n-Refined Neutrosophic Complex Numbers

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Abstract: This paper is dedicated to define and study for the first time the concept of n-refined neutrosophic numbers. These numbers are considered as a new generalization of refined neutrosophic complex numbers and classical complex numbers respectively.

On the other hand, we study many of their elementary properties and illustrate many examples.

Definition 3.1.

We define an n- refine neutrosophic complex number by the following form:

, where $z = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$
$a_o, a_1, a_2, \ldots, a_n, b_o, b_1, b_2, \ldots, b_n$are real coeffients. For example:

$z = (-1 + I_1 + 2I_2 - I_3) + i(1 - 2I_1 - I_2 + 2I_3)$

We recall $a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n$ the real part, then it takes the following standard form $Re(z) = a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n$We recall $b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n$ the imagine part, then it takes the following standard form $Im(z) = b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n$.

Remark 3.1. An n-refined neutrosophic complex number can be defined as follows:

$\boldsymbol{z = a + bI_1 + cI_2 + \cdots + fI_3}$, where $\boldsymbol{a, b, c, \ldots, f}$are complex number. For example:

$\boldsymbol{z = (-1 + i) + iI_1 + (3 - 2i)I_2 - 2iI_3}$.

Remark 3.2.
.$I_n.I_n = I_n$, $I_i.I_j = I_{\min(i,j)}$

Definition 3.2.The conjugate of an n-refined neutrosophic complex number

Let$z = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$ be a refined neutrosophic complex number. We denote the conjugate of an n-refind neutrosophic complex number by $\bar{z}$ and we define it by the following form:

.$\bar{z} = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) - i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$

For example:

, Then $\bar{z} = (-1 - I_1 + I_2 + 4\boldsymbol{I_3}) - i(2 + I_1 + I_2 -$$z = (-1 - I_1 + I_2 + 4\boldsymbol{I_3}) + i(2 + I_1 + I_2 - \boldsymbol{I_3})$
$\boldsymbol{I_3}$).

Definition 3.3. The absolute value of a n- refined neutrosophic complex number:

Suppose that $z = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$ is a refined neutrosophic complex number. The absolute value of a z can be defined by the following form:

$$|z| = \sqrt{(a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n)^2 + (b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)^2}$$

Remark3.3.

$(\mathbf{1}).\overline{(\bar{z})} = \boldsymbol{z}.$

Proof: Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$, then

$\bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n}).$

Now.

$$\overline{(\bar{\boldsymbol{z}})} = \overline{\left((\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) - \imath(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})\right)}$$
$$= (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n}) = \boldsymbol{z}$$

$(\mathbf{2}).\, \boldsymbol{z} + \bar{\boldsymbol{z}} = \mathbf{2}\boldsymbol{Re}(\boldsymbol{z})$

Proof:Let$\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$, then

$\bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n}).$

Now.

$$\boldsymbol{z} + \bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$$
$$+ (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$$

$$\boldsymbol{z} + \bar{\boldsymbol{z}} = \mathbf{2}[(\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n})] = \mathbf{2}\boldsymbol{Re}(\boldsymbol{z})$$

$(3).\, z - \bar{z} = 2Im(z)$

Proof:Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$, then

$\bar{\boldsymbol{z}} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n}).$

Now.

$$z - \bar{z} = (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) + i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)$$
$$- [(\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})]$$

$$z - \bar{z} = (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) + i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)$$
$$- (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) + i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)$$

$$z - \bar{z} = 2i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n) = 2Im(z)$$

$(4).\, \overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$

Proof:

Let$\boldsymbol{z_1} = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$,

$\boldsymbol{z_2} = (\boldsymbol{c_o} + \boldsymbol{c_1 I_1} + \boldsymbol{c_2 I_2} + \cdots + \boldsymbol{c_n I_n}) + \boldsymbol{i}(\boldsymbol{d_o} + \boldsymbol{d_1 I_1} + \boldsymbol{d_2 I_2} + \cdots + \boldsymbol{d_n I_n}).$

Now.

$$z_1 + z_2 = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2 + \cdots + (a_n + c_n)I_n]$$
$$+ i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2 + \cdots + (b_n + d_n)I_n]$$

Then.

$$\overline{z_1+z_2} = [(a_o+c_o)+(a_1+c_1)I_1+(a_2+c_2)I_2+\cdots+(a_n+c_n)I_n] - i[(b_o+d_o)+(b_1+d_1)I_1+(b_2+d_2)I_2+\cdots+(b_n+d_n)I_n]$$

$$\overline{z_1+z_2} = [(\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n}) - i(\boldsymbol{b_o}+\boldsymbol{b_1 I_1}+\boldsymbol{b_2 I_2}+\cdots+\boldsymbol{b_n I_n})] + [(\boldsymbol{c_o}+\boldsymbol{c_1 I_1}+\boldsymbol{c_2 I_2}+\cdots+\boldsymbol{c_n I_n}) - i(\boldsymbol{d_o}+\boldsymbol{d_1 I_1}+\boldsymbol{d_2 I_2}+\cdots+\boldsymbol{d_n I_n})] = \overline{z_1}+\overline{z_2}$$

Definition 3.4. The multiplication of two n-refined neutrosophic real numbers:

Let $w_1 = \boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n}, w_2 = \boldsymbol{b_o}+\boldsymbol{b_1 I_1}+\boldsymbol{b_2 I_2}+\cdots+\boldsymbol{b_n I_n}$ are two n-refined neutrosophic real number, and we put:

$; 1 \le j \le n N_0 = a_0,\ N_j = a_0 + a_j + a_{j+1} + \cdots + a_n$

$; 1 \le j \le n M_0 = b_0,\ M_j = b_0 + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n$

Then a product $w_1.w_2$ is defined by form:

$$w_1.w_2 = N_0 M_0 + \sum_{i=1}^{n} [N_i M_i - N_{i+1} M_{i+1}]\, I_i + [N_n M_n - N_0 M_0] I_n$$

Definition 3.5. The multiplication of two n-refined neutrosophic complex numbers:

Let $\boldsymbol{z_1} = (\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o}+\boldsymbol{b_1 I_1}+\boldsymbol{b_2 I_2}+\cdots+\boldsymbol{b_n I_n}),$

$\boldsymbol{z_2} = (\boldsymbol{c_o}+\boldsymbol{c_1 I_1}+\boldsymbol{c_2 I_2}+\cdots+\boldsymbol{c_n I_n}) + \boldsymbol{i}(\boldsymbol{d_o}+\boldsymbol{d_1 I_1}+\boldsymbol{d_2 I_2}+\cdots+\boldsymbol{d_n I_n}).$

A product $z_1.z_2$ is defined by form:

$$z_1.z_2 = [(\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n}) + \boldsymbol{i}(\boldsymbol{b_o}+\boldsymbol{b_1 I_1}+\boldsymbol{b_2 I_2}+\cdots+\boldsymbol{b_n I_n})][(\boldsymbol{c_o}+\boldsymbol{c_1 I_1}+\boldsymbol{c_2 I_2}+ .\cdots+\boldsymbol{c_n I_n}) + \boldsymbol{i}(\boldsymbol{d_o}+\boldsymbol{d_1 I_1}+\boldsymbol{d_2 I_2}+\cdots+\boldsymbol{d_n I_n})]$$

$$z_1.z_2 = (\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n})(\boldsymbol{c_o}+\boldsymbol{c_1 I_1}+\boldsymbol{c_2 I_2}+\cdots+\boldsymbol{c_n I_n}) - (\boldsymbol{b_o}+\boldsymbol{b_1 I_1}+\boldsymbol{b_2 I_2}+\cdots+\boldsymbol{b_n I_n})(\boldsymbol{d_o}+\boldsymbol{d_1 I_1}+\boldsymbol{d_2 I_2}+\cdots+\boldsymbol{d_n I_n}) + i[(\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n})(\boldsymbol{d_o}+\boldsymbol{d_1 I_1}+ ,\boldsymbol{d_2 I_2}+\cdots+\boldsymbol{d_n I_n}) + (\boldsymbol{c_o}+\boldsymbol{c_1 I_1}+\boldsymbol{c_2 I_2}+\cdots+\boldsymbol{c_n I_n})(\boldsymbol{b_o}+\boldsymbol{b_1 I_1}+\boldsymbol{b_2 I_2}+\cdots+\boldsymbol{b_n I_n})]$$

By using Definition 3.4, we get the product $z_1.z_2$.

Remark3.1.

(1).$\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$

(1).$z.\bar{z} = |z|^2$.

Definition 3.6. Let $w = \boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n}$ be a n-refinedneutrosophicreal number, then the invertible of w defined as follows:

$$w^{-1} = \frac{1}{w} = \frac{1}{\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n}} = (\boldsymbol{a_o}+\boldsymbol{a_1 I_1}+\boldsymbol{a_2 I_2}+\cdots+\boldsymbol{a_n I_n})^{-1}$$

, where:$\frac{1}{w} = (a_0)^{-1} + \sum_{i=1}^{n} [M_i - M_{i+1}]^{-1} I_i + [(M_n)^{-1} - (M_0)^{-1}] I_n$

$; 1 \leq j \leq nM_0 = a_0,\ M_j = \boldsymbol{a_o} + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n$

Definition 3.7. The invertible a n-refined neutrosophic complex number.

Let $z = (\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + i(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})$, then the invertible of z defined as follows:

$$z^{-1} = \frac{1}{z} = \frac{1}{(\boldsymbol{a_o} + \boldsymbol{a_1 I_1} + \boldsymbol{a_2 I_2} + \cdots + \boldsymbol{a_n I_n}) + i(\boldsymbol{b_o} + \boldsymbol{b_1 I_1} + \boldsymbol{b_2 I_2} + \cdots + \boldsymbol{b_n I_n})} = \frac{\bar{z}}{z.\bar{z}} = \frac{\bar{z}}{|z|^2}$$

Example3.1. Let $z = (1 + I_1 - I_2 + 2I_3) + i(2 + 2I_1 - I_2 - I_3)$, $\bar{z} = (1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)$.
then.

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{(1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)}{(1 + I_1 - I_2 + 2I_3)^2 + (2 + 2I_1 - I_2 - I_3)^2}$$
$$= \frac{(1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)}{5 + 9I_1 + 2I_2 + 7I_3}$$

$$z^{-1} = [(1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)](5 + 9I_1 + 2I_2 + 7I_3)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2 + 2I_3)(5 + 9I_1 + 2I_2 + 7I_3)^{-1} - i(2 + 2I_1 - I_2 - I_3)(5 + 9I_1 + 2I_2 + 7I_3)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2 + 2I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right) - i(2 + 2I_1 - I_2 - I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right)$$

By using the Definition 3.4 we get:

$$(1 + I_1 - I_2 + 2I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right) = \frac{1}{5} + \frac{55}{42}I_1 - \frac{44}{210}I_2 + \frac{29}{35}I_3$$

$$(2 + 2I_1 - I_2 - I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right) = \frac{2}{5} + \frac{298}{5}I_1 - \frac{12}{35}I_2 - \frac{2}{35}I_3$$

Hence,

$$.z^{-1} = \left(\frac{1}{5} + \frac{55}{42}I_1 - \frac{44}{210}I_2 + \frac{29}{35}I_3\right) - i\left(\frac{2}{5} + \frac{298}{5}I_1 - \frac{12}{35}I_2 - \frac{2}{35}I_3\right)$$

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