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A NEW TYPE OF QUASI OPEN FUNCTIONS IN NEUTROSOPHIC TOPOLOGICAL ENVIRONMENT

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Abstract

Neutrosophic topological space is an generalization of intuitionistic topological space and each neutrosophic set in neutrosophic topological space is triplet set. Intuitionistic topological set deals membership and non-membership values of each variable in open and closed functions and in neutrosophic topology inderterminacy of the same variable has not discussed in the previous work. This motivates us to propose the new type of quasi open and closed functions in neutrosophic topological space. In this paper, we introduce the notion of neutrosophic *quasi αψ-open* and neutrosophic *quasi αψ-closed* functions. Also we investigate some of its fundamental properties and its characterizations.

Keywords. neutrosophic *αψ-closed* set, neutrosophic *αψ-open* set and neutrosophic *quasi αψ-function*.

AMS(2010) Subject classification : 54A05, 54D10, 54F65, 54G05.

1. Introduction

Atanassov [2] defined the notion of intuitionistic fuzzy sets, which is a generalized form of Zadeh's [19] fuzzy set. Using this concept so many research work are brought in the literature, such as intuitionistic fuzzy semi-generalized closed sets [15]. D. Coker [3] initiated the concept of intuitionistic fuzzy topological spaces. F. Smarandache [17, 18] introduced and studied neutrosophic sets (NS). Later, Salama et al.[13, 14] introduced and studied neutrosophic topology. This approach leads to many investigations in this area. Since then more research

have been identified in the field of neutrosophic topology [7, 10, 11, 12, 13, 14], neutrosophic ideals [6, 8, 9], etc.

Many different terms of open functions have been introduced over the course of years. Various interesting problems arise when one consider openness. Its importance is significant in various areas of Mathematics and related sciences.

The notion of neutrosophic $\alpha\psi$-*closed* set was introduced and studied by M. Parimala et al. [7, 11, 12]. In this paper, we will continue the study of related functions by involving neutrosophic $\alpha\psi$-*open* sets. We introduce and characterize the concept of neutrosophic *quasi $\alpha\psi$-functions*.

Through out this paper, spaces means neutrosophic topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X, \tau) \to (Y, \sigma)$ denotes a neutrosophic function f of a space (X, τ) into a space (Y, σ). Let A be a subset of a space X. The neutrosophic closure and the neutrosophic interior of A are denoted by $ncl(A)$ and $nint(A)$, respectively.

2. Preliminaries

This sections contains the collection of some existing definition in [1, 3, 6, 7, 10, 11, 12, 13, 14, 17, 18] which are helpful for this work.

Definition 2.1. Let X and I are a non-empty set and the interval $[0, 1]$, respectively. An NS A is defined by

$$A = \{\langle e, \mu_A(e), \sigma_A(e), \nu_A(e) \rangle : e \in X\}$$

where the mappings of membership μ_A, indeterminacy σ_A and non-membership ν_A respectively defined from non-empty set X to I , $\forall e \in X$ to the set A and with condition that the sum of $\mu_A(e), \sigma_A(e), \nu_A(e)$ should not exceed 3 and greater than zero, $\forall e \in X$.

Definition 2.2. Let the two NSs be of the form $A = \{\langle e, \mu_A(e), \sigma_A(e), \nu_A(e) \rangle : e \in X\}$ and $B = \{\langle e, \mu_B(e), \sigma_B(e), \nu_B(e) \rangle : e \in X\}$. Then

(i) A is a subset of B if and only if membership of A is less than or equal to membership of B, indeterminacy and non-membership of A are respectively, greater than or equal to indeterminacy and non-membership of B.

(ii) $\overline{A} = \{\langle e, \nu_A(e), \sigma_A(e), \mu_A(e) \rangle : e \in X\}$;

(iii) Union of two NS's A and B is set of all maximum of membership of A and B, minimum of indeterminacy and non-membership of A and B, for each $e \in X$.

(iv) Intersection of two NS's A and B is set of all minimum of membership of A and B, maximum of indeterminacy and non-membership of A and B, for each $e \in X$.

Definition 2.3. A neutrosophic topology (NT) on X is a collection τ of NS's in X holds the following properties

(i) 0 and 1 are in τ.

(ii) Union of any NS of τ in τ.

(iii) Intersection of $A, B \in \tau$ in τ;

Note: Every NS in τ is a neutrosphic open sets (NOS) and its complements are neutrosophic closed sets (NCS).

Definition 2.4. Let A be an NS in NTS (X, τ). Then neutrosophic interior of the given NS A is maximum of all NOS contained in A. Neutrosophic closure of the given NS A is minimum of all NOS contains A. Neutrosophic interior of A is denoted by $nint(A)$ and Neutrosophic closure of A is denoted by $ncl(A)$.

Definition 2.5. Let (X, τ) be a neutrosophic topological space and a subset A of (X, τ) is called

1. a neutrosophic pre-open set, if A subset of neutrosophic interior of neutrosophic closure of A and a neutrosophic pre-closed set if the complement is a neutrosophic pre-open set.

2. a neutrosophic semi-open set, if A is a subset of neutrosophic closure of neutrosophic interior of A and a neutrosophic semi-closed set if the complement is a neutrosophic semi-open set.

3. a neutrosophic α-open set, if A is a subset of neutrosophic interior of neutrosophic closure of neutrosophic interior of A and a neutrosophic α-closed set if complement is a neutrosophic α-open set.

Definition 2.6. A subset A of an neutrosophic topological space (X, τ) is called

1. an neutrosophic semi-generalized closed (briefly, nsg-closed) set if intersection of all neutrosophic semi closed sets which contains A is a subset of U whenever A is a subset of U and U is semi-open in (X, τ).

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2. an neutrosophic ψ-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nsg-open in (X, τ).

3. an neutrosophic $\alpha\psi$-closed (briefly, $n\alpha\psi$-closed) set if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\alpha$-open in (X, τ).

Definition 2.7. Let A be an NS in neutrosophic topological space (X, τ). Then

1. $n\alpha\psi$ interior of A is the minimum of $n\alpha\psi$OS in X contained in A and it is denoted by $n\alpha\psi int(A)$.

2. $n\alpha\psi$-closure of A is the maximum of $n\alpha\psi$CS in X which contains A. and it is denoted by $n\alpha\psi cl(A)$.

3. *On Neutrosophic Quasi $\alpha\psi$-Open Functions*

We introduce the following definition.

Definition 3.1. A neutrosophic function $f : X \to Y$ is said to be neutrosophic *quasi $\alpha\psi$-open* (briefly, nq-$\alpha\psi$-open), if the image of every neutrosophic $\alpha\psi$-open set in X is neutrosophic *open* in Y .

It is evident that, the the concepts of neutrosophic *quasi $\alpha\psi$-openness* and neutrosophic $\alpha\psi$-continuity coinside if the function is a bijection.

Theorem 3.2. A neutrosophic function $f : X \to Y$ is nq-$\alpha\psi$-open if and only if for every subset U of X, $f(n\alpha\psi int(U)) \subset nint(f(U))$.
Proof. Let f be a nq-$\alpha\psi$-open function. Now, we have $nint(U) \subset U$ and $n\alpha\psi int(U)$ is a $n\alpha\psi$-open set. Hence, we obtain that $f(n\alpha\psi int(U)) \subset f(U)$. As $f(n\alpha\psi int(U))$ is open,$f(n\alpha\psi int(U)) \subset nint(f(U))$.

Conversely, assume that U is a $n\alpha\psi$-open set in X. then, $f(U) = f(n\alpha\psi int(U)) \subset nint(f(U))$ but $nint(f(U)) \subset f(U)$. Consequently, $f(U) = nint(f(U))$ and hence f is nq-$\alpha\psi$- open.

Theorem 3.3. If a neutrosophic function $f : X \to Y$ is nq-$\alpha\psi$-open, then $n\alpha\psi int(f^{-1}(G)) \subset f^{-1}(nint(G))$ for every subset G of Y.
Proof. Let G be any arbitrary subset of Y. Then, $n\alpha\psi int(f^{-1}(G))$ is a $n\alpha\psi$-open set in X and f is nq-$\alpha\psi$-open, then $f(n\alpha\psi int(f^{-1}(G))) \subset nint(f(f^{-1}(G))) \subset nint(G)$. Thus, $n\alpha\psi int(f^{-1}(G)) \subset f^{-1}(nint(G))$.

Definition 3.4. A subset A is said to be an *$n\alpha\psi$-neighbourhood* of a point x of X if there exists a *$n\alpha\psi$-open* set U such that $x \in U \subset A$.

Theorem 3.5. For a neutrosophic function $f : X \to Y$, the following are equivalent

(i) f is *nq-$\alpha\psi$-open*;

(ii) for each subset U of X, $f(n\alpha\psi int(U)) \subset nint(f(U))$;

(iii) for each $x \in X$ and each *$n\alpha\psi$-neighbourhood* U of x in X, there exists a neighbourhood V of $f(x)$ in Y such that $V \subset f(U)$.

Proof. (i) $\Rightarrow$ (ii) It follows from Theorem 3.1.
(ii) $\Rightarrow$ (iii) Let $x \in X$ and U be an arbitrary *$n\alpha\psi$-neighbourhood* of $x \in X$. Then, there exists a *$n\alpha\psi$-open* set V in X such that $x \in V \subset U$. Then by (ii), we have $f(V) = f(n\alpha\psi int(V)) \subset nint(f(V))$ and hence $f(V)$ is open in Y such that $f(x) \in f(V) \subset f(U)$.
(iii) $\Rightarrow$ (i) Let U be an arbitrary *$n\alpha\psi$-open* set in X. Then for each $y \in f(U)$, by (iii) there exists a neghbourhood V_y of y in Y such that $V_y \subset f(U)$. As V_y is a neighbourhood of y, there exists an open set W_y in Y such that $y \in W_y \subset V_y$. Thus $f(U) = \bigcup\{W_y : y \in f(U)\}$ which is an open se in Y. This implies that f is *nq-$\alpha\psi$-open* function.

Theorem 3.6. A neutrosophic functon $f : X \to Y$ is *nq-$\alpha\psi$-open* if and only if for any subset B of Y and for any *$n\alpha\psi$-closed* set F of X containing $f^{-1}(B)$, there exists a neutrosophic closed set G of Y containing B such that $f^{-1}(G) \subset F$.
Proof. Suppose f is *nq-$\alpha\psi$-open*. Let $B \subset Y$ and F be a *$n\alpha\psi$-closed* set of X containing $f^{-1}(B)$. Now, put $G = Y - f(X - F)$. It is clear that $f^{-1}(B) \subset F \Rightarrow B \subset G$. Since f is *nq-$\alpha\psi$- open*, we obtain G as a neutrosophic closed set of Y. Moreover, we have $f^{-1}(G) \subset F$.
Conversely, let U be a *$n\alpha\psi$-open* set of X and put $B = Y - f(U)$. Then $X - U$ is a *$n\alpha\psi$-closed* set in X containing $f^{-1}(B)$. By hypothesis, there exists a neutrosophic closed set F of Y such that $B \subset F$ and $f^{-1}(F) \subset X - U$. Hence, we obtain $f(U) \subset Y - F$. On the other hand, it follows that $B \subset F, Y - F \subset Y - B = f(U)$. Thus we obtain $f(U) = Y - F$ which is neutrosophic open and hence f is a *nq-$\alpha\psi$-open* function.

Theorem 3.7 A neutrosophic function $f : X \to Y$ is *nq-$\alpha\psi$-open* if and only

if $f^{-1}(cl(B)) \subset n\alpha\psi cl(f^{-1}(B))$ for every subset B of Y.

Proof. Suppose that f is *nq-αψ-open*. For any subset B of Y, $f^{-1}(B) \subset \alpha\psi$-$cl(f^{-1}(B))$. Therefore, by Theorem 3.5 there exists a neutrosophic closed set F in Y such that $B \subset F$ and $(f^{-1}(F)) \subset n\alpha\psi cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(ncl(B)) \subset (f^{-1}(F)) \subset n\alpha\psi cl(f^{-1}(B))$.

Conversely, let $B \subset Y$ and F be a *nαψ-closed* set of X containing $f^{-1}(B)$. Put $W = ncl_Y(B)$, then we have $B \subset W$ and W is neutrosophic closed and $f^{-1}(W) \subset n\alpha\psi$-$cl(f^{-1}(B)) \subset F$. Then by Theorem 3.6., f is *nq-αψ-open*.

Theorem 3.8. Two neutrosophic function $f : X \to Y$ and $g : Y \to Z$ and $g \circ f : X \to Z$ is *nq-αψ-open*. If g is continuous injective function, then f is *nq-αψ-open*.

Proof. Let U be a *nαψ-open* set in X, then $(g \circ f)(U)$ is open in Z, since $g \circ f$ is *nq-αψ-open*. Again g is an injective continuous function, $f(U) = g^{-1}(g \circ f(U))$ is open in Y. This shows that f is *nq-αψ-open*

4. On Neutrosophic Quasi αψ-Closed Functions

Definition 4.1. A neutrosophic function $f : X \to Y$ is said to be neutrosophic *quasi αψ-closed* (briefly, *nq-αψ-closed*), if the image of every neutrosophic *αψ-closed* set in X is neutrosophic *closed* in Y .

Theorem 4.2. Every *nq-αψ-closed* function is neutrosophic *closed* as well as neutrosophic *αψ-closed*.

Proof. It is obvious.

The converse of the above theorem need not be true by the following example.

Example 4.3. Let a neutrosophic function $f : X \to Y$.

Let $X = \{p, q, r\}$ and $\tau_{N1} = \{0, A, B, C, D, 1\}$ ia a neutrosophic topology on X,
Where

$A = \left\langle x, \left(\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}\right), \left(\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$,

$B = \left\langle x, \left(\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$,

$C = \left\langle x, \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}\right), \left(\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$,

$D = \left\langle x, \left(\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$ and

Let $Y = \{p, q, r\}$ and $\tau_{N2} = \{0, E, F, G, H, 1\}$ is a neutrosophic topology on Y,
Where

$E = \left\langle y, \left(\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}\right)\right\rangle$,

$F = \left\langle y, \left(\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}\right), \left(\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle,$

$G = \left\langle y, \left(\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}\right), \left(\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle,$

$H = \left\langle y, \left(\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}\right)\right\rangle.$

Here $f(p) = p, f(q) = q, f(r) = r$. Then clearly f is $n\alpha\psi$-*closed* as well as neutrosophic *closed* but not nq-$\alpha\psi$-*closed*.

Lemma 4.4. If a neutrosophic function is nq-$\alpha\psi$-*closed*, then $f^{-1}(nint(B)) \subset n\alpha\psi int(f^{-1}(B))$ for every subset B of Y.

Proof. Let B any arbitrary subset of Y. Then, $n\alpha\psi int(f^{-1}(G))$ is a $n\alpha\psi$-*closed* set in X and f is nq-$\alpha\psi$-*closed*, then $f(n\alpha\psi\text{-}nint(f^{-1}(B))) \subset nint(f(f^{-1}(B))) \subset nint(B)$. Thus, $f(n\alpha\psi int(f^{-1}(B))) \subset f^{-1}(nint(B))$.

Theorem 4.5. A neutrosophic function $f : X \to Y$ is nq-$\alpha\psi$-*closed* if and only if for any subset B of Y and for any $n\alpha\psi$-*open* set G of X containing $f^{-1}(B)$, there exists an open set U of Y containing B such that $f^{-1}(U) \subset G$.
Proof This proof is similar to that of theorem 3.6.

Definition 4.6. A neutrosophic function $f : X \to Y$ is called $n\alpha\psi^*$-*closed* if the image of every $n\alpha\psi$-*closed* subset of X is $n\alpha\psi$-*closed* in Y.

Theorem 4.7. If $f : X \to Y$ and $g : Y \to Z$ be any nq-$\alpha\psi$-*closed* functions, then $g \circ f : X \to Z$ is a nq-$\alpha\psi$-*closed* function.
Proof. It is obvious.

Theorem 4.8. Let $f : X \to Y$ and $g : Y \to Z$ be any two neutrosophic functions, then

(i) If f is $n\alpha\psi$-*closed* and g is nq-$\alpha\psi$-*closed*, then $g \circ f$ is neutrosophic *closed*;

(ii) If f is nq-$\alpha\psi$-*closed* and g is nq-$\alpha\psi$-*closed*, then $g \circ f$ is $n\alpha\psi^*$-*closed*;

(iii) If f is $n\alpha\psi^*$-*closed* and g is nq-$\alpha\psi$-*closed*, then $g \circ f$ is nq-$\alpha\psi$-*closed*.

Proof. It is obvious.

Theorem 4.9. Let $f : X \to Y$ and $g : Y \to Z$ be any two neutrosophic functions such that $g \circ f : X \to Z$ is nq-$\alpha\psi$-*closed*.

(i) If f is $n\alpha\psi$-*irresolute* surjective, then g is is neutrosophic *closed*;

(ii) If g is *$n\alpha\psi$-continuous* injective, then f is *$n\alpha\psi^*$-closed*.

Proof. (i) Suppose that F is an arbitrary neutrosophic closed set in Y. As f is *$n\alpha\psi$- irresolute*, $f^{-1}(F)$ is *$n\alpha\psi$-closed* in X. Since $g \circ f$ is *nq-$\alpha\psi$-closed* and f is surjective, $(g \circ f)(f^{-1}(F)) = g(F)$, which is closed in Z. This implies that g is a neutrsophic closed function.

(ii) Suppose F is any *$n\alpha\psi$-closed* set in X. Since $g \circ f$ is *nq-$\alpha\psi$-closed*, $(g \circ f)(F)$ is neutrosophics closed in Z. Again g is a *$n\alpha\psi$-continuous* injective function, $g^{-1}(g \circ f(F)) = f(F)$, which is *$n\alpha\psi$-closed* in Y. This shows that f is *$n\alpha\psi^*$-closed*.

Theorem 4.10. Let X and Y be neutrosophic topological spaces. Then the function $f : X \to Y$ is a *nq-$\alpha\psi$-closed* if and only if $f(X)$ is neutrosophic closed in Y and $f(V) - f(X - V)$ is open in $f(X)$ whenever V is *$n\alpha\psi$-open* in X.

Proof. Necessity: Suppose $f : X \to Y$ is a *nq-$\alpha\psi$-closed* function. Since X is *$n\alpha\psi$-closed*, $f(X)$ is neutrosophic closed in Y and $f(V) - f(X - V) = f(V) \cap f(X) - f(X - V)$ is neutrosophic open in $f(X)$ when V is *$n\alpha\psi$-open* in X.

Sufficiency: Suppose $f(X)$ is neutrosophic closed in Y, $f(V) - f(X - V)$ is neutrosophic open in $f(X)$ when V is *$n\alpha\psi$-open* in X and let C be neutrosophic closed in X. Then $f(C) = f(X) - (f(C - X) - f(C))$ is neutrosophic closed in $f(X)$ and hence neutrosophic closed in Y.

Corollary 4.11. Let X and Y be two neutrosophic topological spaces. Then a surjective function $f : X \to Y$ is *nq-$\alpha\psi$-closed* if and only if $f(V) - f(X - V)$ is open in Y whenever U is *$n\alpha\psi$-open* in X.

Proof. It is obvious.

Theorem 4.12. Let X and Y be neutrosophic topological spaces and let $f : X \to Y$ be *$n\alpha\psi$-continuous* and *nq-$\alpha\psi$-closed* surjective function. Then the topology on Y is $\{f(V) - f(X - V) : V$ is *$n\alpha\psi$-open* in $X\}$.

Proof. Let W be open in Y. Let $f^{-1}(W)$ is *$n\alpha\psi$-open* in X, and $f(f^{-1}(W)) - f(X - f^{-1}(W)) = W$. Hence all open sets an Y are of the form $f(V) - f(X - V)$, V is *$n\alpha\psi$-open* in X. On the other hand, all sets of the form $f(V) - f(X - V)$. V is *$n\alpha\psi$-open* in X, are neutrosophic open in Y from corollary 4.11.

Definition 4.13. A neutrosophic topological space (X, τ) is said to be *$n\alpha\psi$-normal* if for any pair of disjoint *$n\alpha\psi$-closed* subsets F_1 and F_2 of X, there exists

disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 4.14. Let X and Y be a neutrosophic topological spaces with X is $n\alpha\psi$-*normal*. If $f : X \to Y$ is $n\alpha\psi$-*continuous* and nq-$\alpha\psi$-*closed* surjective function. Then Y is *normal*.

Proof. Let K and M be disjoint neutrosophic closed subsets of Y. Then $f^{-1}(K)$, $f^{-1}(M)$ are disjoint $n\alpha\psi$-*closed* subsets of X. Since X is $n\alpha\psi$-*normal*, there exists disjoint neutrosophic open sets V and W such that $f^{-1}(K) \subset V$, $f^{-1}(M) \subset W$. Then $K \subset f(V) - f(X - V)$ and $M \subset f(W) - f(X - W)$, further by corollary 4.11, $f(V) - f(X - V)$ and $f(W) - f(X - W)$ are neutrosophic open sets in Y and clearly $(f(V) - f(X - V)) \cap (f(W) - f(X - W)) = n\phi$. This shows that Y is *normal*.

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