The Analysis Of Refined Neutrosophic Complex Numbers

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Abstract: In this paper, we define refined neutrosophic complex number, by forms cartesian, and some application of it. The main objective is to define powers and roots of neutrosophic complex number, Also, we define the concept of refined neutrosophic complex function, as well as we determine the conditions of Cauchy-Riemann, In addition, we give the algorithm of determining the harmonic conjugate.

Definition 3.1.

We define a refine neutrosophic complex number by the following form:

, where $a_o, a_1, a_2, b_o, b_1, b_2$are real coeffients. For $z = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$ example:

$$z = (1 - I_1 + 2I_2) + i(3 + 2I_1 - I_2)$$

We recall $a_o + a_1 I_1 + a_2 I_2$ the real part, then it takes the following standard form $Re(z) = a_o + a_1 I_1 + a_2 I_2$.

We recall $b_o + b_1 I_1 + b_2 I_2$ the imagine part, then it takes the following standard form $Im(z) = b_o + b_1 I_1 + b_2 I_2$.

Remark 3.2

A refined neutrosophic complex number can be defined as follows:

$\boldsymbol{z = a + bI_1 + cI_2}$where $\boldsymbol{a, b, c}$are complex number. For example:

$$\boldsymbol{z = (1 - i) + (2 + i)I_1 + (3 - 2i)I_2}.$$

4. A neutrosophic complex Function.

Definition 4.1

Let $z = (x + I_1 + I_2) + i(y + I_1 + I_2), w = (u + I_1 + I_2) + i(v + I_1 + I_2)$, Then we call the function:

$$w = f(z) \Rightarrow w = (u + I_1 + I_2) + i(v + I_1 + I_2) = f\big((x + I_1 + I_2) + i(y + I_1 + I_2)\big)$$

Is a refined neutrosophic complex Function.

Example4.2. Let $w = f(z) = |z|^2$ findthe real part and imagine part.

Solution.

Let $\boldsymbol{z = (x + I_1 + I_2) + i(y + I_1 + I_2), w = (u + I_1 + I_2) + i(v + I_1 + I_2)}$, then:

$$w = (u + I_1 + I_2) + i(v + I_1 + I_2) = \left(\sqrt{(x + I_1 + I_2)^2 + (y + I_1 + I_2)^2}\right)^2$$

$$\begin{aligned}\Rightarrow w &= (u + I_1 + I_2) + i(v + I_1 + I_2) \\ &= x^2 + 2xI_1 + I_1 + 2xI_2 + 2I_1 + I_2 + y^2 + 2yI_1 + I_1 + 2yI_2 + 2I_1 + I_2\end{aligned}$$

$$\Rightarrow \boldsymbol{w} = (\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2}) + \boldsymbol{i}(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2}) = \boldsymbol{x^2} + \boldsymbol{y^2} + (\boldsymbol{2x} + \boldsymbol{2y} + \boldsymbol{6})\boldsymbol{I_1} + (\boldsymbol{2x} + \boldsymbol{2y} + \boldsymbol{2})\boldsymbol{I_2}$$

$$(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2}) = \boldsymbol{x^2} + \boldsymbol{y^2} + (\boldsymbol{2x} + \boldsymbol{2y} + \boldsymbol{6})\boldsymbol{I_1} + (\boldsymbol{2x} + \boldsymbol{2y} + \boldsymbol{2})\boldsymbol{I_2}\boldsymbol{I_1}\,, (\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2}) = \boldsymbol{0} + \boldsymbol{0}\boldsymbol{I_1} + \boldsymbol{0}\boldsymbol{I_2}$$

Definition4.3.Cauchy-Riemann conditions.

Suppose that $\boldsymbol{w} = \boldsymbol{f}(\boldsymbol{z})$is a refined neutrosophic complex Function, where $\boldsymbol{z} = (\boldsymbol{x} + \boldsymbol{I_1} + \boldsymbol{I_2}) + \boldsymbol{i}(\boldsymbol{y} + \boldsymbol{I_1} + \boldsymbol{I_2})$

, Cauchy-Riemann conditionsby Cartesian defined by the $w = (u + I_1 + I_2) + i(v + I_1 + I_2)$ following form:

$$\begin{cases} \dfrac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} = \dfrac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{y}} \\ \dfrac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} = -\dfrac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{y}} \end{cases} \dots\dots(\boldsymbol{2})$$

And derivate for function $\boldsymbol{w} = \boldsymbol{f}(\boldsymbol{z})$defined by the following form:

$$\acute{\boldsymbol{f}}(\boldsymbol{z}) = \frac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} + \boldsymbol{i}\frac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} \;\; \boldsymbol{or} \; \acute{\boldsymbol{f}}(\boldsymbol{z}) = \frac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{y}} - \boldsymbol{i}\frac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{y}} \dots\dots(\boldsymbol{3})$$

Example4.4. Let $\boldsymbol{f}(\boldsymbol{z}) = \mathbf{z^2}$, prove $\acute{\boldsymbol{f}}(\boldsymbol{z}) = \boldsymbol{2z}$.

Solution.

Let $\boldsymbol{z} = (\boldsymbol{x} + \boldsymbol{I_1} + \boldsymbol{I_2}) + \boldsymbol{i}(\boldsymbol{y} + \boldsymbol{I_1} + \boldsymbol{I_2}), \boldsymbol{w} = (\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2}) + \boldsymbol{i}(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})$

then:

$(u + I_1 + I_2) + i(v + I_1 + I_2) = x^2 - y^2 + 2(x - y)I_1 + 2(x - y)I_2 + i\,[2xy + 2(x + y + ,3)I_1 + 2(x + y + 1)I_2]$

$$\Rightarrow (u + I_1 + I_2) = x^2 - y^2 + 2(x - y)I_1 + 2(x - y)I_2$$

$$\Rightarrow (v + I_1 + I_2) = 2xy + 2(x + y + 3)I_1 + 2(x + y + 1)I_2$$

Then:

$$\frac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} = 2x + 2I_1 + 2I_2, \frac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial y} = -2y - 2I_1 - 2I_2$$

$$\frac{\partial(v + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} = 2y + 2I_1 + 2I_2, \frac{\partial(v + I_1 + I_2)}{\partial y} = 2x + 2I_1 + 2I_2$$

$$\Rightarrow \begin{cases} \dfrac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} = \dfrac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{y}} \\ \dfrac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} = -\dfrac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{y}} \end{cases}$$

Cauchy-Riemann conditions is satisfytion. Then we have:

$$\acute{\boldsymbol{f}}(\boldsymbol{z}) = \frac{\partial(\boldsymbol{u} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} + \boldsymbol{i}\frac{\partial(\boldsymbol{v} + \boldsymbol{I_1} + \boldsymbol{I_2})}{\partial \boldsymbol{x}} \Rightarrow \acute{\boldsymbol{f}}(\boldsymbol{z}) = \boldsymbol{2x} + \boldsymbol{2I_1} + \boldsymbol{2I_2} + \boldsymbol{i}(\boldsymbol{2y} + \boldsymbol{2I_1} + \boldsymbol{2I_2})$$
$$= \boldsymbol{2}\big(\boldsymbol{x} + \boldsymbol{I_1} + \boldsymbol{I_2} + \boldsymbol{i}(\boldsymbol{y} + \boldsymbol{I_1} + \boldsymbol{I_2})\big) = \boldsymbol{2z}$$

$\Rightarrow \acute{f}(z) = 2z$

5. A refined neutrosophic complex Harmonic Function.

Definition 5. 1.

Suppose that $h = h(x + I_1 + I_2, y + I_1 + I_2)$is a neutrosophic real Function, we say $h(x + I_1 + I_2, y + I_1 + I_2)$ is a neutrosophic harmonic Function, if satisfy the Laplace equation:

$$\frac{\partial 2h}{\partial x^2} + \frac{\partial 2h}{\partial y^2} = 0 \ldots\ldots (4)$$

Definition 5. 2. A harmonic conjugate.

Suppose that $(u + I_1 + I_2), (v + I_1 + I_2)$ is aneutrosophicharmonic Functions, we say $(v + I_1 + I_2)$ is a harmonic conjugate by $(u + I_1 + I_2)$, if $(u + I_1 + I_2), (v + I_1 + I_2)$ are satisfy Cauchy- Riemann conditions.

Example5.3. Let $f(z) = z^2$.

1- Prove $(u + I_1 + I_2), (v + I_1 + I_2)$ are neutrosophic harmonic functions.

2- Find the harmonic conjugate$(v + I_1 + I_2)$.

Solution.

1- Let $z = (x + I_1 + I_2) + i(y + I_1 + I_2), w = (u + I_1 + I_2) + i(v + I_1 + I_2)$, then:

$$\begin{aligned}(u + I_1 + I_2) + i(v + I_1 + I_2) &= x^2 - y^2 + 2(x - y)I_1 + 2(x - y)I_2 \\ &+ i\,[2xy + 2(x + y + 3)I_1 + 2(x + y + 1)I_2]\end{aligned}$$

$$\Rightarrow (u + I_1 + I_2) = x^2 - y^2 + 2(x - y)I_1 + 2(x - y)I_2$$

$$\Rightarrow (v + I_1 + I_2) = 2xy + 2(x + y + 3)I_1 + 2(x + y + 1)I_2$$

Then:

$$\frac{\partial(u + I_1 + I_2)}{\partial x} = 2x + 2I_1 + 2I_2, \frac{\partial(u + I_1 + I_2)}{\partial y} = -2y - 2I_1 - 2I_2$$

$$\frac{\partial(v + I_1 + I_2)}{\partial x} = 2y + 2I_1 + 2I_2, \frac{\partial(v + I_1 + I_2)}{\partial y} = 2x + 2I_1 + 2I_2$$

$$\Rightarrow \begin{cases} \dfrac{\partial 2(u + I_1 + I_2)}{\partial x^2} = 2\,, \dfrac{\partial 2(u + I_1 + I_2)}{\partial y^2} = -2 \\ \dfrac{\partial 2(v + I_1 + I_2)}{\partial x^2} = 0\,, \dfrac{\partial 2(v + I_1 + I_2)}{\partial y^2} = 0 \end{cases}$$

We have:

$$\frac{\partial 2(u + I_1 + I_2)}{\partial x^2} + \frac{\partial 2(u + I_1 + I_2)}{\partial y^2} = 2 - 2 = 0$$

The function $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})$ satisfy Laplac equation, so $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})$is aneutrosophicharmonic Functions.

Similary we have:

$$\frac{\boldsymbol{\partial 2(v+I_1+I_2)}}{\boldsymbol{\partial x^2}}+\frac{\boldsymbol{\partial 2(v+I_1+I_2)}}{\boldsymbol{\partial y^2}}=\boldsymbol{0+0=0}$$

The function $(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})$ satisfy Laplace equation, so $(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})$is a neutrosophic harmonic Functions.

2- We have:

$$\begin{cases}\dfrac{\boldsymbol{\partial(u+I_1+I_2)}}{\boldsymbol{\partial x}}=\dfrac{\boldsymbol{\partial(v+I_1+I_2)}}{\boldsymbol{\partial y}}\\ \dfrac{\boldsymbol{\partial(v+I_1+I_2)}}{\boldsymbol{\partial x}}=-\dfrac{\boldsymbol{\partial(u+I_1+I_2)}}{\boldsymbol{\partial y}}\end{cases}$$

Then $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2}),(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})$ are satisfy Cauchy Riemann conditions, forever $(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})$ is a harmonic conjugate by $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})$.

Example5.4. Let $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})=\boldsymbol{2(x+I_1+I_2)-2(x+I_1+I_2)(y+I_1+I_2)}$.Finde Find the harmonic conjugate$(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})$ and write $\boldsymbol{f(z)}$by z .

Solution.

1- We prove the function $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})$ is a neutrosophic harmonic function.

$$\frac{\boldsymbol{\partial(u+I_1+I_2)}}{\boldsymbol{\partial x}}=2-2(y+\boldsymbol{I_1}+I_2)\Rightarrow\frac{\partial 2(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x^2}=0$$

$$\frac{\partial(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y}=-2(x+\boldsymbol{I_1}+\boldsymbol{I_2})\Rightarrow\frac{\partial 2(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y^2}=0$$

Then:

$$\frac{\partial 2(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x^2}+\frac{\partial 2(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y^2}=0+0=0$$

Then $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})$ is a neutrosophic harmonic Function.

2- We use the first condition of Cauchy Riemann conditions. Then:

$$\frac{\boldsymbol{\partial(u+I_1+I_2)}}{\boldsymbol{\partial x}}=\frac{\boldsymbol{\partial(v+I_1+I_2)}}{\boldsymbol{\partial y}}\Rightarrow\frac{\boldsymbol{\partial(v+I_1+I_2)}}{\boldsymbol{\partial y}}=\boldsymbol{2-2(y+I_1+I_2)}\dots\dots(\boldsymbol{5})$$

3- We integral $(\boldsymbol{5})$ for $(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})$, we have:

$$\int\frac{\boldsymbol{\partial(v+I_1+I_2)}}{\boldsymbol{\partial y}}d(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})=\int\big(2-2(y+\boldsymbol{I_1}+I_2)\big)d(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})+\psi(x+\boldsymbol{I_1}+I_2)$$

$$\Rightarrow(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})=2(y+\boldsymbol{I_1}+\boldsymbol{I_2})-(y+\boldsymbol{I_1}+I_2)^2+\psi(x+\boldsymbol{I_1}+I_2)\dots\dots(6)$$

Where $\boldsymbol{\psi(x+I_1+I_2)}$ is a constant integral.

4- We derivate $(\boldsymbol{6})$ by $(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})$, we have:

$$\frac{\partial(v+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial(x)}=\acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

5- We use the second condition of Cauchy Riemann conditions. Then:

$$\frac{\partial(v+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial(x)}=-\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial(y)}\Rightarrow\acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})=2(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

By integrating the latter, we obtain:

$$\int\acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})dx=\int 2(x+\boldsymbol{I_1}+\boldsymbol{I_2})d(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\Rightarrow\psi(x+\boldsymbol{I_1}+\boldsymbol{I_2})=(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2+a+bI_1+cI_2$$

6- we obtain:

$$(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})=\boldsymbol{2}(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})-(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{2}}+(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{2}}+\boldsymbol{a}+\boldsymbol{bI_1}+\boldsymbol{cI_2}$$

Now:

$$f(z)=(u+\boldsymbol{I_1}+\boldsymbol{I_2})+i(v+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\Rightarrow f(z)=2(x+\boldsymbol{I_1}+\boldsymbol{I_2})-2(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$+i(2(y+\boldsymbol{I_1}+\boldsymbol{I_2})-(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2+\boldsymbol{a}+\boldsymbol{bI_1}+\boldsymbol{cI_2})$$

$$\Rightarrow f(z)=2(x+\boldsymbol{I_1}+\boldsymbol{I_2})-2(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})+i2(y+\boldsymbol{I_1}+\boldsymbol{I_2})-i(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$
$$+i(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2+i(\boldsymbol{a}+\boldsymbol{bI_1}+\boldsymbol{cI_2})$$

$$\Rightarrow f(z)=2\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)+i\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})^2-(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+i2(x+\boldsymbol{I_1}+$$
$$,\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)+i(\boldsymbol{a}+\boldsymbol{bI_1}+\boldsymbol{cI_2})$$

$$.\Rightarrow f(z)=2z+iz^2+i(\boldsymbol{a}+\boldsymbol{bI_1}+\boldsymbol{cI_2})$$

Example5.5. Find the value of $\boldsymbol{\alpha},\boldsymbol{\beta}$ for the function:

$$(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})=\boldsymbol{\alpha}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})^2(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})+\boldsymbol{\beta}(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{2}}-\boldsymbol{3}(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{3}}$$
$$+\boldsymbol{2}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$

is a harmonic function. And find the harmonic conjugate$(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})$ and write $\boldsymbol{f}(\boldsymbol{z})$ by z , and find $\acute{\boldsymbol{f}}(\boldsymbol{z})$.

Solution.

The function $(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})$ is a harmonic function is it satisfay the Laplac equation.

$$\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x^2}+\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y^2}=0$$

Now we have:

$$\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x^2}+\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y^2}=0$$

$$\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x}=2\alpha(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\Rightarrow\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x^2}=2\alpha(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4$$

$$\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y}=\alpha(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2+2\beta(y+\boldsymbol{I_1}+\boldsymbol{I_2})-9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$

$$\Rightarrow\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y^2}=2\beta-18(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x^2}+\frac{\partial 2(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y^2}=0\Rightarrow 2\alpha(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4+2\beta-18(y+\boldsymbol{I_1}+\boldsymbol{I_2})=0$$

$$\Rightarrow(2\alpha-18)(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4+2\beta=0=0(y+\boldsymbol{I_1}+\boldsymbol{I_2})+0$$

Then, we have:

$$\begin{cases}\boldsymbol{2\alpha-18=0}\\ \boldsymbol{4+2\beta=0}\end{cases}\Rightarrow\boldsymbol{\alpha=9},\boldsymbol{\beta=-2}$$

Then:

$$(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})=\boldsymbol{9}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{2}}(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})-\boldsymbol{2}(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{2}}-\boldsymbol{3}(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{3}}+\boldsymbol{2}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})^{\boldsymbol{2}}$$

Now a harmonic conjugate:

$$\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x}=18(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y}=9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2-4(y+\boldsymbol{I_1}+\boldsymbol{I_2})-9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$

1- We use the first condition of Cauchy Riemann conditions. Then:

$$\frac{\boldsymbol{\partial}(\boldsymbol{u}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\boldsymbol{\partial x}}=\frac{\boldsymbol{\partial}(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\boldsymbol{\partial y}}\Rightarrow\frac{\boldsymbol{\partial}(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\boldsymbol{\partial y}}=\boldsymbol{18}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})+\boldsymbol{4}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})\dots\dots(\boldsymbol{7})$$

2- We integral $(\boldsymbol{7})$ for $(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})$, we have:

$$\int\frac{\boldsymbol{\partial}(\boldsymbol{v}+\boldsymbol{I_1}+\boldsymbol{I_2})}{\boldsymbol{\partial y}}d(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})=\int\big(\boldsymbol{18}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})+\boldsymbol{4}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})\big)d(\boldsymbol{y}+\boldsymbol{I_1}+\boldsymbol{I_2})+\psi(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$(v+\boldsymbol{I_1}+\boldsymbol{I_2})=9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2(x+\boldsymbol{I_1}+\boldsymbol{I_2})+4(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})+\psi(x+\boldsymbol{I_1}+\boldsymbol{I_2})\dots\dots(8)$$

Where $\boldsymbol{\psi}(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})$ is a constant integral.

3- We derivate $(\boldsymbol{8})$ by $(\boldsymbol{x}+\boldsymbol{I_1}+\boldsymbol{I_2})$, we have:

$$\frac{\partial(v+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x}=-9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+4(y+\boldsymbol{I_1}+\boldsymbol{I_2})+\acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

4- We use the second condition of Cauchy Riemann conditions. Then:

$$\frac{\partial(v+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x}=-\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial y}$$

$$\Longrightarrow 9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2-4(y+\boldsymbol{I_1}+\boldsymbol{I_2})-9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$
$$=-9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2-4(y+\boldsymbol{I_1}+\boldsymbol{I_2})-\acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\Longrightarrow \acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})=-9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$

By integrating the latter, we obtain:

$$\int \acute{\psi}(x+\boldsymbol{I_1}+\boldsymbol{I_2})d(x+\boldsymbol{I_1}+\boldsymbol{I_2})=\int -9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2 d(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$\Longrightarrow \psi(x+\boldsymbol{I_1}+\boldsymbol{I_2})=-3(x+\boldsymbol{I_1}+\boldsymbol{I_2})^3+a+bI_1+cI_2$$

5- we obtain:

$$(v+\boldsymbol{I_1}+\boldsymbol{I_2})=9(y+I_1+I_2)^2(x+I_1+I_2)+4(x+I_1+I_2)(y+I_1+I_2)-3(x+I_1+I_2)^3$$
$$+a+bI_1+cI_2$$

Now:

$$f(z)=(u+\boldsymbol{I_1}+\boldsymbol{I_2})+i(v+\boldsymbol{I_1}+\boldsymbol{I_2})$$

$$\Longrightarrow f(z)=9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2(y+\boldsymbol{I_1}+\boldsymbol{I_2})-2(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2-3(y+\boldsymbol{I_1}+\boldsymbol{I_2})^3+2(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$
$$+i(9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2(x+\boldsymbol{I_1}+\boldsymbol{I_2})+4(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$-3(x+\boldsymbol{I_1}+\boldsymbol{I_2})^3+a+bI_1+cI_2)$$

$$\Longrightarrow f(z)=9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2(y+\boldsymbol{I_1}+\boldsymbol{I_2})-2(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2-3(y+\boldsymbol{I_1}+\boldsymbol{I_2})^3+2(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$
$$+i9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2(x+\boldsymbol{I_1}+\boldsymbol{I_2})+i4(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$-i3(x+\boldsymbol{I_1}+\boldsymbol{I_2})^3+i(a+bI_1+cI_2)$$

$$\Longrightarrow f(z)=2\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})^2-(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+i(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)-i3(x+\boldsymbol{I_1}+\boldsymbol{I_2})^3$$
$$+i^2 3(y+\boldsymbol{I_1}+\boldsymbol{I_2})^3-i^2 9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$+i9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2(x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(a+bI_1+cI_2)$$

$$\Longrightarrow f(z)=2\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)^2$$
$$-3i\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})^3-i(y+\boldsymbol{I_1}+\boldsymbol{I_2})^3+3i(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$-3i(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2(x+\boldsymbol{I_1}+\boldsymbol{I_2})\big)+i(a+bI_1+cI_2)$$

$$\Longrightarrow f(z)=2\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)^2-3i\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)^3$$
$$+i(a+bI_1+cI_2)$$

$$\Longrightarrow f(z)=2z^2-3iz^3+i(a+bI_1+cI_2)$$

Now:

$$\acute{f}(z)=\frac{\partial(u+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x}+i\frac{\partial(v+\boldsymbol{I_1}+\boldsymbol{I_2})}{\partial x}$$

$$\Longrightarrow \acute{f}(z)=18(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4(x+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$+i(9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+4(y+\boldsymbol{I_1}+\boldsymbol{I_2})-9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2)$$

$$\Longrightarrow \acute{f}(z)=18(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})+4(x+\boldsymbol{I_1}+\boldsymbol{I_2})+i9(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+i4(y+\boldsymbol{I_1}+\boldsymbol{I_2})$$
$$-i9(x+\boldsymbol{I_1}+\boldsymbol{I_2})^2$$

$$\Rightarrow \acute{f}(z) = 4\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big) - i9\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})^2-(y+\boldsymbol{I_1}+\boldsymbol{I_2})^2+2i(x+\boldsymbol{I_1}+\boldsymbol{I_2})(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)$$

$$\Rightarrow \acute{f}(z) = 4\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big) - i9\big((x+\boldsymbol{I_1}+\boldsymbol{I_2})+i(y+\boldsymbol{I_1}+\boldsymbol{I_2})\big)^2$$

$$\Rightarrow \acute{f}(z) = 4z - 9iz^2$$

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