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FUZZY SYSTEMS AND THEIR MATHEMATICS

An introduction to single-valued neutrosophic soft topological structure

Yaser Saber[1,2] · Fahad Alsharari[3] · Florentin Smarandache[4]

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Abstract

Fuzzy soft set theory presented by Maji et al. (J Fuzzy Math 9(3):589–602, 2001) and soft set theory presented by Molodtsov (Comput Math Appl 37(3):19–31, 1999) are important ideas in decision-making problems. They can be used to model uncertainty and make decisions under uncertainty. A single-valued neutrosophic soft set (svnf-set) is a hybrid model of a single-valued neutrosophic set and fuzzy soft set that is shown in this paper. The novel concept of single-valued neutrosophic soft topology (svnft) is defined to discuss topological structure of (svnf-set). Some fundamental properties of svnft and their related results are studied. It is good to use the proposed models of svnf-sets and svnft to figure out how to deal with uncertainty in real life. Thus, svnft is a generalization of fuzzy soft topology and fuzzy intuitionistic soft topology. Moreover, after giving the definition of a single-valued neutrosophic soft base svnf-base, we also added the concept of svnft. Finally, we set up the concept of single-valued neutrosophic soft closure spaces and show that the initial single-valued neutrosophic soft closure structures are real, which is what we did. From this fact, the category **SVNSC** is considered as a topological category over **SET**.

Keywords Single-valued neutrosophic soft set · Single-valued neutrosophic soft topology · Single-valued neutrosophic soft base · Initial single-valued neutrosophic soft topology · Product single-valued neutrosophic soft topology.

1 Introduction

The majority of extant mathematical tools for formal modeling, computing, and reasoning are deterministic, exact, and crisp in nature. In reality, however, challenges in economics, biology, social science, environment science, engineering, and engineering, among other fields, do not necessarily entail precise data. The source of these challenges may be the general insufficiency of the conventional parameterization tool. As a result, Molodtsov (1999) pioneered the idea of soft set theory as a new mathematical technique for dealing with ambiguity and uncertainty that is free of the aforementioned issues. Molodtsov (2001) effectively adapted soft set theory to a variety of fields, including function smoothness, measurement theory, Perron integration, Riemann integration and game theory. Maji et al. (2002) demonstrated how soft sets can be used in real-world decision-making issues. Furthermore, Riaz et al. (2019) introduced multi-criteria group decision-making techniques by means of N-soft set and N-soft topology. The main idea of soft multi-rough set is presented as a hybrid model of rough set, multi-set and soft set, which was defined by Riaz et al. (2021). They have familiarized as well the idea of the fuzzy soft set, a more widespread notion that is a mixture of fuzzy set and soft set, and they secured quite a lot of its attributes. The main idea of fuzzy soft mappings was defined by Ahmed and Kharal (2009). Furthermore, Shabir and Naz (2011) introduced soft topological spaces and initiated some ideas contingent on

✉ Fahad Alsharari
f.alsharari@mu.edu.sa; f.alsharari@ju.edu.sa

Yaser Saber
y.sber@mu.edu.sa

Florentin Smarandache
smarand@unm.edu

1 Department of Mathematics, College of Science and Human Studies, Hotat Sudair, Majmaah University, Majmaah 11952, Saudi Arabia

2 Department of Mathematics, Faculty of Science Al-Azhar University, Assiut 71524, Egypt

3 Department of Mathematics, College of Science and Arts, Jouf University, Gurayat 77455, Saudi Arabia

4 Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA

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soft sets. Tanay and Kandemir (2011) originally presented the idea of fuzzy soft topological space (FSTS) by means of fuzzy soft sets and premeditated the basic concepts by next Chang's fuzzy topology idea Chang (1968). The main idea of fuzzy soft topology in sense of Lowen was defined by Varol and Aygün (2012)) and fuzzy soft topology (FST) in Šostak's sense presented by Aygünoğlu et al. (2014)). Şenel G. (2016, 2017) made a wide research on soft sets and its applications in Şenel (2016, 2017). Numerous applications can be found in works of Abbas et al. (2018), Tripathy and Acharjee (2017), Riaz et al. (2019), Riaz et al. (2019), Maji et al. (2001), Zhang et al. (2021), Nawar et al. (2021), Atef et al. 2021, Atef and Nada (2021) and Feng et al. (2010, 2011).

Smarandache (2007) presented the idea of a neutrusophic set as an intuitionistic fuzzy set generalization. Salama and Alblowi (2012) defined the neutrosophic set theory and neutrosophic crisp set. Correspondingly, Salama and Smarandache (2015) introduced neutrosophic topology as they claimed a number of its characteristics. Others as Wang et al. (2010) defined the single-valued neutrosophic set concept. Alsharari et al. (2021), Saber and Abdel-Sattar (2014), Saber and Alsharari (2018), Saber and Alsharari (2020), Saber et al. (2020), Saber et al. (2020) and Saber et al. (2022) introduced and studied the concepts of single-valued neutrosophic ideal, single-valued neutrosophic ideal open local function, connectedness in single-valued neutrosophic topological spaces, and compactness in single-valued neutrosophic ideal topological spaces.

Thus, the single-valued neutrosophic soft set is a power general formal framework, which generalizes the notion of the classic soft set, fuzzy soft set, interval-valued fuzzy soft set, intuitionistic fuzzy soft set, and interval intuitionistic fuzzy soft set from a philosophical point of view. The application aspects of these types of sets can be further noted. Moreover, it can also be applied to control engineering in average consensus in multi-agent systems with uncertain topologies, multiple time-varying delays, and emergence in random noisy environments (see Shang 2014).

In this work, a general introduction together with a complete survey about the topic is given in the first section. The authors initiate the topological construction of single-valued neutrosophic soft set theory in the second section. In the third section, we present the concept of single-valued neutrosophic soft topology $(\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$, which is a mapping from $\mathcal{E}$ into $\xi^{\widehat{(£,\mathcal{E})}}$ that satisfies the three specified conditions. With respect to this concept, the single-valued neutrosophic soft topology $(\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$ is a single-valued neutrosophic soft set (svnf) on a family of single-valued neutrosophic soft sets $\widetilde{(£, \mathcal{E})}$. Also, since the value of single-valued neutrosophic soft set f_ℓ under the maps $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}$ gives the degree of openness, the degree of indeterminacy, and the degree of non-openness, respectively, of the single-valued neutrosophic soft set with respect to the parameter $\tilde{e} \in \mathcal{E}$, $(\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\delta}}_{\tilde{e}})$, which could be thought of as a single-valued neutrosophic soft topology in the sense of Šostak. In this way, we present the single-valued neutrosophic soft cotopology and offer the significant relations between single-valued neutrosophic soft topology $(\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$ and single-valued neutrosophic soft cotopology $(\tilde{\hbar}^{\tilde{\sigma}}, \tilde{\hbar}^{\tilde{\varsigma}}, \tilde{\hbar}^{\tilde{\delta}})$. We also define the single-valued neutrosophic soft base $(Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}})$. Furthermore, we conclude the notion of single-valued neutrosophic soft topology by using the single-valued neutrosophic soft base on the same set. In addition, we demonstrate the notions of single-valued neutrosophic soft closure spaces (svnf-closure space) in the forth section. We show the existence of initial single-valued neutrosophic soft closure structures. Basing on this premise, the category **SVNSC** is a topological category over **SET**. In particular, an initial structure of single-valued neutrosophic soft topological spaces $(£, \tilde{\top}^{\tilde{\sigma}}_{\mathcal{E}}, \tilde{\top}^{\tilde{\varsigma}}_{\mathcal{E}}, \tilde{\top}^{\tilde{\delta}}_{\mathcal{E}})$ could be obtained by the initial structure of svnf-closure space.

Throughout this work, £ denotes an initial universe, $\xi^£$ is the collection of all single-valued neutrosophic sets (simply, svns) on £ (where $\xi = [0, 1]$, $\xi_0 = (0, 1]$ and $\xi_1 = [0, 1)$) and $\mathcal{E}$ is the set of each parameter on £.

Definition 1 ((Smarandache 2007)) Let £ be a universe set. A neutrosophic set (simply, ns) π of £ was defined as

$$\pi = \{\langle x, \tilde{\sigma}_\pi(x), \tilde{\varsigma}_\pi(x), \tilde{\delta}_\pi(x) \mid x \in £, \tilde{\sigma}_\pi(x), \tilde{\varsigma}_\pi(x), \tilde{\delta}_\pi(x) \in \rfloor^-0, 1^+\lfloor\},$$

where $\tilde{\sigma}_\pi(x)$, $\tilde{\varsigma}_\pi(x)$ and $\tilde{\delta}_\pi(x)$ are the truth, the indeterminacy, and the falsity membership functions, respectively.

Definition 2 [(Wang et al. 2010)] Let £ be a non-null set. The single-valued neutrosophic set (simply, svn-set) π of £ is defined as

$$\pi = \{\langle x, \tilde{\sigma}_\pi(x), \tilde{\varsigma}_\pi(x), \tilde{\delta}_\pi(x) \mid x \in £\}$$

where $\tilde{\sigma}_\pi(x)$, $\tilde{\varsigma}_\pi(x)$ and $\tilde{\delta}_\pi(x) \in [0, 1]$ for every $x \in £$ and

$$0 \le \tilde{\sigma}_\pi(x) + \tilde{\varsigma}_\pi(x) + \tilde{\delta}_\pi(x) \le 3$$

Definition 3 The ideas of intersection, union and inclusion have been defined on svn-sets as follows.

Consider π_1, π_2 to be a svn-sets in £:

(1) Intersection (Yang et al. 2016) of two sets denoted by π_3 is written as:

$$\pi_3 = \pi_1 \cap \pi_2$$
$$\tilde{\sigma}_{\pi_3}(x) = min\{\tilde{\sigma}_{\pi_1}(x), \tilde{\sigma}_{\pi_2}(x)\},$$

$$\tilde{\varsigma}_{\pi_3}(x) = min\{\tilde{\varsigma}_{\pi_1}(x), \tilde{\varsigma}_{\pi_2}(x)\},$$
$$\tilde{\delta}_{\pi_3}(x) = min\{\tilde{\delta}_{\pi_1}(x), \tilde{\delta}_{\pi_2}(x)\}$$

(2) Union (Yang et al. 2016) of two sets denoted by π_3 is defined as:

$$\pi_3 = \pi_1 \cup \pi_2$$
$$\tilde{\sigma}_{\pi_3}(x) = max\{\tilde{\sigma}_{\pi_1}(x), \tilde{\sigma}_{\pi_2}(x)\},$$
$$\tilde{\varsigma}_{\pi_3}(x) = max\{\tilde{\varsigma}_{\pi_1}(x), \tilde{\varsigma}_{\pi_2}(x)\},$$
$$\tilde{\delta}_{\pi_3}(x) = max\{\tilde{\delta}_{\pi_1}(x), \tilde{\delta}_{\pi_2}(x)\}$$

(3) Inclusion (Ye 2014) of two sets ($\pi_1 \subseteq \pi_2$) is defined as:

$$\tilde{\sigma}_{\pi_1}(x) \leq \tilde{\sigma}_{\pi_2}(x), \quad \tilde{\varsigma}_{\pi_1}(x) \geq \tilde{\varsigma}_{\pi_2}(x), \quad \tilde{\delta}_{\pi_1}(x) \geq \tilde{\delta}_{\pi_2}(x)$$

(4) The complemented (Wang et al. 2010) of the set π denoted by π^c is defined as:

$$\tilde{\sigma}_{\pi^c}(x) = \tilde{\delta}_{\pi}(x),$$
$$\tilde{\varsigma}_{\pi^c}(x) = 1 - \tilde{\varsigma}_{\pi}(x),$$
$$\tilde{\delta}_{\pi^c}(x) = \tilde{\sigma}_{\pi}(x).$$

Definition 4 (Atef and Nada 2021) Let £ be the universal set and $\mathcal{E}$ be a set of attributes. We consider the non-null set $A \subseteq \mathcal{E}$. Assume that $\widetilde{P(£)}$ is referred to the set of all fuzzy neutrosophic sets (simply, fn-sets) of £. Then, the aggregation (f_A) is termed to be the fuzzy neutrosophic soft set (simply, fnf-set) on £ where $(f_A) : A \to \widetilde{P(£)}$

2 Single-valued neutrosophic soft sets

In this part, fundamental concepts and notions are introduced.

f_ℓ is a single-valued neutrosophic soft set (simply, svnfs) on £ where $f : \mathcal{E} \to \xi^{£}$, i.e., $f_{\tilde{e}} \triangleq f(\tilde{e})$ is a svns on £, for all $\tilde{e} \in \ell$ and $f(\tilde{e}) = \langle 0, 1, 1\rangle$, if $\tilde{e} \notin \ell$.

The svns $f(\tilde{e})$ is termed as an element of the svnfs f_ℓ. Thus, a svnfs $f_{\mathcal{E}}$ on £ can be defined as:

$$(f, \mathcal{E}) = \left\{(\tilde{e}, f(\tilde{e})) \mid \tilde{e} \in \mathcal{E}, f(\tilde{e}) \in \xi^{£}\right\}$$
$$= \left\{(\tilde{e}, \langle\tilde{\sigma}_f(\tilde{e}), \tilde{\varsigma}_f(\tilde{e}), \tilde{\delta}_f(\tilde{e})\rangle) \mid \tilde{e} \in \mathcal{E}, f(\tilde{e}) \in \xi^{£}\right\},$$

where $\tilde{\sigma}_f : \mathcal{E} \to \xi$ ($\tilde{\sigma}_f$ is termed as a membership function), $\tilde{\varsigma}_f : \mathcal{E} \to \xi$ ($\tilde{\varsigma}_f$ is termed as indeterminacy function), and $\tilde{\delta}_f : \mathcal{E} \to \xi$ ($\tilde{\delta}_f$ is termed as a non-membership function) of svnf set. $\widetilde{(£, \mathcal{E})}$ refers to the collection of all svnfss on £ and is termed svnfs-universe.

A svnfs $f_{\mathcal{E}}$ on £ is termed as a null svnfs (simply, Φ), if $\tilde{\sigma}_f(\tilde{e}) = 0$, $\tilde{\varsigma}_f(\tilde{e}) = 1$ and $\tilde{\delta}_f(\tilde{e}) = 1$, for any $\tilde{e} \in \mathcal{E}$.

A svnf set $f_{\mathcal{E}}$ on £ is termed as an absolute svnf set (simply, $\tilde{\mathcal{E}}$), if $\tilde{\sigma}_f(\tilde{e}) = 1$, $\tilde{\varsigma}_f(\tilde{e}) = 0$ and $\tilde{\delta}_f(\tilde{e}) = 0$, for any $\tilde{e} \in \mathcal{E}$.

A svnfs $f_{\mathcal{E}}$ on £ is termed as an t-absolute svnf set (simply, $\tilde{\mathcal{E}}^t$), if $\tilde{\sigma}_f(\tilde{e}) = t$, $\tilde{\varsigma}_f(\tilde{e}) = 0$ and $\tilde{\delta}_f(\tilde{e}) = 0$, for any $\tilde{e} \in \mathcal{E}$ and $t \in \xi$.

Definition 5 Let $f_\ell, g_\jmath$ be svnf sets over £. Then, the union of svnf sets $f_\ell, g_\jmath$ is a svnf set h_∂, where $\partial = \ell \cup \jmath$ and for any $\tilde{e} \in \partial$ and $\tilde{\sigma}_h : \mathcal{E} \to \xi$ ($\tilde{\sigma}_h$ called truth-membership) $\tilde{\varsigma}_h : \mathcal{E} \to \xi$ ($\tilde{\varsigma}_h$ called indeterminacy), $\tilde{\delta}_h : \mathcal{E} \to \xi$ ($\tilde{\delta}_h$ called falsity membership) of h_∂ are as next:

$$\tilde{\sigma}_{h(\tilde{e})}(\varpi) = \begin{cases} \tilde{\sigma}_{f(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell - \jmath, \\ \tilde{\sigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \jmath - \ell, \\ \tilde{\sigma}_{f(\tilde{e})}(\varpi) \cup \tilde{\sigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell \cup \jmath. \end{cases}$$

$$\tilde{\varsigma}_{h(\tilde{e})}(\varpi) = \begin{cases} \tilde{\varsigma}_{f(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell - \jmath, \\ \tilde{\varsigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \jmath - \ell, \\ \tilde{\varsigma}_{f(\tilde{e})}(\varpi) \cap \tilde{\varsigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell \cap \jmath. \end{cases}$$

$$\tilde{\delta}_{h(\tilde{e})}(\varpi) = \begin{cases} \tilde{\delta}_{f(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell - \jmath, \\ \tilde{\delta}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \jmath - \ell, \\ \tilde{\delta}_{f(\tilde{e})}(\varpi) \cap \tilde{\delta}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell \cap \jmath. \end{cases}$$

Definition 6 The intersection of svnf sets $f_\ell, g_\jmath$ is a svnf set h_∂, where $\partial = \ell \cap \jmath$ and for any $\tilde{e} \in \partial$, $h_{\tilde{e}} = f_{\tilde{e}} \widetilde{\cap} g_{\tilde{e}}$. We write as next:

$$\tilde{\sigma}_{h(\tilde{e})}(\varpi) = \begin{cases} \tilde{\sigma}_{f(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell - \jmath, \\ \tilde{\sigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \jmath - \ell, \\ \tilde{\sigma}_{f(\tilde{e})}(\varpi) \cap \tilde{\sigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell \cap \jmath. \end{cases}$$

$$\tilde{\varsigma}_{h(\tilde{e})}(\varpi) = \begin{cases} \tilde{\varsigma}_{f(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell - \jmath, \\ \tilde{\varsigma}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \jmath - \ell, \\ \tilde{\varsigma}_{f(\tilde{e})}(\varpi) \cup \tilde{\varsigma}_{g(\tilde{e})}(\kappa), & \text{if } \tilde{e} \in \ell \cap \jmath. \end{cases}$$

$$\tilde{\delta}_{h(\tilde{e})}(\varpi) = \begin{cases} \tilde{\delta}_{f(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell - \jmath, \\ \tilde{\delta}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \jmath - \ell, \\ \tilde{\delta}_{f(\tilde{e})}(\varpi) \cup \tilde{\delta}_{g(\tilde{e})}(\varpi), & \text{if } \tilde{e} \in \ell \cap \jmath. \end{cases}$$

Definition 7 Consider f_ℓ and $g_\jmath$ to be a svn-sets in £:

(1) Inclusion of two sets (simply, $f_\ell \widetilde{\subseteq} g_\jmath$) defined as:

$$\tilde{\sigma}_f(\tilde{e}) \leq \tilde{\sigma}_f(\tilde{e}), \quad \tilde{\varsigma}_f(\tilde{e}) \geq \tilde{\varsigma}_f(\tilde{e}), \quad \tilde{\delta}_f(\tilde{e}) \geq \tilde{\delta}_f(\tilde{e}).$$

(2) The complemented of the set f_ℓ denoted by (simply, f_ℓ^c) is defined as:

$$f_\ell^c = \left\{\left(\tilde{e}, \langle\tilde{\delta}_f(\tilde{e}), \tilde{\varsigma}_{f^c}(\tilde{e}), \tilde{\sigma}_f(\tilde{e})\rangle\right) \mid \tilde{e} \in \mathcal{E}\right\}.$$

Theorem 1 *Let $f_\ell, g_\jmath, h_\partial \in \widetilde{(£, \mathcal{E})}$ and $(f_\ell)_j \cong (f_i)_\ell$, $(g_\ell)_j \cong (g_i)_\ell \in \widetilde{(£, \mathcal{E})}$ [$i \in \triangle$, where $\triangle$ is termed to be the index set]. Then,*

(1) $f_\ell \widetilde{\cap} g_\jmath = f_\ell \widetilde{\cap} g_\jmath$ and $f_\ell \widetilde{\cup} f_\ell = f_\ell \widetilde{\cup} g_\jmath$.
(2) $f_\ell \widetilde{\cup}(g_\jmath \widetilde{\cup} h_\partial) = (f_\ell \widetilde{\cup} g_\jmath)\widetilde{\cap} h_\partial$ and $f_\ell \widetilde{\cap}(g_\jmath \widetilde{\cap} h_\partial) = (f_\ell \widetilde{\cap} g_\jmath)\widetilde{\cap} h_\partial$.
(3) $f_\ell \widetilde{\cup}(\widetilde{\bigcap}_{i\in\triangle}[g_\jmath]_i) = \widetilde{\bigcap}_{i\in\triangle}(f_\ell \widetilde{\cup} g_\jmath)$.
(4) $f_\ell \widetilde{\cap}(\widetilde{\bigcup}_{i\in\triangle}[g_\jmath]_i) = \widetilde{\bigcup}_{i\in\triangle}(f_\ell \widetilde{\cap} g_\jmath)$.
(5) $[f_\ell^c]^c = f_\ell$.
(6) If $f_\ell \widetilde{\sqsubseteq} g_\jmath$, then $f_\ell^c \widetilde{\sqsubseteq} g_\jmath^c$.
(7) $f_\ell \widetilde{\cap} f_\ell = f_\ell$ and $f_\ell \widetilde{\cup} f_\ell = f_\ell$.
(8) $\Phi \leq f_\ell \widetilde{\sqsubseteq} \widetilde{\mathcal{E}}$.
(9) $(\widetilde{\bigcup}_{i\in\triangle}[f_\ell]_i)^c = \widetilde{\bigcap}_{i\in\triangle}[f_\ell]_i^c$

Proof It is clear. □

Definition 8 A map $\vartheta_\varphi : \widetilde{(£, \mathcal{E})} \to \widetilde{(\mathcal{G}, \mathcal{Y})}$ is termed as a single-valued neutrosophic soft mapping ($simply$, svnf $-$ map), where $\vartheta : £ \to \mathcal{G}$ and $\varphi : \mathcal{E} \to \mathcal{Y}$ are mappings, with $\mathcal{E}, \mathcal{Y}$ are parameter sets for £, $\mathcal{G}$, respectively.

Definition 9 Assume that f_ℓ and $g_\jmath$ are two svnf sets on £ and $\mathcal{G}$ and $\vartheta_\varphi : \widetilde{(£, \mathcal{E})} \to \widetilde{(\mathcal{G}, \mathcal{Y})}$ is an svnf $-$ map. Then,

(1) The image of f_ℓ under the svnf $-$ map ϑ_φ, referred by $\vartheta_\varphi(f_\ell)$, is the svnf set on $\mathcal{G}$ defined by $\vartheta_\varphi(f_\ell) = \vartheta(f)_{\varphi(\ell)}$, where

$$\vartheta(f)_\pi(\omega) = \begin{cases} \bigvee_{\upsilon\in\vartheta^{-1}(\omega)} \left(\bigvee_{\varepsilon\in\varphi^{-1}(\pi)\wedge\ell} f_\varepsilon(\upsilon) \right), & \text{if } \vartheta^{-1}(\omega) \neq \phi,\ \varphi^{-1}(\pi)\wedge\ell \neq \phi, \\ 0, & \text{otherwise}, \end{cases}$$

for all $\pi \in \mathcal{Y}$ and $\omega \in \mathcal{G}$.

(2) The pre-image of $g_\jmath$ under the svnf $-$ map ϑ_φ, referred by $\vartheta_\varphi^{-1}(g_\jmath)$, is the svnf set on £ defined by $\vartheta_\varphi^{-1}(g_\jmath) = \vartheta^{-1}(f)_{\varphi^{-1}(\jmath)}$, where

$$\vartheta^{-1}(g)_\varepsilon(\upsilon) = \begin{cases} g_{\vartheta(\varepsilon)}(\varphi(\upsilon)), & \text{if } \varphi^{-1}(\varepsilon) \in \jmath \quad \forall \quad \varepsilon \in \mathcal{E} \text{ and } \upsilon \in £, \\ 0, & \text{otherwise}, \end{cases}$$

Example 1 Assume that, $£ = \{x_1, x_2, x_3\}$, $\mathcal{G} = \{y_1, y_2, y_3\}$, $\mathcal{E} = \{\kappa_1, \kappa_2, \kappa_3\}$, $\mathcal{Y} = \{\kappa_1', \kappa_2', \kappa_3'\}$ and $\widetilde{(£, \mathcal{E})}$, $\widetilde{(\mathcal{G}, \mathcal{Y})}$ are svnf classes. Define $\varphi : \mathcal{E} \to \mathcal{Y}$ and $\vartheta : £ \to \mathcal{G}$ as next:

$$\vartheta(x_1) = y_1, \quad \vartheta(x_2) = y_3, \quad \vartheta(x_3) = y_2,$$
$$\varphi(\kappa_1) = \kappa_2', \quad \varphi(\kappa_2) = \kappa_1', \quad \varphi(\kappa_3) = \kappa_1'.$$

Let f_ℓ and $g_\jmath$ be two svnf sets in $\widetilde{(£, \mathcal{E})}$ and $\widetilde{(\mathcal{G}, \mathcal{Y})}$, such that

$$\begin{aligned} f_\ell = \{&\kappa_1, \{\langle x_1 0.4, 0.3, 0.6\rangle, \langle x_2, 0.3, 0.6, 0.4\rangle, \langle x_3, 0.3, 0.5, 0.5\rangle\}, \\ &\kappa_3, \{\langle x_1, 0.3, 0.3, 0.2\rangle, \langle x_2, 0.5, 0.4, 0.4\rangle, \langle x_3, 0.6, 0.4, 0.3\rangle\}, \\ &\kappa_2, \{\langle x_1, 0.5, 0.6, 0.3\rangle, \langle x_2, 0.5, 0.3, 0.6\rangle, \langle x_3, 0.6, 0.4, 0.7\rangle\}\}. \\ g_\jmath = \{&\kappa_1', \{\langle y_1, 0.3, 0.2, 0.1\rangle, \langle y_1, 0.5, 0.6, 0.4\rangle, \langle y_1, 0.3, 0.5, 0.1\rangle\}, \\ &\kappa_1', \{\langle y_2, 0.5, 0.7, 0.4\rangle, \langle y_2, 0.5, 0.2, 0.3\rangle, \langle y_2, 0.6, 0.5, 0.1\rangle\}, \\ &\kappa_2', \{\langle y_3, 0.3, 0.2, 0.4\rangle, \langle y_3, 0.1, 0.7, 0.5\rangle, \langle y_3, 0.1, 0.4, 0.2\rangle\}\}. \end{aligned}$$

Then, the svnf image of f_ℓ under $\vartheta_\varphi : \widehat{(£, \mathcal{E})} \to \widehat{(\mathcal{G}, \mathcal{Y})}$ is gotten as:

$$\begin{aligned} \vartheta(f)_{\kappa_1'}(y_1) &= \bigvee_{x\in\vartheta^{-1}(y_1)} \left(\bigvee_{\varepsilon\in\varphi^{-1}(\kappa_1')\wedge\ell} f_\varepsilon(x) \right) \\ &= \bigvee_{x\in\{x_1\}} \left(\bigvee_{\varepsilon\in\{\kappa_2',\kappa_3'\}} f_\varepsilon(x) \right) \\ &= \langle 0.5, 0.6, 0.3\rangle \vee \langle 0.3, 0.3, 0.2\rangle \\ &= \langle 0.5, 0.45, 0.2\rangle, \\ \vartheta(f)_{\kappa_1'}(y_2) &= \bigvee_{x\in\vartheta^{-1}(y_2)} \left(\bigvee_{\varepsilon\in\varphi^{-1}(\kappa_1')\wedge\ell} f_\varepsilon(x) \right) \\ &= \bigvee_{x\in\{x_3\}} \left(\bigvee_{\varepsilon\in\{\kappa_2',\kappa_3'\}} f_\varepsilon(x) \right) \\ &= \langle 0.6, 0.4, 0.7\rangle \vee \langle 0.6, 0.4, 0.3\rangle \\ &= \langle 0.6, 0.4, 0.3\rangle, \\ \vartheta(f)_{\kappa_1'}(y_3) &= \bigvee_{x\in\vartheta^{-1}(y_3)} \left(\bigvee_{\varepsilon\in\varphi^{-1}(\kappa_1')\wedge\ell} f_\varepsilon(x) \right) \\ &= \bigvee_{x\in\{x_2\}} \left(\bigvee_{\varepsilon\in\{\kappa_2',\kappa_3'\}} f_\varepsilon(x) \right) \\ &= \langle 0.5, 0.3, 0.6\rangle \vee \langle 0.5, 0.4, 0.4\rangle \\ &= \langle 0.5, 0.35, 0.4\rangle, \end{aligned}$$

By similar calculations, consequently, we get

$$\begin{aligned} &\vartheta_\varphi(f_\ell) = \{\kappa_1', \{\langle y_1, 0.5, 0.45, 0.2\rangle, \\ &\langle y_2, 0.6, 0.4, 0.3\rangle, \langle y_3, 0.5, 0.35, 0.4\rangle\}, \\ &\kappa_2', \{\langle y_1, 0.4, 0.3, 0.6\rangle, \langle y_2, 0.3, 0.5, 0.5\rangle, \langle y_3, 0.3, 0.6, 0.4\rangle\}, \\ &\kappa_3', \{\langle y_1, 0.3, 0.3, 0.2\rangle, \langle y_2, 0.6, 0.4, 0.3\rangle, \langle y_3, 0.5, 0.4, 0.4\rangle\}\}. \end{aligned}$$

By similar calculations, we have

$$\begin{aligned} &\vartheta_\varphi(f_\ell) = \{\kappa_1', \{\langle y_1, 0.5, 0.45, 0.2\rangle, \\ &\langle y_2, 0.6, 0.4, 0.3\rangle, \langle y_3, 0.5, 0.35, 0.4\rangle\}, \\ &\kappa_2', \{\langle y_1, 0.4, 0.3, 0.6\rangle, \langle y_2, 0.3, 0.5, 0.5\rangle, \langle y_3, 0.3, 0.6, 0.4\rangle\}, \\ &\kappa_3', \{\langle y_1, 0.3, 0.3, 0.2\rangle, \langle y_2, 0.6, 0.4, 0.3\rangle, \langle y_3, 0.5, 0.4, 0.4\rangle\}\}. \end{aligned}$$

Definition 10 Let $\vartheta_\varphi : \widetilde{(\pounds, \mathcal{E})} \rightarrow \widetilde{(\mathcal{G}, \mathcal{Y})}$ be a map, $f_\ell \in \widetilde{(\pounds, \mathcal{E})}$ and $f_\jmath \in \widetilde{(\mathcal{G}, \mathcal{Y})}$. Then, $\forall\ \pi \in \mathcal{Y}$, $\omega \in \mathcal{G}$, the svnf union and intersection of svnf $-$ *images* $\vartheta_\varphi(f_\ell)$ and $\vartheta_\varphi(g_\jmath)$ in $\widetilde{(\mathcal{G}, \mathcal{Y})}$ are defined as:

$$\left(\vartheta(f\widetilde{\cap}g)\right)_\pi(\omega) = \vartheta(f)_\pi(\omega) \cap \vartheta(g)_\pi(\omega),$$
$$\left(\vartheta(f\widetilde{\cup}g)\right)_\pi(\omega) = \vartheta(f)_\pi(\omega) \cup \vartheta(g)_\pi(\omega)$$

Definition 11 Let $\vartheta_\varphi : \widetilde{(\pounds, \mathcal{E})} \rightarrow \widetilde{(\mathcal{G}, \mathcal{Y})}$ be a map, $f_\ell \in \widetilde{(\pounds, \mathcal{E})}$ and $f_\jmath \in \widetilde{(\mathcal{G}, \mathcal{Y})}$. Then $\forall\ \varepsilon \in \mathcal{E}$, $\upsilon \in \pounds$, the svnf union and intersection of svnf $-$ *inverse images* $\vartheta_\varphi^{-1}(f_\ell)$ and $\vartheta_\varphi^{-1}(g_\jmath)$ in $\widetilde{(\pounds, \mathcal{E})}$ are defined as:

$$\left(\vartheta^{-1}(f\widetilde{\cap}g)\right)_\varepsilon(\upsilon) = \vartheta^{-1}(f)_\varepsilon(\upsilon) \cap \vartheta^{-1}(g)_\varepsilon(\upsilon),$$
$$\left(\vartheta^{-1}(f\widetilde{\cup}g)\right)_\varepsilon(\upsilon) = \vartheta^{-1}(f)_\varepsilon(\upsilon) \cup \vartheta^{-1}(g)_\varepsilon(\upsilon)$$

Theorem 2 *Consider $\vartheta_\varphi : \widetilde{(\pounds, \mathcal{E})} \rightarrow \widetilde{(\mathcal{G}, \mathcal{Y})}$, $\vartheta : \pounds \rightarrow \mathcal{G}$ and $\varphi : \mathcal{E} \rightarrow \mathcal{Y}$ to be a mapping. For svnf sets f_ℓ, $g_\jmath \in \widetilde{(\pounds, \mathcal{E})}$, we have*

(1) $\vartheta_\varphi(\Phi) = \Phi$.
(2) $\vartheta_\varphi(\widetilde{\mathcal{E}}) = \widetilde{\mathcal{E}}$.
(3) $\vartheta_\varphi(f_\ell \widetilde{\cup} g_\jmath) = \vartheta_\varphi(f_\ell) \widetilde{\cup} \vartheta_\varphi(g_\jmath)$.
(4) $\vartheta_\varphi(f_\ell \widetilde{\cap} g_\jmath) \widetilde{\leq} \vartheta_\varphi(f_\ell) \widetilde{\cap} \vartheta_\varphi(g_\jmath)$.
(5) *If $f_\ell \widetilde{\leq} g_\jmath$, then $\vartheta_\varphi(f_\ell) \widetilde{\leq} \vartheta_\varphi(g_\jmath)$.*

Proof We only prove (3)–(5).

(3) We will prove that

$$\left(\vartheta(f\widetilde{\cup}g)\right)_\pi(\omega) = \vartheta(f)_\pi(\omega) \cup \vartheta(g)_\pi(\omega)$$

for every $\pi \in \mathcal{Y}$ and $\omega \in \mathcal{G}$. Thus for left-hand side, consider $\left(\vartheta(f\widetilde{\cup}g)\right)_\pi(\omega) = (\vartheta(h))_\pi(\omega)$. Then,

$$(\vartheta(h))_\pi(\omega) = \begin{cases} \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \partial} h_{\tilde{e}}(\upsilon) \right), & \text{if } \vartheta^{-1}(\omega) \neq \phi,\ \varphi^{-1}(\pi) \wedge \partial \neq \phi, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where $f_{\tilde{e}} \cup g_{\tilde{e}} = h_{\tilde{e}}$ and $\partial = \ell \cup \jmath$ and for any $\tilde{e} \in \partial$.
Seeing only the non-trivial case, we get,

$$(\vartheta(h))_\pi(\omega) = \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge (\ell \cup \jmath)} \left(f_{\tilde{e}} \cup g_{\tilde{e}}\right)(\upsilon) \right).$$

on the other hand, by using Definition 6, we get then,

$$\begin{aligned}
\left(\vartheta(f\widetilde{\cup}g)\right)_\pi(\omega) &= \vartheta(f)_\pi(\omega) \cup \vartheta(g)_\pi(\omega) \\
&= \left(\bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \ell} \left(f_{\tilde{e}}\right)(\upsilon) \right) \right) \\
&\cup \left(\bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \jmath} \left(g_{\tilde{e}}\right)(\upsilon) \right) \right) \\
&= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge (\ell \cup \jmath)} \left(f_{\tilde{e}} \cup g_{\tilde{e}}\right)(\upsilon) \right) \\
&= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \partial} (h_{\tilde{e}})(\upsilon) \right).
\end{aligned}$$

Hence,

$$\vartheta(f)_\pi(\omega) \cup \vartheta(g)_\pi(\omega) = \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \wp} (h_{\tilde{e}})(\upsilon) \right) \tag{2}$$

From Eqs. (1) and (2), we have (3).

(4) For every $\pi \in \mathcal{Y}$ and $\omega \in \mathcal{G}$, and using definition 6, we have ,

$$\begin{aligned}
\left(\vartheta(f\widetilde{\cap}g)\right)_\pi(\omega) &= (\vartheta(h))_\pi(\omega) \\
&= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge (\wp)} (h_{\tilde{e}})(\upsilon) \right) \\
&= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge (\ell \cap \jmath)} \left(f_{\tilde{e}} \cap g_{\tilde{e}}\right)(\upsilon) \right) \\
&= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge (\ell \cap \jmath)} \left(f_{\tilde{e}}(\upsilon)\right) \right. \\
&\left. \cap \left(g_{\tilde{e}}(\upsilon)\right) \right) \\
&\subseteq \left(\bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \ell} \left(f_{\tilde{e}}\right)(\upsilon) \right) \right) \\
&\cap \left(\bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \jmath} \left(g_{\tilde{e}}\right)(\upsilon) \right) \right) \\
&= \vartheta\left(f_\pi(\omega) \cap g_\pi(\omega)\right) = \left(\vartheta(f) \widetilde{\cap} \vartheta(g)\right)_\pi(\omega).
\end{aligned}$$

This proves (4).

(5) Seeing only the non-trivial case, $\forall\, \pi \in \mathcal{Y}, \omega \in \mathcal{G}$ and let $f_\ell \tilde{\leq} g_\jmath$, we obtain

$$(\vartheta(f))_\pi(\omega) = \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \left(\bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \ell} (f_{\tilde{e}}) \right)(\upsilon)$$
$$= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \ell} (f_{\tilde{e}})(\upsilon)$$
$$= \bigvee_{\upsilon \in \vartheta^{-1}(\omega)} \bigvee_{\tilde{e} \in \varphi^{-1}(\pi) \wedge \ell} (g_{\tilde{e}})(\upsilon) = (\vartheta(g))_\pi(\omega).$$

This proves (5). □

Theorem 3 *Let $\vartheta_\varphi : \widetilde{(£, \mathcal{E})} \to \widetilde{(\mathcal{G}, \mathcal{Y})}$, $\vartheta : £ \to \mathcal{G}$ and $\varphi : \mathcal{E} \to \mathcal{Y}$ be mappings. For svnf sets $f_\ell, g_\jmath \in \widetilde{(\mathcal{G}, \mathcal{Y})}$, we have*

(1) $\vartheta_\varphi^{-1}(\Phi) = \Phi$.
(2) $\vartheta_\varphi^{-1}(\widetilde{\mathcal{E}}) = \widetilde{\mathcal{E}}$.
(3) $\vartheta_\varphi^{-1}(f_\ell \widetilde{\cup} g_\jmath) = \vartheta_\varphi^{-1}(f_\ell) \widetilde{\cup} \vartheta_\varphi^{-1}(g_\jmath)$.
(4) $\vartheta_\varphi^{-1}(f_\ell \widetilde{\cap} g_\jmath) = \vartheta_\varphi^{-1}(f_\ell) \widetilde{\cap} \vartheta_\varphi^{-1}(g_\jmath)$.
(5) If $f_\ell \tilde{\leq} g_\jmath$, then $\vartheta_\varphi^{-1}(f_\ell) \tilde{\leq} \vartheta_\varphi^{-1}(g_\jmath)$.

Proof The proof is straightforward. □

3 Single-valued neutrosophic soft topological structure

To communicate our program and overall notions further exactly, we should recall first the notion of fuzzy topological space presented by Šostak (1985), that is a pair $(£, \top)$ such that X is a non-empty set and $\top : \xi^{£} \to \xi$ is a function (satisfying some axioms) that gives to each fuzzy subset of £ a real number, which then indicates [to what degree] this set is open. Rendering to this notion a fuzzy topology $\top$ is a fuzzy set over $\xi^{£}$. This methodology has principled us to define the single-valued neutrosophic soft topology (simply, svnft) which is compatible to the single-valued neutrosophic soft theory. By our characterization, an svnft is a svnfs on the set of all single-valued neutrosophic soft sets $\widehat{(£, \mathcal{E})}$ which refers [to what degree] this set is open according to the parameter set.

Definition 12 Let $(\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$ be a collection of svnfs over £ [where $\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}} : \mathcal{E} \to \xi^{\widehat{(£, \mathcal{E})}}$], then is termed to be svnft on £, if it meets the following criteria, for every $\tilde{e} \in \mathcal{E}$:

$(\top_1)$ $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\Phi) = \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 1$ and $\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\Phi) = \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\Phi) = \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 0$,
$(\top_2)$ $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) \geq \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \cap \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(g_\jmath)$, $\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) \leq \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \cup \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(g_\jmath)$,
$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) \leq \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \cup \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(g_\jmath)$, $\forall f_\ell, g_\jmath \in \widehat{(£, \mathcal{E})}$,
$(\top_3)$ $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\bigcup}_{i \in \triangle}[f_\ell]_i) \geq \bigcap_{i \in \triangle} \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([f_\ell]_i)$, $\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\bigcup}_{i \in \triangle}[f_\ell]_i) \leq \bigcup_{i \in \triangle} \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([f_\ell]_i)$,
$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\bigcup}_{i \in \triangle}[f_\ell]_i) \leq \bigcup_{i \in \triangle} \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}([f_\ell]_i)$, $\forall f_\ell, f_\jmath \in \widehat{(£, \mathcal{E})}$.
The svnft is termed to be stratified if it satisfies the following conditions:
$(\top^s_1)$ $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\tilde{\mathcal{E}}^t) = 1$, $\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\tilde{\mathcal{E}}^t) = 0$ and $\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\tilde{\mathcal{E}}^t) = 0$.

The quadruple $(£, \tilde{\top}^{\tilde{\sigma}}_{\mathcal{E}}, \tilde{\top}^{\tilde{\varsigma}}_{\mathcal{E}}, \tilde{\top}^{\tilde{\delta}}_{\mathcal{E}})$ is termed as the single-valued neutrosophic soft topological spaces (simply, svnfts), representing the degree of openness $(\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell))$, the degree of indeterminacy $(\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell))$, and the degree of non-openness $(\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f_\ell))$; of a svnfs f_ℓ with respect to the parameter $\tilde{e} \in \mathcal{E}$, respectively.

Here, we will write $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$ for $(\tilde{\top}^{\tilde{\sigma}}_{\mathcal{E}}, \tilde{\top}^{\tilde{\varsigma}}_{\mathcal{E}}, \tilde{\top}^{\tilde{\delta}}_{\mathcal{E}})$, and it will be no ambiguity. Let $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$ and $\tilde{\top}^{\star\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$ be svnfts £. We say that $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$ is finer than $\tilde{\top}^{\star\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$ ($\tilde{\top}^{\star\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$ is coarser than $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$), denoted by $\tilde{\top}^{\star\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}} \widetilde{\subseteq} \tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}$, if $\tilde{\top}^{\star\tilde{\sigma}}_{\tilde{e}}(f_\ell) \leq \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell)$, $\tilde{\top}^{\star\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \geq \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell)$ and $\tilde{\top}^{\star\tilde{\delta}}_{\tilde{e}}(f_\ell) \geq \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f_\ell)$ $\forall\, \tilde{e} \in \mathcal{E}; f_\ell \in \widehat{(£, \mathcal{E})}$.

Example 2 Let $(\tau^{\tilde{\sigma}}, \tau^{\tilde{\varsigma}}, \tau^{\tilde{\delta}})$ be a smooth neutrosophic topology on £ presented by Feng et al. (2011), i.e., $\tau^{\tilde{\sigma}} : \xi^{£} \to \xi$, $\tau^{\tilde{\varsigma}} : \xi^{£} \to \xi$ and $\tau^{\tilde{\delta}} : \xi^{£} \to \xi$. Take $\mathcal{E} = \xi$ and define $\tilde{\tau}^{\tilde{\sigma}} : \mathcal{E} \to \xi^{£}$, $\tilde{\tau}^{\tilde{\varsigma}} : \mathcal{E} \to \xi^{£}$, $\tilde{\tau}^{\tilde{\delta}} : \mathcal{E} \to \xi^{£}$ as $\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}} = \{\tau^{\tilde{\sigma}}(\mu) \geq e, \tau^{\tilde{\varsigma}}(\mu) \leq 1 - e, \tau^{\tilde{\delta}}(\mu) \leq 1 - e\}$, which is fuzzy neutrosophic topology of $(\tau^{\tilde{\sigma}}, \tau^{\tilde{\varsigma}}, \tau^{\tilde{\delta}})$ in presented by Salama and Alblowi (2012), for each $e \in \xi$. However, it is well known that each fuzzy topology can be considered as smooth neutrosophic topology by using fuzzifying method. Thus, $\tau^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}(e)$ satisfies $(\top_1)$, $(\top_2)$ and $(\top_3)$. According to this concept and by using the decomposition theorem of neutrosophic sets introduced by Smarandache (2007), if we know the resulting single-valued neutrosophic soft topology, then we can find the first fuzzy neutrosophic topology. Hence, we can say that a fuzzy neutrosophic topology can be uniquely represented as a single-valued neutrosophic soft topology.

Definition 13 Let $(£, \tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})$ and $(\mathcal{G}, \tilde{\top}^{\star\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})$ be svnfts. A svnf − *mapping* $\vartheta_\varphi : \widehat{(£, \mathcal{E})} \to \widehat{(\mathcal{G}, \mathcal{Y})}$ is termed a single-valued neutrosophic soft continuous mapping (simply, svnfc − *map*) if

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_\jmath)) \geq \tilde{\top}^{\star\tilde{\sigma}}_{\vartheta(\tilde{e})}(g_\jmath),$$
$$\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_\jmath)) \leq \tilde{\top}^{\star\tilde{\varsigma}}_{\vartheta(\tilde{e})}(g_\jmath), \quad \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_\jmath))$$

$\leq \tilde{\top}^{\star\tilde{\delta}}_{\vartheta(\tilde{e})}(g_J)\ \forall\ \ g_J \in \widehat{(\mathcal{G}, \mathcal{Y})},\ \ \tilde{e} \in \mathcal{E}.$

Theorem 4 *Let* $\left\{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{j}\right\}_{j\in\Gamma}$ *be a group of single-valued neutrosophic soft topologies on* £*. Then,*

$$\tilde{\top}^{\tilde{\sigma}} = \bigcap_{j\in\Gamma} \tilde{\top}^{\tilde{\sigma}}_{j},\quad \tilde{\top}^{\tilde{\varsigma}} = \bigcup_{j\in\Gamma} \tilde{\top}^{\tilde{\varsigma}}_{j},\quad \tilde{\top}^{\tilde{\delta}} = \bigcup_{j\in\Gamma} \tilde{\top}^{\tilde{\delta}}_{j},$$

is also an svnft on £*, where*

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) = \bigcap_{j\in\Gamma}(\tilde{\top}^{\tilde{\sigma}}_{j})_{\tilde{e}}(f_\ell),$$
$$\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) = \bigcup_{j\in\Gamma}(\tilde{\top}^{\tilde{\varsigma}}_{j})_{\tilde{e}}(f_\ell),$$
$$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) = \bigcup_{j\in\Gamma}(\tilde{\top}^{\tilde{\delta}}_{j})_{\tilde{e}}(f_\ell)\quad \forall\quad f_\ell \in (£, \mathcal{E}),\quad \tilde{e} \in \mathcal{E}.$$

Proof Direct. □

Example 3 Let £ $= \{x_1, x_2\}$ be a universal set, $\mathcal{E} = \{e_1, e_2, e_3, e_4\}$ be a set of parameters for £, $\ell = J = \{e_1, e_2\}$. Let svnf-sets $f_\ell, g_J \in \widetilde{(£, \mathcal{E})}$ as follows

$$f_\ell = \{(e_1, \{\langle x_1, 0.3, 0.3, 0.3\rangle, \langle x_2, 0.3, 0.3, 0.3\rangle\}), (e_2, \{\langle x_1, 0.5, 0.5, 0.5\rangle, \langle x_2, 0.5, 0.5, 0.5\rangle\})\},$$
$$g_J = \{(e_1, \{\langle x_1, 0.4, 0.4, 0.4\rangle, \langle x_2, 0.4, 0.4, 0.4\rangle\}), (e_2, \{\langle x_1, 0.5, 0.5, 0.5\rangle, \langle x_2, 0.5, 0.5, 0.5\rangle\})\},$$

Let us consider the next single-valued neutrosophic soft topologies:

$$\tilde{\top}^{\tilde{\sigma}}_{e}(\Omega) = \begin{cases} 1, & \text{if } \Omega = \Phi,\\ 1, & \text{if } \Omega = \tilde{\mathcal{E}},\\ \frac{2}{3}, & \text{if } \Omega = f_\ell, \end{cases} \qquad \tilde{\top}^{*\tilde{\sigma}}_{e}(\Omega) = \begin{cases} 1, & \text{if } \Omega = \Phi,\\ 1, & \text{if } \Omega = \tilde{\mathcal{E}},\\ \frac{2}{3}, & \text{if } \Omega = f_\ell \ \textit{or}\ g_J, \end{cases}$$

$$\tilde{\top}^{\tilde{\varsigma}}_{e}(\Omega) = \begin{cases} 0, & \text{if } \Omega = \Phi,\\ 0, & \text{if } \Omega = \tilde{\mathcal{E}},\\ \frac{1}{3}, & \text{if } \Omega = f_\ell, \end{cases} \qquad \tilde{\top}^{*\tilde{\varsigma}}_{e}(\Omega) = \begin{cases} 0, & \text{if } \Omega = \Phi,\\ 0, & \text{if } \Omega = \tilde{\mathcal{E}},\\ \frac{1}{3}, & \text{if } \Omega = f_\ell \ \textit{or}\ g_J, \end{cases}$$

$$\tilde{\top}^{\tilde{\delta}}_{e}(\Omega) = \begin{cases} 0, & \text{if } \Omega = \Phi,\\ 0, & \text{if } \Omega = \tilde{\mathcal{E}},\\ \frac{1}{4}, & \text{if } \Omega = f_\ell, \end{cases} \qquad \tilde{\top}^{*\tilde{\delta}}_{e}(\Omega) = \begin{cases} 0, & \text{if } \Omega = \Phi,\\ 0, & \text{if } \Omega = \tilde{\mathcal{E}},\\ \frac{1}{4}, & \text{if } \Omega = f_\ell \ \text{or}\ g_J. \end{cases}$$

The mapping $\vartheta_\varphi : (£, \tilde{\top}^{*\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}}) \to (£, \tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathcal{E}})$ where $\vartheta : £ \to £$ and $\varphi : \mathcal{E} \to \mathcal{E}$ are identity mappings. It's easy to see that ϑ_φ is svnfc $-$ *map*.

Definition 14 A mapping $\tilde{\hbar}^{\tilde{\sigma}}, \tilde{\hbar}^{\tilde{\varsigma}}, \tilde{\hbar}^{\tilde{\delta}} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ is considered as single-valued neutrosophic soft cotopology on £, if it meets the following criteria, for every $\tilde{e} \in \mathcal{E}$

($\hbar_1$) $\tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(\Phi) = \tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 1$ and $\tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(\Phi) = \tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = \tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(\Phi) = \tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 0$,

($\hbar_2$) $\tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell \tilde{\cup} g_J) \geq \tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \cap \tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(g_J)$, $\tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell \tilde{\cup} g_J) \leq \tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \cup \tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(g_J)$,
$\tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(f_\ell \tilde{\cup} g_J) \leq \tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \cup \tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(g_J)$, $\quad \forall f_\ell, g_J \in \widehat{(£, \mathcal{E})}$,

($\hbar_3$) $\tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\bigcap}_{i\in\triangle}[f_\ell]_i) \geq \bigcap_{i\in\triangle} \tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}([f_\ell]_i)$, $\tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\bigcap}_{i\in\triangle}[f_\ell]_i) \leq \bigcup_{i\in\triangle} \tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}([f_\ell]_i)$,
$\tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\bigcap}_{i\in\triangle}[f_\ell]_i) \leq \bigcup_{i\in\triangle} \tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}([f_\ell]_i)$, $\quad \forall f_\ell, f_J \in \widehat{(£, \mathcal{E})}$.

The quadruple $(£, \tilde{\hbar}^{\tilde{\sigma}}_{\mathcal{E}}, \tilde{\hbar}^{\tilde{\varsigma}}_{\mathcal{E}}, \tilde{\hbar}^{\tilde{\delta}}_{\mathcal{E}})$ is termed the single-valued neutrosophic soft cotopological space.

Let $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}$ be a svnft on £, then the mapping $\tilde{\hbar}^{\tilde{\sigma}}, \tilde{\hbar}^{\tilde{\varsigma}}, \tilde{\hbar}^{\tilde{\delta}} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ defined by

$$\tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) = \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f^c_\ell),\quad \tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) = \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f^c_\ell),$$
$$\tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) = \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f^c_\ell),\quad \forall\quad \tilde{e} \in \mathcal{E}.$$

is a single-valued neutrosophic soft cotopology on £, the mapping $\tilde{\hbar}^{\sigma}, \tilde{\hbar}^{\varsigma}, \tilde{\hbar}^{\delta} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ defined by

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) = \tilde{\hbar}^{\tilde{\sigma}}_{\tilde{e}}(f^c_\ell),\quad \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) = \tilde{\hbar}^{\tilde{\varsigma}}_{\tilde{e}}(f^c_\ell),$$
$$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) = \tilde{\hbar}^{\tilde{\delta}}_{\tilde{e}}(f^c_\ell),\quad \forall\quad \tilde{e} \in \mathcal{E},$$

is an svnft on £.

Definition 15 A mapping $Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ is called a single-valued neutrosophic soft base (simply, svnf-base) on £, if it meets the following conditions, $\forall\ \tilde{e} \in \mathcal{E}$:

($Ł_1$) $Ł^{\tilde{\sigma}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 1$ and $Ł^{\tilde{\varsigma}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = Ł^{\tilde{\delta}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 0$,

($Ł_2$) $Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell \tilde{\cap} f_J) \geq Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \cap Ł^{\tilde{\sigma}}_{\tilde{e}}(f_J)$, $Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell \tilde{\cap} f_J) \leq Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \cup Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_J)$, $Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell \tilde{\cap} f_J) \leq Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \cup Ł^{\tilde{\delta}}_{\tilde{e}}(f_J)$, $\forall f_\ell, f_J \in \widehat{(£, \mathcal{E})}$.

Theorem 5 *Let* $(Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}})$ *be an svnf-base on* £*. Define a mapping* $\tilde{\top}^{\tilde{\sigma}}_{Ł}, \tilde{\top}^{\tilde{\varsigma}}_{Ł}, \tilde{\top}^{\tilde{\delta}}_{Ł} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ *as follows:*

$$(\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(f_\ell) = \bigcup \left\{ \bigcap_{j\in J} Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell)_j \mid f_\ell = \widetilde{\bigcup_{j\in J}}(f_\ell)_j \right\},\quad \forall\quad \tilde{e} \in \mathcal{E},$$

$$(\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(f_\ell) = \bigcap \left\{ \bigcup_{j\in J} Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell)_j \mid f_\ell = \widetilde{\bigcup_{j\in J}}(f_\ell)_j \right\},\quad \forall\ \tilde{e} \in \mathcal{E},$$

$$(\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(f_\ell) = \bigcap \left\{ \bigcup_{j\in J} Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell)_j \mid f_\ell = \widetilde{\bigcup_{j\in J}}(f_\ell)_j \right\},\quad \forall\quad \tilde{e} \in \mathcal{E}.$$

Then, $(\tilde{\top}^{\tilde{\sigma}}_{Ł}, \tilde{\top}^{\tilde{\varsigma}}_{Ł}, \tilde{\top}^{\tilde{\delta}}_{Ł})$ is svnft on £ for which

$$(\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(f_\ell) \geq Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell), \quad (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(f_\ell) \leq Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell), \quad (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(f_\ell) \leq Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \quad \forall \quad f_\ell \in \widehat{(£, \mathcal{E})}, \quad \tilde{e} \in \mathcal{E}.$$

Proof $(\top_1)$ From the definition of $Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}}$ we obtain, $Ł^{\tilde{\sigma}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 1$ and $Ł^{\tilde{\varsigma}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = Ł^{\tilde{\delta}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 0$.

$(\top_2)$ Suppose that $\left\{(f_\ell)_j \mid f_\ell = \widetilde{\bigcup_{j\in J}} (f_\ell)_j\right\}$ and $\left\{(g_\jmath)_j \mid g_\jmath = \widetilde{\bigcup_{\kappa\in\Gamma}} (g_\jmath)_\kappa\right\}$, for every $\tilde{e} \in \mathcal{E}$ are families, then there exists $\left\{(f_\ell)_j \widetilde{\bigcap} (g_\jmath)_\kappa\right\}$ such that

$$\begin{aligned} f_\ell \widetilde{\bigcap} g_\jmath &= \left(\bigcup_{j\in J} (f_\ell)_j\right) \widetilde{\bigcap} \left(\bigcup_{\kappa\in\Gamma} (g_\jmath)_\kappa\right) \\ &= \bigcup_{j\in J, \kappa\in\Gamma} \left((f_\ell)_j \widetilde{\cap} (g_\jmath)_\kappa\right). \end{aligned}$$

It implies that

$$\begin{aligned} (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) &\geq \bigcap_{j\in J, \kappa\in\Gamma} Ł^{\tilde{\sigma}}_{\tilde{e}}\left((f_\ell)_j \widetilde{\cap} (g_\jmath)_\kappa\right) \\ &\geq \bigcap_{j\in J, \kappa\in\Gamma} \left(Ł^{\tilde{\sigma}}_{\tilde{e}}\left((f_\ell)_j\right) \cap Ł^{\tilde{\sigma}}_{\tilde{e}}((g_\jmath)_\kappa)\right) \\ &\geq \left(\bigcap_{j\in J,} Ł^{\tilde{\sigma}}_{\tilde{e}}\left((f_\ell)_j\right)\right) \cap \left(\bigcap_{\kappa\in\Gamma} Ł^{\tilde{\sigma}}_{\tilde{e}}\left((g_\jmath)_\kappa\right)\right) \\ &\geq (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(f_\ell) \cap (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(g_\jmath) \\ (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) &\leq \bigcup_{j\in J, \kappa\in\Gamma} Ł^{\tilde{\varsigma}}_{\tilde{e}}\left((f_\ell)_j \widetilde{\cap} (g_\jmath)_\kappa\right) \\ &\leq \bigcup_{j\in J, \kappa\in\Gamma} \left(Ł^{\tilde{\varsigma}}_{\tilde{e}}\left((f_\ell)_j\right) \cap Ł^{\tilde{\varsigma}}_{\tilde{e}}((g_\jmath)_\kappa)\right) \\ &\leq \left(\bigcup_{j\in J,} Ł^{\tilde{\varsigma}}_{\tilde{e}}\left((f_\ell)_j\right)\right) \cap \left(\bigcup_{\kappa\in\Gamma} Ł^{\tilde{\varsigma}}_{\tilde{e}}\left((g_\jmath)_\kappa\right)\right) \\ &\leq \left(\bigcup_{j\in J} Ł^{\tilde{\varsigma}}_{\tilde{e}}\left((f_\ell)_j\right)\right) \cup \left(\bigcup_{\kappa\in\Gamma} Ł^{\tilde{\varsigma}}_{\tilde{e}}\left((g_\jmath)_\kappa\right)\right) \\ &\leq (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(f_\ell) \cup (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(g_\jmath) \\ (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) &\leq \bigcup_{j\in J, \kappa\in\Gamma} Ł^{\tilde{\delta}}_{\tilde{e}}\left((f_\ell)_j \widetilde{\cap} (g_\jmath)_\kappa\right) \\ &\leq \bigcup_{j\in J, \kappa\in\Gamma} \left(Ł^{\tilde{\delta}}_{\tilde{e}}\left((f_\ell)_j\right) \cap Ł^{\tilde{\delta}}_{\tilde{e}}((g_\jmath)_\kappa)\right) \\ &\leq \left(\bigcup_{j\in J} Ł^{\tilde{\delta}}_{\tilde{e}}\left((f_\ell)_j\right)\right) \cap \left(\bigcup_{\kappa\in\Gamma} Ł^{\tilde{\delta}}_{\tilde{e}}\left((g_\jmath)_\kappa\right)\right) \\ &\leq \left(\bigcup_{j\in J} Ł^{\tilde{\delta}}_{\tilde{e}}\left((f_\ell)_j\right)\right) \cup \left(\bigcup_{\kappa\in\Gamma} Ł^{\tilde{\delta}}_{\tilde{e}}\left((g_\jmath)_\kappa\right)\right) \\ &\leq (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(f_\ell) \cup (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(g_\jmath). \end{aligned}$$

$(\top_3)$ Let $\tilde{e} \in \mathcal{E}$ and Υ_i be a family index sets Ω_i such that $\left\{(f_\ell)_{i_\omega} \in \widehat{(£, \mathcal{E})} \mid (f_\ell)_i = \widetilde{\bigcup_{\omega\in\Omega_i}} (f_\ell)_{i_\omega}\right\}$ with $f_\ell = \widetilde{\bigcup_{i\in\Gamma}} (f_\ell)_i = \widetilde{\bigcup_{i\in\Gamma}} \widetilde{\bigcup_{\omega\in\Omega_i}} (f_\ell)_{i_\omega}$. for every $i \in \Gamma$ and $\Re \in \Xi_{i\in\Gamma}\Upsilon_i$ with $\Re(i) = \Omega_i$, we obtain

$$\begin{aligned} (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(f_\ell) &\geq \bigcap_{i\in\Gamma} \left(\bigcap_{\omega\in\Omega_i} Ł^{\tilde{\sigma}}_{\tilde{e}}((f_\ell)_{i\in\omega})\right), \\ (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(f_\ell) &\leq \bigcup_{i\in\Gamma} \left(\bigcup_{\omega\in\Omega_i} Ł^{\tilde{\varsigma}}_{\tilde{e}}((f_\ell)_{i\in\omega})\right), \\ (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(f_\ell) &\leq \bigcup_{i\in\Gamma} \left(\bigcup_{\omega\in\Omega_i} Ł^{\tilde{\delta}}_{\tilde{e}}((f_\ell)_{i\in\omega})\right). \end{aligned}$$

Put $\alpha_i, \Re_i = \bigcap_{\omega\in\Omega_i} (Ł^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\tilde{e}}((f_\ell)_{i\in\omega}))$. Then,

$$\begin{aligned} (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(f_\ell) &\geq \bigcup_{\Re\in\Xi_{i\in\Gamma}\Upsilon_i} \left(\bigcap_{i\in\Gamma} \alpha_i, \Re_i\right) \\ &= \bigcap_{i\in\Gamma} \left(\bigcup_{\eta_i\in\Upsilon_i} \alpha_i, \eta_i\right) \\ &= \bigcap_{i\in\Gamma} \left(\bigcup_{\eta_i\in\Upsilon_i} \left(\bigcap_{\mu\in\eta_i} (Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell)_{i_\mu})\right)\right) = \bigcap_{i\in\Gamma} (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}((f_\ell)_i). \\ (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(f_\ell) &\leq \bigcap_{\Re\in\Xi_{i\in\Gamma}\Upsilon_i} \left(\bigcup_{i\in\Gamma} \alpha_i, \Re_i\right) = \bigcup_{i\in\Gamma} \left(\bigcap_{\eta_i\in\Upsilon_i} \alpha_i, \eta_i\right) \\ &= \bigcup_{i\in\Gamma} \left(\bigcap_{\eta_i\in\Upsilon_i} \left(\bigcup_{\mu\in\eta_i} (Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell)_{i_\mu})\right)\right) = \bigcup_{i\in\Gamma} (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}((f_\ell)_i). \\ (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(f_\ell) &\leq \bigcap_{\Re\in\Xi_{i\in\Gamma}\Upsilon_i} \left(\bigcup_{i\in\Gamma} \alpha_i, \Re_i\right) = \bigcup_{i\in\Gamma} \left(\bigcap_{\eta_i\in\Upsilon_i} \alpha_i, \eta_i\right) \\ &= \bigcup_{i\in\Gamma} \left(\bigcap_{\eta_i\in\Upsilon_i} \left(\bigcup_{\mu\in\eta_i} (Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell)_{i_\mu})\right)\right) = \bigcup_{i\in\Gamma} (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}((f_\ell)_i). \end{aligned}$$

Thus, $(\tilde{\top}^{\tilde{\sigma}}_{Ł}, \tilde{\top}^{\tilde{\varsigma}}_{Ł}, \tilde{\top}^{\tilde{\delta}}_{Ł})$ is svnft on £. □

Theorem 6 *Let $(\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$ be a svnft on £ and $(Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}})$ be a svnf-base on $\mathcal{G}$. Then, the single-valued neutrosophic soft* *mapping* $\vartheta_\varphi : \widetilde{(£, \mathcal{E})} \to \widetilde{(\mathcal{G}, \mathcal{Y})}$ *is svnfc − map if and only if*

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \geq Ł^{\tilde{\sigma}}_{\varphi(\tilde{e})}(g_J), \quad \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \leq Ł^{\tilde{\varsigma}}_{\varphi(\tilde{e})}(g_J),$$
$$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \leq Ł^{\tilde{\delta}}_{\varphi(\tilde{e})}(g_J),$$

for each $g_J \in \widehat{(\mathcal{G}, \mathcal{Y})}$ *and* $\tilde{e} \in \mathcal{E}$.

Proof (⇒) Let $\vartheta_\varphi : (£, \tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}) \to (\mathcal{G}, \tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{Ł})$ be a svnfc-map and $g_J \in \widehat{(\mathcal{G}, \mathcal{Y})}$. Then,

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \geq (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\varphi(\tilde{e})}(g_J) \geq Ł^{\tilde{\sigma}}_{\varphi(\tilde{e})}(g_J),$$
$$\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \leq (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\varphi(\tilde{e})}(g_J) \leq Ł^{\tilde{\varsigma}}_{\varphi(\tilde{e})}(g_J),$$
$$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \leq (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\varphi(\tilde{e})}(g_J) \leq Ł^{\tilde{\delta}}_{\varphi(\tilde{e})}(g_J),$$

(⇐) Let $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \geq Ł^{\tilde{\sigma}}_{\varphi(\tilde{e})}(g_J)$, $\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \leq Ł^{\tilde{\varsigma}}_{\varphi(\tilde{e})}(g_J)$ and $\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi^{-1}(g_J)) \leq Ł^{\tilde{\delta}}_{\varphi(\tilde{e})}(g_J)$ for each $g_J \in \widehat{(\mathcal{G}, \mathcal{Y})}$ and let $h_\partial \in \widehat{(\mathcal{G}, \mathcal{Y})}$ for any family of $\left\{(h_\partial)_j \in \widehat{(\mathcal{G}, \mathcal{Y})} \mid h_\partial = \widetilde{\bigcup_{j\in\Gamma}} (h_\partial)_j\right\}$, we obtain

$$\begin{aligned}
\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(h_\partial)) &= \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}\left(\vartheta_\varphi^{-1}\left(\widetilde{\bigcup_{j\in\Gamma}}(h_\partial)_j\right)\right) = \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}\left(\widetilde{\bigcup_{j\in\Gamma}}\vartheta_\varphi^{-1}((h_\partial)_j)\right) \\
&\geq \bigcap_{j\in\Gamma} \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}\left(\vartheta_\varphi^{-1}((h_\partial)_j)\right) \geq \bigcap_{j\in\Gamma} Ł^{\tilde{\sigma}}_{\varphi(\tilde{e})}((h_\partial)_j) \\
\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(h_\partial)) &= \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}\left(\vartheta_\varphi^{-1}\left(\widetilde{\bigcap_{j\in\Gamma}}(h_\partial)_j\right)\right) \\
&= \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}\left(\widetilde{\bigcap_{j\in\Gamma}}\vartheta_\varphi^{-1}((h_\partial)_j)\right) \\
&\leq \bigcup_{j\in\Gamma} \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}\left(\vartheta_\varphi^{-1}((h_\partial)_j)\right) \leq \bigcup_{j\in\Gamma} Ł^{\tilde{\varsigma}}_{\varphi(\tilde{e})}((h_\partial)_j) \\
&\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi^{-1}(h_\partial)) \\
&= \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}\left(\vartheta_\varphi^{-1}\left(\widetilde{\bigcap_{j\in\Gamma}}(h_\partial)_j\right)\right) = \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}\left(\widetilde{\bigcap_{j\in\Gamma}}\vartheta_\varphi^{-1}((h_\partial)_j)\right) \\
&\leq \bigcup_{j\in\Gamma} \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}\left(\vartheta_\varphi^{-1}((h_\partial)_j)\right) \leq \bigcup_{j\in\Gamma} Ł^{\tilde{\delta}}_{\varphi(\tilde{e})}((h_\partial)_j)
\end{aligned}$$

Thus,

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(h_\partial)) \geq (\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\varphi(\tilde{e})}(h_\partial),$$
$$\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi^{-1}(h_\partial)) \leq (\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\varphi(\tilde{e})}(h_\partial),$$
$$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi^{-1}(h_\partial)) \leq (\tilde{\top}^{\tilde{\delta}}_{Ł})_{\varphi(\tilde{e})}(h_\partial).$$

□

Theorem 7 *Let* $\{(£_i, (\tilde{\top}_i^{\tilde{\sigma}})_{\mathcal{E}_i}, (\tilde{\top}_i^{\tilde{\varsigma}})_{\mathcal{E}_i}, (\tilde{\top}_i^{\tilde{\delta}})_{\mathcal{E}_i})\}$ *be a family of svnfts,* $\mathcal{E}$ *be a parameter set, £ be a non-null set and for every* $i \in \Gamma$, $\vartheta_i : £ \to £_i$ *and* $\varphi_i : \mathcal{E} \to \mathcal{E}_i$ *be a mapping. Define* $Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ *on £ as follows:*

$$Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) = \bigcup \left\{\bigcap_{j=1}^{n} (\tilde{\top}^{\tilde{\sigma}}_{\omega_j})_{\varphi_{\omega_j}(\tilde{e})}((f_\ell)_{\omega_j}) \mid f_\ell = \widetilde{\bigcap_{j=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_j}((f_\ell)_{\omega_j})\right\}$$
$$Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) = \bigcap \left\{\bigcup_{j=1}^{n} (\tilde{\top}^{\tilde{\varsigma}}_{\omega_j})_{\varphi_{\omega_j}(\tilde{e})}((f_\ell)_{\omega_j}) \mid f_\ell = \widetilde{\bigcap_{j=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_j}((f_\ell)_{\omega_j})\right\}$$
$$Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell) = \bigcap \left\{\bigcup_{j=1}^{n} (\tilde{\top}^{\tilde{\delta}}_{\omega_j})_{\varphi_{\omega_j}(\tilde{e})}((f_\ell)_{\omega_j}) \mid f_\ell = \widetilde{\bigcap_{j=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_j}((f_\ell)_{\omega_j})\right\},$$

where $\Omega = \{\omega_1, \omega_2, ..., \omega_n \subseteq \Gamma$. *Then,*

(1) $(Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}})$ *is a svnf-base on £.*
(2) The svnft $(\tilde{\top}^{\tilde{\sigma}}_{Ł}, \tilde{\top}^{\tilde{\varsigma}}_{Ł}, \tilde{\top}^{\tilde{\delta}}_{Ł})$ *generated by* $(Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}})$ *is the svnft on £ for which every* $(\vartheta_\varphi)_i$, $i \in \Gamma$ *are svnfc-maps.*
(3) A map $\vartheta_\varphi : (\mathcal{G}, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}) \to (£, (\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{Ł})_{\mathcal{E}})$ *is a svnfc-map iff for any* $i \in \Gamma$, $(\vartheta_\varphi)_i \circ \vartheta_\varphi : (\mathcal{G}, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}) \to (£_i, (\tilde{\top}_i^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}_i})$ *is a svnfc-map.*

Proof (1) ($Ł_1$) Since $f_\ell = (\vartheta_\varphi)_i^{-1}(f_\ell)$ for every $f_\ell = \{\Phi, \widetilde{\mathcal{E}}\}$, $Ł^{\tilde{\sigma}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\sigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 1$ and $Ł^{\tilde{\varsigma}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\varsigma}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = Ł^{\tilde{\delta}}_{\tilde{e}}(\Phi) = Ł^{\tilde{\delta}}_{\tilde{e}}(\widetilde{\mathcal{E}}) = 0, \forall\, \tilde{e} \in \mathcal{E}$.

($Ł_2$) For each $J = \{j_1, j_i, ..., j_m\}$ and $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$ of γ such that $f_\ell = \bigcap_{i=1}^{n} (\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i})$ and $g_J = \widetilde{\bigcap_{i=1}^{m}} (\vartheta_\varphi)^{-1}_{j_i}((g_J)_{j_i})$, we have

$$\begin{aligned}
f_\ell \widetilde{\cap} g_J = \{\langle &\tilde{\sigma}_{\bigcap_{i=1}^{n}(\vartheta_\varphi)^{-1}_{\omega_i}((f_{\tilde{e}})_{\omega_i})(\tilde{e})} \cap \tilde{\sigma}_{\bigcap_{1=1}^{m}(\vartheta_\varphi)^{-1}_{j_i}((g_{\tilde{e}})_{j_i})(\tilde{e})}, \\
&\tilde{\varsigma}_{\bigcap_{i=1}^{n}(\vartheta_\varphi)^{-1}_{\omega_i}((f_{\tilde{e}})_{\omega_i})(\tilde{e})} \cup \tilde{\varsigma}_{\bigcap_{i=1}^{m}(\vartheta_\varphi)^{-1}_{j_i}((g_{\tilde{e}})_{j_i})(\tilde{e})}, \\
&\tilde{\delta}_{\bigcap_{i=1}^{n}(\vartheta_\varphi)^{-1}_{\omega_i}((f_{\tilde{e}})_{\omega_i})(\tilde{e})} \cup \tilde{\delta}_{\bigcap_{i=1}^{m}(\vartheta_\varphi)^{-1}_{j_i}((g_{\tilde{e}})_{j_i})(\tilde{e})}\rangle\}
\end{aligned}$$

Furthermore, we have for each $\omega \in \Omega \cap J$

$$(\vartheta_\varphi)^{-1}_\omega((f_\ell)_\omega) \cap (\vartheta_\varphi)^{-1}_\omega((g_J)_\omega) = (\vartheta_\varphi)^{-1}_\omega((f_\ell)_\omega \cap (g_J)_\omega).$$

Put $f_\ell \widetilde{\cap} g_J = \{\langle \tilde{\sigma}_{\bigcap_{m_i\in\Omega\cup J}(\vartheta_\varphi)^{-1}_{m_i}(h_\partial)_{(m_i)}}, \tilde{\sigma}_{\bigcup_{m_i\in\Omega\cup J}(\vartheta_\varsigma)^{-1}_{m_i}(h_\partial)_{(m_i)}}, \tilde{\delta}_{\bigcup_{m_i\in\Omega\cup J}(\vartheta_\varsigma)^{-1}_{m_i}(h_\partial)_{(m_i)}}\rangle\}$ where

$$\tilde{\sigma}_{(h_\partial)_{(m_i)}}(r) = \begin{cases} \tilde{\sigma}_{f_{(m_i)}}(r), & \text{if } m_i \in \ell - \jmath, \\ \tilde{\sigma}_{g_{(m_i)}}(r), & \text{if } m_i \in \jmath - \ell, \\ \tilde{\sigma}_{f_{(m_i)}}(r) \cap \tilde{\sigma}_{g_{(m_i)}}(r), & \text{if } m_i \in \ell \cap \jmath. \end{cases}$$

$$\tilde{\varsigma}_{(h_\partial)_{(m_i)}}(r) = \begin{cases} \tilde{\varsigma}_{f_{(m_i)}}(r), & \text{if } m_i \in \ell - \jmath, \\ \tilde{\varsigma}_{g_{(m_i)}}(r), & \text{if } m_i \in \jmath - \ell, \\ \tilde{\varsigma}_{f_{(m_i)}}(r) \cup \tilde{\varsigma}_{g_{(m_i)}}(r), & \text{if } m_i \in \ell \cap \jmath. \end{cases}$$

$$\tilde{\delta}_{(h_\partial)_{(m_i)}}(r) = \begin{cases} \tilde{\delta}_{f_{(m_i)}}(r), & \text{if } m_i \in \ell - \jmath, \\ \tilde{\delta}_{g_{(m_i)}}(r), & \text{if } m_i \in \jmath - \ell, \\ \tilde{\delta}_{f_{(m_i)}}(r) \cup \tilde{\delta}_{g_{(m_i)}}(r), & \text{if } m_i \in \ell \cap \jmath. \end{cases}$$

So, we obtain

$$\begin{aligned} Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) &\geq \bigcap_{j \in \Omega \cup J} (\tilde{\top}^{\tilde{\sigma}}_j)_{(\varphi_j)(\tilde{e})}(h_\partial)_j \\ &\geq \left(\bigcap_{i=1}^{n} (\tilde{\top}^{\tilde{\sigma}}_{\omega_i})_{(\varphi_{\omega_i})(\tilde{e})}((f_\ell)_{\omega_i}) \right) \\ &\quad \cap \left(\bigcap_{i=1}^{m} (\tilde{\top}^{\tilde{\sigma}}_{j_i})_{(\varphi_{j_i})(\tilde{e})}((g_\jmath)_{j_i}) \right) \end{aligned}$$

If we take the sup over the families $f_\ell = \widetilde{\bigcap_{i=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i})$ and $g_\jmath = \widetilde{\bigcap_{i=1}^{m}} (\vartheta_\varphi)^{-1}_{j_i}((g_\jmath)_{j_i})$, then we have

$$Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) \geq Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \cap Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\jmath), \quad \forall \quad \tilde{e} \in \mathcal{E}, \tag{3}$$

and on the other hand,

$$\begin{aligned} Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) &\leq \bigcup_{j \in \Omega \cup J} (\tilde{\top}^{\tilde{\varsigma}}_j)_{(\varphi_j)(\tilde{e})}(h_\partial)_j \\ &\leq \left(\bigcup_{i=1}^{n} (\tilde{\top}^{\tilde{\varsigma}}_{\omega_i})_{(\varphi_{\omega_i})(\tilde{e})}((f_\ell)_{\omega_i}) \right) \\ &\quad \cup \left(\bigcup_{i=1}^{m} (\tilde{\top}^{\tilde{\varsigma}}_{j_i})_{(\varphi_{j_i})(\tilde{e})}((g_\jmath)_{j_i}) \right). \end{aligned}$$

If we take the sup over the families $f_\ell = \widetilde{\bigcup_{i=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i})$ and $g_\jmath = \widetilde{\bigcup_{i=1}^{m}} (\vartheta_\varphi)^{-1}_{j_i}((g_\jmath)_{j_i})$, then we have

$$Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) \leq Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \cup Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\jmath), \quad \forall \quad \tilde{e} \in \mathcal{E}. \tag{4}$$

Also,

$$\begin{aligned} Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) &\leq \bigcup_{j \in \Omega \cup J} (\tilde{\top}^{\tilde{\delta}}_j)_{(\varphi_j)(\tilde{e})}(h_\partial)_j \\ &\leq \left(\bigcup_{i=1}^{n} (\tilde{\top}^{\tilde{\delta}}_{\omega_i})_{(\varphi_{\omega_i})(\tilde{e})}((f_\ell)_{\omega_i}) \right) \\ &\quad \cup \left(\bigcup_{i=1}^{m} (\tilde{\top}^{\tilde{\delta}}_{j_i})_{(\varphi_{j_i})(\tilde{e})}((g_\jmath)_{j_i}) \right) \end{aligned}$$

If we take the sup over the families $f_\ell = \widetilde{\bigcup_{i=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i})$ and $g_\jmath = \widetilde{\bigcup_{i=1}^{m}} (\vartheta_\varphi)^{-1}_{j_i}((g_\jmath)_{j_i})$, then we have

$$Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) \leq Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \cup Ł^{\tilde{\delta}}_{\tilde{e}}(f_\jmath), \quad \forall \quad \tilde{e} \in \mathcal{E}. \tag{5}$$

From Eqs. (3), (4) and (5), we have $Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) \geq Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \cap Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\jmath)$, $Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) \leq Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \cup Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\jmath)$, $Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell \widetilde{\cap} f_\jmath) \leq Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \cup Ł^{\tilde{\delta}}_{\tilde{e}}(f_\jmath)$, $\forall f_\ell, g_\jmath \in \widehat{(£, \mathcal{E})}$.

(2) For any $(f_\ell)_i \in \widehat{(£_i, \mathcal{E})}_i$, one collection $\{(\vartheta_\varphi)^{-1}_i((f_\ell)_i)\}$ and $i \in \Gamma$, for each $\tilde{e} \in \mathcal{E}$, we obtain

$$\begin{aligned} &(\tilde{\top}^{\tilde{\sigma}}_{Ł})_{\tilde{e}}(\vartheta_\varphi)^{-1}_i((f_\ell)_i) \geq Ł^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi)^{-1}_i((f_\ell)_i) \geq (\tilde{\top}^{\tilde{\sigma}}_i)_{(\varphi_i)(\tilde{e})}((f_\ell)_i), \\ &(\tilde{\top}^{\tilde{\varsigma}}_{Ł})_{\tilde{e}}(\vartheta_\varphi)^{-1}_i((f_\ell)_i) \leq Ł^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi)^{-1}_i((f_\ell)_i) \leq (\tilde{\top}^{\tilde{\varsigma}}_i)_{(\varphi_i)(\tilde{e})}((f_\ell)_i), \\ &(\tilde{\top}^{\tilde{\delta}}_{Ł})_{\tilde{e}}(\vartheta_\varphi)^{-1}_i((f_\ell)_i) \leq Ł^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi)^{-1}_i((f_\ell)_i) \leq (\tilde{\top}^{\tilde{\delta}}_i)_{(\varphi_i)(\tilde{e})}((f_\ell)_i). \end{aligned}$$

Therefore, for each $i \in \Gamma$, $(\vartheta_\varphi)_i : (£, (\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{Ł})_{\mathcal{E}}) \to (£_i, (\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_i)_{\mathcal{E}_i})$ is a svnfc-map. Let $(\vartheta_\varphi)_i : (£, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}}) \to (£, (\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_i)_{\mathcal{E}_i})$ be a svnfc-map and $(f_\ell)_i \in \widehat{(£_i, \mathcal{E}_i)}$, $\forall\ i \in \Gamma$ we have,

$$\begin{aligned} &\tilde{\tau}^{\tilde{\sigma}}_{\tilde{e}}((\vartheta_\varphi)^{-1}_i((f_\ell)_i)) \geq (\tilde{\top}^{\tilde{\sigma}}_i)_{(\varphi_i)(\tilde{e})}((f_\ell)_i), \\ &\tilde{\tau}^{\tilde{\varsigma}}_{\tilde{e}}((\vartheta_\varphi)^{-1}_i((f_\ell)_i)) \leq (\tilde{\top}^{\tilde{\varsigma}}_i)_{(\varphi_i)(\tilde{e})}((f_\ell)_i), \\ &\tilde{\tau}^{\tilde{\delta}}_{\tilde{e}}((\vartheta_\varphi)^{-1}_i((f_\ell)_i)) \leq (\tilde{\top}^{\tilde{\delta}}_i)_{(\varphi_i)(\tilde{e})}((f_\ell)_i), \end{aligned}$$

For each $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$ of γ such that $f_\ell = \widetilde{\bigcap_{i=1}^{n}} (\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i})$ we obtain

$$\begin{aligned} &\tilde{\tau}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \geq \bigcap_{i=1}^{n} \tilde{\tau}^{\tilde{\sigma}}_{\tilde{e}}(\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i}) \geq \bigcap_{i=1}^{n} (\tilde{\top}^{\tilde{\sigma}}_{\omega_i})_{(\varphi_{\omega_i})(\tilde{e})}((f_\ell)_{\omega_i}), \\ &\tilde{\tau}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \leq \bigcup_{i=1}^{n} \tilde{\tau}^{\tilde{\varsigma}}_{\tilde{e}}(\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i}) \leq \bigcup_{i=1}^{n} (\tilde{\top}^{\tilde{\varsigma}}_{\omega_i})_{(\varphi_{\omega_i})(\tilde{e})}((f_\ell)_{\omega_i}), \\ &\tilde{\tau}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \leq \bigcup_{i=1}^{n} \tilde{\tau}^{\tilde{\delta}}_{\tilde{e}}(\vartheta_\varphi)^{-1}_{\omega_i}((f_\ell)_{\omega_i}) \leq \bigcup_{i=1}^{n} (\tilde{\top}^{\tilde{\delta}}_{\omega_i})_{(\varphi_{\omega_i})(\tilde{e})}((f_\ell)_{\omega_i}). \end{aligned}$$

It implies for any $\tilde{e} \in \mathcal{E}$

$$\tilde{\tau}^{\tilde{\sigma}}_{\tilde{e}}(f_\ell) \geq Ł^{\tilde{\sigma}}_{\tilde{e}}(f_\ell), \quad \tilde{\tau}^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell) \leq Ł^{\tilde{\varsigma}}_{\tilde{e}}(f_\ell), \quad \tilde{\tau}^{\tilde{\delta}}_{\tilde{e}}(f_\ell) \leq Ł^{\tilde{\delta}}_{\tilde{e}}(f_\ell).$$

By Theorem 5, $\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\tilde{e}} \supseteq \tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{Ł}$.

(3) $\Leftarrow$ suppose that $\vartheta_\varphi : (\mathcal{G}, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}) \to (£, (\tilde{\top}_{Ł}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}})$ is an svnfc-map. For each $i \in \Gamma$ and $(f_\ell)_i \in \widehat{(£_i, \mathcal{E}_i)}$, we obtain

$$\begin{aligned}
&\tilde{\tau}_{\mathcal{F}}^{\tilde{\sigma}}((\varphi_i \circ \varphi)^{-1}_{\vartheta_i \circ \vartheta}((f_\ell)_i))\\
&\quad = \tilde{\tau}_{\mathcal{F}}^{\tilde{\sigma}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_i^{-1}((f_\ell)_i))) \geq (\tilde{\top}_{Ł}^{\tilde{\sigma}})_{\varphi(\mathcal{F})}((\vartheta_\varphi)_i^{-1}((f_\ell)_i))\\
&\quad \geq (\tilde{\top}_i^{\tilde{\sigma}})_{\varphi_i \circ \varphi(\mathcal{F})}((f_\ell)_i),\\
&\tilde{\tau}_{\mathcal{F}}^{\tilde{\varsigma}}((\varphi_i \circ \varphi)^{-1}_{\vartheta_i \circ \vartheta}((f_\ell)_i))\\
&\quad = \tilde{\tau}_{\mathcal{F}}^{\tilde{\varsigma}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_i^{-1}((f_\ell)_i))) \leq (\tilde{\top}_{Ł}^{\tilde{\varsigma}})_{\varphi(\mathcal{F})}((\vartheta_\varphi)_i^{-1}((f_\ell)_i))\\
&\quad \leq (\tilde{\top}_i^{\tilde{\varsigma}})_{\varphi_i \circ \varphi(\mathcal{F})}((f_\ell)_i),\\
&\tilde{\tau}_{\mathcal{F}}^{\tilde{\delta}}((\varphi_i \circ \varphi)^{-1}_{\vartheta_i \circ \vartheta}((f_\ell)_i))\\
&\quad = \tilde{\tau}_{\mathcal{F}}^{\tilde{\delta}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_i^{-1}((f_\ell)_i))) \leq (\tilde{\top}_{Ł}^{\tilde{\delta}})_{\varphi(\mathcal{F})}((\vartheta_\varphi)_i^{-1}((f_\ell)_i))\\
&\quad \leq (\tilde{\top}_i^{\tilde{\delta}})_{\varphi_i \circ \varphi(\mathcal{F})}((f_\ell)_i),
\end{aligned}$$

Thus, $(\varphi_i \circ \varphi)_{\vartheta_i \circ \vartheta} : (\mathcal{G}, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}) \to (£_i, (\tilde{\top}_i^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}_i})$ is an svnfc-map.

$(\Rightarrow)$ For all finite $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$ of Γ such that $f_\ell = \widetilde{\bigcap_{i=1}^{n}} (\vartheta_\varphi)_{\omega_i}^{-1}((f_\ell)_{\omega_i})$ since $(\varphi_i \circ \varphi)_{\vartheta_i \circ \vartheta} : (\mathcal{G}, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}) \to (£_{\omega_i}, (\tilde{\top}_{\omega_i}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}_{\omega_i}})$ is an svnfc-map, for every $p \in \mathcal{F}$

$$\begin{aligned}
&\tilde{\tau}_p^{\tilde{\sigma}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}((f_\ell)_{\omega_i}))) \geq (\tilde{\top}_{\omega_i}^{\tilde{\sigma}})_{(\varphi_i \circ \varphi)(p)}((f_\ell)_{\omega i}),\\
&\tilde{\tau}_p^{\tilde{\varsigma}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}((f_\ell)_{\omega_i}))) \leq (\tilde{\top}_{\omega_i}^{\tilde{\varsigma}})_{(\varphi_i \circ \varphi)(p)}((f_\ell)_{\omega i}),\\
&\tilde{\tau}_p^{\tilde{\delta}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}((f_\ell)_{\omega_i}))) \leq (\tilde{\top}_{\omega_i}^{\tilde{\delta}})_{(\varphi_i \circ \varphi)(p)}((f_\ell)_{\omega i}).
\end{aligned}$$

Hence, we obtain

$$\begin{aligned}
\tilde{\tau}_p^{\tilde{\sigma}}(\vartheta_\varphi^{-1}(f_\ell)) &= \tilde{\tau}_p^{\tilde{\sigma}}(\widetilde{\bigcap_{i=1}^{n}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}(((f_\ell)_{\omega_i})))))\\
&\geq \bigcap_{i=1}^{n} \tilde{\tau}_p^{\tilde{\sigma}}((\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}(((f_\ell)_{\omega_i}))))\\
&\geq \bigcap_{i=1}^{n}(\tilde{\top}_{\omega_i}^{\tilde{\sigma}})_{(\varphi_{\omega_i} \circ \varphi)(p)}(f_\ell)_{\omega_i}),\\
\tilde{\tau}_p^{\tilde{\varsigma}}(\vartheta_\varphi^{-1}(f_\ell)) &= \tilde{\tau}_p^{\tilde{\varsigma}}(\widetilde{\bigcap_{i=1}^{n}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}(((f_\ell)_{\omega_i})))))\\
&\leq \bigcap_{i=1}^{n} \tilde{\tau}_p^{\tilde{\varsigma}}((\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}(((f_\ell)_{\omega_i}))))\\
&\leq \bigcap_{i=1}^{n}(\tilde{\top}_{\omega_i}^{\tilde{\varsigma}})_{(\varphi_{\omega_i} \circ \varphi)(p)}(f_\ell)_{\omega_i}),\\
\tilde{\tau}_p^{\tilde{\delta}}(\vartheta_\varphi^{-1}(f_\ell)) &= \tilde{\tau}_p^{\tilde{\delta}}(\widetilde{\bigcap_{i=1}^{n}}(\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}(((f_\ell)_{\omega_i})))))\\
&\leq \bigcap_{i=1}^{n} \tilde{\tau}_p^{\tilde{\delta}}((\vartheta_\varphi^{-1}((\vartheta_\varphi)_{\omega_i}^{-1}(((f_\ell)_{\omega_i}))))\\
&\leq \bigcap_{i=1}^{n}(\tilde{\top}_{\omega_i}^{\tilde{\delta}})_{(\varphi_{\omega_i} \circ \varphi)(p)}(f_\ell)_{\omega_i}).
\end{aligned}$$

Therefore,

$$\tilde{\tau}_p^{\tilde{\sigma}}(\vartheta_\varphi^{-1}(f_\ell)) \geq Ł_{\varphi(p)}^{\tilde{\sigma}}(f_\ell), \qquad \tilde{\tau}_p^{\tilde{\varsigma}}(\vartheta_\varphi^{-1}(f_\ell)) \leq Ł_{\varphi(p)}^{\tilde{\varsigma}}(f_\ell),$$
$$\tilde{\tau}_p^{\tilde{\delta}}(\vartheta_\varphi^{-1}(f_\ell)) \leq Ł_{\varphi(p)}^{\tilde{\delta}}(f_\ell) \quad \forall \quad p \in \mathcal{F} \ \text{and} \ f_\ell \in \widehat{(£, \mathcal{E})}.$$

By Theorem 6, $\vartheta_\varphi : (\mathcal{G}, (\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}) \to (£, (\tilde{\top}_{Ł}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}})$ is a svnfc-map.

Let $\{(£_i, (\tilde{\top}_i^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}_\rangle})\}_{i \in \Gamma}$ be a collection of svnfts, $\mathcal{E}$ be a parameter set and $\forall\ i \in \Gamma$, $\vartheta_i : £ \to £_i$ and $\varphi_i : \mathcal{E} \to \mathcal{E}_i$ be a mapping. The initial single-valued neutrosophic soft topology $\tilde{\top}_{Ł}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}$ on £ is the coarsest svnfts on £ for which all $(\vartheta_\varphi)_i$, $i \in \Gamma$ are svnfc-maps. □

4 Initial single-valued neutrosophic soft closure spaces

In this segment, we launch the ideas of single-valued neutrosophic soft closure spaces (simply, svnf-closure space). In particular, we prove the existence of initial single-valued neutrosophic soft closure structures. From this fact, the category **SVNSC** is a topological category over **SET**.

Definition 16 A map $\mathbf{C} : \mathcal{E} \times \widehat{(£, \mathcal{E})} \times \xi_1 \to \widehat{(£, \mathcal{E})}$ is considered single-valued neutrosophic soft closure operator on £, if it meets the following conditions for all $\tilde{e} \in \mathcal{E}$, $f_\ell, g_\jmath \in \widehat{(£, \mathcal{E})}$, $r, s \in \xi_1$:

($\mathbf{C}_1$) $\mathbf{C}(\tilde{e}, \Phi, r) = \Phi$,
($\mathbf{C}_2$) $\mathbf{C}(\tilde{e}, f_\ell, r) \widehat{\sqsupseteq} f_\ell$,
($\mathbf{C}_3$) if $f_\ell \widehat{\sqsubseteq} g_\jmath$, then $\mathbf{C}(\tilde{e}, f_\ell, r) \widehat{\sqsubseteq} \mathbf{C}(\tilde{e}, g_\jmath, r)$,
($\mathbf{C}_4$) $\mathbf{C}(\tilde{e}, f_\ell \widehat{\cup} g_\jmath, r) = \mathbf{C}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}(\tilde{e}, g_\jmath, r)$,
($\mathbf{C}_5$) $\mathbf{C}(\tilde{e}, f_\ell, r) \widehat{\sqsubseteq} \mathbf{C}(\tilde{e}, g_\jmath, s)$ if $r \leq s$.

The pair $(£, \mathbf{C})$ is termed svnf-closure space.

An svnfc-closure space $(£, \mathbf{C})$ is addressed as single-valued neutrosophic topological, provided that $\mathbf{C}(\tilde{e}, \mathbf{C}(\tilde{e}, f_\ell, r), r) = \mathbf{C}(\tilde{e}, f_\ell, r)$.

Let $(£, (\mathbf{C}_1)_{\mathcal{E}})$ and $(\mathcal{G}, (\mathbf{C}_2)_{\mathcal{F}})$ be svnf-closure spaces. A mapping $\vartheta_\varphi : (£, (\mathbf{C}_1)_{\mathcal{E}}) \to (\mathcal{G}, (\mathbf{C}_2)_{\mathcal{F}})$ is called single-valued neutrosophic soft **C**-map (simply, svnf**C**-map), if for each $f_\ell \in \widehat{(£, \mathcal{E})}$, $r \in \xi_1$, $\tilde{e} \in \mathcal{E}$,

$$\mathbf{C}_2(\varphi_{\tilde{e}}, \vartheta_\varphi(f_\ell), r) \widehat{\sqsubseteq} \vartheta_\varphi(\mathbf{C}_1(\tilde{e}, f_\ell, r)).$$

Suppose that $(\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}$ and $(\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}}$ are svnfts on £. We say that $(\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}$ is finer than $(\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}}$, $[(\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}}$ is coarser than $(\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}]$, denoted by $(\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}}\widehat{\sqsubseteq}(\tilde{\tau}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{F}}$, if

$$(\tilde{\top}^{\tilde{\sigma}})_{\tilde{e}}(f_\ell) \le (\tilde{\tau}^{\tilde{\sigma}})_{\tilde{e}}(f_\ell), \qquad (\tilde{\top}^{\tilde{\varsigma}})_{\tilde{e}}(f_\ell) \ge (\tilde{\tau}^{\tilde{\varsigma}})_{\tilde{e}}(f_\ell),$$
$$(\tilde{\top}^{\tilde{\delta}})_{\tilde{e}}(f_\ell) \ge (\tilde{\tau}^{\tilde{\delta}})_{\tilde{e}}(f_\ell) \quad \forall \quad \tilde{e} \in \mathcal{E}, \quad f_\ell \in \widehat{(£, \mathcal{E})}, \quad r \in \xi_1.$$

Theorem 8 *Let* (£, $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}$) *be svnfts. For all* $f_\ell \in \xi^{\widehat{(£,\mathcal{E})}}$, $\tilde{e} \in \mathcal{E}$, $r \in \xi_0$, *we define an operator* $\boldsymbol{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}} : \mathcal{E} \times \widehat{(£, \mathcal{E})} \times \xi_0 \to \widehat{(£, \mathcal{E})}$ *as next:*

$$\boldsymbol{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r) = \widehat{\cap}\{g_\jmath \in \widehat{(£, \mathcal{E})} :$$
$$f_\ell \widehat{\sqsubseteq} g_\jmath,\ \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}(g^c_\jmath) \ge r,\ \ \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}(g^c_\jmath) \le 1 - r,\ \ \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}(g^c_\jmath) \le 1 - r\}.$$

Then, (£, $(\boldsymbol{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}})_{\mathcal{E}}$) *is a svnf-closure space.*

Proof $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$ and $\mathbf{C}_5$ follows directly from $\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}$.

$\mathbf{C}_4$: Since $f_\ell \widehat{\sqsubseteq} f_\ell \widehat{\cup} f_\ell$ and $g_\jmath \widehat{\sqsubseteq} f_\ell \widehat{\cup} f_\ell$, we obtain that,

$$\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, g_\jmath, r) \widehat{\sqsubseteq} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell \widehat{\cup} g_\jmath, r).$$

Now we will prove that $\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, g_\jmath, r) \widehat{\sqsupseteq} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell \widehat{\cup} g_\jmath, r)$.

Let (£, $(\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}})_{\mathcal{E}}$) be svnfts. From ($\mathbf{C}_4$), we obtain that

$$f_\ell \widehat{\sqsubseteq} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r),\ \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}}(\tilde{e}, f_\ell, r)]^c)$$
$$\ge r,\ \ \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, f_\ell, r)]^c) \le 1 - r,$$
$$\text{and}\ \ \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, f_\ell, r)]^c) \le 1 - r,$$
$$g_\jmath \widehat{\sqsubseteq} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, g_\jmath, r),\ \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, g_\jmath, r)]^c)$$
$$\ge r,\ \ \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, g_\jmath, r)]^c) \le 1 - r,$$
$$\text{and}\ \ \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, g_\jmath, r)]^c) \le 1 - r,$$

It implies that $f_\ell \widehat{\cup} g_\jmath \widehat{\sqsubseteq} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, g_\jmath, r)$,

$$\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}\ \ ([\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, f_\ell, r) \cup \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, g_\jmath, r)]^c) = \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, f_\ell, r)]^c$$
$$\cap[\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, g_\jmath, r)]^c)$$
$$\ge \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, f_\ell, r)]^c)$$
$$\cap \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}}}(\tilde{e}, g_\jmath, r)]^c) \ge r,$$
$$\tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}\ \ ([\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, f_\ell, r)$$
$$\cup \mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, g_\jmath, r)]^c) = \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, f_\ell, r)]^c$$
$$\cap[\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, g_\jmath, r)]^c)$$
$$\le \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, f_\ell, r)]^c)$$
$$\cup \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\varsigma}}}(\tilde{e}, g_\jmath, r)]^c) \le 1 - r,$$
$$\tilde{\top}^{\tilde{\delta}}_{\tilde{e}}\ \ ([\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, f_\ell, r) \cup \mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, g_\jmath, r)]^c)$$
$$= \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, f_\ell, r)]^c \cap [\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, g_\jmath, r)]^c)$$
$$\le \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, f_\ell, r)]^c)$$
$$\cup \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}([\mathbf{C}_{\tilde{\top}^{\tilde{\delta}}}(\tilde{e}, g_\jmath, r)]^c) \le 1 - r.$$

Hence, $\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, g_\jmath, r) \widehat{\sqsupseteq} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell \widehat{\cup} g_\jmath, r)$. Thus,

$$\mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, g_\jmath, r) = \mathbf{C}_{\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}}(\tilde{e}, f_\ell \widehat{\cup} g_\jmath, r).$$

□

Theorem 9 *Let* (£, $(\boldsymbol{C})_{\mathcal{E}}$) *be a svnf-closure space. Define a mapping* $\tilde{\top}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\boldsymbol{C}} : \mathcal{E} \to \xi^{\widehat{(£,\mathcal{E})}}$ *on £ by: For every* $\tilde{e} \in \mathcal{E}$,

$$(\tilde{\top}^{\tilde{\sigma}}_{\boldsymbol{C}})_{\tilde{e}}(f_\ell) = \bigcup\{r \in \xi : \boldsymbol{C}(\tilde{e}, (f_\ell)^c, r) = (f_\ell)^c\},$$
$$(\tilde{\top}^{\tilde{\varsigma}}_{\boldsymbol{C}})_{\tilde{e}}(f_\ell) = \bigcap\{1 - r \in \xi : \boldsymbol{C}(\tilde{e}, (f_\ell)^c, r) = (f_\ell)^c\},$$
$$(\tilde{\top}^{\tilde{\delta}}_{\boldsymbol{C}})_{\tilde{e}}(f_\ell) = \bigcap\{1 - r \in \xi : \boldsymbol{C}(\tilde{e}, (f_\ell)^c, r) = (f_\ell)^c\}.$$

Then, $\left(\tilde{\top}^{\tilde{\sigma}}_{\boldsymbol{C}}, \tilde{\top}^{\tilde{\varsigma}}_{\boldsymbol{C}}, \tilde{\top}^{\tilde{\delta}}_{\boldsymbol{C}}\right)$ *is a svnfts on £,*

Proof ($\top_1$). Let (£, $(\mathbf{C})_{\mathcal{E}}$) be an svnf-closure space. Since $\mathbf{C}(\tilde{e}, \Phi, r) = \Phi$ and $\mathbf{C}(\tilde{e}, \mathcal{E}, r) = \mathcal{E}$, for each, $\tilde{e} \in \mathcal{E}$, $r \in \xi_0$ we have ($\top_1$).

($\top_2$). Let (£, $(\mathbf{C})_{\mathcal{E}}$) be an svnf-closure space. Assume that there exists $[f_\ell]_1, [f_\ell]_2 \in \xi^{\widehat{(£,\mathcal{E})}}$ such that

$$(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) < (\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1) \cap (\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_2),$$
$$(\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) > (\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1) \cup (\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_2),$$
$$(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) > (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1) \cup (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_2).$$

There exists $r \in \xi_0$ such that

$$(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) < r < (\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1) \cap (\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_2),$$
$$(\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) > 1 - r > (\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1) \cup (\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_2),$$
$$(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) > 1 - r > (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1) \cup (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_2).$$

For each $j \in \{1, 2\}$, there exist $r_i \in \xi_0$ with $\mathbf{C}(\tilde{e}, [f_\ell]^c_i, r_i) = [f_\ell]^c_i$, such that

$$r < r_i \le (\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_i), \quad (\tilde{\top}^{\tilde{\varsigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_i) \le 1 - r_i < 1 - r,$$
$$(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_i) \le 1 - r_i < 1 - r.$$

On the other hand, since $\mathbf{C}(\tilde{e}, [f_\ell]^c_i, r) = [f_\ell]^c_i$ from $\mathbf{C}_2$ and $\mathbf{C}_5$ in Definition 12, for all $i \in \{1, 2\}$, we have

$$\mathbf{C}(\tilde{e}, [f_\ell]^c_1 \widetilde{\cup} [f_\ell]^c_2, r) = [f_\ell]^c_1 \widetilde{\cup} [f_\ell]^c_2.$$

It follows that $(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) \ge r$, $(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2) \le 1 - r$, and $(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}([f_\ell]_1 \widetilde{\cap} [f_\ell]_2)) \le 1 - r$. It is a contradiction. Hence, for all $f_\ell, g_\jmath \in \xi^{\widehat{(£,\mathcal{E})}}$ we have $(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}(f_\ell \widetilde{\cap} g_\jmath) \ge$

$(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) \cap (\tilde{\top}_{\mathbf{C}})^{\tilde{\sigma}}_{\tilde{e}}(g_\jmath)$, $(\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}(f_\ell \tilde{\cap} g_\jmath) \leq (\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) \cup (\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}(g_\jmath)$, $(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}(f_\ell \tilde{\cap} g_\jmath) \leq (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) \cup (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}(g_\jmath)$

$(\top_3)$. Assume that there exists $f_\ell = \bigcup_{i\in\Gamma}(f_\ell)_i \in \xi^{\widehat{(\pounds,\mathcal{E})}}$ such that

$$(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) < \bigcap_{i\in\Gamma}(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i),$$
$$(\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) > \bigcup_{i\in\Gamma}(\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i),$$
$$(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) > \bigcup_{i\in\Gamma}(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i).$$

There exists $r_0 \in \xi_0$ such that

$$(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) < r_0 < \bigcap_{i\in\Gamma}(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i),$$
$$(\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) > 1 - r_0 > \bigcup_{i\in\Gamma}(\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i),$$
$$(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) > 1 - r_0 > \bigcup_{i\in\Gamma}(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i).$$

$\forall\, i \in \Gamma$ and $r_i \in \xi_0$ there exists $\mathbf{C}(\tilde{e}, [f_\ell]^c_i, r_i) = [f_\ell]^c_i$, s.t

$$r_0 < r_i \leq (\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i), \qquad 1 - r_0 > 1 - r_i$$
$$\geq (\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i),$$
$$1 - r_i > 1 - r_0 \geq (\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}((f_\ell)_i).$$

Moreover, since $\mathbf{C}(\tilde{e}, [f_\ell]^c_i, r_0) \leq \mathbf{C}(\tilde{e}, [f_\ell]^c_i, r_i) = [f_\ell]^c_i$ by $\mathbf{C}_2$ in Definition 12, we get that $\mathbf{C}(\tilde{e}, [f_\ell]^c_i, r_i) = [f_\ell]^c_i$. It implies, for all $i \in I$,

$$\mathbf{C}(\tilde{e}, [f_\ell]^c, r_0) \leq \mathbf{C}(\tilde{e}, [f_\ell]^c_i, r_i) = [f_\ell]^c.$$

It follows that $\mathbf{C}(\tilde{e}, [f_\ell]^c, r_0) \leq \bigcap_{i\in\Gamma}[f_\ell]^c_i = [f_\ell]^c$. Hence, $\mathbf{C}(\tilde{e}, [f_\ell]^c, r_0) = [f_\ell]^c$, that is, $(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) \geq r_0$, $(\tilde{\top}^{\tilde{\xi}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) \leq 1 - r_0$ and $(\tilde{\top}^{\tilde{\delta}}_{\mathbf{C}})_{\tilde{e}}(f_\ell) \leq 1 - r_0$. It is a contradiction. Thus, $\left(\tilde{\top}^{\tilde{\sigma}}_{\mathbf{C}}, \tilde{\top}^{\tilde{\xi}}_{\mathbf{C}}, \tilde{\top}^{\tilde{\delta}}_{\mathbf{C}}\right)$ is a svnfts on £. □

Theorem 10 *Allowing $\{(\pounds, (\boldsymbol{C}_i)_{\mathcal{E}_i})\}_{i\in\Gamma}$ to be an aggregation of svnf-closure spaces, let $\vartheta_i : \pounds \to \pounds_i$, $\varphi_i : \mathcal{E} \to \mathcal{E}_i$ be mappings for all $i \in \Gamma$ and £ be a set. Define a map $\boldsymbol{C}_{\tilde{\top}\tilde{\sigma}\tilde{\xi}\tilde{\delta}} : \mathcal{E} \times \widehat{(\pounds, \mathcal{E})} \times \xi_1 \to \widehat{(\pounds, \mathcal{E})}$ over £ as next:*

$$\boldsymbol{C}(\tilde{e}, f_\ell, r) = \widehat{\cap}\left\{\widehat{\bigcup_{i=1}^{n}}(\widehat{\cap}_{i\in\Gamma}(\vartheta_\varphi)^{-1}_i(\boldsymbol{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r)))\right\}.$$

For all $\tilde{e} \in \mathcal{E}$, $f_\ell, g_\jmath \in \widehat{(\pounds, \mathcal{E})}$, $r \in \xi_1$ where the first $\widehat{\cap}$ is taken on all finite aggregations $\{(f_\ell)_i : \quad f_\ell = \widetilde{\bigcup_{i=1}^{n}}((f_\ell)_i)\}$. Then

(1) $\boldsymbol{C}$ is the coarsest single-valued neutrosophic soft closure operator on £, for which all $(\vartheta_\varphi)_i$ are svnf$\boldsymbol{C}$-maps,
(2) if $\{(\pounds, (\boldsymbol{C}_i)_{\mathcal{E}_i})\}_{i\in\Gamma}$ is a aggregation of svnf-closure spaces, then $(\pounds, (\boldsymbol{C})_{\mathcal{E}})$ is a svnfts,
(3) a map $\vartheta_\varphi : (\mathcal{G}, \boldsymbol{C}^*_{\mathcal{F}}) \to (\pounds, \boldsymbol{C}_{\mathcal{E}})$ is a svnf$\boldsymbol{C}$-map iff for all $i \in \Gamma$, $(\vartheta_\varphi)_i \circ \vartheta_\varphi : (\mathcal{G}, \boldsymbol{C}^*_{\mathcal{F}}) \to (\pounds, (\boldsymbol{C}_i)_{\mathcal{E}_i})$ is a svnf$\boldsymbol{C}$-map.

Proof (1) Firstly, we will prove that $\mathbf{C}$ is a single-valued neutrosophic soft closure operator on £.

$\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$ and $\mathbf{C}_5$ follows directly from definition $\mathbf{C}$

$\mathbf{C}_4$: From $(\mathbf{C}_3)$, we get that $\mathbf{C}(\tilde{e}, f_\ell, r)\widehat{\cup}\mathbf{C}(\tilde{e}, g_\jmath, r) \widehat{\sqsubseteq}\mathbf{C}(\tilde{e}, f_\ell\widehat{\cup}g_\jmath, r)$.

Now we show that $\mathbf{C}(\tilde{e}, f_\ell, r)\widehat{\cup}\mathbf{C}(\tilde{e}, g_\jmath, r)\widehat{\sqsupseteq}\mathbf{C}(\tilde{e}, f_\ell\widehat{\cup}g_\jmath, r)$.

For all finite families $\{(f_\ell)_i : \quad f_\ell = \widetilde{\bigcup_{i=1}^{m}}(f_\ell)_i\}$ and $\{(g_\jmath)_i : g_\ell = \widetilde{\bigcup_{i=1}^{n}}(g_\jmath)_i\}$, there exists a finite family $\{(f_\ell)_i, (g_\ell)_i : f_\ell\widetilde{\cup}g_\jmath = (\widetilde{\bigcup_{i=1}^{m}}\left(f_\ell)_i)\widetilde{\cup}(\widetilde{\bigcup_{i=1}^{n}}(g_\jmath)_i\right)\}$, such that

$$\mathbf{C}(\tilde{e}, f_\ell\widetilde{\cup}g_\ell, r)\widehat{\sqsubseteq}\left[\widehat{\bigcup_{i=1}^{m}}(\widehat{\cap}_{i\in\Gamma}(\vartheta_\varphi)^{-1}_i(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r)))\right]\widehat{\cup}$$
$$\left[\widehat{\bigcup_{i=1}^{n}}(\widehat{\cap}_{i\in\Gamma}(\vartheta_\varphi)^{-1}_i(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((g_\jmath)_i), r)))\right].$$

Put $h_\partial = \widehat{\bigcup_{i=1}^{n}}(\widehat{\cap}_{i\in\Gamma}(\vartheta_\varphi)^{-1}_i(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((g_\jmath)_i), r)))$. Then,

$$\begin{aligned}\mathbf{C}(\tilde{e}, f_\ell\widetilde{\cup}g_\ell, r) &\widehat{\sqsubseteq} \widehat{\cap}\left[\widehat{\bigcup_{i=1}^{m}}(\widehat{\cap}_{i\in\Gamma}(\vartheta_\varphi)^{-1}_i(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r))\Big) \;\widehat{\cup}\; h_\partial\right] \\ &= \left[\widehat{\cap}\left(\widehat{\bigcup_{i=1}^{m}}(\widehat{\cap}_{i\in\Gamma}(\vartheta_\varphi)^{-1}_i(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r))))\;\widehat{\cup}\; h_\partial\right] \\ &= \mathbf{C}(\tilde{e}, f_\ell, r) \;\widehat{\cup}\; h_\partial,\end{aligned}$$

where $\widehat{\cup}$ is taken on all finite families $\{(f_\ell)_i : f_\ell = \widetilde{\bigcup_{i=1}^{m}} (f_\ell)_i\}$. Also,

$$\mathbf{C}(\tilde{e}, f_\ell \widetilde{\cup} g_\ell, r) \widehat{\sqsubseteq} \widehat{\cap}\left(\mathbf{C}(\tilde{e}, f_\ell, r)\ \widehat{\cup}\ h_\partial\right)$$
$$= (\mathbf{C}(\tilde{e}, f_\ell, r) \widetilde{\cup} \left[\widehat{\cap}\left[\widehat{\bigcup_{i=1}^{n}} (\widehat{\cap}_{i\in\Gamma} (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}),\right.\right.$$
$$\left.\left.\times(\vartheta_\varphi)_i((g_j)_i), r))))\right]\right]$$
$$= \mathbf{C}(\tilde{e}, f_\ell, r) \widehat{\cup} \mathbf{C}(\tilde{e}, g_j, r),$$

where $\widehat{\cup}$ is taken on all finite families $\{(g_j)_i : f_\ell = \widetilde{\bigcup_{i=1}^{n}} (g_j)_i\}$.

Next, from the definition of $\mathbf{C}$, we have the subsequent regarding the group $\{f_\ell : f_\ell = \widetilde{\bigcup_{i=1}^{n}} (f_\ell)_i\}$,

$$\mathbf{C}(\tilde{e}, f_\ell, r) \ \widehat{\sqsubseteq}\ \widehat{\cap}_{i\in\Gamma} (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r))$$
$$\widehat{\sqsubseteq}\ (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r)),$$

It implies that

$$\vartheta_\varphi(\mathbf{C}(\tilde{e}, f_\ell, r)) \ \widehat{\sqsubseteq}\ (\vartheta_\varphi)_j((\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r)))$$
$$\widehat{\sqsubseteq}\ \mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r),$$

Thus for each $i \in \Gamma$, $(\vartheta_\varphi)_i : (£, \mathbf{C}_{\mathcal{E}}) \to (£_i, (\mathbf{C}_i)_{\mathcal{E}_i})$ is a svnf$\mathbf{C}$-map.

If $(\vartheta_\varphi)_i : (£, \mathbf{C}^*_{\mathcal{F}}) \to (£_i, (\mathbf{C}_i)_{\mathcal{E}_i})$ is a svnf$\mathbf{C}$-map, for every $i \in \Gamma$ and $\tilde{e} \in \mathcal{F}$, then we have

$$(\vartheta_\varphi)_i(\mathbf{C}^*(\tilde{e}, f_\ell, r)) \widehat{\sqsubseteq} \mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r).$$

It implies that

$$\vartheta_\varphi(\mathbf{C}^*(\tilde{e}, f_\ell, r)) \ \widehat{\sqsubseteq}\ (\vartheta_\varphi)_j^{-1}((\vartheta_\varphi)_i(\mathbf{C}^*(\tilde{e}, f_\ell, r)))$$
$$\widehat{\sqsubseteq}\ (\vartheta_\varphi)_j^{-1}(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r)).$$

So we have

$$\vartheta_\varphi(\mathbf{C}^*(\tilde{e}, f_\ell, r)) \ \widehat{\sqsubseteq}\ \widehat{\cap}\ (\vartheta_\varphi)_j^{-1}(\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r)).$$

We have the ensuing in respect of every clusters $\{(f_\ell)_i : f_\ell = \widetilde{\bigcup_{i=1}^{m}} (f_\ell)_i\}$

$$\mathbf{C}(\tilde{e}, f_\ell, r) \ = \ \widehat{\cap}\left[\widehat{\bigcup_{i=1}^{m}} (\widehat{\cap}_{i\in\Gamma} (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}),\right.$$
$$\left.(\vartheta_\varphi)_i((f_\ell)_i), r)))\right]$$
$$\widehat{\sqsupseteq}\ \widehat{\cap}\left[\widehat{\bigcup_{i=1}^{m}} \mathbf{C}^*(\tilde{e}, (f_\ell)_j, r)\right]$$
$$= \widehat{\cap}\left[\mathbf{C}^*(\tilde{e}, \widehat{\bigcup_{i=1}^{m}} (f_\ell)_j, r)\right] = \ \mathbf{C}^*(\tilde{e}, f_\ell, r).$$

Thus, $\mathbf{C}$ is the coarsest single-valued neutrosophic soft closure operator on £.

(2) We will show that $\mathbf{C}(\tilde{e}, \mathbf{C}(\tilde{e}, f_\ell, r), r) = \mathbf{C}(\tilde{e}, f_\ell, r)$, for all $\tilde{e} \in \mathcal{E}, f_\ell, \in \widehat{(£, \mathcal{E})}, r \in \xi_1$.

For all finite families $\{(f_\ell)_i : f_\ell = \widetilde{\bigcup_{i=1}^{m}} (f_\ell)_i\}$, we have

$$\mathbf{C}(\tilde{e}, f_\ell, r) \ = \ \widehat{\cap}\left[\widehat{\bigcup_{i=1}^{m}} (\widehat{\cap}_{i\in\Gamma} (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}),\right.$$
$$\left.(\vartheta_\varphi)_i((f_\ell)_i), r)))\right]$$
$$= \ \widehat{\cap}\left[\widehat{\bigcup_{i=1}^{m}} (\widehat{\cap}_{i\in\Gamma} (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}),\right.$$
$$\left.\mathbf{C}_i(\varphi_i(\tilde{e}), (\vartheta_\varphi)_i((f_\ell)_i), r), r)))\right]$$
$$\widehat{\sqsupseteq}\ \widehat{\cap}\left[\widehat{\bigcup_{i=1}^{m}} (\widehat{\cap}_{i\in\Gamma} (\vartheta_\varphi)_i^{-1} (\mathbf{C}_i(\varphi_i(\tilde{e}),\right.$$
$$\left.(\vartheta_\varphi)_i(\mathbf{C}_i(\tilde{e}, (f_\ell)_i, r)), r)))\right]$$
$$\widehat{\sqsupseteq}\ \mathbf{C}(\tilde{e}, \mathbf{C}(\tilde{e}, f_\ell, r), r).$$

From $(\mathbf{C}_2)$, we have $\mathbf{C}(\tilde{e}, f_\ell, r) = \mathbf{C}(\tilde{e}, \mathbf{C}(\tilde{e}, f_\ell, r), r)$.

(3) Direct. □

The category of svnf-closure spaces and svnf$\mathbf{C}$-maps is denoted by **SVNSC**.

Definition 17 (Alsharari et al. 2021) A category Ç is termed to be a topological category on **SET** with respect to the usual forgetful functor from Ç to **SET** if it meets the following criteria:

($Ç_1$) Existence of initial structures: For every £, any class Γ, and any aggregation $(£_i, \Im_i)_{i\in\Gamma}$ of Ç-object and every aggregation $(f_i : £ \to \Im_i)_{i\in\Gamma}$ of mappings, there exists a unique Ç-structure $\Im$ on £ which is initial with respect to the source $(f_i : £ \to (£_i, \Im_i)_{i\in\Gamma}$, i.e., for a Ç-object $(\mathcal{G}, \Re)$, a mapping $L : (\mathcal{G}, \Re) \to (£, \Im)$ is a Ç-morphism iff for any $i \in \Gamma$, $f_i \circ L : (\mathcal{G}, \Re) \to (£_i, \Im_i)$ is a Ç-morphism.

(Ç$_2$) Fibre smallness: For every set £, the Ç-fibre of £, i.e., the class of all Ç-structure over £, which we denote Ç(£), is a set

Theorem 11 *The forgetful functor* $\aleph$: ***SVNSC*** $\rightarrow$ ***SET*** *defined by* $\aleph(£, \boldsymbol{C}) = £$ *and* $\aleph(\vartheta_\varphi) = \Phi$ *is single-valued neutrosophic topological.*

Proof The proof is straightforward from Theorem 10, and every $\aleph$-structured source $[(\vartheta_\varphi)_i : £ \rightarrow \aleph(£_i, \mathbf{C}_i)_{i\in\Gamma}]$ has a unique $\aleph$-initial left $[(\vartheta_\varphi)_i : (£, \mathbf{C}) \rightarrow \aleph(£_i, \mathbf{C}_i)_{i\in\Gamma}]$, where $\mathbf{C}$ is defined as in Theorem 10. □

Using Theorems 10 and 11, we obtain the following definition.

Definition 18 Let $\{(£_i, (\mathbf{C}_i)_{\mathcal{E}_i})\}_{i\in\Gamma}$ be a family of svnf-closure spaces, for all $i \in \Gamma$, $£ = \prod_{i\in\Gamma} £_i$ and $\mathcal{E} = \prod_{i\in\Gamma} \mathcal{E}_i$. Assume that $P_i : £ \rightarrow £_i$ and $q_i : \mathcal{E} \rightarrow \mathcal{E}_i$ are projection maps for all $i \in \Gamma$. The initial single-valued neutrosophic soft closure operator $\mathbf{C}$ as given in Theorem 10, with respect to the parameter set $\mathcal{E}$, is the coarsest single-valued neutrosophic soft closure operator on £ for which all $(P_q)_i$, $i \in \Gamma$ are svnf$\mathbf{C}$-maps.

5 Conclusions

We have considered the topological structure of single-valued neutrosophic soft set theory. We also have introduced the concept of single-valued neutrosophic soft topology $(\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$ which is a mapping $\tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}} : \mathcal{E} \rightarrow \xi^{\widehat{(£,\mathcal{E})}}$ [where $\mathcal{E}$ is a parameter set] that meet the three specified conditions. Since the value of a single-valued neutrosophic soft set (svnfs) f_ℓ under the mapping $\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\delta}}_{\tilde{e}}$ gives us the degree of openness, the degree of indeterminacy, and the degree of non-openness, respectively, of the svnfs with respect to the parameter $\tilde{e} \in \mathcal{E}$, $(\tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\varsigma}}_{\tilde{e}}, \tilde{\top}^{\tilde{\delta}}_{\tilde{e}})$, which can be thought of as a single-valued neutrosophic soft topology in the sense of Šostak. In this sense, we have presented single-valued neutrosophic soft cotopology and given the relations among single-valued neutrosophic soft topology and single-valued neutrosophic soft cotopology. Then, we have demarcated single-valued neutrosophic soft base $(Ł^{\tilde{\sigma}}, Ł^{\tilde{\varsigma}}, Ł^{\tilde{\delta}})$, and by using a single-valued neutrosophic soft base, we have obtained a single-valued neutrosophic soft topology on the same set. Also, the dual idea of final single-valued neutrosophic soft closure spaces (svnf-closure space) is a typical study and methodically ensured next the same results proved in this paper, and we think that no necessity to improve special study for the final single-valued neutrosophic soft closure structures. Finally, we have showed that the category of single-valued neutrosophic soft topological spaces **SVNTOP** is a topological category on **SET** with respect to the forgetful functor.

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Data Availability Enquiries about data availability should be directed to the authors.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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