diamond}$ from $\overline{OE}_l$ to $\overline{OE}_l^*, \overline{OE}_l^\diamond$

	E_1	E_2	E_3
P_1	0.1046, 0.1821	0.2640, 0.0340	0.2314, 0.1561
P_2	0.1349, 0.1424	0.0277, 0.2719	0.2465, 0.1692
P_3	0.1562, 0.1240	0.2045, 0.1242	0.0883, 0.2793

Table 10 Table for level of closeness

		E_1	E_2	E_3
$OE_d = [p_{il}] =$	P_1	0.6352	0.1141	0.4028
	P_2	0.5135	0.9075	0.4070
	P_3	0.4425	0.3779	0.7598

$$\alpha_1 : (.68 \times .50, .28 + .29 - .28 \times .29, .32 + .20 - .32 \times .20) = (.3400, .4888, .4560)$$
$$\alpha_2 : (.56 \times .40, .51 + .30 - .51 \times .30, .38 + .15 - .38 \times .15) = (.2240, .6570, .4730)$$
$$\alpha_3 : (.62 \times .57, .44 + .40 - .44 \times .40, .27 + .25 - .27 \times .25) = (.3534, .6640, .4525)$$

Step 2 Followings are the positive and negative ideal solutions extracted from weighted opinion tables (Tables 6, 7, 8).

$$\overline{OE}_1^* = \{\alpha_1(.4050, .4888, .4480),\ \alpha_2(.3200, .6570, .4135),\ \alpha_3(.4788, .6580, .4525)\}$$
$$\overline{OE}_1^\diamond = \{\alpha_1(.3400, .7373, .4720),\ \alpha_2(.2240, .6850, .5495),\ \alpha_3(.3534, .7240, .6175)\}$$
$$\overline{OE}_2^* = \{\alpha_1(.3150, .3682, .4675),\ \alpha_2(.3960, .5680, .3276),\ \alpha_3(.3320, .4730, .3920)\}$$
$$\overline{OE}_2^\diamond = \{\alpha_1(.2485, .4618, .7150),\ \alpha_2(.3465, .7192, .6146),\ \alpha_3(.2640, .6600, .4400)\}$$
$$\overline{OE}_3^* = \{\alpha_1(.3496, .4080, .4645),\ \alpha_2(.2580, .4598, .5040),\ \alpha_3(.2520, .4908, .3439)\}$$
$$\overline{OE}_3^\diamond = \{\alpha_1(.2356, .6320, .5835),\ \alpha_2(.1770, .7114, .6240),\ \alpha_3(.1876, .6200, .6679)\}$$

To make out the fact, the calculation for parameter α_1 of $\overline{OE}_3^*$ and $\overline{OE}_3^\diamond$ are proceed here.

$$\overline{OE}_3^* : \max\{.2356, .2774, .3496\}, \min\{.6160, .6320, .4080\}, \min\{.4900, .4645, .5835\} = (.3496, .4080, .4645)$$
$$\overline{OE}_3^\diamond : \min\{.2356, .2774, .3496\}, \max\{.6160, .6320, .4080\}, \max\{.4900, .4645, .5835\} = (.2356, .6320, .5835)$$

Step 3 The distance $d_i^{l*}, d_i^{l\diamond}$ from $\overline{OE}_l$ to $\overline{OE}_l^*, \overline{OE}_l^\diamond$ are calculated in Table 9.

In the present problem, $\{d_1^{1*}, d_2^{1*}, d_3^{1*}\}$ is the set of distances from $\overline{OE}_1^*$ to the plans P_1, P_2, P_3 of $\overline{OE}_1$ respectively. Similarly, the set of distances between $\overline{OE}_1^\diamond$

and the plans P_1, P_2, P_3 of $\overline{OE}_1$ respectively is $\{d_1^{1\diamond}, d_2^{1\diamond}, d_3^{1\diamond}\}$ and so on. The calculation of two entries are executed to convince the Table 9.

$$\begin{aligned} d_1^{1*} =& \sqrt{\Big[\frac{1}{3}\{(.3400 - .4050)^2 + (.4888 - .4888)^2 + (.4560 - .4480)^2|_{\alpha_1}} \\ & + (.2240 - .3200)^2 + (.6570 - .6570)^2 + (.4730 - .4135)^2|_{\alpha_2} \\ & +(.3534 - .4788)^2 + (.6640 - .6580)^2 + (.4525 - .4525)^2|_{\alpha_3}\}\Big] \\ =& 0.1046 \end{aligned}$$

$$\begin{aligned} d_3^{1\diamond} =& \sqrt{\Big[\frac{1}{3}\{(.3700 - .3400)^2 + (.7373 - .7373)^2 + (.4720 - .4720)^2|_{\alpha_1}} \\ & + (.3200 - .2240)^2 + (.6850 - .6850)^2 + (.4135 - .5495)^2|_{\alpha_2} \\ & +(.4332 - .3534)^2 + (.6580 - .7240)^2 + (.5350 - .6175)^2|_{\alpha_3}\}\Big] \\ =& 0.1240 \end{aligned}$$

Step 4 and Step 5 The level of closeness $p_{il} = \frac{d_i^{l\diamond}}{d_i^{l*} \oplus d_i^{l\diamond}}$ is calculated in Table 10.

For instance, $p_{23} = \frac{d_2^{3\diamond}}{d_2^{3*} \oplus d_2^{3\diamond}} = \frac{0.1692}{0.2465+0.1692} = 0.4070.$

Step 6 The distance D_l between $\overline{OE}_l^*, \overline{OE}_l^\diamond$ is now evaluated as follows.

$$\begin{aligned} & D_1(\overline{OE}_1^*, \overline{OE}_1^\diamond) = 0.2178, \\ & D_2(\overline{OE}_2^*, \overline{OE}_2^\diamond) = 0.2733, \\ & D_3(\overline{OE}_3^*, \overline{OE}_3^\diamond) = 0.3095; \text{ and } \sum_l D_l = 0.8006 \end{aligned}$$

One data is worked out here.

$$\begin{aligned} & D_1(\overline{OE}_1^*, \overline{OE}_1^\diamond) \\ =& \sqrt{\Big[\frac{1}{3}\{(.4050 - .3400)^2 + (.4888 - .7373)^2 + (.4480 - .4720)^2|_{\alpha_1}} \\ & + (.3200 - .2240)^2 + (.6570 - .6850)^2 + (.4135 - .5495)^2|_{\alpha_2} \\ & +(.4788 - .3534)^2 + (.6580 - .7240)^2 + (.4525 - .6175)^2|_{\alpha_3}\}\Big] \\ =& 0.2178 \end{aligned}$$

Step 7 The weight $W_{E_l} = \frac{1}{2}(1 - \frac{D_l(\overline{OE}_l^*, \overline{OE}_l^\diamond)}{\sum D_l})$ of each expert is calculated using D_l as: $W_E = (W_{E_1}, W_{E_2}, W_{E_3}) = (0.3640, 0.3293, 0.3067)$.

Table 11 Table for $\overline{OE}_d$

	E_1	E_2	E_3
P_1	0.2312	0.0375	0.1235
P_2	0.1869	0.2988	0.1248
P_3	0.1611	0.1244	0.2330

Step 8 Now $\overline{OE}_d = [W_{E_l} \times p_{il}]$ is designed in Table 11.

Step 9 Then, $\overline{OE}_d^* = (0.2312, 0.2988, 0.2330)$ and $\overline{OE}_d^\diamond = (0.0375, 0.1248, 0.1244)$.

Step 10 The distance λ_i^* and $\lambda_i^\diamond$ of $\overline{OE}_d$ to its respective positive and negative ideal solution $\overline{OE}_d^*, \overline{OE}_d^\diamond$ are now calculated as follows:

$$\lambda_1^* = \sqrt{(0.2312 - 0.2312)^2 + (0.0375 - 0.2312)^2 + (0.1235 - 0.2312)^2} = 0.2216$$
$$\lambda_2^* = \sqrt{(0.1869 - 0.2988)^2 + (0.2988 - 0.2988)^2 + (0.1248 - 0.2988)^2} = 0.2069$$
$$\lambda_3^* = \sqrt{(0.1611 - 0.2330)^2 + (0.1244 - 0.2330)^2 + (0.2330 - 0.2330)^2} = 0.1302$$
$$\lambda_1^\diamond = \sqrt{(0.2312 - 0.0375)^2 + (0.0375 - 0.0375)^2 + (0.1235 - 0.0375)^2} = 0.2119$$
$$\lambda_2^\diamond = \sqrt{(0.1869 - 0.1248)^2 + (0.2988 - 0.1248)^2 + (0.1248 - 0.1248)^2} = 0.1847$$
$$\lambda_3^\diamond = \sqrt{(0.1611 - 0.1244)^2 + (0.1244 - 0.1244)^2 + (0.2330 - 0.1244)^2} = 0.1146$$

Step 11 The score functions are evaluated as follows for $\tau = \eta = 0.88$ and $\xi = 2.25$ (as in Tversky and Kahneman, [22]):

$$S^\diamond(\lambda_1^*) = -\xi(\lambda_1^*)^\tau = -0.5974, \quad S^*(\lambda_1^\diamond) = (\lambda_1^\diamond)^\eta = 0.2553$$
$$S^\diamond(\lambda_2^*) = -\xi(\lambda_2^*)^\tau = -0.5624, \quad S^*(\lambda_2^\diamond) = (\lambda_2^\diamond)^\eta = 0.2262$$
$$S^\diamond(\lambda_3^*) = -\xi(\lambda_3^*)^\tau = -0.3741, \quad S^*(\lambda_3^\diamond) = (\lambda_3^\diamond)^\eta = 0.1486$$

Step 12 The efficiency of wwt-plans is now determined as:

$$B(P_1) = 0.4274, \quad B(P_2) = 0.4022, \quad B(P_3) = 0.3972$$

Hence the waste water treatment plan P_1 is the best selection among three.

5.2 Comparison with existing method

1. Let us solve the problem (Case study 1) by the methodology stated in our previous work (Bera and Mahapatra, [3]). Therein the neutrosophic weight preferences on parameters were not considered. So we apply that methodology on Tables 3, 4 and 5. The final outcome is:

$$S(P_1, P_2, P_3) = \begin{pmatrix} -0.27 & -0.17 & -0.39 \\ 00.37 & 00.10 & -0.31 \\ -0.28 & -0.04 & 00.29 \end{pmatrix}, \quad \text{Total score} = \begin{pmatrix} -0.83 \\ 00.16 \\ -0.03 \end{pmatrix}$$

Thus the efficiency order is: $P_1 < P_3 < P_2$.

Table 12 Fused opinion matrix acted by experts' weight

	α_1	α_2	α_3
P_1	(0.743, 0.350, 0.421)	(0.663, 0.572, 0.458)	(0.660, 0.507, 0.276)
P_2	(0.829, 0.334, 0.320)	(0.806, 0.390, 0.356)	(0.811, 0.445, 0.441)
P_3	(0.812, 0.372, 0.469)	(0.812, 0.449, 0.368)	(0.794, 0.405, 0.264)

Table 13 Fused weighted opinion matrix

	α_1	α_2	α_3
P_1	(0.311, 0.503, 0.535)	(0.258, 0.692, 0.552)	(0.289, 0.629, 0.431)
P_2	(0.347, 0.491, 0.454)	(0.314, 0.561, 0.468)	(0.355, 0.583, 0.561)
P_3	(0.339, 0.520, 0.574)	(0.316, 0.604, 0.478)	(0.348, 0.553, 0.422)

Table 14 Final outcome of alternatives

Alternative	D^{i+}_{Eucl}	D^{i-}_{Eucl}	C^*_i	Order
P_1	0.1236	0.0789	0.3817	
P_2	0.0821	0.1296	0.6122	$P_1 < P_2 < P_3$
P_3	0.0759	0.1238	0.6199	

Table 15 Opinion on wwt-plans by 1st expert

OE_1	α_1	α_2	α_3	α_4
Q_1	(0.4, 0.3, 0.5)	(0.5, 0.5, 0.3)	(0.7, 0.2, 0.2)	(0.8, 0.2, 0.1)
Q_2	(0.6, 0.5, 0.4)	(0.3, 0.6, 0.5)	(0.6, 0.8, 0.1)	(0.5, 0.1, 0.3)
Q_3	(0.7, 0.1, 0.2)	(0.4, 0.5, 0.4)	(0.5, 0.4, 0.3)	(0.6, 0.2, 0.2)

Table 16 Opinion on wwt-plans by 2nd expert

OE_2	α_1	α_2	α_3	α_4
Q_1	(0.6, 0.5, 0.3)	(0.5, 0.6, 0.4)	(0.4, 0.5, 0.6)	(0.3, 0.4, 0.5)
Q_2	(0.8, 0.4, 0.1)	(0.4, 0.6, 0.5)	(0.4, 0.7, 0.3)	(0.5, 0.5, 0.3)
Q_3	(0.5, 0.7, 0.5)	(0.6, 0.3, 0.3)	(0.7, 0.1, 0.2)	(0.4, 0.4, 0.4)

Table 17 Opinion on wwt-plans by 3rd expert

OE_3	α_1	α_2	α_3	α_4
Q_1	(0.5, 0.4, 0.4)	(0.6, 0.3, 0.3)	(0.5, 0.6, 0.4)	(0.8, 0.2, 0.1)
Q_2	(0.6, 0.5, 0.3)	(0.5, 0.4, 0.5)	(0.5, 0.2, 0.5)	(0.7, 0.3, 0.2)
Q_3	(0.4, 0.4, 0.3)	(0.5, 0.2, 0.4)	(0.6, 0.3, 0.2)	(0.6, 0.5, 0.3)

2. Next let us note the TOPSIS methodology developed by Biswas et al. [5]. They used linguistic terms for rating of alternatives and attributes, to put the importance of experts in decision making process. Following this methodology, we shall now find the outcome of Case study 1 (taking also the calculated data for the importance of experts in decision making (see Step 7) i.e., no additional data is allowed for that).

Table 18 Table for $d_i^{l*}, d_i^{l\diamond}$

	E_1	E_2	E_3
Q_1	0.1921, 0.2693	0.3354, 0.1850	0.1841, 0.1652
Q_2	0.3259, 0.0802	0.3614, 0.2963	0.2073, 0.1716
Q_3	0.1203, 0.2928	0.2609, 0.3893	0.1407, 0.2102

Table 19 $\overline{OE}_d$ in neutrosophic sense

	E_1	E_2	E_3
Q_1	0.1975	0.1015	0.1778
Q_2	0.0668	0.1286	0.1703
Q_3	0.2398	0.1710	0.2252

Tables 3, 4 and 5 are now fused by acting experts' weight $(W_{E_1}, W_{E_2}, W_{E_3}) = (0.3640, 0.3293, 0.3067)$ using Eq. (17) of Biswas et al. [5] and is given by Table 12.

Also the fused weight preference of attributes acted on all alternatives is given by:

$(0.418, 0.236, 0.197)_{\alpha_1}, (0.389, 0.281, 0.174)_{\alpha_2}, (0.438, 0.248, 0.214)_{\alpha_3}$.

Following (Table 13) is the weighted opinion matrix.

Now using Eq. [(21), (25), (29)-(31)], the final outcome table (Table 14) is given as:

5.3 Case study 2

Let us now evaluate the efficiency of another three wwt-plans $\{Q_1, Q_2, Q_3\}$ in the parlance of four indicators $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ respectively sought for the degree of primary investment and maintenance cost, degree to meet the environmental aspect, availability of management, scope of employment. Followings (Tables 15, 16, 17) are the opinions of three experts on wwt-plans.

The weight preferences on the indicators by three experts are given as:

$$\rho^1 = \{\alpha_1(0.5, 0.2, 0.2),\ \alpha_2(0.4, 0.3, 0.1),\ \alpha_3(0.2, 0.4, 0.5),\ \alpha_4(0.3, 0.5, 0.6)\}$$
$$\rho^2 = \{\alpha_1(0.3, 0.2, 0.1),\ \alpha_2(0.4, 0.2, 0.3),\ \alpha_3(0.3, 0.1, 0.4),\ \alpha_4(0.5, 0.3, 0.2)\}$$
$$\rho^3 = \{\alpha_1(0.4, 0.3, 0.3),\ \alpha_2(0.5, 0.4, 0.2),\ \alpha_3(0.4, 0.4, 0.1),\ \alpha_4(0.3, 0.5, 0.4)\}$$

5.3.1 Outcome in neutrosophic environment

Table 18 provides the distance $d_i^{l*}, d_i^{l\diamond}$ from $\overline{OE}_l$ to $\overline{OE}_l^*, \overline{OE}_l^\diamond$.

The distances D_l between $\overline{OE}_l^*, \overline{OE}_l^\diamond$ for $l = 1, 2, 3$ are calculated as follows.

Table 20 $d_i^{l*}, d_i^{l\diamond}$ in IF sense

	E_1	E_2	E_3
Q_1	0.2159, 0.1838	0.3043, 0.1778	0.1746, 0.2015
Q_2	0.2285, 0.1296	0.1453, 0.3966	0.2941, 0.0917
Q_3	0.1285, 0.2269	0.3320, 0.2526	0.1487, 0.2739

Table 21 $\overline{OE}_d$ in IF sense

	E_1	E_2	E_3
Q_1	0.1657	0.1079	0.1859
Q_2	0.1304	0.2142	0.0825
Q_3	0.2301	0.1265	0.2248

$$D_1(\overline{OE}_1^*, \overline{OE}_1^\diamond) = 0.3580,$$
$$D_2(\overline{OE}_2^*, \overline{OE}_2^\diamond) = 0.4749,$$
$$D_3(\overline{OE}_3^*, \overline{OE}_3^\diamond) = 0.2747; \text{ and } \sum_l D_l = 1.1076$$

Using these D_l, the experts' weights are worked out as: $W_E = (W_{E_1}, W_{E_2}, W_{E_3}) = (0.3384, 0.2856, 0.3760)$. Table 19 states about $\overline{OE}_d$.

The score functions are evaluated as follows for $\tau = \eta = 0.88$ and $\xi = 2.25$ (as in Tversky and Kahneman, [22]):

$$S^\diamond(\lambda_1^*) = -0.2914, \quad S^*(\lambda_1^\diamond) = 0.1577$$
$$S^\diamond(\lambda_2^*) = -0.3267, \quad S^*(\lambda_2^\diamond) = 0.1553$$
$$S^\diamond(\lambda_3^*) = -0.2175, \quad S^*(\lambda_3^\diamond) = 0.1173$$

The benefit of wwt-plans is now determined as:

$$B(Q_1) = 0.5412, \quad B(Q_2) = 0.4754, \quad B(Q_3) = 0.5393$$

Hence plan Q_1 is the best selection.

5.3.2 Outcome in intuitionistic fuzzy atmosphere

In IF sense, all hesitancy parts in opinion matrices (Tables 15, 16, 17) and in neutrosophic weight preference on indicators are excluded. Table 20 tells about $d_i^{l*}, d_i^{l\diamond}$ in IF sense.

The distances D_l for $l = 1, 2, 3$ are calculated as follows.

$$D_1(\overline{OE}_1^*, \overline{OE}_1^\diamond) = 0.2888,$$
$$D_2(\overline{OE}_2^*, \overline{OE}_2^\diamond) = 0.4287,$$
$$D_3(\overline{OE}_3^*, \overline{OE}_3^\diamond) = 0.3167; \text{ and } \sum_l D_l = 1.0342$$

Table 21 refers $\overline{OE}_d$ in IF sense.

The benefit of wwt-plans in IF atmosphere is calculated as:

$$B(Q_1) = 0.5238, \quad B(Q_2) = 0.4042, \quad B(Q_3) = 0.5891$$

Hence, Q_3 is the best wwt-plan among three.

5.4 Validation of proposed model

On analysis, it is clear that each component of all experimental data in the 1st Case study (Tables 3, 4, 5 and weight preferences) is taken in independent way. The sum of truth, hesitancy and falsity parts for any experimental data is nowhere equal to 1. Moreover, the sum of truth and falsity component of some experimental data (like the all entries in the 2nd and 3rd rows of Table 3, 2nd row of Table 4 and others) are greater than 1. This is possible only in neutrosophic platform, not in IF ground. So, neither any conclusion of 1st Case study can be drawn using IF operations, nor the problem can be reduced in IF sense based on the data structure.

Also the work (Bera and Mahapatra, [3]) did not consider the weight preference of parameters. In real ground, all relevant parameters for a problem may not be effective generally in an equal degree.

Further the methodology of Biswas et al. [5] provides a clear deviation of wwt-plans' order. In the current study, we have used practical data instead of linguistic term as it is not fit suitably everywhere, may be narrow or broad somewhere. Moreover, the importance of expert is worked out from the initial data, no additional data is required on behalf of this. Another fact is that the methodology developed here allows the treatment of expert's opinion individually (i.e., not combined). Hence, the present development and its outcome is more efficient than earlier studies.

Table 22 Comparison for benefit of treatment plans at a glance

ground	nature	$B(Q_1)$	$B(Q_2)$	$B(Q_3)$	effective order
Intuitionistic fuzzy	membership and non-membership dependently	0.5238	0.4042	0.5891	$Q_2 < Q_1 < Q_3$
Neutrosophic	truth, hesitancy and falsity independently	0.5412	0.4759	0.5387	$Q_2 < Q_3 < Q_1$

Next, come to the 2nd Case study. Here, some of the experimental data (e.g., (Q_3, α_1), (Q_3, α_4) in Table 15, (Q_3, α_3) in Table 16, (ρ^2, α_4), (ρ^3, α_1) in weight preferences) are IF set as well as neutrosophic set because the sum of their components are equal to 1. Further, the sum of truth and falsity component of each experimental data is either less than or equal to 1. So the problem can be smoothly reduced into an IF structure by eliminating hesitant values. Now, let us note the final outcome (Table 22) in two different atmospheres for the 2nd Case study at a glance.

Table 22 executes the deviation of order in the effectiveness of wwt-plans based on the experimental data taken here for two atmospheres. May be the order as well as benefit score of wwt-plans in neutrosophic ground changed with the change of hesitant values. But as our motivation, in this study, is to run a decision making in uncertain climate, so it is intelligent to include the hesitancy of experts independently in setting of experimental data. In that perspective, the outcomes in neutrosophic environment for both case studies designed here are realistic and *neuts* theory is the generalised structure of IF set.

6 Conclusion

This study designs a group decision making approach for the selection of wwt-plans alternative in neutrosophic atmosphere. The subject line is one of the timeliness and burning environmental issue in the present situation. Based on a number of relevant criterion, a group of experts initially provide their individual opinion on the plans to be assessed. The opinions are made in term of neutrosophic sets to emphasize the experts' hesitancy independently in today's uncertain platform. A neutrosophic weight factor is also acted on each criteria by individual expert to prefer its degree of importance on the system. The entire methodology is based on TOPSIS technique and prospect theory. A suitable algorithm is developed to sketch that approach. The proposed methodology is demonstrated on three wwt-plans to execute its efficiency. A comparison of the outcome obtained in fuzzy, IF and neutrosophic ground is performed to draw a proper validation.

It is a proposed model. All the numerical data practiced here for the case study are not collected from original sources. These are all experimental data. A slight variation in data setting may call a diversion on the outcome. Hence, proper data setting from experts' corner is strongly expected. Moreover the case study is driven over only three wwt-plans and experts' opinion on the plans are taken with respect to three relevant criterion only. A full deviation on outcomes between the existing studies and the proposed methodology is found. So on all these angles, further research and development are wished. Also to get an instant outcome, a software based algorithm and flow chart are also welcome, we feel.

But the style of applying the TOPSIS approach in neutrosophic atmosphere presented here will bring an advancement of decision making in uncertain scenario, we hope. The model may be widely practiced on different MCDM in industrial sector, manufacturing system, corporate management, supply chain management, etc.

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Declarations

Conflict of interest Both authors declare that they have no any conflict of interest or no any relevant financial or non-financial competing interest or personal relationship that could have appeared to influence the work reported in this text.

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