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OPTIMIZATION

An application of neutrosophic logic on an inventory model with two-level partial trade credit policy for time-dependent perishable products

Kartick Mohanta[1] · Anupam Kumar Jha[1] · Arindam Dey[2] · Anita Pal[1]

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Abstract

Fuzzy and neutrosophic sets are two effective tools to deal with the ambiguities and uncertainties present in real-world problems. To deal with the uncertainties of a real-world scenario, a neutrosophic set is better equipped than a fuzzy set. In this article, we develop a neutrosophic economic order quantity (EOQ) inventory model with the assumption that the market demand is sensitive to the retail price and promotional effort. The supplier and retailer both adopt a partial trade credit policy. We include preservation technology to restrict normal deterioration. We analyse the crisp model first, and then, neutrosophic logic is implemented in the proposed model. De-neutrosophication of total neutrosophic profit is performed based on the removal area approach. The present investigation yields that the de-neutrosophic value of the profit function is convex, that is, a unique solution exists. Mathematical outcomes are generated to efficiently determine the optimal inventory policy for the retailer. To demonstrate numerically and validate the model, a case study is carried out. The findings in this work generalize several existing results, and sensitivity analysis is also reported to support the model.

Keywords Neutrosophic demand · Neutrosophic cost · Neutrosophic profit · Pseudo-concavity · Promotional effort · Preservation technology

1 Introduction

Trade credit is a business-to-commercial enterprise agreement wherein a retailer should buy goods on the account without paying cash in advance, paying the supplier at a later scheduled date. Economic order quantity (EOQ) is a production scheduling model developed by Ford W. Harris in 1913 and has been refined over time. Being a revolutionary model for minimizing inventory costs such as holding costs, shortage costs and order costs, it still draws many of the researcher's attention despite of numerous research that has already been done. The objective of the EOQ formula is to point out the optimal number of product units to order. If achieved, a company can minimize its costs for buying, delivery, and storing units. The EOQ formula can be altered to determine different production levels or order intervals, and corporations with large supply chains and high variable costs use an algorithm in their computer software to determine EOQ. There are many diversity in the research field when it comes to EOQ, trade credit is one of the most realistic field.

In deriving Economic Order Quantity formula, it is tactically assumed that the retailer will pay the amount to supplier as soon as the items are received. However, in most of the case, the supplier will allow a certain fixed period for defraying the amount owed to him for the item supplied. Normally there is no charge if the unsettled amount is settled within the permitted fixed settlement period, following that period some interest is charged. When a supplier allows a fixed time period to settle the account, he is actually giving his customer a loan without interest. During the period before the account has to be settled, the customer can sell the items and continue to collect revenue and earn interest instead of paying off the overdraft. Therefore, it makes economic sense for the cus-

✉ Kartick Mohanta
kartickmath.nit@gmail.com

Anupam Kumar Jha
itsajha@gmail.com

Arindam Dey
arindam84.nit@gmail.com

[1] Department of Mathematics, National Institute of Technology Durgapur, Durgapur, India

[2] VIT-AP Campus, Amaravati, India

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tomer to delay the settlement of the replenishment account up to the last moment of the permissible period by the supplier, this whole method is called upstream trade credit financing. On the other hand, in order to increase the sale and customer retailer also allow a fixed period of time to the consumer to settle the amount and he do not charge if the outstanding amount is settled within the permitted fixed settlement period, beyond this period interest is charged. This is known as downstream trade credit financing.

Inventory is an essential part of our distribution, manufacturing and retail infrastructure where demand plays an important role in choosing the best inventory policy. In the literature referring to models with trade credit ($i.e.$, permissible delay in payment), the demand is mostly treated either as constant or as continuous differentiable function of time. In early stage, demand rate is considered to be constant, as the time progressed, it was seen that demand rate is dependent on many variables like selling price, marketing (promotional effort), downstream trade credit period, etc., and selling price is important contributing factor in estimating demand rate as most of consumer population are economical; hence, slight increase or decrease in selling price may affect the demand rate. Promotional efforts are most common thing a company does in this competitive market and it acts as catalyst when it comes to increase demand rate, and downstream trade credit period allows the consumer to increase their buying power as well it attracts customer; hence, it plays an important role in increasing demand rate.

Many of the goods demand rate remains stagnant throughout the year except some period of time which is also known as seasonal demand, goods like AC, cooler, etc., falls into this category. Goods whose demand rates remain constant throughout the year, like common ailment medicine, spices, etc., fall into the stagnant demand rate category. There are many goods whose demand rate increases exponentially like semiconductor or exponentially decreasing like land phones. Some of the goods follow ramp-type demand rate whose demand increases at a constant rate at first, then remains stagnant for some duration of time and then decreases at constant rate; newly launched garment, automobile, phones, etc., fall into this category.

Deterioration is often called spoilage, obsolescence, loss marginal value of a product, decay, pilferage and damage. Almost each and every commodity deteriorates as the time passes so we can say that it is a function of time. Modern science and technology has managed to decrease the rate of deterioration at some extent, but we cannot eradicate this factor out of equation. The process of decreasing the rate of deterioration is known as preservation. Early methods of preservation are fermentation, refrigeration, and drying. Current methods include the addition of chemicals, irradiation, pasteurization, freezing, and canning . When it comes to food preservation, advanced packaging has decreased the rate of deterioration considerably.

Demand of an item is time-sensitive, and it also depends on the selling price, marketing strategy (promotional effort). It is also common nowadays for the retailer and the supplier to allow some grace period to settle the outstanding amount. Mostly, the items deteriorate with passing time, and salvage value for the item is zero or very less so, many supplier/retailers use preservation technology to increase lifespan to yield maximum profit, and it is not always economical, but in many cases, it yields profit.

1.1 Literature review

The concept of trade credit in inventory control problem was first introduced by S.K Goyal (1985). After that, Shah developed Goyal's model by considering a deteriorating item. In 2002 Teng (2002), come back to Goyal's model and consider the selling price is greater than the purchasing cost and gives an easy analytical closed form solution. Aggarwal and Jaggi (1995) consider an EOQ model with constant deteriorating item allowing delay in payment. Jamal et al. (1997) establish an EOQ model for perishable product with allowable shortage and permissible delay in payment. Sarker et al. (2000) also generalized their model under inflation. Chung et al. (2001) worked on EOQ model for exponentially decay perishable product allowing delay in payment. Teng et al. (2005) modify their old EOQ model by considering selling price (p) is greater than the purchasing price (c) and taking the annual demand rate as a decreasing function of selling price ($D(p) = \alpha p^{-\beta}$, where $\alpha > 0$ and $\beta > 1$) and taking a constant deteriorating item. Chung et al. (2005) further generalized Goyal's 1985 model by considering conditionally permissible delay in payment. Huang (2005) generalized the Goyal's model by adding supplier's partial trade credit. Chung and Huang (2003) built an EPQ model (generalized form of Goyal's model) allowing delay in payment. Huang and Chung (2003) extend Goyal's model with cash discount to reflect the real-life business situations. Ouyang et al. (2007) establish an EOQ model allowing delay in payment for two ware houses under the condition of conditional cash discount policy. Ho et al. (2008) developed an EPQ model under the condition of permissible delay in payment for selling price dependent demand and maximize joint expected total profit. Ouyang et al. (2009) extend Huang and Hsu (2008) model for deteriorating item and cash discount policy link to order quantity. Sana (2008) developed an EOQ model under the condition of permissible delay in payment with selling price and budget for advertisement dependent demand. Skouri et al. (2009) analysed an inventory model for ramp type demand rate, partial backlogging and Weibull deterioration rate. Su (2012) worked on an integrated inventory system with defective items and allowable shortage under trade credit. Teng et al. (2012) developed an EOQ model under trade credit financing and non-decreasing demand ($D(t) = a + bt$). Sarkar (2012) analysed an EOQ model with delay in payments for time varying deteriorating products and time varying demand ($D(t) = a+bt+ct^2$). Ouyang and Chang (2013) proposed an EPQ model with imperfect production process under permissible delay in payments and complete backlogging. Lou and Wang (2013) proposed an EOQ model with demand and default risk linked to credit period. The effect of preservation technology investment on a non-instantaneous deteriorating inventory model was discussed by Dye (2013). Chen et al. (2014) established an EOQ model under the condition of supplier's partial trade credit period link to order quantity. Sarkar et al. (2014) worked on an integrated inventory model with variable lead time, defective units and delay in payments. Wang et al. (2014) proposed an EOQ model for a seller by incorporating the following relevant facts: (1) deteriorating products not only deteriorate continuously but also have their maximum lifetime, and (2) credit period increases not only demand but also default risk. They characterize the seller's optimal credit period and cycle time. Furthermore, they discussed a special case for non-deteriorating items. Ting (2015) developed an EOQ model for deteriorating items with conditional trade credit linked to order quantity. Huang (2003) first generalized Goyal's model by adding the concept of upstream and downstream trade credit period. Liao and Chung (2009) also establish an EOQ model under two levels trade credit policy for deteriorating product. Huang (2006) extend both Huang's 2003 and Teng's 2002 model and handle limited storage space by considering the concept of rented warehouse. Huang and Hsu (2008) extend Huang (2003) model by adding partial upstream trade credit period. Jaggi et al. (2008) derived the concept of credit-linked demand and develops a new inventory model under two levels of trade credit policy to reflect the real-life situations. Teng and Chang (2009) derived an optimal ordering policy for a retailer who offers distinct trade credits to its good and bad credit customers. Mahata and Mahata (2009) establish an EOQ model under two levels trade credit policy for deteriorating item. After that, many research articles like Thangam and Uthayakumar (2010); Kreng and Tan (2011); Ouyang et al. (2013); Liao et al. (2014); Wu et al. (2014); Chen and Teng (2015); Wu et al. (2016); Pal et al. (2016) have been published in the direction of an EOQ model under two-level trade credit policy. Su et al. (2007) establish an EPQ model under the condition that both the supplier and retailer have adopted a trade credit strategy with a demand that is sensitive to the customer's credit period. Huang (2007), Huang (2003) and Huang and Chung (2003) consequently developed an EPQ model under two-level trade credit period. Teng and Chang (2009) also properly work on EPQ model under two levels of trade credit policy. Thangam and Uthayakumar (2008) generalized Su et al. (2007) model by taking that upstream trade credit period is partial. Many researches like Liao (2008); Chung and Liao (2011); Ho (2011); Soni and Patel (2012); Feng et al. (2013); Sarkar et al. (2015); Giri and Sharma (2017) worked on an EPQ model under two-level trade credit finance. Readers are directed to the articles Aazami and Saidi-Mehrabad (2021); Dai et al. (2022); Garg et al. (2022a) for a few recent works on the EOQ model.

1.2 Motivation for neutrosophic EOQ model

Zadeh (1965) introduced the idea of fuzzy sets to capture the uncertainty, vagueness of the real-world phenomenon. After that, several research articles have been developed by numerous eminent researchers by implementing fuzzy sets in various decision-making problems. Researchers like Chang et al. (1998) studied an economic reorder point for fuzzy back-order quantity. Lee and Yao (1998) discussed economic production quantity for fuzzy demand quantity and fuzzy production quantity. Cheng and Wang (2009) presented an inventory model for deteriorating items and trapezoidal type demand rate. Yao and Su (2000) introduced an inventory model with backorder for fuzzy total demand based on interval-valued fuzzy set. Mahata et al. (2005) developed a joint economic lot size model for both the supplier and retailer in fuzzy sense. Mahata and Goswami (2006) established an EPQ model with fuzzy production and fuzzy demand for deteriorating products under trade credit finance. Mahata and Mahata (2011) analysed a fuzzy EOQ model for deteriorating items under partial trade credit finance. Recently, Jaggi et al. (2018) determined a triangular fuzzy EOQ model under delay in payment for perishable products with allowable shortage, and they defuzzify of a total fuzzified cost function using signed distance and centroid method. The fuzzy inventory model has also been enriched with the hands of some other contemporary researchers Shekarian et al. (2016, 2017); Kazemi et al. (2015); Das et al. (2015); Karmakar et al. (2017); Sharma and Govindaluri (2018); Maity et al. (2018). Atanassov (1986) gives an effective extension of Zadeh (1965)'s fuzzy sets by means of introducing the concept of intuitionistic fuzzy sets to address extra appropriately real-life issues. De et al. (2014) considered an EOQ model with promotional and selling price-sensitive demand, and an interpolation by pass to Pareto optimality technique has been implemented under an intuitionistic fuzzy environment. De and Sana (2015) derived an intuitionistic EOQ model for selling price and promotional effort-dependent demand with shortages. Garg et al. (2022a) worked on fuzzy EOQ model.

Fuzzy logic can handle the membership degrees of uncertain parameters of real-world problems, and intuitionistic fuzzy logic can describe the both membership and non-membership degree of different parameters. But fuzzy logic and intuitionistic fuzzy logic cannot properly describe all possible uncertainties of some parameters related to EOQ model and other real-world problems. For instance, when an expert is described an uncertainty of a parameter, he or she may think that the possibility of a true-membership value is equal to 0.6, the possibility of a false-membership value is 0.4, and the degree of their indeterministic value is 0.2. This issue is beyond the scope of FSs and IFSs. To overcome this type of short coming of fuzzy logic and intuitionistic fuzzy logic, Smarandache developed a new concept of neutrosophic logic. After that, many researchers have been many real-life problems (decision making problems) using neutrosophic logic. Pal and Chakraborty (2020) applied the concept of triangular neutrosophic number on an EOQ model for deteriorating products under shortage. Mullai and Surya (2018) developed an EOQ model with price break and obtained its optimum solution by assuming neutrosophic demand and neutrosophic purchasing cost as triangular neutrosophic numbers. Mondal et al. (2020) presented a deterministic single-objective economic order quantity model with limited storage capacity in neutrosophic environment. Deli et al. (2021); Deli (2021) worked on the different types of neutrosophic numbers. Recently, Deli and ÖZTÜRK (2020) analysed a defuzzification method on single-valued trapezoidal neutrosophic number. Recently, Garg et al. (2022b) developed a trapezoidal bipolar neutrosophic EOQ model. For some recent works on neutrosophic environment, readers are referred to the articles Akram and Nawaz (2022); Akram et al. (2021); Ahmed et al. (2021); Habib et al. (2020); Pramanik and Dalapati (2022); Debnath (2022); Smarandache (2022); Kaur and Garg (2022). Neutrosophic logic can deal with the truth membership grade, indeterministic membership grade and falsity membership grade of a parameter. Neutrosophic logic is the most generalized extension of fuzzy logic and intuitionistic fuzzy logic. Triangular single-valued neutrosophic number is a special form of trapezoidal single-valued neutrosophic number. But the number of works in a neutrosophic environment is significantly less. These facts inspire us to build an EOQ model in a neutrosophic environment.

In this study, we first created and analysed a crisp model for time-dependent non-decreasing perishable products to have an expiration date, assuming that market demand is sensitive to the retail price and promotional effort, the retailer and the supplier both adopt a partial trade credit policy and costing for preservation is also taken into account separately. We derived the retailer's total profit function. After that, a neutrosophic logic is implemented in the proposed model, considering demand, retail cost, ordering cost, carrying cost, promotional cost, and cost for preservation technology as a triangular neutrosophic number. Here, we are going to use the removal area method for de-neutrosophication of the total neutrosophic profit. Our main objective is to find the maximum value of the retailer's profit by taking the decision on the selling price and the cycle time. The optimal order quantity and the optimal demand are obtained too. We have introduced some theoretical results to show that the de-neutrosophic value of the total profit function is pseudo-concave. Furthermore, we illustrate the theoretical development with the help of numerical examples. Finally, this paper's results generalize some published results in a crisp sense. We have done a sensitivity analysis to justify the sensitivity of our proposed model.

This paper is structured as follows: Some preliminaries are discussed in Sect. 2. Then, after introducing necessary notations and terminologies in Sects. 3.1 and 3.2, respectively, we formulate the proposed inventory model in crisp sense in Sect. 3.4. In Sect. 4, we develop the total profit in neutrosophic sense. Some new theoretical results concerning the optimal solutions have been established in Sect. 5. Sections 7 and 6 provide a real-life case study and numerical examples, respectively. We further discuss the sensitivity of the proposed model in Sect. 8. We further discuss two already published results in crisp sense in Sect. 9. Finally, we finish this article by illustrating the scope of promising research directions in Sect. 10.

2 Preliminaries

Several definitions and lemmas are discussed in this section. To formulate the proposed model mathematically, these are necessary.

Definition 1 Neutrosophy, a "strong universal formal framework" is thought to include NS. This research focuses on the "origin, nature, and extent of neutralities, as well as their interactions with various ideational spectra." The information about membership and non-membership is measured independently by NS using indeterminacy or reluctance. Thus, it may be said that the idea of NS is a generalization of FS, IFS, and interval-valued sets. Let U_{Neu} be a universe of discourse. Then, a neutrosophic set $\widetilde{H}_{Neu}$ is defined as follows:

$$\widetilde{H}_{Neu} = \Big\{(u,\ T^{\widetilde{H}_{Neu}}(u),\ I^{\widetilde{H}_{Neu}}(u),\ F^{\widetilde{H}_{Neu}}(u)) :\ 0^- \leq T^{\widetilde{H}_{Neu}}(u) + I^{\widetilde{H}_{Neu}}(u) + F^{\widetilde{H}_{Neu}}(u) \leq 3^+,\ \forall\ u \in U_{Neu}\Big\}.$$

Here, $T^{\widetilde{H}_{Neu}}(u) : U_{Neu} \to (0^-, 1^+)$, $I^{\widetilde{H}_{Neu}}(u) : U_{Neu} \to (0^-, 1^+)$ and $F^{\widetilde{H}_{Neu}}(u) : U_{Neu} \to (0^-, 1^+)$ are the truth membership degree, indeterminacy membership degree and falsity membership degree, respectively.

Definition 2 Let U_{Neu} be a universe of discourse. Then, a Single-valued neutrosophic set (SVNS) $\widetilde{H}_{Neu}$ is defined as

Table 1 Overview of recent important publications

Source	Model type	Trade credit		Deterioration rate	Demand rate	PT	PE	Parameters
		US	DS					
Mahata and Mahata (2011)	EOQ	Pa	×	Constant	Constant	×	×	TFN
Liao et al. (2014)	EOQ	F	F	Constant	Constant	×	×	Crisp
Sarkar et al. (2014)	EOQ	F	Pa	$\theta(t) = \frac{1}{1+m-t}$	Constant	×	×	Crisp
Wu et al. (2016)	EOQ	Pa	F	Time varying	Constant	×	×	Crisp
Giri and Sharma (2017)	EPQ	Pa	Pa	×	Selling price sensitive	×	×	Crisp
Tiwari et al. (2018)	EOQ	Pa	Pa	$\theta(t) = \frac{1}{1+m-t}$	Selling price sensitive	×	×	Crisp
Pramanik and Maiti (2019)	EOQ	Pa	Pa	×	DS trade credit period and selling price sensitive	×	×	TFN
Banu and Mondal (2020)	EOQ	F	F	Constant	Time sensitive	×	×	$q-$fuzzy number
Rapolu and Kandpal (2020)	EOQ	F	×	Three parameter Wei-bull deterioration	Promotional effort and selling price sensitive	✓	✓	Crisp
Pramanik and Maiti (2020)	EOQ	F	Pa	×	DS trade credit period, promotional effort and selling price sensitive	×	✓	TFN
Pal et al. (2021)	EPQ	F	F	Time varying	Promotional effort and selling price sensitive	×	✓	Crisp
Bhattacharjee et al. (2021)	EPQ	Pa	F	Time varying	Selling price Sensitive	×	×	Crisp
Das et al. (2021)	EOQ	F	×	Three parameter Weibull distribution	Selling price and stock dependent	✓	×	Crisp
Present paper	EOQ	Pa	Pa	Time varying	Selling price promotional effort and DS trade credit dependent	✓	✓	SVNN

TFN Triangular fuzzy number, m =Expiration date, F Full trade credit, Pa Partial trade credit, $SVNN$ Single valued neutrosophic number, US Upstream trade credit period, DS Down stream trade credit period, PT Preservation technology, PE Promotional effort follows:

$$\widetilde{H}_{Neu} = \Big\{(u,\ T^{\widetilde{H}_{Neu}}(u),\ I^{\widetilde{H}_{Neu}}(u),\ F^{\widetilde{H}_{Neu}}(u)) : 0 \le T^{\widetilde{H}_{Neu}}(u) + I^{\widetilde{H}_{Neu}}(u) + F^{\widetilde{H}_{Neu}}(u) \le 3,\ \forall\, u \in U_{Neu}\Big\}.$$

Here, $T^{\widetilde{H}_{Neu}}(u) : U_{Neu} \to [0,\ 1]$, $I^{\widetilde{H}_{Neu}}(u) : U_{Neu} \to [0,\ 1]$ and $F^{\widetilde{H}_{Neu}}(u) : U_{Neu} \to [0,\ 1]$ are the truth membership degree, indeterminacy membership degree and falsity membership degree, respectively (Fig. 1).

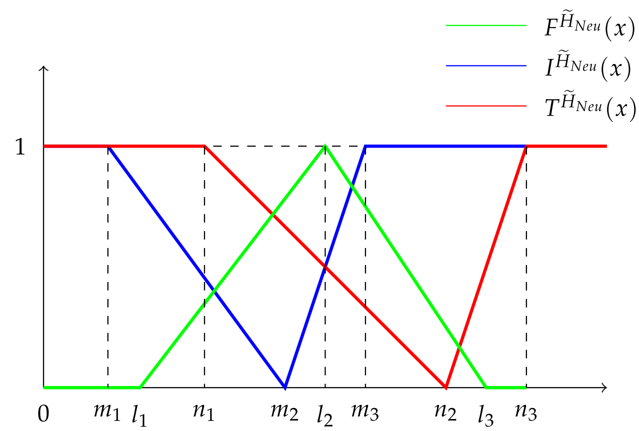

Fig. 1 Triangular SVNN $\widetilde{H}_{Neu} = [(l_1,\ l_2,\ l_4),\ (m_1,\ m_2,\ m_4),\ (n_1,\ n_2,\ n_4)]$

Definition 3 Let $\mathbb{R}$ be a set of all real numbers. Then, a SVNS $\widetilde{H}_{Neu}$ on $\mathbb{R}$ is called a generalized SVNN if its truth membership degree ($T^{\widetilde{H}_{Neu}}(x)$), indeterminacy membership degree ($I^{\widetilde{H}_{Neu}}(x)$) and falsity membership degree ($F^{\widetilde{H}_{Neu}}(x)$) are satisfying the following properties:

1. $T^{\widetilde{H}_{Neu}} : \mathbb{R} \to [0,\ 1]$ is a continuous and concave function,
2. $T^{\widetilde{H}_{Neu}}(x) = 0,\ \forall\, x \in (-\infty,\ l_1]$,
3. $T^{\widetilde{H}_{Neu}}(x)$ is strictly monotonically increasing function on $[l_1,\ l_2]$,
4. $T^{\widetilde{H}_{Neu}}(x) = \alpha_{\widetilde{H}},\ \forall\, x \in [l_2, l_3]$ $(0 < \alpha_{\widetilde{H}} \le 1)$,
5. $T^{\widetilde{H}_{Neu}}(x)$ is strictly monotonically decreasing function on $[l_3,\ l_4]$,
6. $T^{\widetilde{H}_{Neu}}(x) = 0,\ \forall\, x \in [l_4,\ +\infty)$,
7. $I^{\widetilde{H}_{Neu}} : \mathbb{R} \to [0,\ 1]$ is a continuous and convex function,
8. $I^{\widetilde{H}_{Neu}}(x) = 1,\ \forall\, x \in (-\infty,\ m_1]$,
9. $I^{\widetilde{H}_{Neu}}(x)$ is strictly monotonically decreasing function on $[m_1,\ m_2]$,
10. $I^{\widetilde{H}_{Neu}}(x) = \beta_{\widetilde{H}},\ \forall\, x \in [m_2,\ m_3]$ $(0 \le \beta_{\widetilde{H}} < 1)$,
11. $I^{\widetilde{H}_{Neu}}(x)$ is strictly monotonically increasing function on $[m_3,\ m_4]$,
12. $I^{\widetilde{H}_{Neu}}(x) = 1,\ \forall\, x \in [m_4,\ +\infty)$,
13. $F^{\widetilde{H}_{Neu}} : \mathbb{R} \to [0,\ 1]$ is a continuous and convex function.
14. $F^{\widetilde{H}_{Neu}}(x) = 1,\ \forall\, x \in (-\infty,\ n_1]$,
15. $F^{\widetilde{H}_{Neu}}(x)$ is strictly monotonically decreasing function on $[n_1,\ n_2]$,
16. $F^{\widetilde{H}_{Neu}}(x) = \gamma_{\widetilde{H}},\ \forall\, x \in [n_2, n_3]$ $(0 \le \gamma_{\widetilde{H}} < 1)$,
17. $F^{\widetilde{H}_{Neu}}(x)$ is strictly monotonically increasing function on $[n_3,\ n_4]$,
18. $T^{\widetilde{H}_{Neu}}(x) = 0,\ \forall\, x \in [n_4,\ +\infty)$,
19. $0 \le T^{\widetilde{H}_{Neu}}(u) + I^{\widetilde{H}_{Neu}}(u) + F^{\widetilde{H}_{Neu}}(u) \le 3,\ \forall\, u \in \mathbb{R}$,

where $l_1,\ l_2,\ l_3,\ l_4,\ m_1,\ m_2,\ m_3,\ m_4,\ n_1,\ n_2,\ n_3$ and n_4 are real numbers satisfying the inequalities $l_1 \le l_2 \le l_3 \le l_4,\ m_1 \le m_2 \le m_3 \le m_4,\ n_1 \le n_2 \le n_3 \le n_4$. We donate this type of generalized neutrosophic number by $\widetilde{H}_{Neu} = [(l_1,\ l_2,\ l_3,\ l_4;\ \alpha_{\widetilde{H}}),\ (m_1,\ m_2,\ m_3,\ m_4;\ \beta_{\widetilde{H}}),\ (n_1,\ n_2,\ n_3,\ n_4;\ \gamma_{\widetilde{H}})]$.

Definition 4 Let $\widetilde{H}_{Neu} = [(l_1,\ l_2,\ l_3,\ l_4;\ \alpha_{\widetilde{H}}),\ (m_1,\ m_2, m_3,\ m_4;\ \beta_{\widetilde{H}}),\ (n_1,\ n_2,\ n_3,\ n_4;\ \gamma_{\widetilde{H}})]$ be generalized SVNN defined above. So, the generalized SVNN $\widetilde{H}_{Neu}$ is called a trapezoidal SVNN if it satisfies the following conditions:

1. $T^{\widetilde{H}_{Neu}}(x),\ I^{\widetilde{H}_{Neu}}(x)$ and $F^{\widetilde{H}_{Neu}}(x)$ are linear function of x,
2. $\alpha_{\widetilde{H}} = 1,\ \beta_{\widetilde{H}} = 0,\ \gamma_{\widetilde{H}} = 0$.

We represent a trapezoidal SVNN $\widetilde{H}_{Neu}$ by $\widetilde{H}_{Neu} = [(l_1,\ l_2,\ l_3,\ l_4),\ (m_1,\ m_2,\ m_3,\ m_4),\ (n_1,\ n_2,\ n_3,\ n_4)]$. Also, the truth membership function, indeterminacy membership function and falsity membership function of the trapezoidal SVNN are defined as follows:

$$T^{\widetilde{H}_{Neu}}(x) = \begin{cases} \frac{x-l_1}{l_2-l_1}, & l_1 \le x \le l_2 \\ 1, & l_2 \le l_3 \\ \frac{l_4-x}{l_4-l_3}, & l_3 \le x \le l_4 \\ 0, & \text{elsewhere}, \end{cases}$$

$$I^{\widetilde{H}_{Neu}}(x) = \begin{cases} \frac{m_2-x}{m_2-m_1}, & m_1 \le x \le m_2 \\ 0, & m_2 \le m_3 \\ \frac{x-m_3}{m_4-m_3}, & m_3 \le x \le m_4 \\ 1, & \text{elsewhere}, \end{cases}$$

and

$$F^{\widetilde{H}_{Neu}}(x) = \begin{cases} \frac{n_2-x}{n_2-n_1}, & n_1 \le x \le n_2 \\ 0, & n_2 \le n_3 \\ \frac{x-n_3}{n_4-n_3}, & n_3 \le x \le n_4 \\ 1, & \text{elsewhere}. \end{cases}$$

If we consider $l_2 = l_3,\ m_2 = m_3$ and $n_2 = n_3$, the trapezoidal SVNN converted to a triangular SVNN. We represent a triangular SVNN as $\widetilde{H}_{Neu} = [(l_1,\ l_2,\ l_4),\ (m_1,\ m_2,\ m_4),\ (n_1,\ n_2,\ n_4)]$.

Definition 5 *(Score value, Accuracy value and De-neutrosophication of the Triangular SVNN)* Dealing with inaccurate information is a challenge that frequently arises in real-world applications, and uncertainty is a necessary component of all problems that arise in the real world. The neutrosophic numbers are an important modelling tool for these issues because of their capacity to communicate ambiguous information. After defining neutrosophic numbers, ranking and comparison become one of the first crucial concerns. Neutrosophic numbers are difficult to compare and rank, and numerous approaches have been suggested to solve this issue.

Let $\widetilde{H}_{Neu} = [(l_1, l_2, l_3), (m_1, m_2, m_3), (n_1, n_2, n_3)]$ be a triangular SVNN. Then, its score value, accuracy value and de-neutrosophication are denoted by $S(\widetilde{H}_{Neu})$, $A(\widetilde{H}_{Neu})$ and $D(\widetilde{H}_{Neu})$, respectively, is defined by

$$S(\widetilde{H}_{Neu}) = \frac{\left(\begin{array}{r}8 + (l_1 + 2l_2 + l_3) - (m_1 + 2m_2 \\ + m_3) - (n_1 + 2n_2 + n_3)\end{array}\right)}{12},$$

$$A(\widetilde{H}_{Neu}) = \frac{(l_1 + 2l_2 + l_3) - (n_1 + 2n_2 + n_3)}{4}$$

$$\text{and } D(\widetilde{H}_{Neu}) = \frac{\left(\begin{array}{r}l_1 + 2l_2 + l_3 + m_1 + 2m_2 + m_3 \\ + n_1 + 2n_2 + n_3\end{array}\right)}{12}.$$

Let $\widetilde{H}^1_{Neu} = [(l_{11}, l_{12}, l_{13}), (m_{11}, m_{12}, m_{13}), (n_{11}, n_{12}, n_{13})]$ and $\widetilde{H}^2_{Neu} = [(l_{21}, l_{22}, l_{23}), (m_{21}, m_{22}, m_{23}), (n_{21}, n_{22}, n_{23})]$ be two triangular single-valued neutrosophic numbers. Then, according to score and accuracy function, we have ranked the triangular SVNNs as follows:

1. If $S(\widetilde{H}^1_{Neu}) > S(\widetilde{H}^2_{Neu})$, then $\widetilde{H}^1_{Neu}$ is greater than $\widetilde{H}^2_{Neu}$.
2. If $S(\widetilde{H}^1_{Neu}) < S(\widetilde{H}^2_{Neu})$, then $\widetilde{H}^1_{Neu}$ is smaller than $\widetilde{H}^2_{Neu}$.
3. If $S(\widetilde{H}^1_{Neu}) = S(\widetilde{H}^2_{Neu})$, then
   (a) if $A(\widetilde{H}^1_{Neu}) > A(\widetilde{H}^2_{Neu})$, then $\widetilde{H}^1_{Neu}$ is greater than $\widetilde{H}^2_{Neu}$,
   (b) if $A(\widetilde{H}^1_{Neu}) < A(\widetilde{H}^2_{Neu})$, then $\widetilde{H}^1_{Neu}$ is smaller than $\widetilde{H}^2_{Neu}$,
   (c) if $S(\widetilde{H}^1_{Neu}) = S(\widetilde{H}^2_{Neu})$, then $\widetilde{H}^1_{Neu}$ is equivalent to $\widetilde{H}^2_{Neu}$.

2.1 Algebraic operation on triangular single-valued neutrosophic numbers

In this paper, we use the Function Principle to simplify the calculation. Function Principle is used as the computational model avoiding the computations which can be caused by the operations using the Extension Principle. We describe some neutrosophic arithmetical operations as follows: Suppose $\widetilde{H}^1_{Neu} = [(l_{11}, l_{12}, l_{13}), (m_{11}, m_{12}, m_{13}), (n_{11}, n_{12}, n_{13})]$ and $\widetilde{H}^2_{Neu} = [(l_{21}, l_{22}, l_{23}), (m_{21}, m_{22}, m_{23}), (n_{21}, n_{22}, n_{23})]$ are two triangular SVNNs. Then,

1. The addition of $\widetilde{H}^1_{Neu}$ and $\widetilde{H}^2_{Neu}$ is $\widetilde{H}^1_{Neu} \oplus \widetilde{H}^2_{Neu} = (l_{11} + l_{21}, l_{12} + l_{22}, l_{13} + l_{23}; m_{11} + m_{21}, m_{12} + m_{22}, m_{13} + m_{23}; n_{11} + n_{21}, n_{12} + n_{22}, n_{13} + n_{23})$.
2. The addition of $\widetilde{H}^1_{Neu}$ and $\widetilde{H}^2_{Neu}$ is $\widetilde{H}^1_{Neu} \ominus \widetilde{H}^2_{Neu} = (l_{11} - l_{23}, l_{12} - l_{22}, l_{13} - l_{21}; m_{11} - m_{23}, m_{12} - m_{22}, m_{13} - m_{21}; n_{11} - n_{23}, n_{12} - n_{22}, n_{13} - n_{21})$
3. The multiplication of $\widetilde{H}^1_{Neu}$ and $\widetilde{H}^2_{Neu}$ is $\widetilde{H}^1_{Neu} \otimes \widetilde{H}^2_{Neu} = (l_{11}l_{21}, l_{12}l_{22}, l_{13}l_{23}; m_{11}m_{21}, m_{12}m_{22}, m_{13}m_{23}; n_{11}n_{21}, n_{12}n_{22}, n_{13}n_{23})$.
4. The inverse of $\widetilde{H}^1_{Neu} = [(l_{11}, l_{12}, l_{13}), (m_{11}, m_{12}, m_{13}), (n_{11}, n_{12}, n_{13})]$ is

$$\frac{1}{\widetilde{H}^1_{Neu}} = \begin{cases} \left[\left(\frac{1}{l_{13}}, \frac{1}{l_{12}}, \frac{1}{l_{11}}\right), \left(\frac{1}{m_{13}}, \frac{1}{m_{12}}, \frac{1}{m_{11}}\right), \left(\frac{1}{n_{13}}, \frac{1}{n_{12}}, \frac{1}{n_{11}}\right)\right], & \text{if } l_{1i}, m_{1i}, n_{1i} > 0, \ i = 1, 2, 3 \\ \left[\left(\frac{1}{l_{11}}, \frac{1}{l_{12}}, \frac{1}{l_{11}}\right), \left(\frac{1}{m_{11}}, \frac{1}{m_{12}}, \frac{1}{m_{13}}\right), \left(\frac{1}{n_{11}}, \frac{1}{n_{12}}, \frac{1}{n_{13}}\right)\right], & \text{if } l_{1i}, m_{1i}, n_{1i} < 0, \ i = 1, 2, 3. \end{cases}$$

5. The addition and subtraction of $\widetilde{H}^1_{Neu} = [(l_{11}, l_{12}, l_{13}), (m_{11}, m_{12}, m_{13}), (n_{11}, n_{12}, n_{13})]$ with a constant λ are $\widetilde{H}^1_{Neu} \pm \lambda = [(l_{11} \pm \lambda, l_{12} \pm \lambda, l_{13} \pm \lambda), (m_{11} \pm \lambda, m_{12} \pm \lambda, m_{13} \pm \lambda), (n_{11} \pm \lambda, n_{12} \pm \lambda, n_{13} \pm \lambda)]$
6. The multiplication of $\widetilde{H}^1_{Neu} = [(l_{11}, l_{12}, l_{13}), (m_{11}, m_{12}, m_{13}), (n_{11}, n_{12}, n_{13})]$ by a constant λ is

$$\lambda \cdot \widetilde{H}^1_{Neu} = \begin{cases} [(\lambda l_{11},\ \lambda l_{12},\ \lambda l_{13}),\ (\lambda m_{11},\ \lambda m_{12},\ \lambda m_{13}),\ (\lambda n_{11},\ \lambda n_{12},\ \lambda n_{13})], & \text{if } \lambda > 0 \\ [(\lambda l_{13},\ \lambda l_{12},\ \lambda l_{11}),\ (\lambda m_{11},\ \lambda m_{12},\ \lambda m_{13}),\ (\lambda n_{11},\ \lambda n_{12},\ \lambda n_{13})], & \text{if } \lambda < 0. \end{cases}$$

7. The N-th power of $(\widetilde{H}^1_{Neu})^N = [(l_{11})^N,\ (l_{12})^N,\ (l_{13})^N;\ (m_{11})^N,\ (m_{12})^N,\ (m_{13})^N,\ (n_{11})^N,\ (n_{12})^N,\ n_{13})^N]$ is

Theorem 1 *For two triangular SVNNs* $\widetilde{H}^1_{Neu} = [(l_{11},\ l_{12},\ l_{13}),\ (m_{11},\ m_{12},\ m_{13}),\ (n_{11},\ n_{12},\ n_{13})]$ *and* $\widetilde{H}^2_{Neu} = [(l_{21},\ l_{22},\ l_{23}),\ (m_{21},\ m_{22},\ m_{23}),\ (n_{21},\ n_{22},\ n_{23})]$ *satisfies:*

1. $D((a_1 \cdot \widetilde{H}^1_{Neu}) \oplus (a_2 \cdot \widetilde{H}^2_{Neu})) = a_1 D(\widetilde{H}^1_{Neu}) + a_2 D(\widetilde{H}^2_{Neu}), \forall a_1,\ a_2 > 0.$
2. $D((a_1 \cdot \widetilde{H}^1_{Neu}) \ominus (a_2 \cdot \widetilde{H}^2_{Neu})) = a_1 D(\widetilde{H}^1_{Neu}) - a_1 D(\widetilde{H}^2_{Neu}), \forall a_1,\ a_2 > 0.$

Proof We know that $D(\widetilde{H}^1_{Neu}) = \frac{l_{11}+2l_{12}+l_{13}+m_{11}+2m_{12}+m_{13}+n_{11}+2n_{12}+n_{13}}{12}$ and $D(\widetilde{H}^2_{Neu}) = \frac{l_{21}+2l_{22}+l_{23}+m_{21}+2m_{22}+m_{23}+n_{21}+2n_{22}+n_{23}}{12}$. Now,

$$\begin{aligned}
(1)\ & D((a_1 \cdot \widetilde{H}^1_{Neu}) \oplus (a_2 \cdot \widetilde{H}^2_{Neu})) = D(a_1 l_{11} + a_2 l_{21}, \\
& \times a_1 l_{12} + a_2 l_{22},\ a_1 l_{13} + a_2 l_{23};\ a_1 m_{11} + a_2 m_{21},\ a_1 m_{12} \\
& + a_2 m_{22}, a_1 m_{13} + a_2 m_{23};\ a_1 n_{11} + a_2 n_{21},\ a_1 n_{12} \\
& + a_2 n_{22},\ a_1 n_{13} + a_2 n_{23}) \\
& = \frac{1}{12}(a_1 l_{11} + a_2 l_{21} + 2a_1 l_{12} + 2a_2 l_{22} + a_1 l_{13} + a_2 l_{23} \\
& + a_1 m_{11} + a_2 m_{21} + 2a_1 m_{12} + 2a_2 m_{22} + a_1 m_{13} + a_2 m_{23} \\
& \times a_1 n_{11} + a_2 n_{21} + 2a_1 n_{12} + 2a_2 n_{22} + a_1 n_{13} + a_2 n_{23}) \\
& = \frac{1}{12} a_1 (l_{11} + 2l_{12} + l_{13} + m_{11} + 2m_{12} + m_{13} \\
& + n_{11} + 2n_{12} + n_{13}) + \frac{1}{12} a_2 (l_{21} + 2l_{22} + l_{23} \\
& + m_{21} + 2m_{22} + m_{23} + n_{21} + 2n_{22} + n_{23}) \\
& = a_1 D(\widetilde{H}^1_{Neu}) + a_2 D(\widetilde{H}^2_{Neu}),
\end{aligned}$$

and

$$\begin{aligned}
(2)\ & D((a_1 \cdot \widetilde{H}^1_{Neu}) \ominus (a_2 \cdot \widetilde{H}^2_{Neu})) = D(a_1 l_{11} - a_2 l_{23}, \\
& \times a_1 l_{12} - a_2 l_{22},\ a_1 l_{13} - a_2 l_{21};\ a_1 m_{11} - a_2 m_{23},\ a_1 m_{12} \\
& - a_2 m_{22},\ a_1 m_{13} - a_2 m_{21};\ a_1 n_{11} - a_2 n_{23}, a_1 n_{12} \\
& - a_2 n_{22},\ a_1 n_{13} - a_2 n_{21}) \\
& = \frac{1}{12}(a_1 l_{11} - a_2 l_{23} + 2a_1 l_{12} - 2a_2 l_{22} + a_1 l_{13} - a_2 l_{21} \\
& + a_1 m_{11} - a_2 m_{23} + 2a_1 m_{12} - 2a_2 m_{22} + a_1 m_{13} - \\
& \times a_2 m_{21} a_1 n_{11} - a_2 n_{23} + 2a_1 n_{12} - 2a_2 n_{22} + a_1 n_{13} \\
& - a_2 n_{21}) \\
& = \frac{1}{12} a_1 (l_{11} + 2l_{12} + l_{13} + m_{11} + 2m_{12} + m_{13} \\
& + n_{11} + 2n_{12} + n_{13}) - \frac{1}{12} a_2 (l_{21} + 2l_{22} + l_{23} \\
& + m_{21} + 2m_{22} + m_{23} + n_{21} + 2n_{22} + n_{23}) \\
& = a_1 D(\widetilde{H}^1_{Neu}) - a_2 D(\widetilde{H}^2_{Neu}).
\end{aligned}$$

□

3 Proposed model in crisp sense

In this study, we have considered the following notations and assumptions for developing our proposed model.

3.1 Notations

Parameters	Description
c_H	Holding cost per unit item per year
c_O	Ordering cost per order
c_P	Purchasing cost per unit item
c_{PE}	Cost for promotional effort
i_{IE}	Percentage of interest earned by the retailer per unit time
i_{IC}	Percentage of interest charged by the retailer per unit time
η	Preservation cost per order
κ	Parameter of promotional effort
b	Parameter of preservation technology
a	Parameter of demand rate
n	A natural number, elasticity factor of pro--motional cost
U	Upstream trade credit period offered by the suppler to retailer
V	Downstream trade credit period offered by the retailer to customer
λ_1	The portion of the items allowed to permis--sible delay for the retailer ($0 \leq \lambda_1 \leq 1$)
λ_2	The portion of the purchasing cost paid by the customer at the time of placing an order ($0 \leq \lambda_2 \leq 1$)
$\zeta(t)$	Deterioration rate
$\Psi(t)$	Inventory level at time t
$\Pi(F, c_S)$	Total profit of the retailer per unit time
Dependent variables	
Q	Order quantity per cycle
Ω	Demand rate depend on promotional effort, selling price and downstream trade credit period

Decision variables	
c_s	Selling price per unit item ($c_s > c_p$)
F	Replenishment cycle time

3.2 Assumptions

1. Restocking happens instantly.
2. Time horizon is infinite.
3. Demand function $\Omega(\kappa, V, c_s) = Ae^{(V+\kappa-ac_s)}$ $(0 \leq a < 1)$ is depend on downstream trade credit period, promotional effort and selling price. Here, V and κ are finite non-negative real numbers. When $c_s \to +\infty, \Omega \to 0$.
4. We assume that initially ($i.e.$, at $t = 0$) deterioration rate is zero. Deterioration rate $\zeta(t)$ is monotonically non-decreasing function and near expiration date it has maximum value i.e., $\forall\, t \in [0, m], 0 \leq \zeta(t) \leq 1,\ \zeta'(t) > 0,\ \zeta(m) = 1$. In this paper, we assumed that the expiry date m is higher than the time of the replenishment period F.
5. The deterioration rate is reduced by effect of preservation technology η. The proportion of reduced deterioration rate is $\tau(t, \eta) = \zeta(t)\left(1 - e^{-b\eta}\right)$, where $b > 0$, $\eta > 0$ and b is the simulation coefficient representing the percentage increase in $\tau(t, \eta)$ per dollar increase in η, is a continuous, concave, increasing function of retailer's capital investment, where $\tau(t, 0) = 0$ and $\lim_{\eta\to\infty} \tau(t, \eta) = \zeta(t)$. imply the diminishing marginal productivity of capital.
6. The retailer pays $1-\lambda_1$ portion of purchase amount upon receiving the order quantity at time 0 and gets a short term trade credit period up to time V for the remaining purchase amount. On the other hand, the customer must pay λ_2 portion of the item that he bought while postpone paying the remaining portion ($i.e.$, $1 - \lambda_2$) of the goods.
7. The seller funds the initial sum with a bank loan on Price i_{IC} percent and then settle at $F + U$. The retailer pays interest to suppliers at a rate of i_{IC} and earned interest from his/her customer at a rate of i_{IE}.

3.3 Model description

This paper treats a supplier-retailer-customer three echelon inventory model with time sensitive perishable products, and selling price and promotional effort-dependent demand. The concepts of preservation technology and promotional effort have been included in order to lessen the effects of deteriorating rate and enhance market demand, respectively. The retailer gives a partial downstream trade credit periods to customer and received a partial upstream trade credit period from supplier. The retailer pays $(1 - \lambda_1)QC_P$ to supplier at the time $t = 0$ and hold the payment for the item $\lambda_1 Q$ up to time U. Again, the retailer received λ_2 portion of the payment of selling items from customer and the customer paid remain portion of the payment within time period V. This paper seeks to maximize the retailer's total profit per unit time by taking decisions on the selling price C_P^*, the replenishment cycle time F^*.

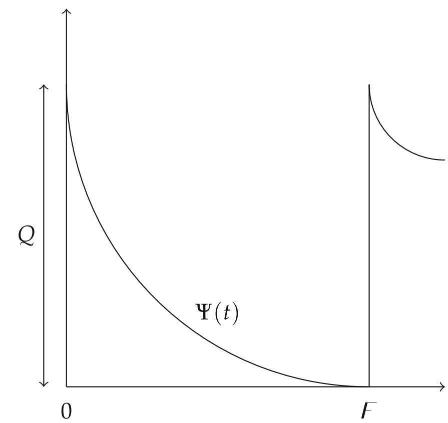

Fig. 2 Inventory level at the time t

3.4 Model formulation

Depletion of inventory (shown in Fig. 2) occurs due to deterioration of items and customer demand. Also, for the preservation technology η, date of expiration is increased. So the effective deterioration rate is

$$\zeta(t) - \tau(t, \eta) = \zeta(t)e^{-b\eta}$$

Hence, the governing differential equation is

$$\frac{d\Psi(t)}{dt} + \Psi(t)\zeta(t)e^{-b\eta} = -\Omega, \quad 0 \leq t \leq F, \tag{1}$$

with boundary condition $\Psi(F) = 0$. Solving Eq.(1), we get

$$\Psi(t) = e^{-\xi(t)} \int_t^F \Omega e^{\xi(u)} du, \quad 0 \leq t \leq F, \quad \text{where } \xi(t) = e^{-b\eta}\int_0^t \zeta(s)ds. \tag{2}$$

Using the condition $\Psi(0) = Q$, we have

$$Q = \int_0^F \Omega e^{\xi(u)} du. \tag{3}$$

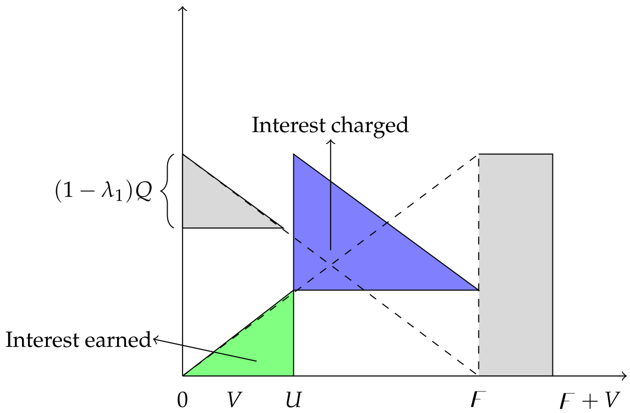

Fig. 3 Instant payment $U \geq V$ and $U \leq F$

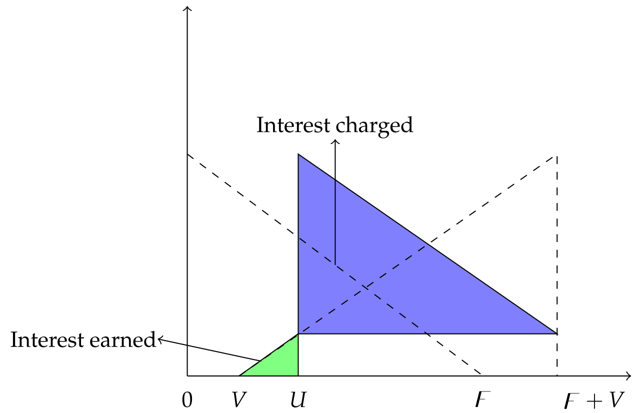

Fig. 4 Delayed payment $U \geq V$ and $U \leq F$

Now, we have considered the following cost parameters for the retailer's total profit.

1. Ordering cost per order $= c_O$.
2. Cost for PE per cycle $= c_{PE}\kappa^n$
3. Purchasing cost per cycle

$$c_P Q = c_P \int_0^F \Omega e^{\xi(u)} du. \tag{4}$$

4. Selling price per cycle

$$c_S \int_0^F \Omega dt = c_S \Omega F. \tag{5}$$

5. Inventory carrying cost per cycle

$$c_H \int_0^F \Psi(t) dt = c_H \int_0^F e^{-\xi(t)} \left(\int_t^F \Omega e^{\xi(u)} du \right) dt. \tag{6}$$

The retailer received and charged interest depending on the downstream trade credit period V and upstream trade credit duration U. So we took into consideration two major scenarios $V \leq U$ and $U \leq V$.

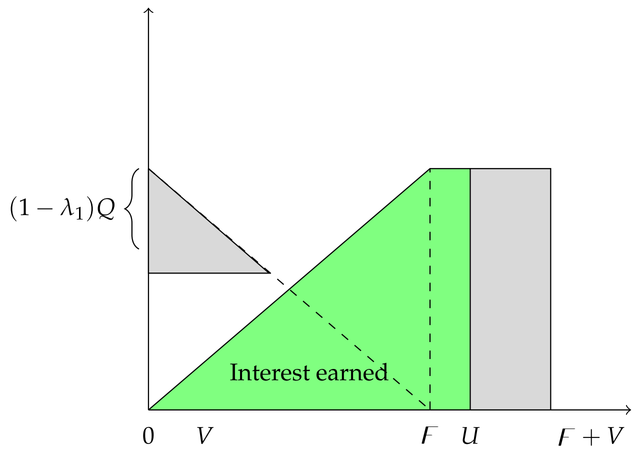

Fig. 5 Instant payment $U \geq V$ and $F \leq U \leq F + V$

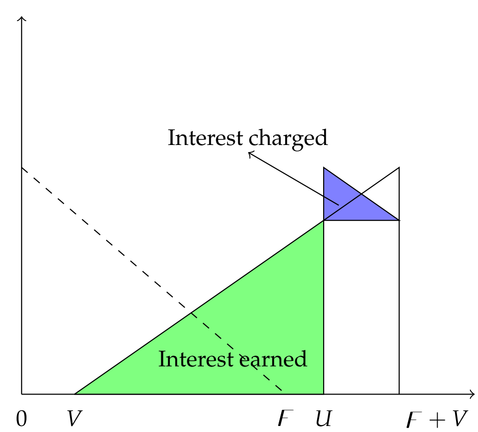

Fig. 6 Delayed payment $U \geq V$ and $F \leq U \leq F + V$

3.4.1 Case-1 ($V \leq U$)

In this case, we obtain three sub-cases according to U, V, and F values as follows:

Sub-case1.1 ($V \leq U \leq F$) :

Under this situation, the retailer accumulates income and receives interest, (i) from immediate payment over the duration $[0, U]$, and (ii) from pending payment over the period $[V, U]$. So, the retailer's interest earn (shown in Figs. 3, 4) per cycle is

$$\begin{aligned} & c_S i_{IE} \left(\lambda_2 \Omega \int_0^U t dt + (1 - \lambda_2) \Omega \int_V^U (t - V) dt \right) \\ & = \frac{c_S i_{IE} \Omega}{2} \left(\lambda_2 U^2 + (1 - \lambda_2)(U - V)^2 \right). \end{aligned} \tag{7}$$

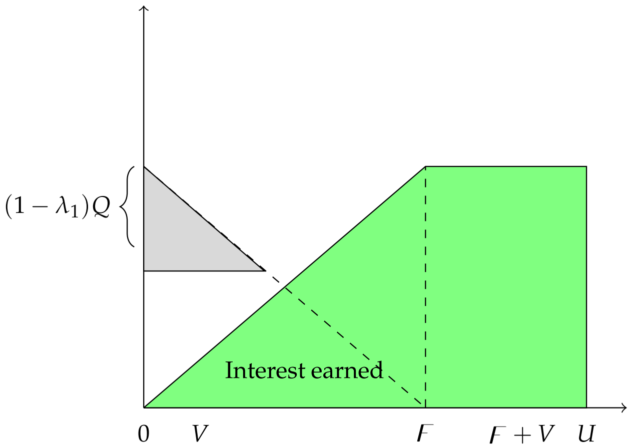

Fig. 7 Instant payment $U \geq V$ and $U \geq F + V$

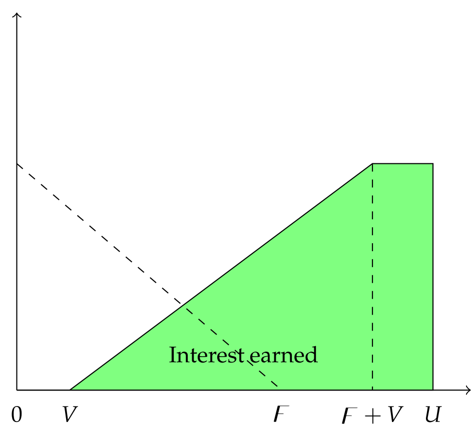

Fig. 8 Delayed payment $U \geq V$ and $U \geq F + V$

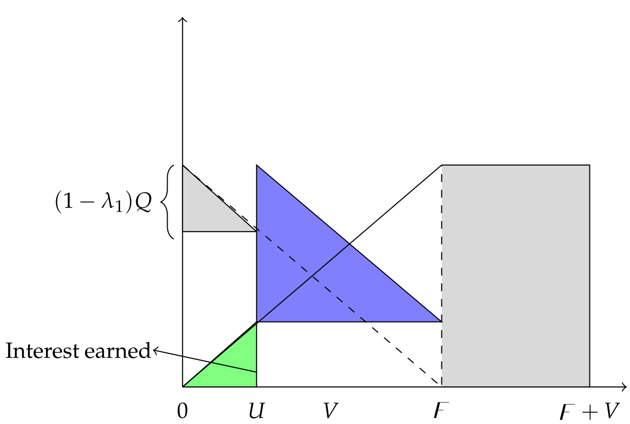

Fig. 9 Instant payment $U \leq V$ and $U \leq F$

At time $t = 0$, the retailer funds the amount \$ $(1 - \lambda_1)c_p Q$ with a bank loan and the amount is settled by him/her at time $F + V$. Also, the retailer start paying interest to his/her supplier, (i) from U to F for the goods stock in hand for both immediate payment and delayed payment, and (ii) from F to $F + V$ for the items that sell but yet not payment. Hence,

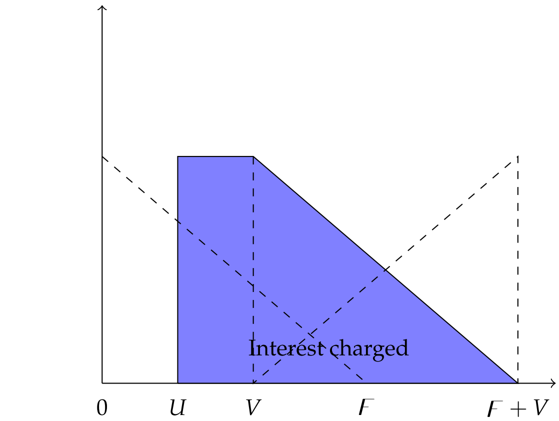

Fig. 10 Delayed payment $U \leq V$ and $U \leq F$

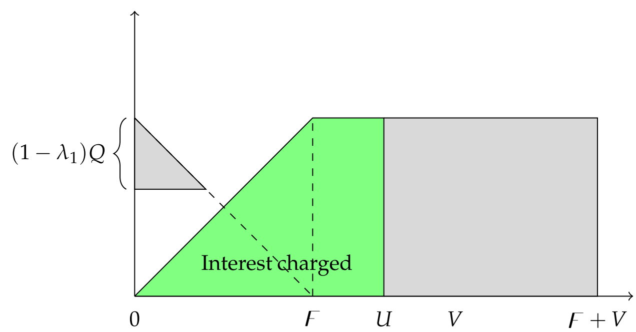

Fig. 11 Instant payment $U \leq V$ and $U \geq F$

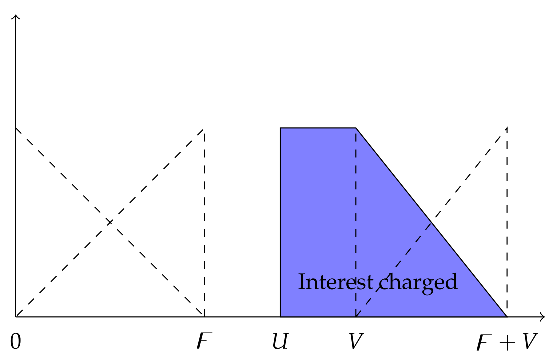

Fig. 12 Delayed payment $U \leq V$ and $U \geq F$

the retailer's total interest charged (shown in Figs. 3, 4) per cycle is given by

$$c_p i_{IC} \left((1 - \lambda_1) Q(F + V) + \frac{(1 - \lambda_1)^2 QF}{2} + \lambda_2 \Omega \int_U^F (t - U)dt + (1 - \lambda_2)\Omega \int_U^{F+V} (t - U)dt \right). \tag{8}$$

Sub-case1.2$(F \leq U \leq F + V)$:

The retailer's total interest earn (shown in Figs. 5, 6) in this sub-case is

$$c_s i_{IE}\left((\lambda_2)\left(\Omega\int_0^F t dt + \Omega F(U-F)\right) + (1-\lambda_2)\Omega\int_V^U (t-V)dt\right). \quad (9)$$

The retailer's interest paid (shown in Figs. 5, 6) in this sub-case is given by

$$c_p i_{IC}\left((1-\lambda_1)Q(F+V) + \frac{(1-\lambda_1)^2 QF}{2} + (1-\lambda_2)\Omega\int_U^{F+V}(t-U)dt\right). \quad (10)$$

Sub-case 1.3($F+V \le U$)

The retailer's total interest earned (shown in Figs. 7, 8) in this sub-case is

$$c_s i_{IE}\left(\lambda_2\left(\Omega\int_0^F t dt + \Omega F(U-F)\right) + (1-\lambda_2)\left(\Omega\int_V^{F+V}(t-V)dt + \Omega F(U-F-V)\right)\right). \quad (11)$$

The retailer's interest paid (shown in Figs. 7, 8) in this sub-case is given by

$$c_p i_{IC}\left((1-\lambda_1)Q(F+V) + \frac{(1-\lambda_1)^2 QF}{2}\right). \quad (12)$$

3.4.2 Case-2 ($U \le V$)

According to value of U, V and F, in this case, we have two sub-cases as follows:

Sub-case 2.1 ($U \le F$) :

The retailer's interest earned (shown in Figs. 9, 10) in this sub-case is given by

$$c_s i_{IE}\lambda_2\Omega\int_0^U t dt = \frac{1}{2}c_s i_{IE}\lambda_2\Omega U^2. \quad (13)$$

The retailer's total interest paid (shown in Figs. 9, 10) in this sub-case is given by

$$c_p i_{IC}\left((1-\lambda_1)Q(F+V) + \frac{(1-\lambda_1)^2 QF}{2} + \lambda_2\Omega\int_U^F (t-U)dt + (1-\lambda_2)\left(\Omega F(V-U) + \Omega\int_V^{F+V}(t-V)dt\right)\right). \quad (14)$$

Sub-case 2.2 ($U \ge F$) :

The retailer's interest earned (shown in Figs. 11, 12) in this sub-case is given by

$$c_s i_{IE}\left(\lambda_2\Omega\int_0^F t dt + \lambda_2\Omega F(U-F)\right). \quad (15)$$

The retailer's total interest paid (shown in Figs. 11, 12) in this sub-case is given by

$$c_p i_{IC}\left((1-\lambda_1)Q(F+V) + \frac{(1-\lambda_1)^2 QF}{2} + (1-\lambda_2)\left(\Omega F(V-U) + \Omega\int_V^{F+V}(t-V)dt\right)\right). \quad (16)$$

3.4.3 Retailer's profit

Therefore, based on case-1 ($V \le U$) and case-2 ($U \le V$), the retailer's total profit is given by

$$\begin{aligned}\Pi(c_s, F) &= \frac{1}{F}(\text{selling price} - \text{ordering cost} - \text{PE} \\ &\quad -\text{purchasing cost} - \text{holding cost} - \text{interest charged} \\ &\quad +\text{interest earned} - \text{cost for PT}) \\ &= \begin{cases} \Pi_{11}(c_s, F), & U \le F \text{ and } V \le U \\ \Pi_{12}(c_s, F), & F \le U \le F+V \text{ and } V \le U \\ \Pi_{13}(c_s, F), & F+V \le U \text{ and } V \le U \\ \Pi_{21}(c_s, F), & U \le F \text{ and } U \le V \\ \Pi_{22}(c_s, F), & F \le U \text{ and } U \le V, \end{cases}\end{aligned} \quad (17)$$

where

$$\Pi_{11}(F, c_s) = \frac{1}{F}\Big(c_s\Omega F - c_O - c_{PE}\kappa^n - c_P\int_0^F \Omega e^{\xi(u)}du - c_H\int_0^F e^{-\xi(t)}\left(\int_t^F \Omega e^{\xi(u)}du\right)dt - c_p i_{IC}((1-\lambda_1)Q(F$$

$$
\begin{aligned}
&+V)+\frac{(1-\lambda_1)^2 QF}{2}+\frac{1}{2}\lambda_2\Omega(F\\
&-U)^2+\frac{1}{2}(1-\lambda_2)\Omega(F+V\\
&-U)^2\Big)+\frac{c_S i_{IE}\Omega}{2}\Big(\lambda_2 U^2+(1-\lambda_2)(U\\
&-V)^2\Big)-\eta\Big),
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\Pi_{12}(F,c_S)=\frac{1}{F}\Bigg[&c_S\Omega F-c_O-c_{PE}\kappa^n\\
&-c_P\int_0^F \Omega e^{\xi(u)}du-c_H\int_0^F e^{-\xi(t)}\left(\int_t^F \Omega e^{\xi(u)}du\right)dt\\
&-c_P i_{IC}\left\{(1-\lambda_1)Q(F+V)+\frac{(1-\lambda_1)^2 QF}{2}\right.\\
&\left.+\frac{1}{2}(1-\lambda_2)\Omega(F+V-U)^2\right\}+c_S i_{IE}\Big\{\lambda_2\\
&\Big\{\frac{1}{2}\Omega F^2+\Omega F(U-F)\Big\}+\frac{1}{2}(1-\lambda_2)\Omega(U\\
&-V)^2\Big\}-\eta\Bigg],
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\Pi_{13}(F,c_S)=\frac{1}{F}\Bigg[&c_S\Omega F-c_O-c_{PE}\kappa^n\\
&-c_P\int_0^F \Omega e^{\xi(u)}du-c_H\int_0^F e^{-\xi(t)}\left(\int_t^F \Omega e^{\xi(u)}du\right)dt\\
&-c_P i_{IC}\left\{(1-\lambda_1)Q(F+V)+\frac{(1-\lambda_1)^2 QF}{2}\right\}\\
&+c_S i_{IE}\Big\{\lambda_2\Big\{\frac{1}{2}\Omega F^2+\Omega F(U-F)\Big\}+(1-\lambda_2)\\
&\Big\{\frac{1}{2}\Omega F^2+\Omega F(U-F-V)\Big\}\Big\}-\eta\Bigg]
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\Pi_{21}(F,c_S)=\frac{1}{F}\Bigg[&c_S\Omega F-c_O-c_{PE}\kappa^n\\
&-c_P\int_0^F \Omega e^{\xi(u)}du-c_H\int_0^F e^{-\xi(t)}\left(\int_t^F \Omega e^{\xi(u)}du\right)dt\\
&-c_P i_{IC}\left\{(1-\lambda_1)Q(F+V)+\frac{(1-\lambda_1)^2 QF}{2}\right.\\
&+\frac{1}{2}\lambda_2\Omega(F-U)^2+(1-\lambda_2)\{\Omega F(V-U)\\
&\left.+\frac{1}{2}\Omega F^2\}\right\}+\frac{1}{2}c_S i_{IE}\lambda_2\Omega U^2-\eta\Bigg]
\end{aligned}
\tag{21}
$$

and
$$
\begin{aligned}
\Pi_{22}(F,c_S)=\frac{1}{F}\Bigg[&c_S\Omega F-c_O-c_{PE}\kappa^n\\
&-c_P\int_0^F \Omega e^{\xi(u)}du-c_H\int_0^F e^{-\xi(t)}\left(\int_t^F \Omega e^{\xi(u)}du\right)dt\\
&-c_P i_{IC}\left\{(1-\lambda_1)Q(F+V)+\frac{(1-\lambda_1)^2 QF}{2}\right.\\
&\left.+(1-\lambda_2)\{\Omega F(V-U)+\frac{1}{2}\Omega F^2\}\right\}+c_S i_{IE}\\
&\left\{\frac{1}{2}\lambda_2\Omega F^2+\lambda_2\Omega F(U-F)\right\}-\eta\Bigg]
\end{aligned}
\tag{22}
$$

4 Neutrosophic total profit

In real-life problems, it is observed that most of the cost parameters have some uncertainty due to insufficient past data. Let us consider the ordering cost (c_o), carrying cost (c_H), purchasing price (c_P), the cost for promotional effort (c_{PE}) and the parameters $A,\ \kappa,\ a$ as triangular neutrosophic numbers. Then, the total profit is converted to a triangular neutrosophic number or neutrosophic profit as follows:

$$
\widetilde{\Pi}_{Neu}(c_S,F)=\begin{cases}
(\widetilde{\Pi}_{Neu})_{11}(c_S,F), & U\le F \text{ and } V\le U\\
(\widetilde{\Pi}_{Neu})_{12}(c_S,F), & F\le U\le F+V \text{ and } V\le U\\
(\widetilde{\Pi}_{Neu})_{13}(c_S,F), & F+V\le U \text{ and } V\le U\\
(\widetilde{\Pi}_{Neu})_{21}(c_S,F), & U\le F \text{ and } U\le V\\
(\widetilde{\Pi}_{Neu})_{22}(c_S,F), & F\le U \text{ and } U\le V,
\end{cases}
\tag{23}
$$

where

$$
\begin{aligned}
(\widetilde{\Pi}_{Neu})_{rj}(F,c_S)=&(E_{11})_{rj}\cdot(\widetilde{X}^{11}_{Neu})_{rj}\ominus(E_{12})_{rj}\\
&\cdot(\widetilde{X}^{12}_{Neu})_{rj}\ominus(E_{13})_{rj}\cdot(\widetilde{X}^{13}_{Neu})_{rj}\\
&\ominus(E_{14})_{rj}\cdot(\widetilde{X}^{14}_{Neu})_{rj}\ominus(E_{15})_{rj}\\
&\cdot(\widetilde{X}^{15}_{Neu})_{rj}\ominus(E_{15})_{rj}\cdot(\widetilde{X}^{15}_{Neu})_{rj}\\
&\ominus(E_{16})_{rj}\cdot(\widetilde{X}^{16}_{Neu})_{rj}\oplus(E_{17})_{rj}\\
&\cdot(\widetilde{X}^{17}_{Neu}))_{rj}\ominus(E_{18})_{rj}\cdot(\widetilde{X}^{18}_{Neu})_{rj},\\
&r=1,2,\ \ j=1,2,3.
\end{aligned}
\tag{24}
$$

For all $r=1,2\ \ j=1,2,3,$

$$
(E_{11})_{rj}=c_S e^V,\ (E_{12})_{rj}=\frac{1}{F},\ (E_{13})_{rj}=\frac{1}{F},
$$

$$
(E_{14})_{rj}=\frac{e^V}{F}\int_0^F e^{-\xi(t)}du,\ (E_{18})_{rj}=\frac{\eta}{F},
$$

$$
(E_{15})_{rj}=\frac{e^V}{F}\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt;
$$

$$
\begin{aligned}
(E_{16})_{11}=\frac{i_{IC}e^V}{F}\Bigg\{&(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du\\
&+\frac{1}{2}(1-\lambda_1)^2F\int_0^F e^{\xi(u)}du+\frac{1}{2}\lambda_2(F-U)^2\\
&+\frac{1}{2}(1-\lambda_2)(F+V-U)^2\Bigg\},
\end{aligned}
$$

$$
(E_{17})_{11}=\frac{c_S i_{IE}e^V}{2F}\left\{\lambda_2U^2+(1-\lambda_2)(U-V)^2\right\},
$$

$$(E_{16})_{12} = \frac{i_{IC}e^V}{F}\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2 F\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_2)(F+V-U)^2\right\}$$

$$(E_{17})_{12} = \frac{c_s i_{IE}e^V}{2F}\left\{\lambda_2 F^2 + 2\lambda_2 F(U-F) + (1-\lambda_2)(U-V)^2\right\},$$

$$(E_{16})_{13} = \frac{i_{IC}e^V}{F}\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2 F\int_0^F e^{\xi(u)}du\right\}$$

$$(E_{17})_{13} = \frac{c_s i_{IE}e^V}{2F}\left\{\lambda_2 F^2 + 2\lambda_2 F(U-F) + (1-\lambda_2)F^2 + 2(1-\lambda_2)F(U-F-V)\right\}$$

$$(E_{16})_{21} = \frac{i_{IC}e^V}{F}\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2 F\int_0^F e^{\xi(u)}du + \frac{1}{2}\lambda_2(F-U)^2 + (1-\lambda_2)(F+V-U)^2 + \frac{1}{2}(1-\lambda_2)F^2\right\}$$

$$(E_{17})_{21} = \frac{c_s i_{IE}e^V}{2F}\lambda_2 U^2$$

$$(E_{16})_{22} = \frac{i_{IC}e^V}{F}\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2 F\int_0^F e^{\xi(u)}du + (1-\lambda_2)(F+V-U)^2 + \frac{1}{2}(1-\lambda_2)F^2\right\}$$

$$(E_{17})_{22} = \frac{c_s i_{IE}e^V}{2F}\left(\lambda_2 F^2 + 2\lambda_2 F(U-F)\right)$$

For $r = 1, 2, \quad j = 1, 2, 3$

$$\begin{aligned}(\widetilde{X}^{11}_{Neu})_{rj} &= \widetilde{A}_{Neu} \otimes e^{\widetilde{\kappa}_{Neu}\ominus(c_s\cdot\widetilde{a}_{Neu})}\\ &= (A_{11}e^{\kappa_{11}-c_s a_{13}},\ A_{12}e^{\kappa_{12}-c_s a_{12}},\ A_{13}e^{\kappa_{13}-c_s a_{11}};\\ &\quad A_{21}e^{\kappa_{21}-c_s a_{23}},\ A_{22}e^{\kappa_{22}-c_s a_{22}},\ A_{23}e^{\kappa_{23}-c_s a_{21}};\\ &\quad A_{31}e^{\kappa_{31}-c_s a_{33}},\ A_{32}e^{\kappa_{32}-c_s a_{32}},\ A_{33}e^{\kappa_{33}-c_s a_{31}}),\end{aligned}$$

$$\begin{aligned}(\widetilde{X}^{12}_{Neu})_{rj} &= \widetilde{c}_{O\,Neu}\\ &= (c_{O\,11},\ c_{O\,12},\ c_{O\,13};\ c_{O\,21},\ c_{O\,22},\ c_{O\,23};\\ &\quad c_{O\,31},\ c_{O\,32},\ c_{O\,33}),\end{aligned}$$

$$\begin{aligned}(\widetilde{X}^{13}_{Neu})_{rj} &= \widetilde{c_{PE\,Neu}} \otimes (\widetilde{\kappa}_{Neu})^N\\ &= (c_{PE\,11}(\kappa_{11})^N,\ c_{PE\,12}(\kappa_{12})^N,\ c_{PE\,13}(\kappa_{13})^N;\\ &\quad c_{PE\,21}(\kappa_{21})^N,\ c_{PE\,22}(\kappa_{22})^N, c_{PE\,23}(\kappa_{23})^N;\\ &\quad c_{PE\,31}(\kappa_{31})^N,\ c_{PE\,32}(\kappa_{32})^N,\ c_{PE\,33}(\kappa_{33})^N),\end{aligned}$$

$$\begin{aligned}(\widetilde{X}^{14}_{Neu})_{rj} &= \widetilde{c}_{P\,Neu} \otimes \widetilde{A}_{Neu} \otimes e^{\widetilde{\kappa}_{Neu}\ominus(c_s\cdot\widetilde{a}_{Neu})}\\ &= (c_{P\,11}A_{11}e^{\kappa_{11}-c_s a_{13}},\ c_{P\,12}A_{12}e^{\kappa_{12}-c_s a_{12}},\\ &\quad c_{P\,13}A_{13}e^{\kappa_{13}-c_s a_{11}}; c_{P\,21}A_{21}e^{\kappa_{21}-c_s a_{23}},\\ &\quad c_{P\,22}A_{22}e^{\kappa_{22}-c_s a_{22}},\ c_{P\,23}A_{23}e^{\kappa_{23}-c_s a_{21}};\\ &\quad c_{P\,31}A_{31}e^{\kappa_{31}-c_s a_{33}},\ c_{P\,32}A_{32}e^{\kappa_{32}-c_s a_{32}},\\ &\quad c_{P\,33}A_{33}e^{\kappa_{33}-c_s a_{31}}),\end{aligned}$$

$$\begin{aligned}(\widetilde{X}^{15}_{Neu})_{rj} &= \widetilde{c}_{H\,Neu} \otimes \widetilde{A}_{Neu} \otimes e^{\widetilde{\kappa}_{Neu}\ominus(c_H\cdot\widetilde{a}_{Neu})}\\ &= (c_{H\,11}A_{11}e^{\kappa_{11}-c_s a_{13}},\ c_{H\,12}A_{12}e^{\kappa_{12}-c_s a_{12}},\\ &\quad c_{H\,13}A_{13}e^{\kappa_{13}-c_s a_{11}};\ c_{H\,21}A_{21}e^{\kappa_{21}-c_s a_{23}},\\ &\quad c_{H\,22}A_{22}e^{\kappa_{22}-c_s a_{22}},\ c_{H\,23}A_{23}e^{\kappa_{23}-c_s a_{21}};\\ &\quad c_{H\,31}A_{31}e^{\kappa_{31}-c_s a_{33}},\ c_{H\,32}A_{32}e^{\kappa_{32}-c_s a_{32}},\\ &\quad c_{H\,33}A_{33}e^{\kappa_{33}-c_s a_{31}}),\end{aligned}$$

$(\widetilde{X}^{16}_{Neu})_{rj} = \widetilde{X}^{14}_{Neu}$, $(\widetilde{X}^{17}_{Neu})_{rj} = \widetilde{X}^{11}_{Neu}$, $(\widetilde{X}^{18}_{Neu})_{rj} = (1,\ 1,\ 1;\ 1,\ 1,\ 1;\ 1,\ 1,\ 1)$.
Now for $r = 1, 2, \quad j = 1, 2, 3$, we rewrite $(\widetilde{\Pi}_{Neu})_{rj}(F, c_s)$ as follows:

$$\begin{aligned}(\widetilde{\Pi}_{Neu})_{rj}(F, c_s) =&((\Pi_{11})_{rj}(F, c_s),\ (\Pi_{11})_{rj}(F, c_s),\\ &(\Pi_{11})_{rj}(F, c_s); (\Pi_{11})_{rj}(F, c_s),\\ &(\Pi_{11})_{rj}(F, c_s),\ (\Pi_{11})_{rj}(F, c_s);\\ &(\Pi_{11})_{rj}(F, c_s),\ (\Pi_{11})_{rj}(F, c_s),\\ &(\Pi_{11})_{rj}(F, c_s))\end{aligned}$$

4.1 De-neutrosophication of the neutrosophic total profit

As $\widetilde{\Pi}_{Neu}(c_s, F)$ is a triangular neutrosophic number, de-neutrosophication has been done using removal area method mentioned in Definition 5 as follows:

$$D(\widetilde{\Pi}_{Neu}(c_s, F)) = \begin{cases} D\left((\widetilde{\Pi}_{Neu})_{11}(c_s), F\right), & U \le F \text{ and } V \le U\\ D\left((\widetilde{\Pi}_{Neu})_{12}(c_s, F)\right), & F \le U \le F+V \text{ and } V \le U\\ D\left((\widetilde{\Pi}_{Neu})_{13}(c_s, F)\right), & F+V \le U \text{ and } V \le U\\ D\left((\widetilde{\Pi}_{Neu})_{21}(c_s, F)\right), & U \le F \text{ and } U \le V\\ D\left((\widetilde{\Pi}_{Neu})_{22}(c_s, F)\right), & F \le U \text{ and } U \le V, \end{cases} \quad (25)$$

$$D((\widetilde{\Pi}_{Neu})_{rj}(F,c_S)) = D((E_{11})_{rj}\cdot(\widetilde{X}^{11}_{Neu})_{rj} \ominus (E_{12})_{rj}\cdot (\widetilde{X}^{12}_{Neu})_{rj} \ominus (E_{13})_{rj}\cdot(\widetilde{X}^{13}_{Neu})_{rj} \ominus (E_{14})_{rj}\cdot(\widetilde{X}^{14}_{Neu})_{rj}\ominus (E_{15})_{rj}\cdot(\widetilde{X}^{15}_{Neu})_{rj} \ominus (E_{15})_{rj}\cdot(\widetilde{X}^{15}_{Neu})_{rj} \ominus (E_{16})_{rj}\cdot (\widetilde{X}^{16}_{Neu})_{rj} \oplus (E_{17})_{rj}\cdot(\widetilde{X}^{17}_{Neu})_{rj} \ominus (A_{18})_{rj}\cdot(\widetilde{X}^{18}_{Neu})_{rj}) = (E_{11})_{rj}D((\widetilde{X}^{11}_{Neu})_{rj}) - (E_{12})_{rj}D((\widetilde{X}^{12}_{Neu})_{rj}) - (E_{13})_{rj}D((\widetilde{X}^{13}_{Neu})_{rj}) - (E_{14})_{rj}D((\widetilde{X}^{14}_{Neu})_{rj}) - (E_{15})_{rj}D((\widetilde{X}^{15}_{Neu})_{rj}) - (E_{16})_{rj}D((\widetilde{X}^{16}_{Neu})_{rj}) + (E_{17})_{rj}D((\widetilde{X}^{17}_{Neu})_{rj}) - (E_{18})_{rj}D((\widetilde{X}^{18}_{Neu})_{rj}),\quad r=1,2,\ j=1,2,3. \tag{26}$$

Here,

$$D((\widetilde{X}^{11}_{Neu})_{11}) = \frac{1}{12}\Big\{A_{11}e^{\kappa_{11}-c_S a_{13}} + 2A_{12}e^{\kappa_{12}-c_S a_{12}} + A_{13}e^{\kappa_{13}-c_S a_{11}} + A_{21}e^{\kappa_{21}-c_S a_{23}} + 2A_{22}e^{\kappa_{22}-c_S a_{22}} + A_{23}e^{\kappa_{23}-c_S a_{21}} + A_{31}e^{\kappa_{31}-c_S a_{33}} + 2A_{32}e^{\kappa_{32}-c_S a_{32}} + A_{33}e^{\kappa_{33}-c_S a_{31}}\Big\}$$

$$D((\widetilde{X}^{14}_{Neu})_{11}) = D((\widetilde{X}^{16}_{Neu})_{11}) = \frac{1}{12}\Big\{c_{P\,11}A_{11}e^{\kappa_{11}-c_S a_{13}} + 2c_{P\,12}A_{12}e^{\kappa_{12}-c_S a_{12}} + c_{P\,13}A_{13}e^{\kappa_{13}-c_S a_{11}} + c_{P\,21}A_{21}e^{\kappa_{21}-c_S a_{23}} + 2c_{P\,22}A_{22}e^{\kappa_{22}-c_S a_{22}} + c_{P\,23}A_{23}e^{\kappa_{23}-c_S a_{21}} + c_{P\,31}A_{31}e^{\kappa_{31}-c_S a_{33}} + 2c_{P\,32}A_{32}e^{\kappa_{32}-c_S a_{32}} + c_{P\,33}A_{33}e^{\kappa_{33}-c_S a_{31}}\Big\}$$

$$D((\widetilde{X}^{15}_{Neu})_{11}) = \frac{1}{12}\Big\{c_{H\,11}A_{11}e^{\kappa_{11}-c_S a_{13}} + 2c_{H\,12}A_{12}e^{\kappa_{12}-c_S a_{12}} + c_{H\,13}A_{13}e^{\kappa_{13}-c_S a_{11}} + c_{H\,21}A_{21}e^{\kappa_{21}-c_S a_{23}} + \times 2c_{H\,22}A_{22}e^{\kappa_{22}-c_S a_{22}} + c_{H\,23}A_{23}e^{\kappa_{23}-c_S a_{21}} + \times c_{H\,31}A_{31}e^{\kappa_{31}-c_S a_{33}} + 2c_{H\,32}A_{32}e^{\kappa_{32}-c_S a_{32}} + \times c_{H\,33}A_{33}e^{\kappa_{33}-c_S a_{31}}\Big\}$$

are the function of c_S and remaining $D((\widetilde{X}^{1j}_{Neu})_{11})$, for $j=2,3,7,8$ are constant.

$$\frac{dD((\widetilde{X}^{1j}_{Neu})_{11})}{dc_S} = D((\widetilde{X}^{1j}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})),$$

$$\frac{d^2D((\widetilde{X}^{1j}_{Neu})_{11})}{dc_S^2} = D((\widetilde{X}^{1j}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu})).$$

5 Some theoretical results

Theorem 2 *If $F(x)$ is a non-negative differentiable strictly concave function and $G(x)$ is a positive, differentiable convex function, then the real valued function $H(x)=\frac{F(x)}{G(x)}$ is a strictly pseudo-concave function.*

Theorem 3 *For any given C_S,*

1. *$D((\widetilde{\Pi}_{Neu})_{11}(F,c_S))$ is a strictly pseudo-concave function in F, and hence $\exists$ a unique maximum solution F^*_{11}.*
2. *If $U \le F^*_{11}$, then $D((\widetilde{\Pi}_{Neu})_{11}(F,c_S))$ has maximum value at F^*_{11}.*
3. *If $U \ge F^*{}_{11}$, then $D((\widetilde{\Pi}_{Neu})_{11}(F,c_S))$ has maximum value at U.*

Proof See Appendix A. □

To find F^*_{11} for any given c_S, taking first order derivative of $D((\widetilde{\Pi}_{Neu})_{11}(F,c_S))$ with respect to F and equate to zero, we get

$$\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{11}(F,c_S)) = \frac{1}{F^2}D((\widetilde{X}^{12}_{Neu})_{11}) + \frac{1}{F^2}D((\widetilde{X}^{13}_{Neu})_{11}) + \frac{e^V}{F^2}\left\{\left(\int_0^F e^{-\xi(t)}du\right) - Fe^{\xi(F)}\right\}D((\widetilde{X}^{14}_{Neu})_{11}) + \frac{e^V}{F^2}\left\{\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right) - F\left(\int_0^F e^{-\xi(t)+\xi(F)} dt\right)\right\}D((\widetilde{X}^{15}_{Neu}) + \frac{i_{IC}e^V}{F^2}\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2F\int_0^F e^{\xi(u)}du + \frac{1}{2}\lambda_2(F-U)^2 + \frac{1}{2}(1-\lambda_2)(F+V-U)^2\right\}D((\widetilde{X}^{16}_{Neu}) - \frac{e^V i_{IC}}{F}\left\{\left\{V+F+\frac{1}{2}F(1-\lambda_1)\right\}(1-\lambda_1)e^{\xi(F)} + \frac{1}{2}(3-\lambda_1)(1-\lambda_1)\int_0^F e^{\xi(t)}dt + (F-U+V)(1-\lambda_2) + (F-U)\lambda_2\right\}D((\widetilde{X}^{16}_{Neu}) - \frac{c_S i_{IE}e^V}{2F^2}\Big\{\lambda_2U^2$$

$$+ (1-\lambda_2)(U-V)^2\Big\} D((\widetilde{X}^{17}_{Neu}) + \frac{\eta}{F^2} D((\widetilde{X}^{18}_{Neu}).$$

Theorem 4 *For any given F, if $\Theta_{11} > c_s e^V \dfrac{d^2 D((\widetilde{X}^{1j}_{Neu})11)}{dc_s^2}$, then $D((\widetilde{\Pi}_{Neu})_{11}(F, c_s))$ is a strictly pseudo-concave function in c_s.*

Proof See Appendix B □

We now obtain a theorem that completely describes the pseudo-concavity of the function $D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$. We omit the proof, as it is trivial in view of Theorem 3.

Theorem 5 *For any given C_S,*

1. *$D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$ is a strictly pseudo-concave function in F, and hence $\exists$ a unique maximum solution F^*_{12}.*
2. *If $U \le F^*_{12} + V$, then $D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$ has maximum value at F^*_{12}.*
3. *If $U \ge F^*_{12} + V$, then $D((\widetilde{\Pi}_{Neu})_{11}(F, c_s))$ has maximum value at $U - V$.*

For any given c_s, to find the F^*_{12}, taking the first order derivative of $D((\widetilde{\Pi}_{Neu})_{12}(F)) = D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$ with respect to F and equate it to zero, we get

$$\frac{d}{dF} D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$$
$$= \frac{1}{F^2} D((\widetilde{X}^{12}_{Neu})12) + \frac{1}{F^2} D((\widetilde{X}^{13}_{Neu})12) + \frac{e^V}{F^2}$$
$$\Bigg\{ \left(\int_0^F e^{-\xi(t)} du \right) - F e^{\xi(F)} \Bigg\} D((\widetilde{X}^{14}_{Neu})12) + \frac{e^V}{F^2}$$
$$\Bigg\{ \left(\int_0^F e^{-\xi(t)} \left(\int_t^F e^{\xi(u)} du \right) dt \right) - F(\int_0^F e^{-\xi(t)+\xi(F)}$$
$$dt) \Bigg\} D((\widetilde{X}^{15}_{Neu})12) + \frac{i_{IC} e^V}{F^2} \Bigg\{ (1-\lambda_1)(F+V)$$
$$\int_0^F e^{\xi(u)} du + \frac{1}{2}(1-\lambda_1)^2 F \int_0^F e^{\xi(u)} du + \frac{1}{2}(1-\lambda_2)$$
$$(F+V-U)^2 \Bigg\} D((\widetilde{X}^{16}_{Neu})12) - \frac{e^V i_{IC}}{F} \Bigg\{ \{V + F$$
$$+ \frac{1}{2} F(1-\lambda_1)\}(1-\lambda_1)e^{\xi(F)} + \frac{1}{2}(3-\lambda_1)(1-$$
$$\lambda_1) \int_0^F e^{\xi(t)} dt + (F-U+V)(1-\lambda_2) \Bigg\}$$
$$D((\widetilde{X}^{16}_{Neu})12) - \frac{c_s i_{IE} e^V}{2F^2} \Bigg\{ \lambda_2 F^2 + 2\lambda_2 F(U-F)$$
$$+ (1-\lambda_2)(U-V)^2 \Bigg\} D((\widetilde{X}^{17}_{Neu})12) + \frac{e^V}{F}\lambda_2(U$$
$$-F) D((\widetilde{X}^{17}_{Neu})12) + \frac{\eta}{F^2} D((\widetilde{X}^{18}_{Neu})12).$$

Theorem 6 *For any given F, if $\Theta_{12} > c_s e^V \dfrac{d^2 D((\widetilde{X}^{11}_{Neu})12)}{dc_s^2}$, then $D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$ is a strictly pseudo-concave function in c_s.*

Proof See Appendix C. □

As expected, the following theorem shows that the function $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$ is strictly pseudo-concave. The proof can be achieved by proceeding in the same direction, as in Theorem 3.

Theorem 7 *For any given C_S,*

1. *$D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$ is a strictly pseudo-concave function in F, and hence, $\exists$ a unique maximum solution F^*_{13}.*
2. *If $U \ge F^*_{13} + V$, then $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$ has maximum value at F^*_{13}.*
3. *If $U \le F^*13 + V$, then $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$ has maximum value at $U - V$.*

Theorem 8 *For any given F, if $\Theta_{13} > c_s e^V \dfrac{d^2 D((\widetilde{X}^{11}_{Neu})13)}{dc_s^2}$, then $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$ is a strictly pseudo-concave function in c_s.*

Proof See Appendix D □

Theorem 9 *For any given C_S,*

1. *if $\Delta_{13}(U-V) < 0$, then retailer optimal cycle time is F^*_{13},*
2. *if $\Delta_{13}(U-V) = 0$, then retailer optimal cycle time is $U - V$,*
3. *if $\Delta_{13}(U-V) > 0$, then*
   - *(a) if $\Delta_{12}(U) < 0$, then retailer optimal cycle time is F^*_{12}*
   - *(b) if $\Delta_{12}(U) = 0$, then retailer optimal cycle time is U,*
   - *(c) if $\Delta_{12}(U) > 0$, then retailer optimal cycle time is F^*_{11}.*

Here $\Delta_{12}(F) = \frac{d}{dF} D((\widetilde{\Pi}_{Neu})12(F, c_s))$ and $\Delta_{13}(F) = \frac{d}{dF} D((\widetilde{\Pi}_{Neu})13(F, c_s))$.

Proof See Appendix E. □

In the following two theorems, we characterize the pseudo-concavity of functions $D((\widetilde{\Pi}_{Neu})_{22}(F, c_s))$ and $D((\widetilde{\Pi}_{Neu})_{21}(F, c_s))$, respectively. The proofs of the theorems follow from Theorem 3.

Theorem 10 *For any given* C_S,

1. $D((\widetilde{\Pi}_{Neu})_{21}(F, c_s))$ *is a strictly pseudo-concave function in* F, *and hence,* $\exists$ *a unique maximum solution* F^*_{21}.
2. *If* $U \leq F^*_{21}$, *then* $D((\widetilde{\Pi}_{Neu})_{21}(F, c_s))$ *has maximum value at* F^*_{21}.
3. *If* $U \geq F^*{21}$, *then* $D((\widetilde{\Pi}_{Neu})_{21}(F, c_s))$ *has maximum value at* U.

Theorem 11 *For any given* C_S,

1. $D((\widetilde{\Pi}_{Neu})_{22}(F, c_s))$ *is a strictly pseudo-concave function in* F, *and hence* $\exists$ *a unique maximum solution* F^*_{22}.
2. *If* $U \geq F^*_{22}$, *then* $D((\widetilde{\Pi}_{Neu})_{22}(F, c_s))$ *has maximum value at* F^*_{22}.
3. *If* $U \leq F^*{22}$, *then* $D((\widetilde{\Pi}_{Neu})_{22}(F, c_s))$ *has maximum value at* U.

In the next theorem, we explore the retailer's optimal cycle time, under the constrained $U \leq V$. The proof of the theorem can be completed by applying the analogous arguments, as in Theorem 9.

Theorem 12 *For any given* C_S,

1. *if* $\Delta_{22}(U) < 0$, *then retailer optimal cycle time is* F^*_{22},
2. *if* $\Delta_{22}(U) = 0$, *then retailer optimal cycle time is* U,
3. *if* $\Delta_{22}(U) > 0$, *then retailer optimal cycle time is* F^*_{21}.

Here $\Delta_{22}(F) = \dfrac{d}{dF} D((\widetilde{\Pi}_{Neu})_{22}(F, c_s))$.

6 Case study

We went to a neighbourhood grocery store in Benachity, Durgapur, West Bengal, India. After extensive conversations with the manager, we learned that the store is only making a small profit, and the management is making an effort to minimize its expenditure. Due to demand and deterioration, the stocks are becoming depleted. From prior experience, it has been noted that few items degrade at a steady rate. The existence of four different types of expenses and parameters are shown in Table 1. Finally, we revealed that the store faces the following issue:

1. How many units should be ordered to keep the profit at a maximum?
2. Due to the quick changes in market data, the launch of a new product, the introduction of new policies by rival companies, etc., the cost parameters and market demand may change (Table 2).
3. What should the store's product selling price and replenishment cycle be to maximize profits and maintain customer satisfaction?
4. Due to the influence of the environment, the pace of deterioration may differ.

All of the expenses and specifications could change because of the unpredictability of market data and other factors. To address the aforementioned issues, we treat all of the costs and some parameters as triangular single-valued neutrosophic numbers.

7 Numerical examples

We use the data set collected from the case study in Tables 3 and 4 to produce the desired optimal result, which is shown in Table 5. We apply Theorems 3–9 to arrive at the optimal solution. To reduce the complexity of the calculation, we utilize the software Mathematica 11.3.

8 Sensitivity analysis

We take a sensitivity for the numerical example presented in the previous section of the parameters (i_{IC}, i_{IE} λ_1 λ_2 b U V ζ) ranging from (−30% to +30%), as shown in Table 2. The sensitive analysis is carried out to investigate the effects of under-or overestimating the parameters on the de-neutrosophic values of the total neutrosophic profit ($D(\widetilde{\Pi}_{Neu})$, order quantity ($D(\widetilde{Q}_{Neu})$, market demand ($D(\widetilde{\Omega}_{Neu})$, replenishment time ($F$), and product price ($c_s$). Table 2 displays the following computational outcomes.

1. The retailer's optimum order quantity, replenishment cycle and selling price of the products increase as the percentage of allowable payment delay (λ_1) for the retailer increases. This suggests that the store will place more orders in order to fully take advantage of the supplier's partial trade credit (U). Similarly, the retailer's ideal order quantity and replenishment cycle increase, the selling price decreases, and the optimum total profit per unit of time increases as the portion of the customer's purchase cost (λ_2) that must be paid to the retailer at the time of placing an order increases.
2. Additionally, a higher interest earned rate (i_{IE}) generates more money, less quantity ordered, and an increase in demand, which raises overall profit. A more increased interest charge rate (i_{IC}) increases overall expenses and reduces earnings.
3. The parameter b for preservation technology is quite sensitive. As b increases, profit and replenishment times rise, the selling price falls, and demand rises as a result.

Table 2 Sensitivity analysis of the neutrosophic EOQ model

Parameters	% change in parameters	Change in optimal values				
		$(D(\widetilde{\Pi}_{Neu}))^*$	$(D(\widetilde{\Omega}_{Neu}))^*$	$(D(\widetilde{Q}_{Neu}))^*$	F^*	c_S^*
i_{IC}	−30%	2330.40	297.458	375.262	1.18112	17.1638
	−20%	2309.20	296.348	367.348	1.16338	17.2014
	+20%	2228.43	292.109	341.823	1.10056	17.3463
	+30%	2209.15	291.095	336.049	1.08659	17.3813
i_{IE}	−30%	2267.08	294.092	354.411	1.13143	17.2782
	−20%	2267.40	294.190	354.200	1.13110	17.2771
	+20%	2268.70	294.257	354.078	1.12980	17.2726
	+30%	2269.03	294.290	354.010	1.12951	17.2715
λ_1	−30%	2186.89	289.465	331.282	1.07775	17.4377
	−20%	2214.63	291.083	338.444	1.09515	17.3817
	+20%	2318.40	297.110	369.737	1.16609	17.1756
	+30%	2342.28	298.492	377.482	1.18382	17.1289
λ_2	−30%	2263.60	293.720	354.055	1.13170	17.2910
	−20%	2265.09	293.877	354.107	1.13129	17.2856
	+20%	2271.02	294.505	354.315	1.12965	17.2641
	+30%	2272.51	294.662	354.367	1.12924	17.2588
b	−30%	2225.97	292.323	347.141	1.09343	17.3389
	−20%	2241.32	293.004	349.693	1.10667	17.3155
	+20%	2290.20	295.176	358.028	1.15095	17.2412
	+30%	2299.80			1.16004	17.2267
U	−30%	2257.09	293.023	353.787	1.13343	17.3149
	−20%	2260.68	293.409	353.942	1.13249	17.3016
	+20%	2275.67	294.986	354.427	1.12825	17.2477
	+30%	2279.58	295.388	354.514	1.12707	17.2340
	−30%	2185.62	284.926	348.731	1.14804	17.2950
	−20%	2212.80	287.983	350.551	1.14216	17.2882
	+20%	2324.54	300.526	357.896	1.11888	17.2617
	+30%	2353.25	303.742	359.748	1.11313	17.2553
ζ	−30%	2304.84	295.829	360.587	1.16488	17.2190
	−20%	2292.50	295.279	358.420	1.15312	17.2378
	+20%	2243.89	293.118	350.123	1.10891	17.3116
	+30%	2231.91	292.587	348.125	1.09852	17.3299

Table 3 Cost vectors and associate parameters value

Cost parameters	Values	Parameters	Values
Holding cost (c_H)	\$ 2/unit/unit time	a	0.1
Ordering cost (c_O)	\$ 500/order	b	0.005
Purchasing cost (c_P)	\$ 5/unit	λ_1	0.7
Advertisement cost (c_{PE})	\$ 40 per unit effort	λ_2	0.6
Preservation technology cost (η)	\$ 200/order	κ	2
Interest charged (c_{IC})	15%	n	1
Interest earned (c_{IE})	10%	A	200
Upstream trade credit (U)	0.15 years		
Downstream trade credit(V)	0.10 years		
Deterioration rate ($\zeta(t)$)	0.1		

Table 4 Parameters in triangular SVNN form

Parameters	Components of Triangular SVNN.								
	l_1	l_2	l_3	m_1	m_2	m_3	n_1	n_2	n_3
c_O	475	500	525	484.5	510	535.5	465.5	490	514.5
c_H	1.9	2	2.1	1.938	2.04	2.142	1.862	1.96	2.058
A	190	200	210	193.8	204	214.2	186.2	196	205.8
c_P	4.75	5	5.25	4.845	5.1	5.355	4.655	4.9	5.145
κ	1.90	2.00	2.10	1.938	2.04	2.142	1.862	1.96	2.058
a	0.095	0.100	0.105	0.0969	0.102	0.1071	0.0931	0.098	0.1029
c_{PE}	38	40	42	38.76	40.80	42.84	37.24	39.20	41.16

Table 5 Optimal solutions for different deterioration rates

$\xi(t)$	$\Delta_{13}(U-V)$	$\Delta_{12}(U-V)$	$\left(D(\widetilde{\Pi}_{Neu})\right)^*$	$\left(D(\widetilde{\Omega}_{Neu})\right)^*$	$\left(D(\widetilde{Q}_{Neu})\right)^*$	F^*	C_S^*
0.3	>0	>0	\$ 2268.05	294.191 units	354.211 units	1.13047 years	\$ 17.2749/unit
$\frac{1}{2-t}$	>0	>0	\$ 2145.59	291.391 units	324.147 units	0.994208 years	\$ 17.3710/unit
$0.6t$	>0	>0	\$ 2293.47	308.292 units	336.867 units	1.07443 years	\$ 17.0685/unit

4. Table 2 shows that the optimal order quantity and retailer profit both rise with a longer down-stream credit period. As a result, V has a favourable effect on consumer demand. The retailer's optimum profit also rises with the length of the credit period, U that the supplier offers.

9 Particular cases

In this section, we find some previously developed well known result of the other author as special cases.

9.1 Goyal's model

If $V=0$, $\widetilde{\kappa}_{Neu}=\mathbf{0}$, $\widetilde{a}_{Neu}=\mathbf{0}$, then $\widetilde{\Omega}_{Neu}=[(A,A,A),(A,A,A),(A,A,A)]$. Also, we consider $\widetilde{c_H}_{Neu}=[(c_H,c_H,c_H),(c_H,c_H,c_H),(c_H,c_H,c_H)]$, $\widetilde{C_P}_{Neu}=[(c_P,c_P,c_P),(c_P,c_P,c_P),(c_P,c_P,c_P)]$, $\widetilde{C_O}_{Neu}=[(c_O,c_O,c_O),(c_O,c_O,c_O),(c_O,c_O,c_O)]$, $b=0$, $\lambda_1=1,\lambda_2=1$, deterioration rate $\zeta(t)=0$, that is all the cost parameters are taken in crisp sense. In Goyal's model $c_S=c_P$. Then, the total profit function becomes

$$D(\widetilde{\Pi}_{Neu}(F)) = \begin{cases} D\left((\widetilde{\Pi}_{Neu})_{11}(F)\right), & U \le F \\ D\left((\widetilde{\Pi}_{Neu})_{13}(F)\right), & F \le U, \end{cases} \tag{27}$$

where

$$D((\widetilde{\Pi}_{Neu})_{11}(F)) = -\frac{c_O}{F} - \frac{C_H AF}{2} - \frac{1}{2F}Ac_P i_{IC}(F-U)^2 + \frac{1}{2F}c_S i_{IE}U^2 A$$

$$D((\widetilde{\Pi}_{Neu})_{13}(F)) = -\frac{c_O}{F} - \frac{C_H AF}{2} + \frac{1}{2F}c_S i_{IE} A(F^2 + F(U-F))$$

For the Goyal's model, we have consider total variable cost function as

$$\begin{aligned} TVC_1(F) &= -D((\widetilde{\Pi}_{Neu})_{11}(F)) \\ &= \frac{c_O}{F} + \frac{C_H AF}{2} + \frac{1}{2F}Ac_P i_{IC}(F-U)^2 - \frac{1}{2F}c_S i_{IE}U^2 A, \\ TVC_2(F) &= -D((\widetilde{\Pi}_{Neu})_{13}(F)) \\ &= \frac{c_O}{F} + \frac{C_H AF}{2} - \frac{1}{2F}c_S i_{IE} A(F^2 + F(U-F)). \end{aligned}$$

$$\therefore\ F_{11}^* = \sqrt{\frac{2c_O + AU^2 c_P(i_{IC}-i_{IE})}{A(c_H + c_P i_{IC})}} \text{ and } F_{13}^* = \sqrt{\frac{2c_O}{A(c_H + c_P i_{IC})}}.$$

9.2 Teng's model

If $\widetilde{\kappa}_{Neu}=\mathbf{0}$, $\widetilde{a}_{Neu}=[(\frac{V}{c_S},\frac{V}{c_S},\frac{V}{c_S}),(\frac{V}{c_S},\frac{V}{c_S},\frac{V}{c_S}),(\frac{V}{c_S},\frac{V}{c_S},\frac{V}{c_S})]$, then $\widetilde{\Omega}_{Neu}=[(A,A,A),(A,A,A),(A,A,A)]$. Also, we consider $\widetilde{c_H}_{Neu}=[(c_H,c_H,c_H),(c_H,c_H,c_H),(c_H,c_H,c_H)]$, $\widetilde{C_P}_{Neu}=[(c_P,c_P,c_P),(c_P,c_P,c_P),(c_P,c_P,c_P)]$, $\widetilde{C_O}_{Neu}=[(c_O,c_O,c_O),(c_O,c_O,c_O),(c_O,c_O,c_O)]$, $b=0$, $\lambda_1=1$, $0\le\lambda_2\le1$, deterioration rate $\zeta(t)=0$, that is all the cost parameters are taken in crisp sense. In Teng's model $c_S < c_P$. Then, the total profit function becomes

$$D(\widetilde{\Pi}_{Neu}(F)) = \begin{cases} D\left((\widetilde{\Pi}_{Neu})_{11}(F)\right), & U \le F \ \&\ V \le U \\ D\left((\widetilde{\Pi}_{Neu})_{12}(F)\right), & F \le U \le F+V \ \&\ V \le U \\ D\left((\widetilde{\Pi}_{Neu})_{13}(F)\right), & F+V \le U \ \&\ V \le U \\ D\left((\widetilde{\Pi}_{Neu})_{21}(F)\right), & U \le F \ \&\ U \le V \\ D\left((\widetilde{\Pi}_{Neu})_{22}(F)\right), & F \le U \ \&\ U \le V \end{cases} \tag{28}$$

where

$$D((\widetilde{\Pi}_{Neu})_{11}(F)) = Ac_S - Ac_P - \frac{c_O}{F} - \frac{C_H AF}{2} - \frac{1}{2F}Ac_P i_{IC}\{\lambda_2(F-U)^2 + (1-\lambda_2)(F+V-U)^2\} + \frac{1}{2F}c_S i_{IE} A\{\lambda_2 U^2 + (1-\lambda_2)(U-V)^2\}$$

$$D((\widetilde{\Pi}_{Neu})_{12}(F)) = Ac_S - Ac_P - \frac{c_O}{F} - \frac{C_H AF}{2} - \frac{1}{2F}Ac_P i_{IC}(1-\lambda_2)(F+V-U)^2 + \frac{1}{2F}c_S i_{IE} A\{\lambda_2 U^2 + (1-\lambda_2)(U-V)^2\}$$

$$D((\widetilde{\Pi}_{Neu})_{13}(F)) = Ac_S - Ac_P - \frac{c_O}{F} - \frac{C_H AF}{2} + \frac{1}{2F}c_S i_{IE} A[\lambda_2\{F^2 + F(U-F)\} + (1-\lambda_2)F(U-F-V)]$$

$$D((\widetilde{\Pi}_{Neu})_{21}(F)) = Ac_S - Ac_P - \frac{c_O}{F} - \frac{C_H AF}{2} - \frac{1}{2F}Ac_P i_{IC}\{\lambda_2(F-U)^2 + (1-\lambda_2)F(V-U) + (1-\lambda_2)F^2\} + \frac{1}{2F}c_S i_{IE} A\lambda_2 U^2$$

$$D((\widetilde{\Pi}_{Neu})_{22}(F)) = Ac_S - Ac_P - \frac{c_O}{F} - \frac{C_H AF}{2} - \frac{1}{2F}Ac_P i_{IC}(1-\lambda_2)\{F(U-V) + F^2\} + \frac{1}{2F}c_S i_{IE} A\lambda_2\{F^2 + F(U-F)\}$$

For the Teng's model, we have consider total relevant cost function as:

$$TRC_{11}(F) = Ac_S - Ac_P - D((\widetilde{\Pi}_{Neu})_{11}(F)) = \frac{c_O}{F} + \frac{C_H AF}{2} + \frac{1}{2F}Ac_P i_{IC}\{\lambda_2(F-U)^2 + (1-\lambda_2)(F+V-U)^2\} - \frac{1}{2F}c_S i_{IE} A\{\lambda_2 U^2 + (1-\lambda_2)(U-V)^2\}$$

$$TRC_{12}(F) = Ac_S - Ac_P - D((\widetilde{\Pi}_{Neu})_{12}(F)) = \frac{c_O}{F} + \frac{C_H AF}{2} + \frac{1}{2F}Ac_P i_{IC}(1-\lambda_2)(F+V-U)^2 - \frac{1}{2F}c_S i_{IE} A\{\lambda_2 U^2 + (1-\lambda_2)(U-V)^2\}$$

$$TRC_{13}(F) = Ac_S - Ac_P - D((\widetilde{\Pi}_{Neu})_{13}(F)) = \frac{c_O}{F} + \frac{C_H AF}{2} + \frac{1}{2F}c_S i_{IE} A[\lambda_2\{F^2 + F(U-F)\} + (1-\lambda_2)F(U-F-V)]$$

$$TRC_{21}(F) = Ac_S - Ac_P - D((\widetilde{\Pi}_{Neu})_{21}(F)) = \frac{c_O}{F} + \frac{C_H AF}{2} + \frac{1}{2F}Ac_P i_{IC}\{\lambda_2(F-U)^2 + \frac{1}{2F}Ac_P i_{IC}\{\lambda_2(F-U)^2 + (1-\lambda_2)F(V-U) + (1-\lambda_2)F^2\} - \frac{1}{2F}c_S i_{IE} A\lambda_2 U^2$$

$$TRC_{22}(F) = Ac_S - Ac_P - D((\widetilde{\Pi}_{Neu})_{22}(F)) = \frac{c_O}{F} + \frac{C_H AF}{2} + \frac{1}{2F}Ac_P i_{IC}\{\lambda_2(F-U)^2 + \frac{1}{2F}Ac_P i_{IC}(1-\lambda_2)\{F(U-V) + F^2\} - \frac{1}{2F}c_S i_{IE} A\lambda_2\{F^2 + F(U-F)\}$$

$$F_{11}^* = \sqrt{\frac{(c_P i_{IC} - c_S i_{IE})\{2c_O + \lambda_2 AU^2 + (1-\lambda_2)A(U-V)^2\}}{A(c_H + c_P i_{IC})}},$$
$$F_{12}^* = \sqrt{\frac{(c_P i_{IC} - c_S i_{IE})\{2c_O + (1-\lambda_2)A(U-V)^2\}}{A(c_H + (1-\lambda_2)c_P i_{IC} + \lambda_2 c_P i_{IE})}},$$
$$F_{13}^* = \sqrt{\frac{2c_O}{A(c_H + c_S i_{IE})}}, F_{21}^* = \sqrt{\frac{2c_O + (c_P i_{IC} - c_S i_{IE}) + \lambda_2 AU^2}{A(c_H + c_P i_{IC})}},$$
$$F_{13}^* = \sqrt{\frac{2c_O}{A(c_H + (1-\lambda_2)c_P i_{IC} + \lambda_2 c_S i_{IE})}}$$

10 Conclusion and future scope

In this paper, we have developed an EOQ-based inventory model for time sensitive deteriorating items to determine the optimal ordering policies of a retailer under two levels of partial trade credit to reflect the supply chain management situation in the neutrosophic sense. Furthermore, selling price and promotional-dependent demand have been introduced to the proposed model. Also, preservation technology has been included to restrict the deterioration rate. It is assumed that the retailer maintains a powerful position and can obtain the full trade credit offered by the supplier and the retailer just offers partial trade credit to customers. Furthermore, the demand rate, holding cost, ordering cost, purchasing cost and promotional cost have been considered as triangular neutrosophic numbers. Based on the above situations, we have investigated the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory policy under the supply chain management in the neutrosophic sense. De-neutrosophication of the total annual neutrosophic profit for the retailer has been done using the removal area method.

From the viewpoint of the costs, decision rules to find the optimal cycle time ($\digamma^*$) and optimal selling price (c_s^*) contain five cases. To obtain the optimal ordering policy, we have proposed eleven theorems. Using these results, it has been proved that the neutrosophic value of the total annual profit for the retailer is strictly pseudo-concave for selling price and cycle time. The significant contributions of the present study are: (i) new demand function of the selling price, downstream trade credit period and neutrosophic advertising effort, (ii) neutrosophic ordering quantity, and (iii) solved numerical examples whose data are taken from a real-life case study. We have handled the uncertainty of market demand, cost parameters and other parameters in a neutrosophic sense. Table 5 demonstrates that when a product follows the Weibull deterioration rate, the overall neutrosophic profit ($\zeta(t) = 0.6t$) takes on higher values. Additionally, the subcase-1.1 ($V \leq U \leq \digamma$) achieves the best result throughout our analysis. With the partial trade credit policy and marketing initiatives, the retailer's profit can increase, and the market demand can rise.

The proposed model can be generalized in several ways. For instance, we could generalize the model to allow for shortages, quantity discounts, advanced cash payment policy and others.

Author Contributions k.m., a.k.j., a.d., and a.p. conceived the presented idea. k.m., and a.d. developed the theory and performed the computations. k.m. and a.k.j. verified the analytical methods. k.m. contributed reagents/materials/analysis tools. k.m., a.k.j., a.d., and a.p wrote the paper.

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Declarations

Conflict of interest All authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual. participants included in the study.

A Proof of Theorem 3

$$
\begin{aligned}
&D((\widetilde{\Pi}_{Neu})_{11}(\digamma, c_s)) \\
&= D((E_{11})_{11} \cdot (\widetilde{X}^{11}_{Neu})_{11} \ominus (E_{12})_{11} \cdot (\widetilde{X}^{12}_{Neu})_{11} \ominus (E_{13})_{11} \\
&\cdot (\widetilde{X}^{13}_{Neu})_{11} \ominus (E_{14})_{11} \cdot (\widetilde{X}^{14}_{Neu})_{11} \ominus (E_{15})_{11} \cdot (\widetilde{X}^{15}_{Neu})_{11} \ominus \\
&(E_{16})_{11} \cdot (\widetilde{X}^{16}_{Neu})_{11} \oplus (E_{17})_{11} \cdot (\widetilde{X}^{17}_{Neu})_{11} \ominus (E_{18})_{11} \cdot \\
&(\widetilde{X}^{18}_{Neu})_{11}) \\
&= (E_{11})_{11}\ D((\widetilde{X}^{11}_{Neu})_{11}) - (E_{12})_{11}\ D\ ((\widetilde{X}^{12}_{Neu})_{11}) \\
&\quad - (E_{13})_{11}\ D((\widetilde{X}^{13}_{Neu})_{11}) - (E_{14})_{11}\ D((\widetilde{X}^{14}_{Neu})_{11}) \\
&\quad - (E_{15})_{11}\ D((\widetilde{X}^{15}_{Neu})_{11}) - (E_{16})_{11}\ D((\widetilde{X}^{16}_{Neu})_{11}) \\
&\quad + (E_{17})_{11}\ D((\widetilde{X}^{17}_{Neu})_{11}) - (A_{18})_{11}\ D((\widetilde{X}^{18}_{Neu})_{11}) \\
&= c_s\ e^V D((\widetilde{X}^{11}_{Neu})_{11}) - \frac{1}{\digamma}\ D((\widetilde{X}^{12}_{Neu})_{11}) - \frac{1}{\digamma} \\
&D((\widetilde{X}^{13}_{Neu})_{11}) - \frac{e^V}{\digamma}\left(\int_0^{\digamma} e^{-\xi(t)}du\right) D((\widetilde{X}^{14}_{Neu})_{11}) - \frac{e^V}{\digamma} \\
&\left(\int_0^{\digamma} e^{-\xi(t)}(\int_t^{\digamma} e^{\xi(u)}du)dt\right) D((\widetilde{X}^{15}_{Neu}) - \frac{i_{IC}e^V}{\digamma}\left\{(1-\lambda_1)\right. \\
&(\digamma + V)\int_0^{\digamma} e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2\digamma\int_0^{\digamma} e^{\xi(u)}du + \frac{1}{2}\lambda_2 \\
&\left.(\digamma - U)^2 + \frac{1}{2}(1-\lambda_2)(\digamma + V - U)^2\right\} D((\widetilde{X}^{16}_{Neu}) \\
&+ \frac{c_s i_{IE} e^V}{2\digamma}\left\{\lambda_2 U^2 + (1-\lambda_2)(U-V)^2\right\} \\
&D((\widetilde{X}^{17}_{Neu}) - \frac{\eta}{\digamma} D((\widetilde{X}^{18}_{Neu})`
\end{aligned}
$$

Let $D((\widetilde{\Pi}_{Neu})_{11}(\digamma)) = \frac{f(\digamma)}{g(\digamma)}$, where

$$
\begin{aligned}
f(\digamma) &= c_s e^V \digamma D((\widetilde{X}^{11}_{Neu})_{11}) - D((\widetilde{X}^{12}_{Neu})_{11}) \\
&\quad - D((\widetilde{X}^{13}_{Neu})_{11}) - e^V\left(\int_0^{\digamma} e^{-\xi(t)}du\right)
\end{aligned}
$$

$$D((\widetilde{X}^{14}_{Neu})_{11}) - e^{V}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)$$

$$D((\widetilde{X}^{15}_{Neu}) - i_{IC}e^{V}\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du\right.$$

$$+\frac{1}{2}(1-\lambda_1)^2F\int_0^F e^{\xi(u)}du + \frac{1}{2}\lambda_2(F-U)^2 +$$

$$\left.\frac{1}{2}(1-\lambda_2)(F+V-U)^2\right\}D((\widetilde{X}^{16}_{Neu})$$

$$+\frac{c_s i_{IE}e^{V}}{2}\left\{\lambda_2U^2 + (1-\lambda_2)(U-V)^2\right\}$$

$$D((\widetilde{X}^{17}_{Neu}) - \eta D((\widetilde{X}^{18}_{Neu})$$

and $g(F) = F$.

$$\frac{df(F)}{dF} = c_s e^{V} D((\widetilde{X}^{11}_{Neu})_{11}) - e^{V}e^{\xi(F)}D((\widetilde{X}^{14}_{Neu})_{11})$$

$$-e^{V}\int_0^F e^{-\xi(t)+\xi(F)}dt\ D((\widetilde{X}^{15}_{Neu})_{11})$$

$$+i_{IC}e^{V}D((\widetilde{X}^{16}_{Neu})_{11})\left\{e^{\xi(F)}(V+x)(1-\lambda_1)\right.$$

$$+\frac{1}{2}x(1-\lambda_1)^2e^{\xi(F)} + \frac{1}{2}(1-\lambda_1)(3-\lambda_1)\int_0^F e^{\xi(t)}dt$$

$$\left.+(F-U+V)(1-\lambda_2) + (F-U)\lambda_2\right\}$$

$$\frac{d^2f(F)}{dF^2} = \frac{1}{2}e^{V}\left[-2\left\{D((\widetilde{X}^{15}_{Neu})_{11}) + i_{IC}D((\widetilde{X}^{16}_{Neu})_{11})\right.\right.$$

$$\left.+e^{\xi(F)}D((\widetilde{X}^{16}_{Neu})_{11})(3-4\lambda_1+\lambda_1^2)\right\}$$

$$-2D((\widetilde{X}^{15}_{Neu})_{11})\int_0^F e^{-b\eta-\xi(t)+\xi(F)}\zeta(F)dt$$

$$-e^{-b\eta+\xi(F)}\left\{2D((\widetilde{X}^{14}_{Neu})_{11}) + D((\widetilde{X}^{15}_{Neu})_{11})\right.$$

$$\left.\left.(-2V+F(-3+\lambda_1))(\lambda_1-1)\right\}\zeta(F)\right]$$

Here, $\frac{d^2f(F)}{dF^2} < 0$. Hence, $f(F)$ is a strictly concave function. Again $g(F) = F$ is positive, differentiable and convex function of F on $(0, \infty)$. So, by the theorem 2, $D((\widetilde{\Pi}_{Neu})_{11}(F))$ is a strictly pseudo-concave function of F on $(0, \infty)$.

B Proof of the Theorem 4

For any F, differentiating $D((\widetilde{\Pi}_{Neu})_{11}(c_s)) = D((\widetilde{\Pi}_{Neu})_{11}(F, c_s))$ with respect to c_s, we get

$$\frac{dD((\widetilde{\Pi}_{Neu})_{11}(F, c_s))}{dc_s}$$

$$= e^{V}D((\widetilde{X}^{11}_{Neu})_{11}) + c_s e^{V}D((\widetilde{X}^{11}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu}))$$

$$-\frac{e^{V}}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu}))$$

$$-\frac{e^{V}}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)D((\widetilde{X}^{15}_{Neu})_{11}$$

$$\otimes(-\widetilde{a}_{Neu})) - \frac{i_{IC}e^{V}}{F}\left\{(1-\lambda_1)(F+V)\right.$$

$$\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2F\int_0^F e^{\xi(u)}du + \frac{1}{2}$$

$$\left.\lambda_2(F-U)^2 + \frac{1}{2}(1-\lambda_2)(F+V-U)^2\right\}$$

$$D((\widetilde{X}^{16}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})) + \frac{i_{IE}e^{V}}{2F}\left\{\lambda_2U^2\right.$$

$$\left.+(1-\lambda_2)(U-V)^2\right\}D((\widetilde{X}^{17}_{Neu}).$$

$$\frac{d^2D((\widetilde{\Pi}_{Neu})_{11}(F, c_s))}{dc_s^2}$$

$$= 2e^{V}D((\widetilde{X}^{11}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})) + c_s e^{V}$$

$$D((\widetilde{X}^{11}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu}))-$$

$$\frac{e^{V}}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})$$

$$\otimes(-\widetilde{a}_{Neu})) - \frac{e^{V}}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)$$

$$D((\widetilde{X}^{15}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu})) - \frac{i_{IC}e^{V}}{F}$$

$$\left\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du + \frac{1}{2}(1-\lambda_1)^2F\right.$$

$$\int_0^F e^{\xi(u)}du + \frac{1}{2}\lambda_2(F-U)^2 + \frac{1}{2}(1-\lambda_2)(F$$

$$\left.+V-U)^2\right\}D((\widetilde{X}^{16}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu})).$$

Let

$$\Theta_{11} = -2e^{V}D((\widetilde{X}^{11}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu}))$$

$$+\frac{e^{V}}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})\otimes$$

$$(-\widetilde{a}_{Neu})) + \frac{e^{V}}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)D((\widetilde{X}^{15}_{Neu})_{11}$$

$$\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu}))+\frac{i_{IC}e^{V}}{F}\Big\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_1)^2F\int_0^F e^{\xi(u)}du+\frac{1}{2}\lambda_2(F-U)^2+\frac{1}{2}(1-\lambda_2)(F+V-U)^2\Big\}D((\widetilde{X}^{16}_{Neu})_{11}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu})).$$

So, $\dfrac{d^2D((\widetilde{\Pi}_{Neu})_{11}(F,c_S))}{dc_S^2}<0$,

if $\Theta_{11}>c_Se^V\dfrac{d^2D((\widetilde{X}^{11}_{Neu})_{11})}{dc_S^2}$.

C Proof of Theorem 6

$$\begin{aligned}
&D((\widetilde{\Pi}_{Neu})_{12}(F,c_S))\\
&=c_Se^VD((\widetilde{X}^{11}_{Neu})_{12})-\frac{1}{F}D((\widetilde{X}^{12}_{Neu})_{12})\\
&-\frac{1}{F}D((\widetilde{X}^{13}_{Neu})_{12})-\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{12})\\
&-\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)D((\widetilde{X}^{15}_{Neu})_{12})-\frac{i_{IC}e^V}{F}\\
&\Big\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_1)^2F\\
&\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_2)(F+V-U)^2\Big\}D((\widetilde{X}^{16}_{Neu})_{12})\\
&+\frac{c_Si_{IE}e^V}{2F}\Big\{\lambda_2F^2+2\lambda_2F(U-F)+(1-\lambda_2)(U-\\
&V)^2\Big\}D((\widetilde{X}^{17}_{Neu})_{12})-\frac{\eta}{F}D((\widetilde{X}^{18}_{Neu})_{12}).
\end{aligned}$$

$$\begin{aligned}
&\frac{dD((\widetilde{\Pi}_{Neu})_{12}(F,c_S))}{dc_S}\\
&=e^VD((\widetilde{X}^{11}_{Neu})_{12})+c_Se^VD((\widetilde{X}^{11}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu}))\\
&-\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu}))\\
&-\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)D((\widetilde{X}^{15}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu}))\\
&-\frac{i_{IC}e^V}{F}\Big\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_1)^2F\\
&\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_2)(F+V-U)^2\Big\}\\
&D((\widetilde{X}^{16}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu}))+\frac{i_{IE}e^V}{2F}\Big\{\lambda_2F^2+2\lambda_2\\
&F(U-F)+(1-\lambda_2)(U-V)^2\Big\}D((\widetilde{X}^{17}_{Neu})_{12}).
\end{aligned}$$

$$\begin{aligned}
&\frac{d^2D((\widetilde{\Pi}_{Neu})_{12}(F,c_S))}{dc_S^2}\\
&=2e^VD((\widetilde{X}^{11}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu}))+c_Se^VD((\widetilde{X}^{11}_{Neu})_{12}\\
&(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu}))\\
&-\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{12}\\
&\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu}))\\
&-\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)\\
&D((\widetilde{X}^{15}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu}))\\
&-\frac{i_{IC}e^V}{F}\Big\{(1-\lambda_1)\\
&(F+V)\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_1)^2F\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\\
&\lambda_2)(F+V-U)^2\Big\}D((\widetilde{X}^{16}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu})).
\end{aligned}$$

Let

$$\begin{aligned}
\Theta_{12}=&-2e^VD((\widetilde{X}^{11}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu}))\\
&+\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}du\right)D((\widetilde{X}^{14}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu})\\
&\otimes(-\widetilde{a}_{Neu}))+\frac{e^V}{F}\left(\int_0^F e^{-\xi(t)}\left(\int_t^F e^{\xi(u)}du\right)dt\right)\\
&D((\widetilde{X}^{15}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu}))+\frac{i_{IC}e^V}{F}\\
&\Big\{(1-\lambda_1)(F+V)\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_1)^2F\\
&\int_0^F e^{\xi(u)}du+\frac{1}{2}(1-\lambda_2)(F+V-U)^2\Big\}\\
&D((\widetilde{X}^{16}_{Neu})_{12}\otimes(-\widetilde{a}_{Neu})\otimes(-\widetilde{a}_{Neu})).
\end{aligned}$$

So, $\dfrac{d^2D((\widetilde{\Pi}_{Neu})_{12}(F,c_S))}{dc_S^2}<0$,

if $\Theta_{12}>c_Se^V\dfrac{d^2D((\widetilde{X}^{11}_{Neu})_{12})}{dc_S^2}$.

D Proof of Theorem 8

For any given c_S, to find the F^*_{13}, taking the first-order derivative of $D((\widetilde{\Pi}_{Neu})_{13}(F))=D((\widetilde{\Pi}_{Neu})_{13}(F,c_S))$ with respect to F and equate it to zero, we get

$$\Delta_{13}(F) = \frac{d}{dF} D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$$
$$= \frac{1}{F^2} D((\widetilde{X}^{12}_{Neu})_{13}) + \frac{1}{F^2} D((\widetilde{X}^{13}_{Neu})_{13}) + \frac{e^V}{F^2}$$
$$\left\{ \left(\int_0^F e^{-\xi(t)} du \right) - F e^{\xi(F)} \right\} D((\widetilde{X}^{14}_{Neu})_{13}) + \frac{e^V}{F^2}$$
$$\left\{ \left(\int_0^F e^{-\xi(t)} \left(\int_t^F e^{\xi(u)} du \right) dt \right) - F \left(\int_0^F e^{-\xi(t)+\xi(F)} dt \right) \right\}$$
$$D((\widetilde{X}^{15}_{Neu})_{13}) + \frac{i_{IC} e^V}{F^2} \left\{ (1-\lambda_1)(F+V) \int_0^F e^{\xi(u)} du \right.$$
$$\left. + \frac{1}{2}(1-\lambda_1)^2 F \int_0^F e^{\xi(u)} du \right\} D((\widetilde{X}^{16}_{Neu})_{13}) - \frac{e^V i_{IC}}{F}$$
$$\left\{ \left\{ V + F + \frac{1}{2} F(1-\lambda_1) \right\} (1-\lambda_1) e^{\xi(F)} + \frac{1}{2}(3-\lambda_1) \right.$$
$$\left. (1-\lambda_1) \int_0^F e^{\xi(t)} dt \right\} D((\widetilde{X}^{16}_{Neu})_{13}) - \frac{c_s i_{IE} e^V}{2F^2} \left\{ F^2 + \right.$$
$$\left. 2\lambda_2 F(U-F) + 2(1-\lambda_2) F(U-F-V) \right\}$$
$$D((\widetilde{X}^{17}_{Neu})_{13}) + \frac{e^V}{F} 7(U - F - V(1-\lambda_2))$$
$$D((\widetilde{X}^{17}_{Neu})_{12}) + \frac{\eta}{F^2} D((\widetilde{X}^{18}_{Neu})_{13}.$$

So, $\dfrac{d^2 D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))}{dc_s^2} < 0,$
 if $\Theta_{13} > c_s e^V \dfrac{d^2 D((\widetilde{X}^{11}_{Neu})_{13})}{dc_s^2}.$

E Proof of Theorem 9.

1.

$$\lim_{F \to 0} \Delta(F) = \infty \tag{29}$$

If $\Delta_{13}(U-V) < 0$, then by intermediate value property (IVP) and theorem 7, there exists a unique $F^*_{13} \in (0,\ U - V)$ such that $\Delta(F^*_{13}) = 0$. Hence, $D((\widetilde{\Pi}_{Neu})_{13}\ (F), c_s)$ is maximize at a unique F^*_{13}. Now

$$\Delta_{13}(U-V) = \Delta_{12}(U-V).$$

So, if $\Delta_{13}(U-V) < 0$, then $\Delta_{12}(U-V) < 0$ and by the theorem 4, we get

$$\frac{d}{dF} D((\widetilde{\Pi}_{Neu})_{12}(F, c_s)) < 0, \quad \forall F \in [U-V,\ U].$$

So, for all $F \in [U-V,\ U]$, $D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$ is decreasing and maximize at $U-V$. Again,

$$\Delta_{12}(U) = \Delta_{11}(U).$$

Since $\Delta_{12}(U) < 0$, so $\Delta_{11}(U) < 0$ and by the theorem 3, we have

$$\frac{d}{dF} D((\widetilde{\Pi}_{Neu})_{11}(F, c_s)) < 0, \quad \forall F \in [U,\ \infty].$$

So, for all $F \in [U,\ \infty]$, $D((\widetilde{\Pi}_{Neu})_{11}(F, c_s))$ is decreasing and maximize at U. So if $\Delta_{13}(U-V) < 0$, then

$$\begin{aligned} D((\widetilde{\Pi}_{Neu})_{13}(F^*_{13}, c_s) &\geq D((\widetilde{\Pi}_{Neu})_{13}(U-V, c_s) \\ &= D((\widetilde{\Pi}_{Neu})_{12}(U-V, c_s) \\ &> D((\widetilde{\Pi}_{Neu})_{12}(F, c_s) \\ &\geq D((\widetilde{\Pi}_{Neu})_{12}(U, c_s), \\ &\qquad \forall F \in [U-V, U]. \end{aligned}$$

Again,

$$\begin{aligned} D((\widetilde{\Pi}_{Neu})_{12}(U, c_s) &= D((\widetilde{\Pi}_{Neu})_{11}(U, c_s) \\ &\geq D((\widetilde{\Pi}_{Neu})_{11}(F, c_s) \ \forall F \in [U, \infty]. \end{aligned}$$

Hence, the for $U \leq V$ and for any c_s, if $\Delta_{13}(U-V) < 0$, then $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s)$ is maximize at F^*_{13}.

2. If $\Delta_{13}(U-V) = 0$, then from Eq. (29) and Theorem 7, we conclude that $U - V \in (0, U-V]$ is the unique point such that $\Delta_{13}(U-V) = 0$. Hence, $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s)$ is maximized at $U-V$. Again,

$$\Delta_{13}(U-V) = \Delta_{12}(U-V) = 0.$$

From Theorem5, we have $U - V \in [U-V, U]$ is the unique point such that

$$\frac{d}{dF} D((\widetilde{\Pi}_{Neu})_{12}(U-V, c_s)) = 0.$$

Hence, $D((\widetilde{\Pi}_{Neu})_{12}(F, c_s)$ is maximized at $U-V$. Further, we can write

$$\frac{d}{dF} D((\widetilde{\Pi}_{Neu})_{12}(U-V, c_s)) < 0, \quad \forall F \in (U-V, U].$$

So,

$$\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{12}(U, c_s)) = \frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{11}(U, c_s)) < 0.$$

Hence, by Theorem 3, we have

$$\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{11}(F, c_s)) < 0\, \forall F \in [U, \infty).$$

Thus, $D((\widetilde{\Pi}_{Neu})_{11}(F, c_s)$ is strictly decreasing and maximize at U. Therefore,

$$\begin{aligned} D((\widetilde{\Pi}_{Neu})_{13}(U - V, c_s) &= D((\widetilde{\Pi}_{Neu})_{12}(U - V, c_s) \\ &\geq D((\widetilde{\Pi}_{Neu})_{12}(U, c_s) \\ &= D((\widetilde{\Pi}_{Neu})_{11}(U, c_s). \end{aligned}$$

Hence, if $\Delta_{13}(U - V) = 0$, then for $U \leq V$ De-neutrosophic value of retailer optimal profit is maximized at $U - V$.

3. If $\Delta_{13}(U - V) > 0$, then by Theorem 7 and Eq. (29), we can write

$$\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{13}(F, c_s)) > 0,\ \forall F \in (0, U - V].$$

Hence, $D((\widetilde{\Pi}_{Neu})_{13}(F, c_s))$ is monotonically strictly increasing function and is maximized at $U - V$. Again,

$$\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{13}(U - V, c_s))$$

$$= \frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{12}(U - V, c_s)) > 0.$$

Now

(a) if $\Delta_{12}(U) < 0$, then by Theorem 4, there exits a unique point $F^*_{12} \in (U - V, U)$ such that $\Delta_{12}(F^*_{12}) = 0$. Hence, $D((\widetilde{\Pi}_{Neu})_{12}(F, c_s))$ is maximize at F^*_{12}. Also,

$$\begin{aligned} &\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{12}(U, c_s)) \\ &\quad = \frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{11}(U, c_s)) < 0. \end{aligned}$$

By Theorem 3, we have

$$\frac{d}{dF}D((\widetilde{\Pi}_{Neu})_{11}(F, c_s)) < 0,\ \forall F \in [U, \infty].$$

Hence, $D((\widetilde{\Pi}_{Neu})_{11}(F, c_s))$ is strictly decreasing and is maximize at U. Now

$$\begin{aligned} D((\widetilde{\Pi}_{Neu})_{12}(F^*_{12}, c_s)) &\geq D((\widetilde{\Pi}_{Neu})_{12}(U - V, c_s)) \\ &= D((\widetilde{\Pi}_{Neu})_{13}(U - V, c_s)) \end{aligned} \tag{30}$$

and

$$\begin{aligned} D((\widetilde{\Pi}_{Neu})_{12}(F^*_{12}, c_s)) &\geq D((\widetilde{\Pi}_{Neu})_{12}(U, c_s)) \\ &= D((\widetilde{\Pi}_{Neu})_{11}(U, c_s)). \end{aligned} \tag{31}$$

Thus, from Eq.(30) and Eq.(31), we conclude that De-neutrosophic value of retailer optimal profit is maximized at F^*_{12}.

(b) The proof is similar to 3a.

(c) The proof is similar to 3a.

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