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FUZZY SYSTEMS AND THEIR MATHEMATICS

Design of acceptance sampling plans based on interval valued neutrosophic sets

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Abstract

Acceptance sampling plans (ASPs) are conducted by inspecting a small set of items instead of all outputs. Although traditional ASPs use certain plan parameters, it is clear that quality characteristics or definitions may not be certain in some real case applications because of uncertainties. The fuzzy set theory (FST) is a popular technique to model uncertainty in the engineering problems. It is known that ASPs have been successfully formulated based on FST in the literature. However, the uncertainty is generally more complex in cases including human evaluations. Neutrosophic sets (NSs) that is one of the fuzzy set extensions bring some advantages to manage more complicated uncertainties in quality problems especially uncertainty based on human's hesitancy. Since the NSs include three terms as truthiness (t), indeterminacy (i), and falsity (f), they can successfully model the human thinking and inspectors' evaluations under uncertainty. In this paper, traditional attribute ASPs have been extended based on interval NSs to combine the computational and interpretational advantages of the interval statistics with the advantages of NSs. Additionally, two well-known distributions for ASPs called Binomial and Poisson distributions are redesigned by using NSs. For this aim, NSs are converted to interval NSs by using α-cut technique and some characteristic functions of ASPs such as acceptance probability (P_a), average sample number (ASN), and average total inspection (ATI) have been designed for single and double ASPs based on interval NSs. The proposed ASPs based on NSs have been tested on some numerical applications from a manufacturing process, and results obtained based on real cases have been compared.

Keywords Acceptance sampling plans · Binomial distribution · Fuzzy sets · Interval neutrosophic sets · Neutrosophic sets · Poisson distribution

1 Introduction

Compliance with expected quality specifications is critically important in manufacturing systems nevertheless testing whole items causes high cost in inspection process. Acceptance sampling plans (ASPs) offer to inspect a small set of items instead of all of population to minimize inspection cost and harmful effects of the process. The quality of the produced items is decided statistically if a specified producer's risk (α) and consumer's risk (β) are in acceptable level for a product. ASPs are the set of certain rules for accepting and rejecting the parties depending on the result of the inspection of a sample (Montgomery 2009). Plan parameters sample size (n) and allowed defective item count (c) are determined depending on the risks α and β. The parameter α is defined as the probability of rejecting a lot in which the defective ratio (p) of the lot is the same as the producer's acceptable quality level (AQL), and the parameter β is also defined as the probability of accepting a lot, while p is over the allowed proportion defective (Chun and Rinks 1998). We also know that one type of ASPs named double acceptance sampling plans (DASPs) is used for reaching lower risks with small sample sizes (Montgomery 2009).

The defective ratio of the parties and the plan parameters is handled as certain values in traditional ASPs. Unfortunately, this approach does not give accurate outputs

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in case of uncertainty in real case applications. It is also clear that the process can include many uncertainties because of many reasons such as human's hesitancies, process' vagueness, measurement systems. The fuzzy set theory (FST) is a completely effective approach to model and manage these uncertainties. This modeling is built on the term "membership function" by using a continuous variable between 0 and 1. On the other hand, statements are answered by using two absolute values; 0 and 1 in classical logic: that is the main difference of the classical logic and fuzzy logic (Zadeh 1965). Fuzzy Sets (FSs) have been extended recently to model uncertainty better depending on the nature of the process. One of these extensions called Neutrosophic Sets (NSs) that having three terms for membership, non-membership, and indeterminacy and handle the inconsistent information case has been successfully used to model uncertainty. These definitions make the Neutrosophic logic that is similar to human thinking thereby it gives good results to model uncertainty related with human factors (Smarandache 2005). Having indeterminacy at the same time as membership and non-membership is the characteristic attribute of the NSs.

There are limited studies into literature related with ASPs analyzing based on NSs. Most of these studies consider variable ASPs and employ the same sampling model with some differences related to the population distributions. These studies were constructed on interval valued random variables, and three main characteristics of NSs such as truthiness, falsity, and indeterminacy are not considered together at problem definition and to construct model. Since they used interval valued numbers with deterministic limits, these models have been generally used in interval type-1 fuzzy environments. In real case problems, attribute ASPs are generally preferred as a part of quality control operations. There are also some studies in the literature based on NSs for Weibull, Binomial, and Poisson distributions. One of the studies suggested single ASP based on NSs for the problems that were not originally Neutrosophic by adding a preliminary step to calculate the lot related probabilities with the help of standard deviation. However, this approach may not be suitable for the items having several types of defects that means classifying on defect status. In real case problems, attribute ASPs are generally realized according to the Poisson distribution. In the literature, attribute single and double ASPs were developed for Poisson distribution based on single-valued NSs. Since truthiness, indeterminacy and falsity can be real numbers, intervals, or functions in NS theory, the mentioned studies do not cover NS theory entirely. In real case problems, especially engineering applications, interval valued FSs have been more preferred because of their advantages about computation and interpretation. However, traditional interval valued FSs are not capable of modeling uncertainties causing for multiple reasons. The 3-tuple structure of the NS that consists of truthiness, falsity, and indeterminacy is advantageous for more realistic modeling of the inspection process because of better representing the human thoughts and evaluations. This structure provides an important advantage especially in modeling the indecision situations of the inspector or quality engineer. Also, in the sampling inspection, the inspector may experience uncertainty in defining the defect condition of product. If the modeling of this uncertainty depending on the degree of uncertainty cannot be done exactly, the results obtained will not be correct. In this respect, NSs provide a successful modeling capability. Considering the definition of uncertainty as a range will also make an important contribution such as flexibility, including more information and sensitiveness. Additionally, neutrosophic evaluations in the form of intervals can provide computational advantage and ease of interpretation. From this point of view, extending the ASPs based on interval NSs can increase the real case applicability.

This paper aims to design and analyze the single and double ASPs based on INSs. Two of well-known statistical distributions, Poisson, and Binomial are analyzed and reformulated based on INSs. The obtained formulations are also analyzed for triangular and trapezoidal NSs by converting them to INSs by using α-cut technique. The redesigned statistical distributions are inserted into main procedure of ASPs, and a design that can model the uncertainty better is proposed.

The rest of this paper is organized as follows: Sect. 2 includes brief information about traditional single and double ASPs. Section 3 comprises brief information about FST and NSs. A literature analysis-based uncertainty on ASPs by using FST is presented into Sect. 4. The design procedure of ASPs based on INSs is detailed by explaining statistical distributions on NSs into Sect. 5. The suggested ASPs based on INSs are analyzed with some numerical examples into Sect. 6. The limitations of the proposed plans are discussed in Sect. 7. The conclusions and future research suggestions are presented in Sect. 8.

2 Acceptance sampling plans

ASPs consist of certain set of sampling rules and acceptance criteria to reach a specified quality level by inspecting only a small sample having size n instead of all items in a lot that indicates as N. ASPs can be classified in two groups based on quality metrics: (i) variable and (ii) attribute. Variable ASPs tackle with the quality characteristics such as length modeled with a statistical distribution. On the other hand, attribute ASPs are interested with the defectiveness of the inspected items by considering the inspection operation as a probabilistic event. They are formulated as Binomial or Poisson distributions by using lot defectiveness metric (Montgomery 2009). This paper analyzes the main characteristics of attribute ASPs based on INSs. For this aim, Binomial and Poisson distributions have been redesigned by using INSs and the main characteristic functions of attribute ASPs such as probability of accepting a lot (P_a), average outgoing quality (AOQ), average total inspection (ATI), and average sample number (ASN) have been derivate based on INSs. Totally, flowchart and stages of ASPs have been re-considered by using INSs.

2.1 Single acceptance sampling plans

For Binomial attribute ASPs, plan parameters are sample size (n) and maximum allowed defective item count (c) which are determined to reach a specified quality level with minimum cost. Lot acceptance or rejection decision can be decided by inspecting only one sample in Single Acceptance Sampling Plans (SASPs). The operator randomly selects only one sample from a lot and inspects all of the items of this sample. The main characteristics of the ASPs can be summarized as in below:

The P_a is formulated as shown in Eq. (1), while observed defective item count is represented with the variable d for the Binomial distribution (Montgomery 2009):

$$P_a = P\{d \le c\} = \sum_{d=0}^{c} \binom{n}{d} \times p^d \times (1-p)^{(n-d)}. \tag{1}$$

The probability of accepting a party P_a is formulated for Poisson distribution as shown in Eq. (2), while defect frequency (λ) is equal to ($n \times p$)(Montgomery 2009):

$$P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{\lambda^d \times e^{-\lambda}}{d!}. \tag{2}$$

Operating characteristic (OC) can be stated with a curve in a graph between p and P_a axes. Plan parameters are determined by using OC curve, α and β as shown in Fig. 1. This is because ASPs should have P_a as near as α to minimize the inspection cost (Montgomery 2009).

The AOQ value represents the long-term average fraction defective that the consumer will encounter for a given value of p. The AOQ value is calculated as shown in Eq. (3) (Montgomery 2009):

$$AOQ = P_a \times p = P_a \times \lambda / n. \tag{3}$$

The ATI value is the average number of inspections per lot if 100% inspection is performed for rejected lots, and it can be calculated with the Eq. (4) (Montgomery 2009):

$$ATI = n + (1 - P_a) \times (N - n). \tag{4}$$

2.2 Double acceptance sampling plans

Double Acceptance Sampling Plans (DASPs) are two-step ASPs having two sets of plan parameters. For the DASPs, if the first limit for allowable defect count (c_1) is not exceeded in first sample size (n_1) in the first step, the lot is accepted. Similarly, if the second threshold value for defect count (c_2) is exceeded in the first step, the lot is rejected without continuing the second step. In addition to that, the second inspection step is applied with a new sample size (n_2) if the defective item count is observed between c_1 and c_2 in the first step (Montgomery 2009).

Binomial distribution can be converted to Poisson distribution by converting p into the defect frequency. There are two sample sizes for DASPs; thus, there should be two defect frequencies for two steps. Defect frequencies can be calculated as shown in Eq. (5):

$$\lambda_1 = n_1 \times p, \lambda_2 = n_2 \times p. \tag{5}$$

A lot can be accepted either in the first step or the second step, so the lot acceptance probability is calculated as sum of these two probabilities. P_a can be formulated as shown in Eq. (6), while observed defective item count in step 1 and 2 is d_1 and d_2, respectively (Montgomery 2009):

$$\begin{aligned} P_a &= P\{d_1 \le c_1\} + P\{d_1 + d_2 \le c_2 | c_1 < d_1 \le c_2\} \\ P_{a_{(\text{Binomial})}} &= \sum_{d_1=0}^{c_1} \left(\binom{n_1}{d_1} p^{d_1} \times (1-p)^{(n_1-d_1)} \right) \\ &\quad + \sum_{d_1=c_1+1}^{c_2} \left(\binom{n_1}{d_1} p^{d_1} \times (1-p)^{(n_1-d_1)} \right. \\ &\quad \left. \times \left[\sum_{d_2=0}^{c_2-d1} \left(\binom{n_2}{d_2} p^{d_2} \times (1-p)^{(n_2-d_2)} \right) \right] \right) \\ P_{a_{(\text{Poisson})}} &= \sum_{d_1=0}^{c_1} \left(\frac{\lambda_1^{d_1}}{d_1!} \times e^{-\lambda_1} \right) \\ &\quad + \sum_{d_1=c_1+1}^{c_2} \left(\frac{\lambda_1^{d_1}}{d_1!} \times e^{-\lambda_1} \times \left[\sum_{d_2=0}^{c_2-d1} \left(\frac{\lambda_2^{d_2}}{d_2!} \times e^{-\lambda_2} \right) \right] \right). \end{aligned} \tag{6}$$

Although the OC curve for DASPs is obtained similar to Fig. 1, it becomes steeper because of the acceptance chance yielded by the second step of the plan. The AOQ is independent of sampling steps and plan parameters, so it calculated with same formula with the single ASPs as shown in Eq. (3). It also can be re-written with by using defect frequencies as shown in Eq. (7):

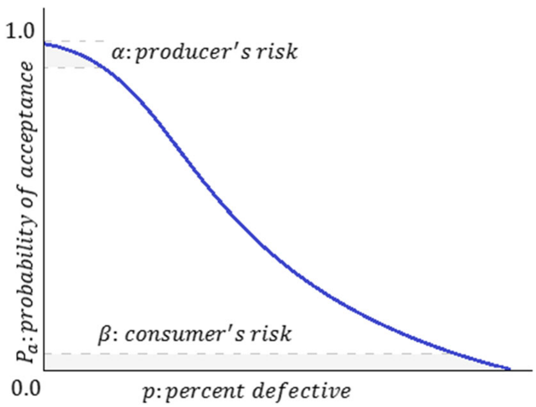


Fig. 1 An example OC curve

$$AOQ = P_a \times \left(\frac{\lambda_1}{n_1}\right) = P_a \times \left(\frac{\lambda_2}{n_2}\right). \tag{7}$$

There are three cases for *ATI* as follows: (i) the lot can be accepted in the first step with the probability P_a^I, (ii) or can be accepted the second step with the probability P_a^{II}, or (iii) it can be rejected with the probability $(1 - P_a)$. So, the ATI value can be calculated as shown in Eq. (8) (Montgomery 2009):

$$ATI = \left(n_1 \times P_a^I\right) + \left((n_1 + n_2) \times P_a^{II}\right) + (N \times (1 - P_a)). \tag{8}$$

The ASN value to be inspected per lot in reaching decisions to accept or reject is calculated as shown in Eq. (9), while P^I represents the probability of terminating the sampling in the first step as either accept or reject (Montgomery 2009):

$$ASN = n_1 + \left(n_2 \times \left(1 - P^I\right)\right). \tag{9}$$

3 The fuzzy set theory

The modeling of the uncertainty in the fuzzy set theory (FST) is built on the term "membership function" $(\mu(x))$ by using a continuous variable x inside $[0, 1]$ interval in a space X. In classical logic, a variable is either a full member or a full non-member to a set. However, it can be both member and non-member at the same time in FST approach. If the uncertainty is high, the membership value decreases, and if the uncertainty is low, the membership value increases. A simple fuzzy set is stated as in Eq. (10) (Zadeh 1965):

$$\widetilde{A} = \left\{\left(x, \mu_{\widetilde{A}}(x) | x \epsilon X\right)\right\}. \tag{10}$$

The uncertainty level is represented with membership degree: 1 means full membership and 0 means full non-membership. The non-membership function $(\vartheta(x))$ can be formulated as in Eq. (11) (Zadeh 1965):

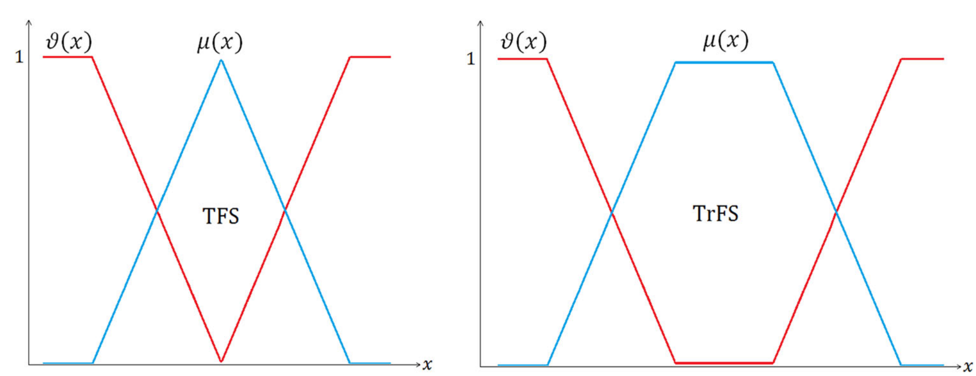
$$\vartheta(x) = 1 - \mu(x). \tag{11}$$

Membership functions of FSs can have different shapes. Depending on its shape, the most popular FSs are triangular (TFS) and trapezoidal (TrFS) FSs. Figure 2 shows membership and non-membership functions of a TFS and TrFS (Zadeh 1965).

3.1 Neutrosophic sets

Summation of the membership and non-membership degrees for set elements is equal to 1 for traditional FSs. This is named as "complete information case". However, the uncertainty can be more complicated in some real case problems. It is not possible to evaluate all conditions in complete information cases. For example, inadequacy on capability of collecting information about membership and non-membership functions can obstruct to satisfy the complete information case. To tackle with the "incomplete information case", Intuitionistic fuzzy sets (IFSs) are offered (Atanassov 2003) and has been used to model especially real case problems. Uncertainty caused by incomplete information is dependent to membership and non-membership functions in IFS and represented as $(1 - (\mu(x) + \vartheta(x)))$. Neutrosophic Sets (NSs) are the generalized form of the IFSs. The uncertainty caused by incomplete information is named as "indeterminacy" and

Fig. 2 Membership and Non-membership Functions of a TFS and TrFS



represented with an independent term in NSs. NSs handle membership (truthiness), non-membership (falsity), and indeterminacy cases independent from each other. This leads up to the exemption about the sum of these three terms. The independency of membership, non-membership, and indeterminacy terms makes possible to use inconsistent data in modeling. NSs can be formulated by using the terms membership/truthiness (t), non-membership/falsity (f), and indeterminacy (i) as shown in Eq. (12) (Smarandache 2005):

$$(t, i, f) = (\text{truthiness, indeterminacy, falsity}) \quad 0 \le t + i + f \le 3, t, i, f \in [0, 1]. \tag{12}$$

3.2 Interval neutrosophic sets

If t, i, and f are interval valued numbers such as $[t_{x_L}, t_{x_U}]$, $[i_{x_L}, I_{x_U}]$, $[f_{x_L}, f_{x_U}]$, respectively, the set is named as interval NSs (INSs), and it is represented with three intervals. INSs are useful in engineering problems because it is practical to work with interval valued numbers instead of continuous functions. They also provide convenience in the calculation stages. Summation of the biggest upper limits (*sup*) of these three intervals must between 0 and 3. Representation of INSs is shown in Eq. (13) (Wang et al. 2005):

$$\tilde{\ddot{x}} = \langle [t_{x_L}, t_{x_U}], [i_{x_L}, I_{x_U}], [f_{x_L}, f_{x_U}] \rangle \quad t_x, i_x, f_x \in [0, 1] \quad 0 \le sup t_x + sup f_x + sup i_x \le 3. \tag{13}$$

3.3 α-cut of NSs

In α-cut approach, a threshold value is decided in terms of membership degree, and the FS is cut from this level horizontally. The upper side is assumed as full member, and the lower membership degrees are ignored (Zadeh 1975). In this way, an interval valued set is produced, and the membership value can be stated as a 1–0 crisp value. NSs can also be converted to INSs by using α-cut technique. It can be applied to any shape NSs such as triangular NSs (TNSs) and trapezoidal NSs (TrNSs). For an NS of $\widetilde{A} = \langle \mu_{\widetilde{A}}(x), \sigma_{\widetilde{A}}(x), \vartheta_{\widetilde{A}}(x) \rangle$ where $\mu(x)$ is truthiness, $\sigma(x)$ is indeterminacy, and $\vartheta(x)$ is falsity, α–cut is stated with one of the statements shown in Eq. (14) for an $\alpha \in [0, 1]$ (Salama and Smarandache 2015):

$$Type1 : \tilde{\ddot{A}}_\alpha = \left\{ x : x \epsilon X, \left(\mu_{\widetilde{A}}(x), \sigma_{\widetilde{A}}(x) \ge \alpha \right) \vee \left(\vartheta_{\widetilde{A}}(x) \le 1 - \alpha \right) \right\}$$
$$Type2 : \tilde{\ddot{A}}_\alpha = \left\{ x : x \epsilon X, \left(\mu_{\widetilde{A}}(x) \ge \alpha, \sigma_{\widetilde{A}}(x) \le \alpha \right) \vee \left(\vartheta_{\widetilde{A}}(x) \le 1 - \alpha \right) \right\}. \tag{14}$$

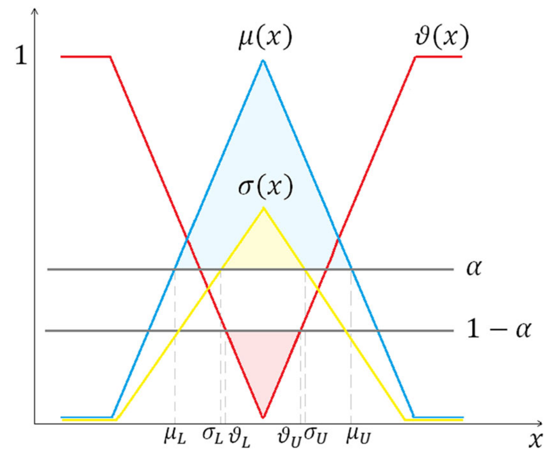


Fig. 3 An example type-1 α-cut for a TNSs

In Eq. (14), $\mu_{\widetilde{A}}(x) \ge \alpha$ condition dictates the condition $\vartheta_{\widetilde{A}}(x) \le 1 - \alpha$, so α-cut can be simplified as shown in Eq. (15) (Salama and Smarandache 2015):

$$\tilde{\ddot{A}}_\alpha = \left\{ x : x \epsilon X, \vartheta_{\widetilde{A}}(x) \le 1 - \alpha \right\}. \tag{15}$$

An example type-1 α-cut for a TNS is seen in Fig. 3. When α-cut is applied to an NS, three intervals are obtained. When it is taken into consideration that INSs are represented with three intervals as introduced in Eq. (13), it can be said that α-cut turns NSs into INSs.

3.4 Operations on α-cut intervals

FSs are represented with an interval having two end points a_1 anda_3 and a peak point a_2 as $[a_1, a_2, a_3]$. If α-cut is applied to a FS of$\widetilde{A}$, $\widetilde{A}_\alpha = [a_{1\alpha}, a_{3_\alpha}]$ is obtained as show in Fig. 4 (Gao et al. 2009). It can be mathematically stated as shown in Eq. (16) (Garai et al. 2020):

$$\widetilde{A}_\alpha = [a_{1\alpha}, a_{3_\alpha}] = \left[a_1 + \frac{\alpha(a_2 - a_1)}{w_{\widetilde{A}}}, a_3 - \frac{\alpha(a_3 - a_2)}{w_{\widetilde{A}}} \right]. \tag{16}$$

Interval operations can be applied on α-cut intervals. For given two intervals $K = [k_1, k_3]$,$L = [l_1, l_3]$ and a scalar λ satisfying $\forall k_1, k_3, l_1, l_3, \lambda \in \mathbb{R}^+$, addition, subtraction, multiplication, division, and multiplication with scalar operations are given in Eqs. (17–22), respectively (Gao, Zhang, & Cao, 2009):

$$[k_1, k_3] \oplus [l_1, l_3] = [k_1 + l_1, k_3 + l_3], \tag{17}$$

$$[k_1, k_3] \ominus [l_1, l_3] = [k_1 - l_3, k_3 - l_1], \tag{18}$$

$$[k_1, k_3] \otimes [l_1, l_3] = [k_1 \times l_1, k_3 \times l_3], \tag{19}$$

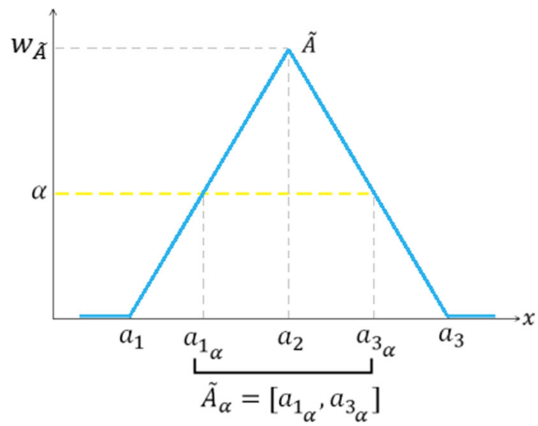


Fig. 4 α-cut for a TFS

$$[k_1, k_3] \varnothing [l_1, l_3] = [k_1/l_3, k_3/l_1], \tag{20}$$

$$\lambda \otimes [k_1, k_3] = [\lambda \times k_1, \lambda \times k_3], \tag{21}$$

$$[k_1, k_3]^\lambda = \left[(k_1)^\lambda, (k_3)^\lambda\right]. \tag{22}$$

INSs are represented with three independent intervals. The interval operations presented in Eqs. (17–22) are usable for INSs too.

4 A literature analysis on acceptance sampling plans used the fuzzy set theory

In this paper, the design of ASPs has been analyzed with NSs. The ASPs have been investigated into literature based on many perspectives and different tools. Some studies were carried out in the literature on attribute ASPs with the help of traditional fuzzy sets with α-cut approach to transform the fuzzy numbers to interval valued numbers (Divya 2012; Jamkhaneh et al. 2011). Similar studies were also conducted for DASPs with α–cut by Kahraman and Kaya (2010) and Jamkhaneh and Gildeh (2012). Attribute ASPs were also enhanced based on FS extensions, but these studies are very limited. Işık and Kaya (2021a) redesigned single and double attribute ASPs based on intuitionistic fuzzy linguistic term sets. Later on, they offered Hesitant and Pythagorean fuzzy linguistic approaches to enable to use this intuitionistic ASPs under hesitant and Pythagorean fuzzy environments (Işık and Kaya 2022a, b). Attribute ASPs were also extended for interval type-2 FSs (Işık and Kaya 2021b, c).

ASPs had been also studied based on NSs but most of these studies consider variable ASPs and use the same sampling model. The difference between them is related to the population distributions used and whether the parameters of the population distributions are known or not. The appropriate test statistic was decided by considering the conditions of each problem and the available parameters. For this reason, the test statistics used in the studies differ from each other. Aslam et al. (2019) developed a variable ASP for a population with specifications expressed by random variables that fit a Neutrosophic Pareto distribution, the mean and standard deviation of which were also expressed as Neutrosophic intervals. The lower and upper specification limits were considered as deterministic values and the sample mean, and variance was used as unbiased estimators of the population. The random number of the probability distribution was expressed as an interval valued number. A similar variable ASP was designed by Aslam (2018a). In this plan, the mean and variance of the population distribution considered were unknown, and a process loss index was calculated with the help of the arithmetic mean of the specification limits. Chi-square test statistic was used for sample acceptance as the process loss index conformed to the Neutrosophic Chi-square distribution. Aslam et al. (2022) considered process loss using Neutrosophic statistics for two-stage ASPs. Aslam (2018b) designed a variable single ASP for cases where the population fits Neutrosophic exponential distribution, and the ASP parameters are interval valued numbers. In this plan, the operating characteristic is obtained as an interval. Aslam and Al-Marshadi (2018) suggested variable acceptance sampling for a population distributed by Neutrosophic normal distribution whose mean was known, but whose variance is unknown. Neutrosophic correlation and regression estimators were used for acceptance testing. Aslam (2019b), created a variable sampling plan for a population fit a Neutrosophic normal distribution whose mean and standard deviation are Neutrosophic. The correlation between correct observation and Neutrosophic values was determined by using the probability distribution function of the population, population parameters and Neutrosophic random error. The observation error was calculated by means of this correlation value. Neither the definition of the problem nor the model mention indeterminacy. Aslam (2019c) suggested a similar variable ASP for a normally distributed population. Unlike other studies, indeterminacy was considered in problem formulation. However, by making an assumption, truthiness and falsity conditions were ignored and the model was developed using only a random number with an interval value. Other variables mentioned as Neutrosophic were also expressed as singular intervals with deterministic limits. Azam et al. (2022) modified these ASPs to have successive two occasions based on Neutrosophic interval number by considering two scenarios: standard deviation of population was (i) known and (ii) unknown based on normal distribution. Rao and Aslam (2023) studied various time-truncated single and repetitive ASPs using gamma distribution based on indeterminacy. In common in all of these mentioned studies on Variable ASP, the random variable of probability distribution used in the proposed model was expressed as an interval valued number. However, truthiness, falsity, and indeterminacy are not considered together either in the definition of the problem or in the model. That is, there are only interval valued sets with constant boundaries. Thus, the models seem to be more suitable for use under interval type-1 fuzzy environments.

In the literature, there are also a few studies on attributing ASPs based on NSs. Jeyadurga and Balamurali (2021) proposed an attribute ASP by using Weibull distributed lifetime assurance based on Neutrosophic statistics to cover the indeterminacy about the failure probability of the products. Aslam (2019c) designed an attribute ASP for normally distributed populations having no quality specifications. Essentially, the problem being addressed was not originally Neutrosophic. By adding a preliminary step to the ASP, the probability of defectiveness, non-defectiveness, and indeterminacy was calculated as disjoint sets by using the standard deviation ($\mathcal{S}$) as a transformation factor. Around the mean, the items in interval $[-\mathcal{S}, +\mathcal{S}]$ were considered non-defective, and the items in interval $(-\infty, -3\mathcal{S}) \cup (+3\mathcal{S}, +\infty)$ were considered defective, and the items in $[-3\mathcal{S}, -\mathcal{S}) \cup (+\mathcal{S}, +3\mathcal{S}]$ were considered indeterminate. This approach had an important limitation of the proposed model, as it may not be possible to calculate these probabilities with the proposed method for the items that require manual/visual/automatic inspection. An uncertainty threshold was needed to use the Neutrosophic binomial distribution. Such a value was not mentioned in the mentioned study, and the allowable number of defects was used instead of the uncertainty threshold. These two are different concepts; using the allowable number of defects as the uncertainty threshold value is often not an acceptable assumption. These are the limitations of the mentioned study. Since defect probability values are very small in real case problem and the sample size is preferred relatively high, attribute ASPs are usually performed in accordance with the Poisson distribution. To take the indeterminacy situation into account in such applications, attribute single ASPs were developed for Poisson distribution based on single-valued NSs (Işık and Kaya 2020a) and a similar study was organized for double ASPs based on single-valued NSs for Binomial distribution (Işık and Kaya 2020b). By covering these two studies, an inclusive design of single and double ASPs based on single-valued NSs for Poisson and Binomial distributions had been suggested. The main characteristic functions of ASPs such as AOQ, ATI and ASN were also formulated by handling the truthiness, falsity, and indeterminacy as real numbers in formulations (Işık and Kaya 2022d). In these ASPs, probability values were not derived from the normal distribution. So, these were usable in a Neutrosophic environment. There were different parameters for the allowable number of defects and the allowable number of indeterminate items, and the plan has three possible outputs: acceptance, rejection, indecision. It should be noted that these terms can be real numbers, intervals, or functions in NS theory. From this point of view, it can be said that the mentioned studies do not stand for NSs entirely and can be extended. In real case problems and especially in engineering applications, interval fuzzy sets are more preferred because of their advantages such as ease of computation and easy interpretation of results obtained. However, classical interval fuzzy sets are insufficient for modeling uncertainties arising from different reasons. The 3-tuple structure of the NSs is well suited for more realistic modeling of the sample inspection, as it better reflects human thoughts and evaluations. We know that many of uncertainties come from hesitancy of inspector or quality engineer. Since considering Neutrosophic evaluations in the form of intervals will provide computational advantage and ease of interpretation, expanding the ASPs for interval NSs will be very useful in terms of real case applicability. So, in this paper, single and double ASPs have been analyzed based on interval valued NSs. For this aim, two well-known and mostly used distribution that are Binomial and Poisson have been designed on interval valued NSs.

5 Design of acceptance sampling plans based on interval valued neutrosophic sets

NSs are useful methodology to model the uncertainty in real case especially for engineering problems. Since they have three parts such as truthiness (t), indeterminacy (i), and falsity (f), it brings a huge advantage to model uncertainties that especially occur by human hesitancy or process characteristics. So, it would be beneficial to develop ASPs that include a critical decision based on product's quality level for INSs. The attribute ASP recommendations in the literature were developed using single-valued NS. In this study, unlike other studies, ASPs in which the probability values about the defectiveness of items are expressed as membership functions are

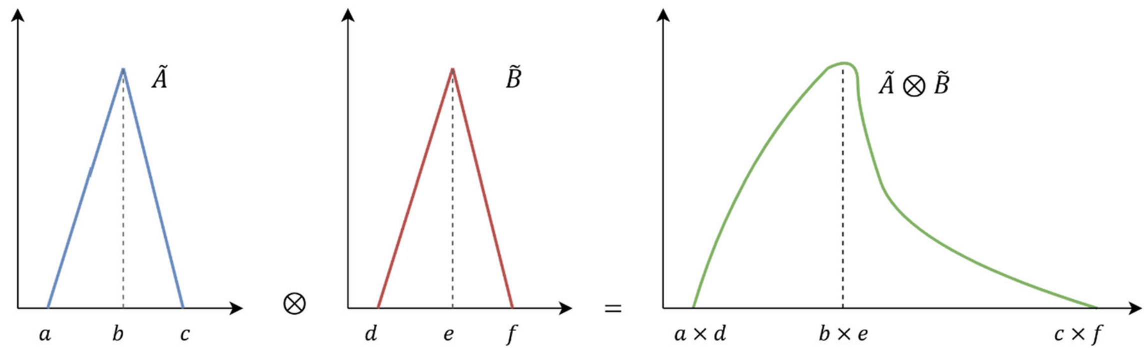


Fig. 5 Multiplication of two triangular fuzzy sets

developed. The main formulations for NSs are first developed, and then they are converted into INS with the help of α-cut. So, the derivations and obtained formulations can be used for INSs and serves NSs too. From this perspective, this paper proposes some new mathematical formulations of ASPs based on INSs. For this aim, main principles of ASPs such as Pa, AOQ, AOQL, ATI, and ASN have been redesigned by considering new design principles of Binomial and Poisson Distributions based on INSs.

ASPs based on INS give more useful and understandable results than ASPs based on NS. There are various reasons for this inference. One of the most important reasons is that the multiplication operation is inevitably used in ASP calculations. Since the multiplication between two fuzzy membership functions does not guarantee that the shape of the set is preserved. For example, the new set obtained from the multiplication of two triangular fuzzy sets represented in a 3-tuple structure (a, b, c) and (d, e, f) does not have to be a triangle. Therefore, although the obtained multiplication set is continued to be symbolized in a 3-tuple structure as $(a \times d, b \times e, c \times f)$, this representation does not correspond to a triangular fuzzy set. Therefore, this 3-tuple representation has no additional explanatory power other than showing the core and support points. This phenomenon is symbolized in Fig. 5. So, INS-based ASPs are more advantageous than NS-based ASPs, which do not have a significant added value but cause visual and operational complexity.

Suppose that the α-cut approach is applied to a given NS of $\dddot{A} = \langle \tilde{t}, \tilde{i}, \tilde{f} \rangle = \langle \mu_{\tilde{A}}(x), \sigma_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x) \rangle$, then, the INS shown in Eq. (20) is obtained. $\tilde{f}$ is complementing term, so it is cut by $(1 - \alpha)$ depending on Eq. (14). After transforming a NS into an INS by applying the α-cut approach, the interval operations shown in Eqs. (17–22) become applicable for the intervals $t_\alpha = [t_L, t_U]$, $i_\alpha = [i_L, i_U]$ and $f_{1-\alpha} = [f_L, f_U]$ as shown in Eq. (23):

$$\begin{aligned}\dddot{A}_\alpha &= \langle t_\alpha, i_\alpha, f_{1-\alpha} \rangle = \langle \mu_{\tilde{A}_\alpha}(x), \sigma_{\tilde{A}_\alpha}(x), \vartheta_{\tilde{A}_{1-\alpha}}(x) \rangle \\ &= \langle [\mu_{L_\alpha}(x), \mu_{U_\alpha}(x)], [\sigma_{L_\alpha}(x), \sigma_{U_\alpha}(x)], [\vartheta_{L_{1-\alpha}}(x), \vartheta_{U_{1-\alpha}}(x)] \rangle \\ &= \langle [t_L, t_U], [i_L, i_U], [f_L, f_U] \rangle, t_L, t_U, i_L, i_U, f_L, f_U \in [0, 1].\end{aligned} \tag{23}$$

In this study, the mathematical formulations of ASPs are extended for INSs yielded by α-cut of NSs. In the offered plans, the inspection process has indeterminacy outcome that considered as intervals in addition to acceptance and rejection decisions. For this aims, Neutrosophic Binomial distribution offered by Smarandache (2014) and Neutrosophic Poisson distribution offered by Işık and Kaya (2022d) are used as baselines in formulations. However, ASPs were offered by Işık and Kaya (2022d) for lots having Neutrosophic defectiveness information such that $\dddot{A} = \{t, i, f\} = \{(P(S), P(I), P(F))\}$, while $P(S)$ (non-defective probability), $P(I)$ (indeterminacy probability), and $P(F)$ (defective probability) are represented with real numbers. In this study, lots having Neutrosophic defect information as fuzzy membership functions such that $\dddot{A} = \{\tilde{t}, \tilde{i}, \tilde{f}\} = \{(\tilde{P}(S), \tilde{P}(I), \tilde{P}(F))\}$ are considered in design of ASP and main formulations. The equation $\tilde{t} \oplus \tilde{i} \oplus \tilde{f} = 1$ [1]should be satisfied according to Neutrosophic Binomial distribution and Neutrosophic ASPs. Since $\tilde{f}$ is complementing term, $\tilde{t} \oplus \tilde{i} \oplus \tilde{f} = 1$ equity is satisfied with the help of $\tilde{f}$ as shown in Eq. (24) by using the interval summation and extraction operations presented in Eqs. (17–18):

[1] To distinguish operations between fuzzy numbers from ordinary arithmetic operations, $\oplus, \ominus, \otimes, \oslash$ symbols have been preferred for fuzzy numbers.

$$\begin{aligned}
1 &= \widetilde{t} \oplus \widetilde{i} \oplus \widetilde{f} \\
\widetilde{f} &= 1 \ominus \left(\widetilde{t} \oplus \widetilde{i}\right) \\
\widetilde{f}_\alpha &= 1 \ominus \left(\widetilde{t}_\alpha \oplus \widetilde{i}_\alpha\right) \\
[f_L, f_U] &= [1,1] - ([t_L, t_U] + [i_L, i_U]) \\
[f_L, f_U] &= [1,1] - [t_L + i_L, t_U + i_U] \\
[f_L, f_U] &= [1 - (t_U + i_U), 1 - (t_L + i_L)] \\
1 &= t_U + i_U + f_L = t_L + i_L + f_U.
\end{aligned} \tag{24}$$

The ASP formulations offered by Işık and Kaya (2022d) use summation and multiplication operations between t, i and f. Fortunately, Eq. (17) and Eq. (18) allow making operations for the upper and lower limits of the intervals separately for summation and multiplication operations. Owing to this, the following sub-sections introduce the formulations by handling interval limits separately as real numbers. Additionally, to satisfy $t + i + f = 1$, the real number groups $\{t_U, i_U, f_L\}$ and $\{t_L, i_L, f_U\}$ are used.

5.1 Design of single ASPs based on INSs

When the defectiveness information made up of fuzzy membership functions such that $\widetilde{\ddot{A}} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\} = \left\{\left(\widetilde{P}(S), \widetilde{P}(I), \widetilde{P}(F)\right)\right\}$ is used in formulations, results of the ASPs should also be reached as a Neutrosophic set such that $\widetilde{\ddot{B}} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\} = \left\{\left(\widetilde{P}_a, \widetilde{P}_i, \widetilde{P}_r\right)\right\}$ where $\widetilde{P}_a$ is lot acceptance probability, $\widetilde{P}_r$ is lot rejection probability, and $\widetilde{P}_i$ is indeterminacy probability. There is an extra plan parameter named maximum allowed indeterminate item count (I) used in the formulations because of NSs.

It is hard to use fuzzy membership functions directly in formulations. For this reason, α-cut technique is used to covert to membership functions into interval valued numbers similar to the common approach applied by Divya (2012), Jamkhaneh et al. (2011), Jamkhaneh and Gildeh (2012) and Kahraman and Kaya (2010). In these studies, two of main parameters for ASPs that are n and c are also considered as traditional FSs. It is easy to apply α-cut for these too, but NSs have three independent membership functions for t,i and f. Considering n,c, and I as NSs makes the formulation more complicated. To simplify the related formulations, these plan parameters are considered as real numbers in this paper. If α-cut is applied to $\widetilde{\ddot{A}}$, three intervals given in Eq. (25) are obtained as follows:

$$\begin{aligned}
\widetilde{\ddot{A}}_\alpha &= \left\{\widetilde{t}_\alpha, \widetilde{i}_\alpha, \widetilde{f}_{1-\alpha}\right\} = \{[t_L, t_U], [i_L, i_U], [f_L, f_U]\} \\
&= \left\{\left[P(S)_L, P(S)_U\right], \left[P(I)_L, P(I)_U\right], \left[P(F)_L, P(F)_U\right]\right\}.
\end{aligned} \tag{25}$$

For Poisson distribution, defect and indeterminacy probabilities are converted to defect frequency ($\widetilde{\lambda}_F$) and indeterminacy frequency ($\widetilde{\lambda}_I$) by using Eq. (26):

$$\widetilde{\lambda}_F = n \otimes \widetilde{P}(F), \widetilde{\lambda}_I = n \otimes \widetilde{P}(I). \tag{26}$$

For a lot having N items, $\widetilde{P}_a$ is calculated as shown in Eq. (27). These formulations include fuzzy membership functions $\left(\widetilde{t}, \widetilde{i}, \widetilde{f}\right)$ instead of real numbers (t, i, f) as defect information:

$$\begin{aligned}
&\widetilde{P}_a(\widetilde{t}, \widetilde{i}, \widetilde{f}) = \widetilde{P}\{d \le c, i \le I\} \\
&\widetilde{P}_{a_{\text{Binomial}}} = \sum_{d=0}^{c}\left(\binom{n}{d} \otimes \widetilde{P}(F)^d \otimes \left[\sum_{i=0}^{\min(I,n-d)}\left(\binom{n-d}{i} \otimes \right.\right.\right. \\
&\qquad \left.\left.\left.\widetilde{P}(I)^i \otimes \widetilde{P}(S)^{(n-i-d)}\right)\right]\right) \simeq \widetilde{P}_{a_{\text{Poisson}}} \\
&\qquad = \sum_{d=0}^{c}\left(\left(\frac{\widetilde{\lambda}_F^d}{d!}\right) \otimes \left[\sum_{i=0}^{\min(I,n-d)}\left(\left(\frac{\widetilde{\lambda}_I^i}{i!}\right) \otimes e^{\ominus\left(\widetilde{\lambda}_I \oplus \widetilde{\lambda}_F\right)}\right)\right]\right).
\end{aligned} \tag{27}$$

Acceptance probability of lot after α-cut ($\widetilde{P}_{a_\alpha}$) becomes an INS and it is determined by Eq. (28). In the equation, $P_a(t, i, f)$ means using Eq. (24) with real numbers (t, i, f) as defect information:

$$\begin{aligned}
\widetilde{P}_{a_\alpha} &= [P_{a_L}, P_{a_U}] \\
&= \begin{bmatrix} min\{P_a(t,i,f) | (t,i,f) \in \{(t_L, i_L, f_U), (t_U, i_U, f_L)\}\} \\ , max\{P_a(t,i,f) | (t,i,f) \in \{(t_L, i_L, f_U), (t_U, i_U, f_L)\}\} \end{bmatrix}.
\end{aligned} \tag{28}$$

$\widetilde{P}_r$ is calculated as seen in Eq. (29):

$$\begin{aligned}
&\widetilde{P}_r(\widetilde{t}, \widetilde{i}, \widetilde{f}) = \widetilde{P}\{d > c\} \\
&\widetilde{P}_{r_{\text{Binomial}}} = \sum_{d=c+1}^{n}\left(\binom{n}{d} \otimes \widetilde{P}(F)^d \otimes \left[\sum_{i=0}^{n-d}\left(\binom{n-d}{i} \otimes \right.\right.\right. \\
&\qquad \left.\left.\left.\widetilde{P}(I)^i \otimes \widetilde{P}(S)^{(n-i-d)}\right)\right]\right) \simeq \widetilde{P}_{r_{\text{Poisson}}} \\
&\qquad = \sum_{d=c+1}^{n}\left(\left(\frac{\widetilde{\lambda}_F^d}{d!}\right) \otimes \left[\sum_{i=0}^{n-d}\left(\left(\frac{\widetilde{\lambda}_I^i}{i!}\right) \otimes e^{\ominus\left(\widetilde{\lambda}_I \oplus \widetilde{\lambda}_F\right)}\right)\right]\right).
\end{aligned} \tag{29}$$

Reject probability after α-cut ($\widetilde{P}_{r_\alpha}$) becomes an INS, and it is obtained by Eq. (30):

$$\widetilde{P}_{r_\alpha} = [P_{r_L}, P_{r_U}] = \begin{bmatrix} min\{P_r(t,i,f)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \\ ,max\{P_r(t,i,f)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \end{bmatrix}. \quad (30)$$

$\widetilde{P}_i$ is clarified as shown in Eq. (31):

If the items in the lot are considered as non-defective in case of indeterminacy, $\widetilde{ATI}_{\text{opt}}$ is obtained as shown in Eqs. (35–36) for $\dddot{\widetilde{A}}$ and $\dddot{\widetilde{A}}_\alpha$. If all the items are tested in case of indeterminacy, $\widetilde{ATI}_{\text{pes}}$ is calculated as shown as in Eqs. (37–38):

$$\widetilde{ATI}_{\text{opt}} = n \oplus \widetilde{P}_r\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \otimes (N-n), \quad (35)$$

$$\widetilde{ATI}_{\text{opt}_\alpha} = \left[\begin{array}{l} min\{n + P_r(t,i,f) \times (N-n)|(t,i,f) \\ \quad \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\}, max\{n + P_r(t,i,f) \times (N-n)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \end{array}\right], \quad (36)$$

$$\widetilde{ATI}_{\text{pes}} = n \oplus \left(\widetilde{P}_r\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \oplus \widetilde{P}_i\left(\widetilde{t},\widetilde{i},\widetilde{f}\right)\right) \otimes (N-n), \quad (37)$$

$$\widetilde{ATI}_{\text{pes}_\alpha} = \begin{bmatrix} min\{n + (P_r(t,i,f) + P_i(t,i,f)) \times (N-n)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \\ ,max\{n + (P_r(t,i,f) + P_i(t,i,f)) \times (N-n)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \end{bmatrix}. \quad (38)$$

$$\begin{aligned} \widetilde{P}_i(\widetilde{t},\widetilde{i},\widetilde{f}) &= \widetilde{P}\{d \le c, i > I\} \\ \widetilde{P}_{i_{\text{Binomial}}} &= \sum_{i=I+1}^{n}\left(\binom{n}{i} \otimes \widetilde{P}(I)^i \otimes \left[\sum_{d=0}^{\min(c,n-i)}\left(\binom{n-i}{d} \otimes \widetilde{P}(F)^d \otimes \widetilde{P}(S)^{(n-i-d)}\right)\right]\right) \simeq \widetilde{P}_{i_{\text{Poisson}}} \\ &= \sum_{i=I+1}^{n}\left(\left(\frac{\widetilde{\lambda}_I^i}{i!}\right) \otimes \left[\sum_{d=0}^{\min(c,n-i)}\left(\left(\frac{\widetilde{\lambda}_F^d}{d!}\right) \otimes e^{\ominus\left(\widetilde{\lambda}_I \oplus \widetilde{\lambda}_F\right)}\right)\right]\right). \end{aligned} \quad (31)$$

Indeterminate probability after α-cut ($\widetilde{P}_{i_\alpha}$) becomes an INS, and it is derived by Eq. (32):

$$\widetilde{P}_{i_\alpha} = [P_{i_L}, P_{i_U}] = \begin{bmatrix} min\{P_i(t,i,f)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \\ ,max\{P_i(t,i,f)|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \end{bmatrix}. \quad (32)$$

$\widetilde{AOQ}$ is calculated for NS $\dddot{\widetilde{A}}$ and INS $\dddot{\widetilde{A}}_\alpha$ as in Eqs. (33) and (34), respectively:

$$\begin{aligned} \widetilde{AOQ} &= \widetilde{P}_a\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \otimes \widetilde{t} = \widetilde{P}_a\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \otimes \widetilde{P}(F) \\ &= \widetilde{P}_a\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \otimes \left(\frac{\widetilde{\lambda}_F}{n}\right), \end{aligned} \quad (33)$$

$$\widetilde{AOQ}_\alpha = \begin{bmatrix} min\{P_a(t,i,f) \times t|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \\ ,max\{P_a(t,i,f) \times t|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \end{bmatrix}. \quad (34)$$

5.2 Design of double ASPs based on INSs

Similar to the previous sub-section, lots having Neutrosophic defect information as $\dddot{\widetilde{A}} = \left\{\widetilde{t},\widetilde{i},\widetilde{f}\right\} = \left\{\left(\widetilde{P}(S),\widetilde{P}(I),\widetilde{P}(F)\right)\right\}$ are also re-considered in design of double ASPs into this sub-section. DASPs have extra plan parameters in the formulations such that sample size for the first and second steps (n_1, n_2), maximum allowed defective item count for the first and second steps (c_1, c_2) and maximum allowed indeterminate item count for the first and second steps (I_1, I_2). Because of having two steps, acceptance probability of the lot in the first step $(\widetilde{P}_a^I(\widetilde{t},\widetilde{i},\widetilde{f}))$, acceptance probability of the lot in the second step $(\widetilde{P}_a^{II}(\widetilde{t},\widetilde{i},\widetilde{f}))$, and termination probability of the sampling in the first step $(\widetilde{P}^I)$ should be defined. For Poisson distribution, $\widetilde{P}(F)$ and $\widetilde{P}(I)$ are converted to defect frequencies for the first and the second steps $(\widetilde{\lambda}_{F_1} = n_1 \otimes \widetilde{P}(F), \widetilde{\lambda}_{F_2} = n_2 \otimes \widetilde{P}(F))$, and indeterminacy frequencies for the first and the second steps$(\widetilde{\lambda}_{I_1} = n_1 \otimes \widetilde{P}(I), \lambda_{I_2} = n_2 \otimes \widetilde{P}(I))$.

For a lot having N items, $\widetilde{P}_a$ is calculated as shown in Eq. (39):

$$\begin{aligned}
\widetilde{P}_a(\widetilde{t},\widetilde{i},\widetilde{f}) &= \widetilde{P}\{d_1 \le c_1, i_1 < I_1\}\\
&\oplus \widetilde{P}\{d_1 + d_2 \le c_2, i_1 + i_2 \le I_2 | c_1 < d_1 \le c_2, i_1 \le I_2\}\\
&\oplus \widetilde{P}\{d_1 + d_2 \le c_2, i_1 + i_2 \le I_2 | d_1 \le c_1, I_1 < i_1 \le I_2\}\\
\widetilde{P_{a\,\text{Binomial}}} &= \sum_{d_1=0}^{c_1}\left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=0}^{I_1}\left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)}\right)\right]\right)\\
&\oplus \sum_{d_1=c_1+1}^{c_2}\left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=0}^{I_2}\left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)}\right.\right.\right.\\
&\otimes \left.\left.\left.\left[\sum_{d_2=0}^{c_2-d_1}\left(\binom{n_2}{d_2} \otimes \widetilde{P}(F)^{d_2} \otimes \left[\sum_{i_2=0}^{I_2-i_1}\left(\binom{n_2-d_2}{i_2} \otimes \widetilde{P}(I)^{i_2} \otimes \widetilde{P}(S)^{(n_2-i_2-d_2)}\right)\right]\right)\right]\right)\right]\right)\\
&\oplus \sum_{d_1=0}^{c_1}\left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=I_1+1}^{I_2}\left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)}\right.\right.\right.\\
&\otimes \left.\left.\left.\left[\sum_{d_2=0}^{c_2-d_1}\left(\binom{n_2}{d_2} \otimes \widetilde{P}(F)^{d_2} \otimes \left[\sum_{i_2=0}^{I_2-i_1}\left(\binom{n_2-d_2}{i_2} \otimes \widetilde{P}(I)^{i_2} \otimes \widetilde{P}(S)^{(n_2-i_2-d_2)}\right)\right]\right)\right]\right)\right]\right)\\
&\simeq \widetilde{P}_{a_{\text{Poisson}}} = \sum_{d_1=0}^{c_1}\left(\left(\widetilde{\lambda_{F1}}^{d_1} ø d_1!\right) \otimes \left[\sum_{i_1=0}^{I_1}\left(\left(\widetilde{\lambda}_{I1}^{i_1} ø i_1!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F_1}\right)}\right)\right]\right)\\
&\oplus \sum_{d_1=c_1+1}^{c_2}\left(\left(\widetilde{\lambda_{F1}}^{d_1} ø d_1!\right) \otimes \left[\sum_{i_1=0}^{I_2}\left(\left(\widetilde{\lambda}_{I1}^{i_1} ø i_1!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F_1}\right)}\right.\right.\right.\\
&\otimes \left.\left.\left.\left[\sum_{d_2=0}^{c_2-d_1}\left(\left(\widetilde{\lambda_{F2}}^{d_2} ø d_2!\right) \otimes \left[\sum_{i_2=0}^{I_2-i_1}\left(\left(\widetilde{\lambda}_{I2}^{i_2} ø i_2!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I2} \oplus \widetilde{\lambda}_{F_2}\right)}\right)\right]\right)\right]\right)\right]\right)\\
&\oplus \sum_{d_1=0}^{c_1}\left(\left(\widetilde{\lambda_{F1}}^{d_1} ø d_1!\right) \otimes \left[\textstyle\sum_{i_1=I_1+1}^{I_2}\left(\left(\widetilde{\lambda}_{I1}^{i_1} ø i_1!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F_1}\right)} \otimes\right.\right.\right.\\
&\left.\left.\left.\left[\sum_{d_2=0}^{c_2-d_1}\left(\left(\widetilde{\lambda_{F2}}^{d_2} ø d_2!\right) \otimes \left[\sum_{i_2=0}^{I_2-i_1}\left(\left(\lambda\widetilde{I}_2^{i_2} ø i_2!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I2} \oplus \widetilde{\lambda}_{F_2}\right)}\right)\right]\right)\right]\right)\right]\right).
\end{aligned} \tag{39}$$

$\widetilde{P}_{a_\alpha}$ is determined by Eq. (28) whereby using Eq. (39) for $P_a(t,i,f)$ calculation. $\widetilde{P}_r$ is derived as seen in Eq. (40):

$\widetilde{P}_{r_\alpha}$ is obtained by Eq. (30) through using Eq. (40) for $P_r(t,i,f)$ calculation.

$\widetilde{P}_i$ is found as shown in Eq. (41):

$$\begin{aligned}
\widetilde{P}_r(\widetilde{t},\widetilde{i},\widetilde{f}) = {} & \widetilde{P}\{d_1 > c_2\} \\
& \oplus \widetilde{P}\{d_1 + d_2 > c_2 | c_1 < d_1 \le c_2, i_1 \le I_2\} \\
& \oplus \widetilde{P}\{d_1 + d_2 > c_2 | d_1 \le c_1, I_1 < i_1 \le I_2\} \\
\widetilde{P}_{r_{\text{Binomial}}} = {} & \sum_{d_1=c_2+1}^{n_1} \left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=0}^{n_1-d_1} \left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)} \right) \right] \right) \\
& \oplus \sum_{d_1=c_1+1}^{c_2} \left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=0}^{I_2} \left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)} \right. \right. \right. \\
& \left. \left. \left. \otimes \left[\sum_{d_2=c_2-d_1+1}^{n_2} \left(\binom{n_2}{d_2} \otimes \widetilde{P}(F)^{d_2} \otimes \left[\sum_{i_2=0}^{n_2-d_2} \left(\binom{n_2-d_2}{i_2} \otimes \widetilde{P}(I)^{i_2} \otimes \widetilde{P}(S)^{(n_2-i_2-d_2)} \right) \right] \right) \right] \right) \right] \right) \\
& \oplus \sum_{d_1=0}^{c_1} \left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=I_1+1}^{I_2} \left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)} \right. \right. \right. \\
& \left. \left. \left. \otimes \left[\sum_{d_2=c_2-d_1+1}^{n_2} \left(\binom{n_2}{d_2} \otimes \widetilde{P}(F)^{d_2} \otimes \left[\sum_{i_2=0}^{n_2-d_2} \left(\binom{n_2-d_2}{i_2} \otimes \widetilde{P}(I)^{i_2} \otimes \widetilde{P}(S)^{(n_2-i_2-d_2)} \right) \right] \right) \right] \right) \right] \right) \\
\simeq \widetilde{P}_{r_{\text{Poisson}}} = {} & \sum_{d_1=c_2+1}^{n_1} \left(\left(\widetilde{\lambda_{F1}}^{d_1} \text{ø} d_1! \right) \otimes \left[\sum_{i_1=0}^{n_1-d_1} \left(\left(\widetilde{\lambda}_{I1}^{i_1} \text{ø} i_1! \right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F1}\right)} \right) \right] \right) \\
& \oplus \sum_{d_1=c_1+1}^{c_2} \left(\left(\widetilde{\lambda_{F1}}^{d_1} \text{ø} d_1! \right) \otimes \left[\sum_{i_1=0}^{I_2} \left(\left(\widetilde{\lambda}_{I1}^{i_1} \text{ø} i_1! \right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F1}\right)} \right. \right. \right. \\
& \left. \left. \left. \otimes \left[\sum_{d_2=c_2-d_1+1}^{n_2} \left(\left(\widetilde{\lambda_{F2}}^{d_2} \text{ø} d_2! \right) \otimes \left[\sum_{i_2=0}^{n_2-d_2} \left(\left(\widetilde{\lambda}_{I2}^{i_2} \text{ø} i_2! \right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I2} \oplus \widetilde{\lambda}_{F2}\right)} \right) \right] \right) \right] \right) \right] \right) \\
& \oplus \sum_{d_1=0}^{c_1} \left(\left(\left(\widetilde{\lambda_{F1}}^{d_1} \text{ø} d_1! \right) \right) \otimes \left[\sum_{i_1=I_1+1}^{I_2} \left(\left(\widetilde{\lambda}_{I1}^{i_1} \text{ø} i_1! \right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F1}\right)} \otimes \right. \right. \right. \\
& \left. \left. \left. \left[\sum_{d_2=c_2-d_1+1}^{n_2} \left(\left(\widetilde{\lambda_{F2}}^{d_2} \text{ø} d_2! \right) \otimes \left[\sum_{i_2=0}^{n_2-d_2} \left(\left(\widetilde{\lambda}_{I2}^{i_2} \text{ø} i_2! \right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I2} \oplus \widetilde{\lambda}_{F2}\right)} \right) \right] \right) \right] \right) \right] \right).
\end{aligned} \tag{40}$$

$$\begin{aligned}
\widetilde{P}_i(\widetilde{t},\widetilde{i},\widetilde{f}) &= \widetilde{P}\{i_1 > I_2, d_1 \le c_1\} \\
&\oplus \widetilde{P}\{d_1+d_2 \le c_2, i_1+i_2 > I_2 | c_1 < d_1 \le c_2, i_1 \le I_2\} \\
&\oplus \widetilde{P}\{d_1+d_2 \le c_2, i_1+i_2 > I_2 | d_1 \le c_1, I_1 < i_1 \le I_2\} \\
\widetilde{P}_{i_{\text{Binomial}}} &= \sum_{d_1=0}^{c_2}\left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=I_2+1}^{n_1-d_1}\left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)}\right)\right]\right) \\
&\oplus \sum_{d_1=c_1+1}^{c_2}\left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=0}^{I_2}\left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)}\right.\right.\right. \\
&\otimes \left[\sum_{d_2=0}^{c_2-d_1}\left(\binom{n_2}{d_2} \otimes \widetilde{P}(F)^{d_2} \otimes \left[\sum_{i_2=I_2-i_1+1}^{n_2-d_2}\left(\binom{n_2-d_2}{i_2} \otimes \widetilde{P}(I)^{i_2} \otimes \widetilde{P}(S)^{(n_2-i_2-d_2)}\right)\right]\right)\right]\Bigg)\Bigg]\Bigg) \\
&\oplus \sum_{d_1=0}^{c_1}\left(\binom{n_1}{d_1} \otimes \widetilde{P}(F)^{d_1} \otimes \left[\sum_{i_1=I_1+1}^{I_2}\left(\binom{n_1-d_1}{i_1} \otimes \widetilde{P}(I)^{i_1} \otimes \widetilde{P}(S)^{(n_1-i_1-d_1)}\right.\right.\right. \\
&\otimes \left[\sum_{d_2=0}^{c_2-d_1}\left(\binom{n_2}{d_2} \otimes \widetilde{P}(F)^{d_2} \otimes \left[\sum_{i_2=I_2-i_1+1}^{n_2-d_2}\left(\binom{n_2-d_2}{i_2} \otimes \widetilde{P}(I)^{i_2} \otimes \widetilde{P}(S)^{(n_2-i_2-d_2)}\right)\right]\right)\right]\Bigg)\Bigg]\Bigg) \\
\simeq \widetilde{P}_{i_{\text{Poisson}}} &= \sum_{d_1=0}^{c_2}\left(\left(\widetilde{\lambda_{F1}}^{d_1} ø d_1!\right) \otimes \left[\sum_{i_1=I_2+1}^{n_1-d_1}\left(\widetilde{\lambda}_{I1}^{i_1} ø i_1!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F1}\right)}\right]\right) \\
&\oplus \sum_{d_1=c_1+1}^{c_2}\left(\left(\widetilde{\lambda_{F1}}^{d_1} ø d_1!\right) \otimes \left[\sum_{i_1=0}^{I_2}\left(\left(\widetilde{\lambda}_{I1}^{i_1} ø i_1!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F1}\right)}\right.\right.\right. \\
&\otimes \left[\sum_{d_2=0}^{c_2-d_1}\left(\left(\widetilde{\lambda_{F2}}^{d_2} ø d_2!\right) \otimes \left[\sum_{i_2=I_2-i_1+1}^{n_2-d_2}\left(\left(\widetilde{\lambda}_{I2}^{i_2} ø i_2!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I2} \oplus \widetilde{\lambda}_{F2}\right)}\right)\right]\right)\right]\Bigg)\Bigg]\Bigg) \\
&\oplus \sum_{d_1=0}^{c_1}\left(\left(\widetilde{\lambda_{F1}}^{d_1} ø d_1!\right) \otimes \left[\textstyle\sum_{i_1=I_1+1}^{I_2}\left(\left(\widetilde{\lambda}_{I1}^{i_1} ø i_1!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I1} \oplus \widetilde{\lambda}_{F1}\right)}\right.\right.\right. \\
&\otimes \left[\sum_{d_2=0}^{c_2-d_1}\left(\left(\widetilde{\lambda_{F2}}^{d_2} ø d_2!\right) \otimes \left[\sum_{i_2=I_2-i_1+1}^{n_2-d_2}\left(\left(\widetilde{\lambda}_{I2}^{i_2} ø i_2!\right) \otimes e^{\ominus\left(\widetilde{\lambda}_{I2} \oplus \widetilde{\lambda}_{F2}\right)}\right)\right]\right)\right]\Bigg)\Bigg]\Bigg).
\end{aligned} \tag{41}$$

$\widetilde{P}_{i_\alpha}$ is derived by Eq. (32) via using Eq. (41) for $P_i(t,i,f)$ calculation. $\widetilde{AOQ}$ is calculated for an NS $\dddot{A}$ as in Eq. (42) and for an INS $\widetilde{A}_\alpha$ as in Eq. (34):

$$\begin{aligned}
\widetilde{AOQ} &= \widetilde{P}_a(\widetilde{t},\widetilde{i},\widetilde{f}) \otimes \widetilde{t} = \widetilde{P}_a(\widetilde{t},\widetilde{i},\widetilde{f}) \otimes \widetilde{P}(F) \\
&= \widetilde{P}_a(\widetilde{t},\widetilde{i},\widetilde{f}) \otimes \widetilde{\lambda_{F1}} ø n_1 = \widetilde{P}_a(\widetilde{t},\widetilde{i},\widetilde{f}) \otimes \widetilde{\lambda_{F2}} ø n_2
\end{aligned} \tag{42}$$

To calculate $\widetilde{ATI}$, $\widetilde{P}_a^I(\widetilde{t},\widetilde{i},\widetilde{f})$ and $\widetilde{P}_a^{II}(\widetilde{t},\widetilde{i},\widetilde{f})$ are found by splitting Eq. (39) into parts shown in Eq. (43):

$$\begin{aligned}
\widetilde{P}_a^I\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) &= \widetilde{P}\{d_1 \le c_1, i_1 < I_1\} \\
\widetilde{P}_a^{II}\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) &= \widetilde{P}\{d_1+d_2 \le c_2, i_1+i_2 \le I_2 | c_1 < d_1 \le c_2, i_1 \le I_2\} \\
&\oplus \widetilde{P}\{d_1+d_2 \le c_2, i_1+i_2 \le I_2 | d_1 \le c_1, I_1 < i_1 \le I_2\}.
\end{aligned} \tag{43}$$

$\widetilde{ATI}_{\text{opt}}$ and $\widetilde{ATI}_{pes}$ are calculated as shown in Eqs. (44–47):

$$\begin{aligned}
\widetilde{ATI}_{\text{opt}} &= n_1 \otimes \widetilde{P}_a^I\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \oplus (n_1+n_2) \otimes \widetilde{P}_a^{II}\left(\widetilde{t},\widetilde{i},\widetilde{f}\right) \oplus N \\
&\otimes \widetilde{P}_r\left(\widetilde{t},\widetilde{i},\widetilde{f}\right),
\end{aligned} \tag{44}$$

Table 1 Comparisons of the results of the ASPs based on interval statistics for risk-averse and risk-taking operators

	Risk-Averse Operator		Risk-Taking Operator	
	Single ASP	Double ASP	Single ASP	Double ASP
$\widetilde{P}_a$	[25.03%, 64.73%]	[27.65%, 70.13%]	[42.53%, 98.22%]	[46.77%, 99.23%]
$\widetilde{P}_r$	[35.27%, 74.97%]	[29.87%, 72.35%]	[1.78%, 57.47%]	[0.77%, 53.23%]
$\widetilde{AOQ}$	[0.025, 0.039]	[0.028, 0.042]	[0.020, 0.039]	[0.020, 0.042]
$\widetilde{ATI}$	[208.713, 387.368]	[31.442, 360.875]	[57.991, 308.617]	[35.794, 276.615]
$\widetilde{ASN}$	–	[49.265, 52.485]	–	[36.275, 52.131]

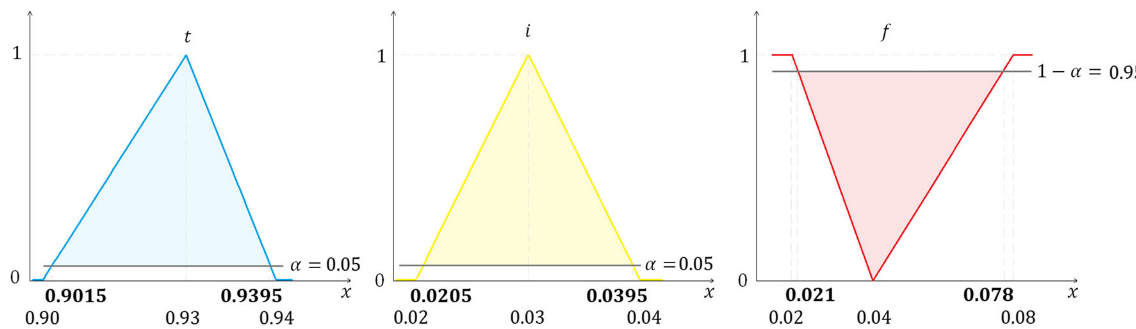


Fig. 6 α-Cut of the neutrosophic defectiveness information

Table 2 Comparisons of the Results of the ASPs based on INS and NS

	Interval neutrosophic plan		Neutrosophic plan	
	Single ASP	Double ASP	Single ASP	Double ASP
$\widetilde{P}_a$	[60.15%, 70.05%]	[63.14%, 70.36%]	(38.89%, 66.41%, 69.53%)	(41.18%, 66.73%, 70.24%)
$\widetilde{P}_i$	[5.73%, 31.05%]	[8.32%, 32.02%]	(3.64%, 16.56%, 31.82%)	(5.97%, 20.32%, 32.55%)
$\widetilde{P}_r$	[2.08%, 34.12%]	[0.88%, 28.54%]	(1.78%, 13.91%, 57.47%)	(0.72%, 9.44%, 52.84%)
$\widetilde{AOQ}$	[0.014, 0.035]	[0.014, 0.037]	(0.013, 0.028, 0.031)	(0.013, 0.028, 0.033)
$\widetilde{ATI}_{opt}$	[59.369, 216.471]	[30.001, 188.878]	(117.533, 166.424, 308.617)	(87.079, 140.793, 283.415)
$\widetilde{ATI}_{pes}$	[184.781, 229.308]	[177.709, 212.996]	(187.128, 201.175, 324.975)	(178.726, 191.653, 313.284)
$\widetilde{ASN}$	–	[42.441, 50.229]	–	(42.261, 46.484, 52.613)

$$\widetilde{ATI}_{\text{opt}_\alpha} = \left[\begin{array}{l} min\left\{ n_1 \times P_a^I(t,i,f) + (n_1+n_2) \times P_a^{II}(t,i,f) + N \times P_r(t,i,f) | (t,i,f) \in \left\{ \begin{array}{l} (t_L, i_L, f_U), \\ (t_U, i_U, f_L) \end{array} \right\} \right\} \\ , max\left\{ n_1 \times P_a^I(t,i,f) + (n_1+n_2) \times P_a^{II}(t,i,f) + N \times P_r(t,i,f) | (t,i,f) \in \left\{ \begin{array}{l} (t_L, i_L, f_U), \\ (t_U, i_U, f_L) \end{array} \right\} \right\} \end{array} \right], \quad (45)$$

$$\widetilde{ATI}_{\text{pes}} = n_1 \otimes \widetilde{P}_a^{I}\left(\tilde{t},\tilde{i},\tilde{f}\right) \oplus (n_1+n_2) \otimes \widetilde{P}_a^{II}\left(\tilde{t},\tilde{i},\tilde{f}\right) \oplus N \otimes \left(\widetilde{P}_r\left(\tilde{t},\tilde{i},\tilde{f}\right) \oplus \widetilde{P}_i\left(\tilde{t},\tilde{i},\tilde{f}\right)\right), \tag{46}$$

$$\widetilde{ATI}_{\text{pes}_\alpha} = \left[\begin{array}{l} min\left\{\begin{array}{c} n_1 \times P_a^I(t,i,f) + (n_1+n_2) \times P_a^{II}(t,i,f) + N \times (P_r(t,i,f) + P_i(t,i,f)) \\ |(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\} \end{array}\right\} \\ ,max\left\{\begin{array}{c} n_1 \times P_a^I(t,i,f) + (n_1+n_2) \times P_a^{II}(t,i,f) + N \times (P_r(t,i,f) + P_i(t,i,f)) \\ |(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\} \end{array}\right\} \end{array}\right]. \tag{47}$$

To calculate $\widetilde{ASN}$, termination probability in the first step ($\widetilde{P}^I\left(\tilde{t},\tilde{i},\tilde{f}\right)$) is found by splitting parts from Eqs. (39–41) shown in Eq. (48):

$$\widetilde{P}^I\left(\tilde{t},\tilde{i},\tilde{f}\right) = \widetilde{P}_a{}^I\left(\tilde{t},\tilde{i},\tilde{f}\right) \oplus \widetilde{P}_r{}^I\left(\tilde{t},\tilde{i},\tilde{f}\right) \oplus \widetilde{P}_i{}^I\left(\tilde{t},\tilde{i},\tilde{f}\right) = \widetilde{P}\{d_1 \le c_1, i_1 < I_1\} \oplus \widetilde{P}\{d_1 > c_2\} \oplus \widetilde{P}\{i_1 > I_2, d_1 \le c_1\}. \tag{48}$$

$\widetilde{ASN}$ is calculated as shown in Eqs. (49–50):

$$\widetilde{ASN} = n_1 \oplus n_2 \otimes \left(1 \ominus \widetilde{P}^I\left(\tilde{t},\tilde{i},\tilde{f}\right)\right), \tag{49}$$

$$\widetilde{ASN}_\alpha = \left[\begin{array}{l} min\{n_1 + n_2(1-P^I(t,i,f))|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \\ ,max\{n_1 + n_2(1-P^I(t,i,f))|(t,i,f) \in \{(t_L,i_L,f_U),(t_U,i_U,f_L)\}\} \end{array}\right]. \tag{50}$$

6 Some numerical applications based on real cases

In this section, some numerical applications have been analyzed for suggested ASPs based on INSs. The application case has been managed into a manufacturing company. The company supplies pistons from a supplier and applies ASPs for these pistons. Pistons can have various types of issues such as “surface roughness,” “structural deformities,” “dimensional differences,” and “material-related inconsistencies,” In addition, the size of the issue can be different for each item. For example, some pistons may have roughness only in the middle of the piston rod, while others may have roughness in the piston ring area. Whether or not these issues are considered defects varies greatly from case to case, and in some cases, the operator may hesitate and remain undecided while deciding on the defectiveness of items. In such a case, the result of the inspection directly depends on the character of the operator involved. Because an operator with a low-risk orientation tends to regard the part as defective, while an operator with a high-risk orientation tends to regard the part as flawless. This may lead to high inspection costs and occasional disputes with the supplier. When the previous batches were examined, the defective item ratio was determined between 0.9 and 0.94 with the most common value 0.93, and the indeterminate (the issues causing hesitation) ratio was determined as 0.03 ± 0.01. Based on this, the Neutrosophic defectiveness has been obtained in the form of a triangular fuzzy set as follows: $\left(\tilde{t},\tilde{i},\tilde{f}\right) = \langle(0.90, 0.93, 0.94), \quad (0.02, 0.03, 0.04), (0.02, 0.04, 0.08)\rangle$. The company prefers to use interval statistics instead of triangular fuzzy sets to minimize computational and interpretability problems and uses MIL-STD-105E normal inspection plan with an Acceptance Quality Limit (AQL) of 2.5%. According to MIL-STD-105E standards, plan parameters are determined as $n = 50, c = 3$ for single ASP, and $n_1 = 32, n_2 = 32, c_1 = 1, c_2 = 4$ for double ASP while the lot size is 500 items. The results of this ASP for both a risk-averse operator (who tends to label as defective in case of indecision) and risk-taking operator (who tends to label as non-defective in case of indecision) are given in Table 1.

According to the results, single and double ASPs give similar acceptance probabilities and the AOQ values, but the double ASPs are more advantageous in terms of ATI. The most significant finding is that the character of the operator affects the results dramatically. The acceptance probability of the risk-taking operator is nearly double of the acceptance probability of the risk-averse operator. In single ASP, it is observed between approximately 25% and 65% for the risk-averse operator when it is observed between 42 and 98%. Also, the lower limit of average total inspection nearly quadrupled. This means a significant increase in the inspection cost. It is not desirable to experience such a cost increase just because of the character of the operator. Another important problem is that the reliability of ASP modeling is negatively affected because the metrics calculated on paper and the metrics encountered in real life are different from each other. All these negativities are due to the fact that classical ASPs do not consider indeterminacy and therefore, do not handle the triple structure that occurs by being defective, non-defective and indeterminate.

As discussed above, the practical results of the classical ASPs vary depending on the character of the operator. The company aims to use an ASP variation that considers the indeterminacy to prevent the mentioned problems. For this aim, the application of ASPs based on INSs is investigated. Thus, it has been decided to operate a similar plan to MIL-STD-105E normal inspection plan, with an Acceptance Quality Limit (AQL) of 2.5% for defective items and the AQL of 4% for indeterminate and defective items together, by modifying it for NSs. As a result of this tendency, the plan parameters are determined as $n = 50, c = 3, I = 2$ for single ASP, and $n_1 = 32, n_2 = 32, c_1 = 1, c_2 = 4, I_1 = 1, I_2 = 2$ for double ASP, while the lot size is 500 items. Expressing these data in triangular form are not preferred because it does not give an idea at first glance about the intermediate values of plan parameters such as AOQ, ASN, causes computational difficulties, and reduces interpretability. Thence, it is desired to express the sampling plan using numbers with interval values. In this way, operators will also be able to be guided more easily. It was considered appropriate to perform this transformation at a 95% confidence level. According to Eq. (14), $\widetilde{t}$ and $\widetilde{i}$ are cut by $\alpha = 0.05$, and $\widetilde{f}$ is cut by $(1 - \alpha = 0.95)$. The INSs are obtained as follows:

$$\begin{aligned}\dddot{A}_{\alpha=0.05} &= \langle \widetilde{t}_{\alpha=0.05}, \widetilde{i}_{\alpha=0.05}, \widetilde{f}_{1-\alpha=0.95} \rangle \\ &= \langle [t_L, t_U], [i_L, i_U], [f_L, f_U] \rangle \\ &= \langle [0.9015, 0.9395], [0.0205, 0.0395], [0.021, 0.078] \rangle.\end{aligned}$$

This is illustrated as shown in Fig. 6.

The $\widetilde{P}_{a_{\alpha=0.05}}, \widetilde{P}_{r_{\alpha=0.05}}, \widetilde{P}_{i_{\alpha=0.05}}, \widetilde{AOQ}_{\alpha=0.05}, \widetilde{ATI}_{opt_{\alpha=0.05}}, \widetilde{ATI}_{pes_{\alpha=0.05}}$, and $\widetilde{ASN}_{\alpha=0.05}$ values are calculated and presented as shown in Table 1. Results of ASPs based on INS have been compared with the results of the ASPs based on NSs in the table. Similar results are obtained for both Binomial and Poisson distributions with a deviation under 1%. To earn from space, only Binomial distribution results are presented into Table 2.

The results show that the probability of acceptance in INS-based ASPs is more narrowly distributed compared to both classical ASPs and NS-based ASPs. It can now easily be said that an acceptance probability of approximately 60 to 70% will be encountered, regardless of the character of the operator. Accordingly, the amount of uncertainty observed in other ASP characteristics also decreased. In the proposed INS-based ASPs, when indeterminacy prevails as a result of the inspection, two different paths can be followed. If an optimistic approach is adopted and a full inspection is not performed for the lot, the average total inspection is observed between 60 and 217 for single ASP and between 30 and 189 for double ASP. On the other hand, if a pessimistic approach is adopted and full inspection is applied, then, the average total inspection is observed between 185 and 230 for single ASP and between 178 and 213 for double ASP.

The interpretability of the results of the ASPs based on INS is quite higher than the ASPs based on NSs—especially for the plan characteristics AOQ, ATI, and ASN—because all output values are in interval form. In this respect, the proposed plans have also a significant advantage in terms of applicability and calculational simplicity. Although the ASPs based on NS seem to be more inclusive and more informative, they do not provide sufficient information about the membership degrees of the intermediate values of the variables, since the FS shapes of the output variables are not the same as the input variables. Therefore, knowing the core points of the FSs increases the complexity of the results, but has no functional benefit.

When the results in Table 2 are compared, it can be inferred that ASPs based on INSs are better in terms of simplicity and accuracy because working with intervals instead of functions ensures ease of calculation, perception, and interpretation over the results. So, ASPs based on INSs give more useful and understandable results than ASPs based on NS. On the other hand, the ASPs based on NSs are more inclusionary because of giving insights about the core and support points of the fuzzy sets but hard to evaluate. However, the shapes of the input fuzzy sets are not preserved in output variables since the multiplication operation is used in ASP calculations. Although both the input triangular fuzzy sets and the output fuzzy sets are represented are represented in 3-tuple (ex: (a, b, c)) structure, this representation does not guarantee that the output variables are also triangular fuzzy sets. The representation of the results in 3-tuple structure can be misleading and has low interpretability. Thus, this 3-tuple representation of ASPs based on NSs has limited explanatory power than the ASPs based on INSs. The proposed ASPs based on INSs are more advantageous than the ASPs based on NSs in terms of visual and operational complexity. Consequently, while the ASPs based on NS are more suitable for theoretical studies, the ASPs based on INSs are more favorable and preferable for engineering problems.

7 Discussion

In classical ASPs, if there are situations during the inspection process where the operator has a hesitancy about deciding on the defectiveness of items, it often depends on the character of the operator whether the item is labeled as defective. If the operator tends to avoid risk, he/she will label the item as defective in cases where it is difficult to decide. Therefore, such an operator may result in increased inspection costs. On the other hand, another more optimistic operator may decide the item as non-defective, which can cause problems with the supplier or other departments. In both cases, there will be differences between the calculations on paper and the probability of lot acceptance encountered in real cases. The proposed ASPs in this paper largely eliminate this problem. Since the operator can now label the item as indeterminate, she/he does not need to take the initiative regarding the decision and can perform the inspection process in a more comfortable and stress-free way. In this way, the difference between the calculations on paper and the values encountered in real life is significantly eliminated.

However, the proposed ASPs have also some assumptions and limitations. Firstly, it is assumed that a rough presumption can be made as to which types of errors are considered defects and which types of errors can cause indeterminacy. Accordingly, the plans consider the probability of item defectiveness and the probability of indeterminacy about item defectiveness as input variables. Neutrosophic ASPs may not produce meaningful outputs if there is such a high level of uncertainty that no error type can be determined as a defect and all errors cause indeterminacy. Secondly, although the formulas presented are adaptable for scenarios where the probabilities about the defectiveness of items are given in the form of membership functions, the scenario that the study focuses on is the scenario where these values are in the form of interval numbers. A similar limitation is also valid for the plan parameters, since it is also assumed that the plan parameters can be determined clearly. In other words, the analysis was made by considering the scenario where the plan parameters are deterministic. If the plan parameters have also some vagueness, the proposed plans may not be applicable.

8 Conclusions and future research suggestions

Coming up to the right quality level for the produced items is very important in manufacturing systems. The received and produced items are inspected to check the quality level. ASPs are common quality control techniques for accepting and rejecting the parties by inspecting only a small set of items to reach specified consumer's and producer's risks. Although the traditional ASPs use certain mass quality metrics, they may not be known as certain in some cases especially in real case problems. For this aim, the FST has been successfully used to redesign of the ASPs with fuzzy defectiveness information in the literature. Nevertheless, the studies using FST extensions in ASPs are very limited. NS theory is an efficient approach on modeling the problems having uncertainty related with human factor. Concerning indeterminacy and covering inconsistent data, the case makes it similar to human thinking and brings an advantage to model uncertainty. INSs have advantages on engineering problems due to the simplicity in problem formulation and calculations for uncertainties. It is clear that this ability can be successfully used for ASPs.

In this study, single and double ASPs having INSs defection statuses are formulated to merge the advantageous of NSs and interval statistics. The sampling procedures of uncertainty cases have been analyzed, and the related traditional formulations have been converted for INSs. Thus, more precise and more flexible results are obtained for inspection processes. These formulations are also applicable for generalized NSs by using α-cut technique. Additionally, two of well-known statistical distributions called Binomial and Poisson distributions are analyzed based on INSs and single and double ASPs have been redesigned based on interval NSs. The main formulations of ASPs have been derived based on these distributions. The proposed ASPs based on INSs have a very powerful procedure to reflect inspectors' hesitancy and risk perception status that are effect inspection stage. If the operator or inspector has difficulty deciding on the defectiveness of items in some cases during the inspection process, practical results of the classical ASPs vary depending on the tendency of the operator to avoid risk, and the calculated plan characteristics do not reflect the real case. The proposed plans have a clear advantage to prevent this issue. The proposed ASPs based on INSs are also more advantageous than the ASPs based on NSs in terms of visual and operational complexity. While the ASPs based on NS are more suitable for theoretical studies, the ASPs based on INSs are more favorable and preferable for engineering problems.

As a future study, the manuscript can be extended to determine standard sampling plans that include indeterminacy. In this way, a framework structure can be proposed for the determination of Neutrosophic ASP parameters. Such a framework will guide the determination of a plan that will maximize the probability and minimize the producer/consumer risks, depending on the probability of acceptance.

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Declarations

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