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SPECIAL ISSUE

Multi-criteria decision making in linguistic values of neutrosophic trapezoidal fuzzy multi-numbers

K. Arun Prakash[1] · M. Suresh[1]

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Abstract
Neutrosophic trapezoidal fuzzy multi-numbers(NTFMN) are a particular case of neutrosophic fuzzy numbers on a real number set, which are used to predict the solution for multi-criteria decision making problems. In this paper we initiate the definition for NTFMN and formulate a multiple criteria decision-making problems in which the alternatives are NTFMN. Expected value is used for ranking neutrosophic trapezoidal fuzzy multi-numbers and for selection of best alternative. Finally, a numerical example is given to justify the practicality and efficiency of the proposed method.

Keywords Neutrosophic fuzzy set · Neutrosophic trapezoidal fuzzy number · Neutrosophic fuzzy multiset · Neutrosophic trapezoidal fuzzy multi-numbers

1 Introduction

The study of LPP has been of considerable interest ever since the birth of optimization techniques. Optimization techniques continue to be one of the most extensive techniques of operations research. Progress and development of these techniques, both in methodology and in applications, are growing. Innovative analytic treatments towards their theoretical development are being advanced and newer areas of application are emerging. In many real world problems, due to insufficiency of the data available, the evaluation of membership values is not possible up to one's satisfaction. In addition, the evaluation of non-membership values are not always possible and there remains an indeterministic part in which hesitation survives. A fuzzy number plays a vital role in the representation of unknown quantity. Fuzzy numbers are a special kind of fuzzy sets which are of importance in solving Fuzzy Linear Programming Problems (FLPPs).

Ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. In order to rank fuzzy numbers, one fuzzy number needs to be compared with the others, but it is difficult to determine clearly which of them is smaller or larger. Numerous methods have been proposed in literature to rank fuzzy numbers. As a generalization of fuzzy numbers, an IFN seems to fit more suitably to describe uncertainty. Many research works have been carried out in defining and studying IFNs. Further, several interesting properties of various types of IFNs are investigated. Recently, the research on IFNs has received high attention as it is more suitable for solving IFLPPs. Many ranking methods for ordering IFNs have been introduced by some researchers. In day-to-day life, many decisions are being made from various criteria, the decision can be made by providing weights to different criteria and all the weights are obtained from expert groups. It is important to determine the structure of the problem and evaluate multi criteria explicitly. There are not simply very complex issues involving multi criteria, some criteria may have an effect towards some problem. However on the whole to have an optimum solution, all the alternatives must have common criteria which clearly lead to more informed and better decisions. The classical decision making methods generally assume that all criteria and their respective weights are expressed in crisp values and, thus, that the rating and the ranking of the alternatives can be carried out without any problem. In a real-world decision situation, the application of the classical decision making method may face serious practical constraints from the criteria perhaps containing imprecision in the information. In many cases, the performance of the criteria can only be expressed qualitatively or by using linguistic terms, which certainly demands a more appropriate method. The most

✉ Suresh M
mathssuresh84@gmail.com

1 Department of Mathematics, Kongu Engineering College, Erode, Tamil Nadu, India

preferable situation for a decision making problem is when all the ratings of the criteria and their degree of importance are known precisely, which makes it possible to arrange them in a crisp ranking. However, many of the decision making problems in the real world take place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. As a result, the best condition for a classic decision making problem may not be satisfied when the decision situation involves both fuzzy and crisp data. The classical decision making methods cannot handle such problems effectively because they are suitable only for dealing with problems in which all the performances of the criteria are represented by crisp numbers. The application of the fuzzy set theory in the field of decision making is justified when the intended goals or their attainment cannot be defined or judged crisply, but only as fuzzy sets. The fuzzy multi criteria techniques have been applied in various fields such as banking sectors, issues such as urban distribution centers, water shed allocation, safety assessment, and performance evolution of business organizations. The use of fuzzy is to analyze the quantitative and qualitative data for any application. The different methods under FMCDM help to perform may subtasks between evaluation and ranking done by different methods. Each method has its own uniqueness. Many methods exist to handle fuzzy multi-criteria decision making problems. Multiple Attribute Decision Making (MADM) has been one of the fastest growing areas during the last decades depending on the changes in the business sector.

2 Literature review

Over the last two decades many problems involving vagueness and uncertainty are settled by the concept of Fuzzy Sets proposed by Zadeh [29] and its extension given by Attanassov as Intuitionistic Fuzzy Sets [2]. Later, a neutral membership function was added to Intuitionistic Fuzzy Sets by [7] and proposed the Picture Fuzzy Set. Nowadays, there are critical problems in uncertainty which cannot be settled by FS, IFS and PFS. The interaction operators,weighted interaction operators in terms of intuitionistic fuzzy numbers were proposed by Liu et.al. Also, Liu et.al settled problems on multi attribute decision making problems using Dombi geometric Bonferroni mean operator. The ranking of Intuitionistic fuzzy numbers in terms of comparison of likelihood relations was proposed by Chen et.al. Also, Chen et al. settled problems of intuitionistic fuzzy multi attribute decision making. In the combination of triangular fuzzy numbers and intuitionistic fuzzy sets Liu et.al developed triangular intuitionistic fuzzy numbers. Fuzzy number intuitionistic fuzzy weighted operator and fuzzy number intuitionistic fuzzy weighted averaging operator ranking methods were proposed and hybrid operators on this division were developed to deal problems of multi attribute by Wang et.al. To represent the situation of the proposition Movie X would be hit, a precise answer cannot be given by human brain. Such situations are clearly handle by neutrosophic fuzzy sets. In 2012,Wang et al., [23] proposed single valued Neutrosophic sets and extended the work to introduce interval- valued neutrosophic sets. Many research has been done in the field of neutrosophic sets and neutrosophic numbers. Evidently simplified neutrosophic sets and its aggregation operation were given by Ye [28]. Multi valued neutrosophic sets and its operations were proposed by Wang and Li [24], Yang and Pang [26]. Bipolar Neutrosophic sets was studied in Deli et al., [8]. Few researchers settled the problems on decision making under Neutrosophic fuzzy environment. Pramanik et al., [13]; Dey et al., [10]; Pramanik et al., [14, 15]; Deli et al., [9]; Ali et al., [1]; Tian et al., [21]. Trapezoidal Neutrosophic fuzzy sets were initiated by Biswas et al.,in [3]. Decision making models under Trapezoidal Neutrosophic Numbers environment were proposed and solved in Ye 2015 [27]; Tan et al., [20]; Pramanik et al., [13]; Biswas et al., [4–6]; Jana et al., [12].

A Set is a collection of well defined objects which are pair wise different. If we form a structure of a set by allowing repetition of any element which is termed as Multisets. Yager [25] proposed the concept of fuzzy multiset, Shinoj and John [17, 18] proposed intuitionistic fuzzy multisets. Single valued neutrosophic fuzzy multiset was studied by Ye in 2014[28]. Ulucay, et al., [22] proposed an application to solve MCDM problems with Neutrosophic Multisets. Recently many researches are being carried out in the field of Neutrosophic multi sets. Throughout the existing researches, evidently there is no study on Neutrosophic trapezoidal fuzzy multi numbers. The innovation of uncertainty theory assumes an essential job in the definition of a genuine logical scientific model, basic demonstrating in designing space, multi standards situated clinical diagnosis issue, and so forth. Let us consider an example of the poling result in an election, a particular candidate got thirty percent of the result as a favor, twenty percent are against, ten percent give up and forty percent are undecided. To address this problem employing IFS there is a lack of surety in distinguishing the criteria give up and undecided. This undesirable problem can be addressed by Neutrosophic Set (NS) which was framed by Smarandache (1999), which contains three characteristic functions truth membership, indeterminacy membership, and falsity membership. It is seen that lone neutrosophic sets can handle the inaccuracy, faltering and truthiness of an uncertain number, which is progressively solid, legitimate, and reasonable for a decision maker. For the past few years, the ambiguous data are handled by fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and many such structures. Recently, the introduction of neutrosophic sets proves to be more suited to handle vagueness than the existing set-theoretical structure. The fuzzy number can measure only uncertainty, intuitionistic, and interval-valued intuitionistic fuzzy number can measure uncertainty and vagueness, not hesitation. Only a neutrosophic number can measure all three parameters effectively. Thus trapezoidal neutrosophic number attracts more attention and paves the path for new research.

2.1 Novelty and motivation

The following table throws an insight into the novelty of proposed research work. The current research available in the literature lacks to deal indeterminacy of fuzzy numbers having more number of membership and non-membership values. So the introduction of Neutrosophic fuzzy multi-numbers, plays a vital role in dealing real life problems involving truth, indeterminacy and falsity functions having more than one membership functions (Table 1).

Consider there is a panel of four possible alternatives to invest the money namely, A_1 is a car company, A_2 is a food company, A_3 is a computer company, and A_4 is a television company. The investment company takes a decision according to the following three criteria C_1 is the risk analysis, C_2 is the growth analysis, C_3 is the environmental impact analysis. For the company A_1 corresponding to C_1, we are considering Neutrosophic trapezoidal fuzzy multi-number as follows:

$$\langle [0.3, 0.4, 0.5, 0.6], (0.3, 0.2, 0.4, 0.6), (0.6, 0.3, 0.5, 0.2), (0.5, 0.6, 0.7, 0.3) \rangle$$

Here the 2nd element represents truth membership, 3rd element represents indeterminacy and last element represents falsity membership values. In each element, there are four values, which refers to risk analysis due to various factors, but here we consider only 4 factors such as (a) evaluation given by existing stock holders (b) suggestion given by experts in that field (c) our own evaluation and (d) capability to take risk. The membership values for these 4 factors corresponding to truth, indeterminacy and falsity were expressed above. To the best our knowledge until October 2020, no work has been published on neutrosophic trapezoidal fuzzy multi-numbers. So, the present research will help the researchers in various fields to apply the proposed ranking to solve their problems, which deals multi-numbers.

2.2 Significance of present work

Neutrosophic trapezoidal fuzzy multi-numbers is a generalization of neutrosophic fuzzy numbers. There was no work carried out in the proposed research area to the best of our knowledge. In single valued trapezoidal fuzzy numbers, the truth, indeterminacy and falsity functions takes single values, but NTFMNs takes multiple values, which is an added advantage that enables one to deal more crucial real time problems. Thus, this research work throws an insight into the concept of NTFMN, which will open door for future researchers to handle imprecise information in more effective manner.

2.3 Structure of the paper

In Sect. 2, some basic definitions of neutrosophic multiset and neutrosophic fuzzy multi—numbers are provided. Section 3, provides the definition of neutrosophic trapezoidal fuzzy multi number, some basic operations on NTFMN, ranking of NTFMN based on expected value, its validation

Table 1 Insight of novelty of the proposed method

S.No	Type of numbers	Indeterminacy	Ambiguity	Uncertainty
1	Crisp value	Unable to handle	Unable to handle	Unable to handle
2	Fuzzy numbers	Unable to handle	Unable to handle	Able to handle
3	Interval Fuzzy Number	Unable to handle	Unable to handle	Able to handle
4	Intuitionistic Fuzzy Number (IFN)	Inadequate to handle	Adequate to handle	Adequate to handle
5	Interval Intuitionistic Fuzzy Number	Inadequate to handle	Adequate to handle more clearly than IFN's	Adequate to handle more clearly than IFN's
6	Neutrosophic Number	Able to handle	Able to handle	Able to handle
7	Interval Neutrosophic Number	Able to handle more accurately than Neutrosophic numbers	Able to handle	Able to handle
8	Intuitionistic fuzzy multi-numbers	Unable to handle	Able to handle, if the data contains more than one memberships and non-memberships functions	Able to handle, if the data contains more than one memberships and non-memberships functions
9	Neutrosophic fuzzy Multi-Number	Able to handle, if the data contains more than one memberships and non-memberships functions	Able to handle, if the data contains more than one memberships and non-memberships functions	Able to handle, if the data contains more than one memberships and non-memberships functions

and an algorithm to obtain the solution of MCDM problem has been proposed and an real case problem was given to justify the algorithm. The final section ends with conclusion and future research work that can be extended from present study.

3 Preliminaries of neutrosophic trapezoidal fuzzy multi number

This section starts with basic concepts of multi-fuzzy set, intuitionistic fuzzy multiset, neutrosophic multiset and trapezoidal neutrosophic fuzzy number.

Definition 1 [7] An IFS A in X is given by.

$$A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ define, the degree of membership and degree of non-membership of the element $x \in X$ to the set A respectively, which is a subset of X, and for every $x \in X, 0 \le \mu_A(x) + \nu_A(x) \le 1$.

Obviously, every fuzzy set has the form $\{(x, \mu_A(x), \mu_{A^c}(x)), x \in X\}$.For each IFS A in X, $\Pi_A(x) = 1 - \mu(x) - \nu(x)$, is called the intuitionistic fuzzy index of x in A. It is obvious that $0 \le \Pi_A(x) \le 1, \forall x \in X$.

Definition 2 [28] An IFS $A = \{(x, \mu_A(x), \nu_A(x) | x \in X)\}$ is called IF-normal, if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, \nu_A(x_1) = 1$. It is easily seen that the given IFS A will be IF-normal if there is at least one point that surely belongs to A and atleast one point which does not belong to A.

Definition 3 [28] An IFS $A = \{(x, \mu_A(x), \nu_A(x) | x \in X)\}$ of the real line is called IF-convex, if $\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1]$.

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge \mu_A(x_1) \wedge \mu_A(x_2)$$

$$\gamma_A(\lambda x_1 + (1-\lambda)x_2) \ge \gamma_A(x_1) \wedge \gamma_A(x_2)$$

Definition 4 [28] An IFS $A = \{(x, \mu_A(x), \nu_A(x) | x \in X)\}$ of the real line is called an IFN if.

(i) A is IF-normal,
(ii) A is IF-convex,
(iii) μ_A is upper semicontinuous and γ_A is lower semicontinuous and.
(iv) $A = \{(x \in X | \nu_A(x) < 1)\}$ is bounded.

Definition 5 Let X be a universe of discourse, then a neutrosophic set $\tilde{N}$ in X is given by.

$$\tilde{N} = \{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x) \rangle | x \in X\}, \tag{1}$$

where $T_{\tilde{N}}(x), I_{\tilde{N}}(x)$ *and* $F_{\tilde{N}}(x)$ are called truth-membership function, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by $T_{\tilde{N}}(x) \subset]^-0, 1^+[$, $I_{\tilde{N}}(x) \subset]^-0, 1^+[$, and $F_{\tilde{N}}(x) \subset]^-0, 1^+[$ such that $0^- \le T_{\tilde{N}}(x) + I_{\tilde{N}}(x) + F_{\tilde{N}}(x) \le 3^+$.

Definition 6 A neutrosophic trapezoidal fuzzy number is denoted by.

$\tilde{n} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ in a universe of discourse X. The parameters satisfy the relation $t_1 \le t_2 \le t_3 \le t_4$, $i_1 \le i_2 \le i_3 \le i_4$ and $f_1 \le f_2 \le f_3 \le f_4$ Its truth-membership function, indeterminacy-membership function and falsity membership function are defined as follows

$$T_{\tilde{N}}(x) = \begin{cases} \frac{x-t_1}{t_2-t_1}; & t_1 \le x \le t_2 \\ 1; & t_2 \le x \le t_3 \\ \frac{t_4-x}{t_4-t_3}; & t_3 \le x \le t_4 \\ 0; & otherwise \end{cases} \tag{2}$$

$$I_{\tilde{N}}(x) = \begin{cases} \frac{i_2-x}{i_2-i_1}; & i_1 \le x \le i_2 \\ 0; & i_2 \le x \le i_3 \\ \frac{x-i_3}{i_4-i_3}; & i_3 \le x \le i_4 \\ 1 & otherwise \end{cases} \tag{3}$$

and

$$F_{\tilde{N}}(x) = \begin{cases} \frac{f_2-x}{f_2-f_1}; & f_1 \le x \le f_2 \\ 0; & f_2 \le x \le f_3 \\ \frac{x-f_3}{f_4-f_3}; & f_3 \le x \le f_4 \\ 1; & otherwise \end{cases} \tag{4}$$

As a special case, in the above definition when $t_2 = t_3$, $i_2 = i_3$ and $f_2 = f_3$ then we will get a neutrosophic triangular fuzzy number.

Definition 7 [16] Let X be a non-empty set. A multi-fuzzy set A on X is given by.

$$A = \{(x, \mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^p(x), x \in E)\}, \tag{5}$$

where the functions $\mu_A^i(x) : X \to [0,1]$ for all $i \in \{1, 2, \ldots, p\}$ such that $\mu_A^1(x) \ge \mu_A^2(x) \ge \ldots \ge \mu_A^p(x)$, for all $x \in E$.

Definition 8 [17, 18] Let X be a non-empty set. A intuitionistic fuzzy multiset A on X is given by,

$$A=\left\{\left(x,\left(\mu_A^1(x),\mu_A^2(x),\ldots,\mu_A^p(x)\right),\left(\nu_A^1(x),\nu_A^2(x),\ldots,\nu_A^p(x)\right)\,:\,x\in X\right)\right\}, \tag{6}$$

where the functions $\mu_A^i(x)\,:\,X\to[0,1]$ and $\nu_A^i(x)\,:\,X\to[0,1]$ such that $0\le\mu_A^i(x)+\nu_A^i(x)\le 1$ for all $i\in\{1,2,\ldots,p\}$. Also the membership sequence $\left(\mu_A^1(x),\mu_A^2(x),\ldots,\mu_A^p(x)\right)$ is decreasingly ordered sequence of elements, $\mu_A^1(x)\ge\mu_A^2(x)\ge\ldots\ge\mu_A^p(x)$, for all $x\in X$ and the corresponding non-membership sequence $\left(\nu_A^1(x),\nu_A^2(x),\ldots,\nu_A^p(x)\right)$ is neither decreasing nor increasing function.

Definition 9 [19] Let X be a universe of discourse, then a neutrosophic set $\tilde{N}$ in X is given by.

$$\tilde{N}=\left\{\langle x,\,T_{\tilde{N}}(x),\,I_{\tilde{N}}(x),F_{\tilde{N}}(x)\,\rangle|x\in X\right\}, \tag{7}$$

where $T_{\tilde{N}}(x)$, $I_{\tilde{N}}(x)$ and $\mathrm{F}_{\tilde{N}}(x)$ are called truth-membership function, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by $T_{\tilde{N}}(x)\subset\,]^-0,1^+[\,,\,I_{\tilde{N}}(x)\subset\,]^-0,1^+[\,,$ and $\mathrm{F}_{\tilde{N}}(x)\subset\,]^-0,1^+[$ such that $0^-\le T_{\tilde{N}}(x)+I_{\tilde{N}}(x)+\mathrm{F}_{\tilde{N}}(x)\le 3^+$.

Definition 10 [27] Let X be a nonempty set with generic elements in X denoted by x. The single valued neutrosophic multiset (SVNM) A drawn from X is characterized by the three functions: count truth membership of $\mathrm{CT_A}$, count indeterminacy-membership of $\mathrm{CI_A}$, and count falsity-membership of $\mathrm{CF_A}$ such that $CT_A(x)\,:\,X\to R, CI_A(x)\,:\,X\to R, CF_A(x)\,:\,X\to R$ where R is the set of all real number multisets in the real unit interval [0, 1]. Then, a SVNM A is denoted by,

$$\begin{aligned}A=\Big\{\Big\langle x,\left(T_A^1(x),T_A^2(x),\ldots,T_A^p(x)\right),\left(I_A^1(x),I_A^2(x),\ldots,\right.\\ \left.I_A^p(x)\right),\left(F_A^1(x),F_A^2(x),\ldots,F_A^p(x)\right)\Big\rangle\,:\,x\in X\Big\}\end{aligned} \tag{8}$$

where the truth-membership sequence $\left(T_A^1(x),T_A^2(x),\ldots,T_A^p(x)\right)$, the indeterminacy-membership sequence $\left(I_A^1(x),I_A^2(x),\ldots,I_A^p(x)\right)$, and the falsity membership sequence $\left(F_A^1(x),F_A^2(x),\ldots,F_A^p(x)\right)$ may be in decreasing or increasing order, and the sum of $T_A^i(x),I_A^i(x),F_A^i(x)\in[0,1]$ satisfies the condition $0\le\sup T_A^i(x)+\sup I_A^i(x)+\sup F_A^i(x)\le 3$, for $x\in X$ and $i=1,2,\ldots,q$.

In simplified form a SVNM set A is denoted by $A=\left\{\left\langle x,T_A^i(x),I_A^i(x),F_A^i(x)\right\rangle\,:\,x\in X,i=1,2,\ldots,q\right\}$.

Definition 11 [3] A neutrosophic trapezoidal fuzzy number is denoted by,

$\tilde{n}=\left\langle\left(t_1,t_2,t_3,t_4\right),\left(i_1,i_2,i_3,i_4\right),\left(f_1,f_2,f_3,f_4\right)\right\rangle$ in a universe of discourse X. The parameters satify the relation $t_1\le t_2\le t_3\le t_4,\ i_1\le i_2\le i_3\le i_4$ and $\mathrm{f}_1\le f_2\le f_3\le f_4$. Its truth-membership function, indeterminacy-membership function and falsity membership function are defined as follows

$$T_{\tilde{N}}(x)=\begin{cases}\frac{x-t_1}{t_2-t_1}; & t_1\le x\le t_2\\ 1; & t_2\le x\le t_3\\ \frac{t_4-x}{t_4-t_3}; & t_3\le x\le t_4\\ 0 & otherwise\end{cases}$$

$$I_{\tilde{N}}(x)=\begin{cases}\frac{i_2-x}{i_2-i_1}; & i_1\le x\le i_2\\ 0; & i_2\le x\le i_3\\ \frac{x-i_3}{i_4-i_3}; & i_3\le x\le i_4\\ 1; & otherwise\end{cases}$$

and

$$F_{\tilde{N}}(x)=\begin{cases}\frac{f_2-x}{f_2-f_1}; & f_1\le x\le f_2\\ 0; & f_2\le x\le f_3\\ \frac{x-f_3}{f_4-f_3}; & f_3\le x\le f_4\\ 1; & otherwise\end{cases}$$

As a special case, in the above definition when $t_2=t_3\,,i_2=i_3$ and $f_2=f_3$ then we will get a neutrosophic triangular fuzzy number.

Definition 12 [11]. Let $\tilde{n}=\left\langle\left(t_1,t_2,t_3,t_4\right),\left(i_1,i_2,i_3,i_4\right)\right\rangle$ be an intuitionistic fuzzy number in the set of real numbers R. Then the expected value of $\tilde{n}$ is given by.

$$EV(\tilde{n})=\frac{E_*(A)+E^*(A)}{2}$$

4 The proposed methodology

In this section, we introduce the definition of neutrosophic trapezoidal fuzzy multi-number and some of basic operations related to it. Also expected value for the neutrosophic trapezoidal fuzzy multi—number was given and ranking of neutrosophic trapezoidal fuzzy multi numbers based on expected value was proposed.

4.1 Neutrosophic trapezoidal fuzzy multi number and ranking by expected values

Definition 7 The neutrosophic trapezoidal fuzzy multi number (NTFMN) is given by.

$$\tilde{n} = \left\langle [a,b,c,d];\left(\alpha^1,\alpha^2,...,\alpha^p\right), \left(\beta^1,\beta^2,...,\beta^p\right),\left(\gamma^1,\gamma^2,...,\gamma^p\right)\right\rangle$$

where $\alpha^i,\beta^i,\gamma^i \in [0,1],\ i=1,2,...,p$ and $a,b,c,d \in \Re$.

Its truth-membership function is defined by

$$T_{\tilde{n}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}\alpha_A^i; & a \le x \le b \\ \alpha_A^i; & b \le x \le c \\ \frac{(d-x)}{(d-c)}\alpha_A^i; & c \le x \le d \\ 0 & otherwise \end{cases}$$

The indeterminacy membership function is given by

$$I_{\tilde{n}}(x) = \begin{cases} \frac{(b-a)+\beta_A^i(x-a_1)}{(b-a_1)}; & a_1 \le x \le b \\ \beta_A^i; & b \le x \le c \\ \frac{(x-c)+\beta_A^i(d_1-x)}{(d_1-c)}; & c \le x \le d_1 \\ 1 & otherwise \end{cases}$$

and the falsity membership function is given by

$$F_{\tilde{n}}(x) = \begin{cases} \frac{(b-a)+\gamma^i(x-a_1)}{(b-a_1)}; & a_1 \le x \le b \\ \gamma^i; & b \le x \le c \\ \frac{(x-c)+\gamma^i(d_1-x)}{(d_1-c)}; & c \le x \le d_1 \\ 1; & otherwise \end{cases}$$

Definition 8 Let $\tilde{n}_1 = \left\langle [a_1,b_1,c_1,d_1];\left(\alpha_{n_1}^1,\alpha_{n_1}^2,...,\alpha_{n_1}^p\right),\left(\beta_{n_1}^1,\beta_{n_1}^2,...,\beta_{n_1}^p\right),\left(\gamma_{n_1}^1,\gamma_{n_1}^2,...,\gamma_{n_1}^p\right)\right\rangle$ and $\tilde{n}_2 = \left\langle [a_2,b_2,c_2,d_2];\left(\alpha_{n_2}^1,\alpha_{n_2}^2,...,\alpha_{n_2}^p\right),\left(\beta_{n_2}^1,\beta_{n_2}^2,...,\beta_{n_2}^p\right),\left(\gamma_{n_2}^1,\gamma_{n_2}^2,...,\gamma_{n_2}^p\right)\right\rangle$ $\tilde{n}_2 = \left\langle [a_2,b_2,c_2,d_2];\left(\alpha_{n_2}^1,\alpha_{n_2}^2,...,\alpha_{n_2}^p\right),\left(\beta_{n_2}^1,\beta_{n_2}^2,...,\beta_{n_2}^p\right),\left(\gamma_{n_2}^1,\gamma_{n_2}^2,...,\gamma_{n_2}^p\right)\right\rangle$ be any two NTFMNs and $\lambda \neq 0$ be any real number. Then

1. $$\tilde{n}_1 \oplus \tilde{n}_2 = \left\langle \begin{array}{l} [a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];\left(\alpha_{n_1}^1 \wedge \alpha_{n_2}^1,\alpha_{n_1}^2 \wedge \alpha_{n_2}^2,...,\alpha_{n_1}^p \wedge \alpha_{n_2}^p\right), \\ \left(\beta_{n_1}^1 \vee \beta_{n_2}^1,\beta_{n_1}^2 \vee \beta_{n_2}^2,...,\beta_{n_1}^p \vee \beta_{n_2}^p\right),\left(\gamma_{n_1}^1 \vee \gamma_{n_2}^1,\gamma_{n_1}^2 \vee \gamma_{n_2}^2,...,\gamma_{n_1}^p \vee \gamma_{n_2}^p\right) \end{array} \right\rangle.$$

2. $$\tilde{n}_1 \otimes \tilde{n}_2 = \begin{cases} \left\langle \begin{array}{l} [a_1a_2,b_1b_2,c_1c_2,d_1d_2];\left(\alpha_{n_1}^1 \wedge \alpha_{n_2}^1,\alpha_{n_1}^2 \wedge \alpha_{n_2}^2,...,\alpha_{n_1}^p \wedge \alpha_{n_2}^p\right), \\ \left(\beta_{n_1}^1 \vee \beta_{n_2}^1,\beta_{n_1}^2 \vee \beta_{n_2}^2,...,\beta_{n_1}^p \vee \beta_{n_2}^p\right),\left(\gamma_{n_1}^1 \vee \gamma_{n_2}^1,\gamma_{n_1}^2 \vee \gamma_{n_2}^2,...,\gamma_{n_1}^p \vee \gamma_{n_2}^p\right) \end{array} \right\rangle, & d_1 > 0,\ d_2 > 0 \\ \left\langle \begin{array}{l} [a_1d_2,b_1c_2,c_1b_2,d_1a_2];\left(\alpha_{n_1}^1 \wedge \alpha_{n_2}^1,\alpha_{n_1}^2 \wedge \alpha_{n_2}^2,...,\alpha_{n_1}^p \wedge \alpha_{n_2}^p\right), \\ \left(\beta_{n_1}^1 \vee \beta_{n_2}^1,\beta_{n_1}^2 \vee \beta_{n_2}^2,...,\beta_{n_1}^p \vee \beta_{n_2}^p\right),\left(\gamma_{n_1}^1 \vee \gamma_{n_2}^1,\gamma_{n_1}^2 \vee \gamma_{n_2}^2,...,\gamma_{n_1}^p \vee \gamma_{n_2}^p\right) \end{array} \right\rangle, & d_1 < 0,\ d_2 > 0, \\ \left\langle \begin{array}{l} [d_1d_2,c_1c_2,b_1b_2,a_1a_2];\left(\alpha_{n_1}^1 \wedge \alpha_{n_2}^1,\alpha_{n_1}^2 \wedge \alpha_{n_2}^2,...,\alpha_{n_1}^p \wedge \alpha_{n_2}^p\right), \\ \left(\beta_{n_1}^1 \vee \beta_{n_2}^1,\beta_{n_1}^2 \vee \beta_{n_2}^2,...,\beta_{n_1}^p \vee \beta_{n_2}^p\right),\left(\gamma_{n_1}^1 \vee \gamma_{n_2}^1,\gamma_{n_1}^2 \vee \gamma_{n_2}^2,...,\gamma_{n_1}^p \vee \gamma_{n_2}^p\right) \end{array} \right\rangle, & d_1 < 0,\ d_2 < 0 \end{cases}$$

3. $$\lambda\tilde{n}_1 = \begin{cases} \left\langle [\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1];\left(\alpha_{n_1}^1,\alpha_{n_1}^2,...,\alpha_{n_1}^p\right),\left(\beta_{n_1}^1,\beta_{n_1}^2,...,\beta_{n_1}^p\right),\left(\gamma_{n_1}^1,\gamma_{n_1}^2,...,\gamma_{n_1}^p\right)\right\rangle, \lambda > 0 \\ \left\langle [\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1];\left(\alpha_{n_1}^1,\alpha_{n_1}^2,...,\alpha_{n_1}^p\right),\left(\beta_{n_1}^1,\beta_{n_1}^2,...,\beta_{n_1}^p\right),\left(\gamma_{n_1}^1,\gamma_{n_1}^2,...,\gamma_{n_1}^p\right)\right\rangle, \lambda < 0 \end{cases}$$

Example 1 Let us consider two NTFMNs.

$$\begin{array}{l} \tilde{n}_1 = \langle [2,4,5,7];(0.3,0.6,...,0.8),(0.7,0.4,...,0.2),(0.5.0.3....,0.1)\rangle \\ \tilde{n}_2 = \langle [1,3,5,6];(0.7,0.2,...,0.9),(0.4,0.8,...,0.1),(0.3.0.5....,0.01)\rangle \end{array}, \text{Then}$$

1. $\tilde{n}_1 \oplus \tilde{n}_2 = \langle [3,7,10,13];(0.3,0.2,...,0.8),(0.7,0.4,...,0.2),(0.5,0.3,...,0.1)\rangle$
2. $\tilde{n}_1 \otimes \tilde{n}_2 = \langle [2,12,25,42];(0.3,0.2,...,0.8),(0.7,0.4,...,0.2),(0.5,0.3,...,0.1)\rangle,$
3. $4.\tilde{n}_1 = \langle [8,16,20,32];(0.3,0.6,...,0.8),(0.7,0.4,...,0.2),(0.5.0.3....,0.1)\rangle,$

Theorem 1 *Let* $\tilde{n}_1 = \left\langle [a_1,b_1,c_1,d_1];\left(\alpha_{n_1}^1,\alpha_{n_1}^2,...,\alpha_{n_1}^p\right),\left(\beta_{n_1}^1,\beta_{n_1}^2,...,\beta_{n_1}^p\right),\left(\gamma_{n_1}^1,\gamma_{n_1}^2,...,\gamma_{n_1}^p\right)\right\rangle$, $\tilde{n}_2 = \left\langle [a_2,b_2,c_2,d_2];\left(\alpha_{n_2}^1,\alpha_{n_2}^2,...,\alpha_{n_2}^p\right),\ \left(\beta_{n_2}^1,\beta_{n_2}^2,...,\beta_{n_2}^p\right),\ \left(\gamma_{n_2}^1,\gamma_{n_2}^2,...,\gamma_{n_2}^p\right)\right\rangle$ *and* $\tilde{n}_3 = \left\langle [a_3,b_3,c_3,d_3];\left(\alpha_{n_3}^1,\alpha_{n_3}^2,...,\alpha_{n_3}^p\right),\left(\beta_{n_3}^1,\beta_{n_3}^2,...,\beta_{n_3}^p\right),\left(\gamma_{n_3}^1,\gamma_{n_3}^2,...,\gamma_{n_3}^p\right)\right\rangle$. *Then the following relations holds good.*

1. $\hat{n}_1 \oplus \hat{n}_2 = \hat{n}_2 \oplus \hat{n}_1$,
2. $(\hat{n}_1 \oplus \hat{n}_2) \oplus \hat{n}_3 = \hat{n}_1 \oplus (\hat{n}_2 \oplus \hat{n}_3)$,
3. $\hat{n}_1 \otimes \hat{n}_2 = \hat{n}_2 \otimes \hat{n}_1$,
4. $(\hat{n}_1 \otimes \hat{n}_2) \otimes \hat{n}_3 = \hat{n}_1 \otimes (\hat{n}_2 \otimes \hat{n}_3)$,
5. $\lambda(\hat{n}_1 \oplus \hat{n}_2) = \lambda\hat{n}_2 \oplus \lambda\hat{n}_1; \lambda \geq 0$,
6. $\lambda_1\hat{n} \oplus \lambda_2\hat{n} = (\lambda_1 + \lambda_2)\hat{n}; \lambda_1, \lambda_2 \geq 0$.

Proof 1. The following proof is based on the definition 3.2,

$$\tilde{n}_1 \oplus \tilde{n}_2 = \left\langle \begin{array}{c} [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \left(\alpha^1_{n_1} \wedge \alpha^1_{n_2}, \alpha^2_{n_1} \wedge \alpha^2_{n_2}, ..., \alpha^p_{n_1} \wedge \alpha^p_{n_2}\right), \\ \left(\beta^1_{n_1} \vee \beta^1_{n_2}, \beta^2_{n_1} \vee \beta^2_{n_2}, ..., \beta^p_{n_1} \vee \beta^p_{n_2}\right), \left(\gamma^1_{n_1} \vee \gamma^1_{n_2}, \gamma^2_{n_1} \vee \gamma^2_{n_2}, ..., \gamma^p_{n_1} \vee \gamma^p_{n_2}\right) \end{array} \right\rangle,$$

$= \tilde{n}_2 \oplus \tilde{n}_1$

2. The following proof is based on the definition 3.2,

$$(\tilde{n}_1 \oplus \tilde{n}_2) \oplus \tilde{n}_3 = \left\langle \begin{array}{c} [(a_1 + a_2) + a_3, (b_1 + b_2) + b_3, (c_1 + c_2) + c_3, (d_1 + d_2) + d_3]; \\ \left(\left(\alpha^1_{n_1} \wedge \alpha^1_{n_2}\right) \wedge \alpha^1_{n_3}, \left(\alpha^2_{n_1} \wedge \alpha^2_{n_2}\right) \wedge \alpha^2_{n_3}, ..., \left(\alpha^p_{n_1} \wedge \alpha^p_{n_2}\right) \wedge \alpha^p_{n_3}\right), \\ \left(\left(\beta^1_{n_1} \vee \beta^1_{n_2}\right) \vee \beta^1_{n_3}, \left(\beta^2_{n_1} \vee \beta^2_{n_2}\right) \vee \beta^2_{n_3}, ..., \left(\beta^p_{n_1} \vee \beta^p_{n_2}\right) \vee \beta^p_{n_3}\right), \\ \left(\left(\gamma^1_{n_1} \vee \gamma^1_{n_2}\right) \vee \gamma^1_{n_3}, \left(\gamma^2_{n_1} \vee \gamma^2_{n_2}\right) \vee \gamma^2_{n_3}, ..., \left(\gamma^p_{n_1} \vee \gamma^p_{n_2}\right) \vee \gamma^p_{n_3}\right) \end{array} \right\rangle,$$

$= \tilde{n}_1 \oplus (\tilde{n}_2 \oplus \tilde{n}_3)$

In this way the proofs of rest of the properties from 3 to 4 can be obtained.

The expected value of neutrosophic trapezoidal fuzzy multi number can be obtained as follows.

Theorem 2 *Let* $\tilde{n} = \langle [a, b, c, d]; (\alpha^1, \alpha^2, ..., \alpha^p), (\beta^1, \beta^2, ..., \beta^p), (\gamma^1, \gamma^2, ..., \gamma^p) \rangle$ *be a NTFMN. Then.*

$$\begin{aligned} EX(\tilde{n}) &= \frac{EX_T + EX_I + EX_F}{3} \times \left(\frac{\min(\alpha^i) + \max(\beta^i, \gamma^i)}{\max(\alpha^i) + \min(\beta^i, \gamma^i)}\right) \\ &= \frac{1}{3}\left(\frac{a + b + c + d}{4}\right) \times \left(\frac{\min(\alpha^i) + \max(\beta^i, \gamma^i)}{\max(\alpha^i) + \min(\beta^i, \gamma^i)}\right) \end{aligned} \tag{9}$$

If.

$\tilde{n}_1 = \left\langle [a_1, b_1, c_1, d_1]; \left(\alpha^1_{n_1}, \alpha^2_{n_1}, ..., \alpha^p_{n_1}\right), \left(\beta^1_{n_1}, \beta^2_{n_1}, ..., \beta^p_{n_1}\right), \left(\gamma^1_{n_1}, \gamma^2_{n_1}, ..., \gamma^p_{n_1}\right) \right\rangle$,
then $EX(\hat{n}_1) = \frac{1}{12}(a_1 + b_1 + c_1 + d_1) * \frac{\min\left(\alpha^1_{n_1}, \alpha^2_{n_1}, ..., \alpha^p_{n_1}\right) + \max\left[\left(\beta^1_{n_1}, \beta^2_{n_1}, ..., \beta^p_{n_1}\right), \left(\gamma^1_{n_1}, \gamma^2_{n_1}, ..., \gamma^p_{n_1}\right)\right]}{\max\left(\alpha^1_{n_1}, \alpha^2_{n_1}, ..., \alpha^p_{n_1}\right) + \min\left[\left(\beta^1_{n_1}, \beta^2_{n_1}, ..., \beta^p_{n_1}\right), \left(\gamma^1_{n_1}, \gamma^2_{n_1}, ..., \gamma^p_{n_1}\right)\right]}$

Let S be the set of fuzzy quantities, and M be an ordering approach then,

A1: For an arbitrary finite sub set of S, $\tilde{a} \in A, \tilde{a} \geq \tilde{a}$ by M on A.

A2: For an arbitrary finite subset A of S, $(\tilde{a}, \tilde{b}) \in A^2, \tilde{a} \geq \tilde{b}\, and\, \tilde{b} \geq \tilde{a}\, by$ M on A. We should have $\tilde{a} \approx \tilde{b}$ by M on A.

A3: For an arbitrary finite subset A of S, $(\tilde{a}, \tilde{b}, \tilde{c}) \in A^3, \tilde{a} \geq \tilde{b}\, and\, \tilde{b} \geq \tilde{c}$ by M on A. We should have $\tilde{a} \approx \tilde{c}$ by M on A.

A4: For an arbitrary finite subset A of S, $(\tilde{a}, \tilde{b}) \in A^2$, inf supp$(\tilde{a}) \geq$ sup supp$(\tilde{b})$. We should have $\tilde{a} \geq \tilde{b}$ by M on A.

A5: Let S and S ' be two arbitrary finite sets of fuzzy quantities in which M can be applied and $\tilde{a}\, and\, \tilde{b}\, are\, in\, S \cap S'$, we obtain the ranking ordering $\tilde{a} > \tilde{b}\, by\, M\, on\, S'\, iff\, \tilde{a} > \tilde{b}$ by M on S.

A6: Let $\tilde{a}, \tilde{b}, \tilde{a} + \tilde{c}, \tilde{b} + \tilde{c}$ be the elements of S, if $\tilde{a} \geq \tilde{b}$ by M on $\{\tilde{a}, \tilde{b}\}$ then $\tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{c}$ by M on $(\tilde{a} + \tilde{c}, \tilde{b} + \tilde{c})$.

A6′: Let $\tilde{a}, \tilde{b}, \tilde{a} + \tilde{c}, \tilde{b} + \tilde{c}$ be the elements of S if $\tilde{a} > \tilde{b}$ by M on $\{\tilde{a}, \tilde{b}\}$, then $\tilde{a} + \tilde{c} > \tilde{b} + \tilde{c}$ by M on $(\tilde{a} + \tilde{c}, \tilde{b} + \tilde{c})$.

$$EX(\tilde{n}_1) = \frac{1}{12}(a_1 + b_1 + c_1 + d_1) \times \frac{\min\left(\alpha^1_{n1}, \alpha^2_{n1}, ..., \alpha^p_{n1}\right) + \max\left[\max\left(\beta^1_{n1}, \beta^2_{n1}, ..., \beta^p_{n1}\right), \left(\gamma^1_{n1}, \gamma^2_{n1}, ..., \gamma^p_{n1}\right)\right]}{\max\left(\alpha^1_{n1\max}, \alpha^2_{n1}, ..., \alpha^p_{n1}\right) + \min\left[\left(\beta^1_{n1}, \beta^2_{n1}, ..., \beta^p_{n1}\right), \left(\gamma^1_{n1}, \gamma^2_{n1}, ..., \gamma^p_{n1}\right)\right]} \tag{10}$$

5 Theorem 3 (Rationality validation of proposed ranking)

The expected value defined in theorem 3.5 satisfies the following properties.

A7: Let $\tilde{a}, \tilde{b}, \tilde{a}\tilde{c}, \tilde{b}\tilde{c}$ be the elements of S if $\tilde{a} \geq \tilde{b}$ by M on $\{\tilde{a}, \tilde{b}\}$, then $\tilde{a}\tilde{c} \geq \tilde{b}\tilde{c}$ by M on $(\tilde{a}\tilde{c}, \tilde{b}\tilde{c})$.

For the sake of completeness, the proof of properties A4 and A6.

Theorem 4 *Let $\tilde{n}_1$ and $\tilde{n}_2$ be two NTFMN, if $a_{\tilde{n}_1 1} \succ a_{\tilde{n}_2 4}$ and $b_{\tilde{n}_1 1} \succ b_{\tilde{n}_2 4}$ then $\tilde{n}_1 \succ \tilde{n}_2$.*

Proof It is known that

$T_{\tilde{n}_1} \succ a_{\tilde{n}_1 1}$ and $T_{\tilde{n}_1} \succ a_{\tilde{n}_1 4}; T_{\tilde{n}_2} \succ b_{\tilde{n}_2 1}$ and $T_{\tilde{n}_1} \succ b_{\tilde{n}_1 4}$

Also

$I_{\tilde{n}_1} \succ a_{\tilde{n}_1 1}$ and $I_{\tilde{n}_1} \succ a_{\tilde{n}_1 4}; I \succ b_{\tilde{n}_2 1}$ and $I_{\tilde{n}_1} \succ b_{\tilde{n}_1 4}$

$F_{\tilde{n}_1} \succ a_{\tilde{n}_1 1}$ and $F_{\tilde{n}_1} \succ a_{\tilde{n}_1 4}; F \succ b_{\tilde{n}_2 1}$ and $F_{\tilde{n}_1} \succ b_{\tilde{n}_1 4}$

Therefore $EX(\tilde{n}_1) > EX(\tilde{n}_2) \Rightarrow \tilde{n}_1 \succ \tilde{n}_2$.

Theorem 5 *Let $\tilde{n}_1$ and $\tilde{n}_2$ be two NTFMN, then $EX(\tilde{n}_1 + \tilde{n}_3) > EX(\tilde{n}_2 + \tilde{n}_3) \Rightarrow \tilde{n}_1 + \tilde{n}_3 \succ \tilde{n}_2 + \tilde{n}_3$.*

Proof

$EX(\tilde{n}_1 + \tilde{n}_2) = EX(\tilde{n}_1) + EX(\tilde{n}_2)$

similarly

$EX(\tilde{n}_2 + \tilde{n}_3) = EX(\tilde{n}_2) + EX(\tilde{n}_3)$

if, $\tilde{n}_1 \succ \tilde{n}_2$

$EX(\tilde{n}_1 + \tilde{n}_3) > EX(\tilde{n}_2 + \tilde{n}_3) \Rightarrow \tilde{n}_1 + \tilde{n}_3 \succ \tilde{n}_2 + \tilde{n}_3$

Proposition 1 *Let $\tilde{n}_1 = \left\langle [a_1, b_1, c_1, d_1]; \left(\alpha^1_{n_1}, \alpha^2_{n_1}, ..., \alpha^p_{n_1}\right), \left(\beta^1_{n_1}, \beta^2_{n_1}, ..., \beta^p_{n_1}\right), \left(\gamma^1_{n_1}, \gamma^2_{n_1}, ..., \gamma^p_{n_1}\right) \right\rangle$ and $\tilde{n}_2 = \left\langle [a_2, b_2, c_2, d_2]; \left(\alpha^1_{n_2}, \alpha^2_{n_2}, ..., \alpha^p_{n_2}\right), \left(\beta^1_{n_2}, \beta^2_{n_2}, ..., \beta^p_{n_2}\right), \left(\gamma^1_{n_2}, \gamma^2_{n_2}, ..., \gamma^p_{n_2}\right) \right\rangle$ be any two NTFMNs. Then* **t***he Expected value based ranking satisfies the following properties.*

1.$0 \le EX(\tilde{n}) \le 1$.

2. $EX(\tilde{n}_1) = EX(\tilde{n}_2)$ *iff* $\tilde{n}_1 \approx \tilde{n}_2$.

3.$EX(\tilde{n}_1) > EX(\tilde{n}_2)$ *then* $\tilde{n}_1 \succ \tilde{n}_2$

4.$EX(\tilde{n}_1) < EX(\tilde{n}_2)$ *then* $\tilde{n}_1 \prec \tilde{n}_2$

5.1 Steps of the proposed methodology to solve multicriteria decision making problem

In this section, we apply the proposed ranking of NTFMNs to solve a multi criteria decision making problem involving neutrosophic trapezoidal fuzzy multi numbers.

Let $A = (a_1, a_2, ..., a_m)$ be a set of alternatives with the criteria $C = (c_1, c_2, ..., c_n)$. The value of an alternative on a criterion $c_j, j = 1, 2, ..., n$ is a neutrosophic trapezoidal fuzzy multi number

$$\begin{aligned} \left[\tilde{n}_{ij}\right] = & \left\langle \left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]; \left(\alpha^1_{ij}, \alpha^2_{ij}, ..., \alpha^p_{ij}\right), \right. \\ & \left. \left(\beta^1_{ij}, \beta^2_{ij}, ..., \beta^p_{ij}\right), \left(\gamma^1_{ij}, \gamma^2_{ij}, ..., \gamma^p_{ij}\right) \right\rangle, \\ & i = 1, 2, ..., m; j = 1, 2, ..., n. \end{aligned} \tag{11}$$

Therefore we can form a decision matrix $D = \left[\tilde{n}_{ij}\right]$ in which the terms are expressed as NTFMN. The weight criterion w_i is also a NTFMN. The expected weight value is computed by the Eq. (9). Then the normalized expected weight value is computed by the following relation

$$N_j = \frac{EX(w_j)}{\sum_{j=1}^{n} EX(w_j)} \tag{12}$$

Therefore, the weighted expected value for an alternative $A_i, i = 1, 2, ..., m$ is given by

$$WEX(A_j) = \sum_{j=1}^{n} N_j EX(\tilde{n}_{ij}) \tag{13}$$

Using the above expression we can compute the weighted expected value for an alternative to rank alternatives and then to select the best in all the alternatives.

Now, we summarize the procedure as follows:

Step 1 Construct the NTFMN multi attribute decision matrix $\left[\tilde{n}_{ij}\right]$.

Step 2 Calculate the expected weight value for a criterion $c_j, j = 1, 2, ..., n$

Step 3 Calculate the weighted expected value for an alternative $A_i, i = 1, 2, ..., m$

Step 4 Rank the alternatives and select the best with respect to the weighted expected.

6 A real case application

6.1 Problem description

In this section, we illustrate a multicriteria decision making problem under neutrosophic fuzzy multi number environment in a realistic scenario. An investor wants to select the most appropriate company to enhance profit from the money invested. Consider there are four possible alternatives to invest the money namely, A1 is a car company, A2 is a food company, A3 is a computer company, and A4 is a television company. The investment company take a decision according to the following three criteria C1 is the risk analysis, C2 is the growth analysis and C3 is the environmental impact analysis. The four possible alternatives are to be evaluated under the above three criteria by corresponding to linguistic values of neutrosophic trapezoidal fuzzy multi numbers for linguistic terms as shown in Table 2.

Table 2 Linguistic scale of neutrosophic trapezoidal fuzzy multi numbers for linguistic terms

Linguistic terms	Linguistic values
Absolutely low	< [0.0,0.0,0.0,0.0],(0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) >
Low	< [0.0,0.1,0.2,0.3],(0.6,0.3,0.5,0.7), (0.1,0.5,0.4,0.1), (0.2,0.3,0.4,0.1) >
Fairly low	< [0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1) >
Medium	< [0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.8,0.1), (0.5,0.6,0.7,0.3) >
Fairly high	< [0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2) >
High	< [0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4) >
Absolutely high	< [1.0,1.0,1.0,1.0],(1.0,1.0,1.0,1.0), (1.0,1.0,1.0,1.0), (1.0,1.0,1.0,1.0) >

6.2 Problem solution

Step 1. The neutrosophic trapezoidal fuzzy multi number $\tilde{n}$ in the decision matrix and weights can be calculated by

$$\tilde{n} = \left\langle \begin{matrix} \left[\frac{\sum_{k=1}^{5} a_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij5}(k)}{5} \right], \left(\frac{\sum_{k=1}^{5} \alpha_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij5}(k)}{5} \right), \\ \left(\frac{\sum_{k=1}^{5} \beta_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij5}(k)}{5} \right), \left(\frac{\sum_{k=1}^{5} \gamma_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij5}(k)}{5} \right) \end{matrix} \right\rangle$$

Therefore the decision matrix is given by.

$$D = \begin{bmatrix} \left\langle \begin{matrix} [0.26, 0.36, 0.46, 0.56], (0.28, 0.34, 0.32, 0.6), \\ (0.54, 0.36, 0.58, 0.26), (0.46, 0.5, 0.54, 0.20) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.34, 0.44, 0.54, 0.64], (0.32, 0.3, 0.42, 0.48), \\ (0.42, 0.42, 0.48, 0.38), (0.36, 0.46, 0.56, 0.22) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.12, 0.22, 0.32, 0.42], (0.3, 0.4, 0.24, 0.74), \\ (0.56, 0.34, 0.66, 0.12), (0.5, 0.48, 0.46, 0.14) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.5, 0.6, 0.7, 0.8], (0.42, 0.38, 0.52, 0.34), \\ (0.2, 0.48, 0.32, 0.54), (0.2, 0.4, 0.5, 0.26) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.5, 0.6, 0.7, 0.8], (0.42, 0.38, 0.52, 0.34), \\ (0.2, 0.48, 0.32, 0.54), (0.2, 0.4, 0.5, 0.26) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.34, 0.44, 0.54, 0.64], (0.32, 0.22, 0.44, 0.52), \\ (0.5, 0.36, 0.46, 0.3), (0.42, 0.54, 0.66, 0.28) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.38, 0.48, 0.58], (0.34, 0.24, 0.48, 0.44), \\ (0.4, 0.42, 0.42, 0.4), (0.34, 0.48, 0.62, 0.26) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.54, 0.64, 0.74, 0.84], (0.44, 0.4, 0.56, 0.26), \\ (0.1, 0.54, 0.28, 0.64), (0.12, 0.34, 0.46, 0.24) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.26, 0.36, 0.46, 0.56], (0.28, 0.26, 0.34, 0.64), \\ (0.62, 0.3, 0.56, 0.18), (0.52, 0.58, 0.64, 0.26) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.66, 0.76, 0.86, 0.96], (0.56, 0.7, 0.44, 0.44), \\ (0.1, 0.36, 0.22, 0.46), (0.18, 0.46, 0.34, 0.36) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.62, 0.72, 0.82, 0.92], (0.52, 0.6, 0.48, 0.38), \\ (0.1, 0.42, 0.24, 0.52), (0.16, 0.42, 0.38, 0.32) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.18, 0.28, 0.38, 0.48], (0.24, 0.38, 0.22, 0.72), \\ (0.66, 0.3, 0.68, 0.14), (0.56, 0.54, 0.52, 0.18) \end{matrix} \right\rangle \end{bmatrix}$$

The weight value is given by

$$w = \begin{Bmatrix} \langle [0.38, 0.48, 0.58, 0.68], (0.34, 0.24, 0.48, 0.44), (0.4, 0.42, 0.42, 0.4), (0.34, 0.48, 0.62, 0.26) \rangle, \\ \langle [0.22, 0.32, 0.42, 0.52], (0.26, 0.32, 0.28, 0.68), (0.64, 0.3, 0.62, 0.16), (0.54, 0.56, 0.58, 0.22) \rangle \\ \langle [0.46, 0.56, 0.66, 0.76], (0.38, 0.28, 0.56, 0.28), (0.2, 0.54, 0.34, 0.6), (0.18, 0.36, 0.54, 0.22) \rangle \end{Bmatrix}$$

Step 2. Compute the expected weight value for the criterions by using the Eqs. (5), we get EX(w1) = 0.20532, EX(w2) = = 0.1321, EX(w3) = 0.2418 and the normalized weight values are N1=0.3544, N2 = 0.2281, N3 = 0.4174.

Step 3. Compute the weighted expected values of the alternatives by using the Eqs. (9), we have WEX(A1) = 0.1372, WEX(A2) =0.2293, WEX(A3) = 0.2069, WEX(A4) = 0.2309.

Step 4. The alternatives are ranked as $A_2 \succ A_4 \succ A_3 \succ A_1$. Thus the optimal alternative is A_2 (Tables 3 and 4).

Table 3 Preference scale of alternatives and criteria weights given by five experts by linguistic scale

	k	C1	C2	C3
A1	1	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	2	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.0,0.1,0.2,0.3],(0.6,0.3,0.5,0.7), (0.1,0.5,0.4,0.1), (0.2,0.3,0.4,0.1)>
	3	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	5	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
A2	1	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	2	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	3	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	5	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
A3	1	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	2	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	3	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	5	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
A4	1	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	2	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	3	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	5	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
Weights	1	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	2	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6)(0.6, 0.3,0.5,0.2),(0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	3	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	5	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>

Table 4 Comparison of the study with existing methodologies

S. No	Methodology	Accuracy of Result %
1	Vector valued similarity measure	90
2	MFC classifier	98.15
3	Decision tree J48	98.38
4	Multilayer perceptron	98.22
5	Naive Bayes	96.55
6	Lazy IBK	97.17
7	Aggregation and score function, similarity measure	97
8	TNNWAA	98.28
9	TNNWGA	98.48
10	Expected value	98

7 Conclusion

During the literature review, ranking of IFNs which is a major task for solving optimization problems in uncertainty nature was identified. Accordingly, a collection of reference were discussed in detail. Also they need for neutrosophic fuzzy sets and extension for neutrosophic fuzzy numbers were debated. A new form of neutrosophic fuzzy multi set and numbers were introduced. The expected based ranking method was employed to neutrosophic fuzzy multi number which is incentive based, fast responsive and light weighted which is independent and easy to deploy for solving any optimization problems. An approach to solve Multi attribute decision making problems is used for validation of the proposed ranking of method with suitable illustrations. In this frame work, an investor tries to evaluate the most appropriate company to invest the money in the panel of four possible alternatives with three criteria. In this paper, we defined NTFMN which is a generalization of neutrosophic fuzzy numbers. Due to lack of information available in real life situation problems, it is comfort to go for NTFMN than NTFN. Therefore it is essential to go one step forward to the task of multi numbers under neutrosophic environment. Ranking of multi numbers are more difficult,here we established the ranking by means of expected values and developed an algorithmic approach to handle multi criteria decision making problem. In future, we will put forth new ranking techniques to order neutrosophic multi numbers and extend our work to solve existing real life problems in the field of medicine, defense, investments and related fields where the necessity occurs. Multi—criteria decision making problem is an essential issue in complexity of socio—monetary situations. In real life problems, the data that exists are constantly vague and imprecise, so the ancient strategies don't seem to be helpful for handling these issues. Hence, we use neutrosophic trapezoidal fuzzy multi numbers to handle such crucial problems. Upto now, not many research efforts have been devoted to rank neutrosophic fuzzy multi numbers using a single formula. The present research provides initial work in this respect. As a very new and promising research area, there are several interesting and important future directions in this work, one can Expected ranking method for ranking of neutrosophic fuzzy multi numbers are proposed in the present work. The work can be extended to evaluate new ranking techniques based on the proposed approach. The work proposed to solve Neutrosophic fuzzy multi number multi criteria problems focus only on optimization problems. This can be extended to solve any mathematical problem involving neutrosophic fuzzy environment. In order to provide better ranking performance, it is necessary to analyze and categorize ranking algorithms proposed so far suitably to the current scenario. A software can be developed to solve optimization problems in uncertainty nature.

8 Future work

The proposed methodology is applicable to any multi-criteria decision making problem, pattern recognition problems and real world classification problems, particularly for the problems with more than one decision makers. Therefore, this new methodology is a useful tool in multi criteria decision making problems such as problems in robot selection, analyzing of software selection, green suppliers selection or solid waste landfill site selection problems etc. using expected ranking methodology. In the future, some applications of the proposed methodology will rule the problems in real life not limited to medical diagnosis, pattern recognition etc. involving more than one decision attributes.

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Declarations

Conflict of interest The authors declare no competing interests.

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