SOME MIXED NEUTROSOPHIC SETS

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S. JAFARI, G. NORDO AND N. RAJESH

Abstract. In this paper, we introduce and study some subsets in mixed neutrosophic topological spaces.

1. Introduction

Theory of Fuzzy sets [6], Theory of Intuitionistic fuzzy sets [1], Theory of Neutrosophic sets [4] and the theory of Interval Neutrosophic sets [2] can be considered as tools for dealing with uncertainities. However, all of these theories have their own difficulties which are pointed out in [4]. In 1965, Zadeh [6] introduced fuzzy set theory as a mathematical tool for dealing with uncertainities where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov [1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The neutrosophic set was introduced by Smarandache [4] and explained, neutrosophic set is a generalization of Intuitionistic fuzzy set. In 2012, Salama and Alblowi [3], introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element.

2. Preliminaries

Definition 2.1. [5] *Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A =< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >$, where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ represent the degree of member ship function, the degree of indeterminacy, and the degree of non-membership, respectively of each element $x \in X$ to the set A.*

Definition 2.2. [3] *A neutrosophic topology on a nonempty set X is a family τ of neutrosophic subsets of X which satisfies the following three conditions:*

(1) $0, 1 \in \tau$,
(2) *If* $g, h \in \tau$, *their* $g \wedge h \in \tau$,
(3) *If* $f_i \in \tau$ *for each* $i \in I$, *then* $\vee_{i \in I} f_i \in \tau$.

The pair (X, τ) is called a neutrosophic topological space.

Definition 2.3. *Members of τ are called neutrosophic open sets and complement of neutrosophic open sets are called neutrosophic closed sets, where the complement of a neutrosophic set A, denoted by A^c, is $1 - A$.*

3. Some mixed neutrosophic sets

Definition 3.1. *Let (X, τ_1) and (X, τ_2) be two neutrosophic topological spaces. Then the system (X, τ_1, τ_2) is called a mixed neutrosophic topological space.*

Definition 3.2. *A subset A of a mixed neutrosophic topological space (X, τ_1, τ_2) is said to be:*

(1) *(τ_i, τ_j)-regular open if $A = \mathrm{Int}_i(\mathrm{Cl}_j(A))$;*
(2) *(τ_i, τ_j)-semiopen if $A \subset \mathrm{Cl}_j(\mathrm{Int}_i(A))$;*
(3) *(τ_i, τ_j)-preopen if $A \subset \mathrm{Int}_i(\mathrm{Cl}_j(A))$;*
(4) *(τ_i, τ_j)-α-open if $A \subset \mathrm{Int}_i(\mathrm{Cl}_j(\mathrm{Int}_i(A)))$;*
(5) *(τ_i, τ_j)-b-open if $A \subset \mathrm{Int}_i(\mathrm{Cl}_j(A)) \cup \mathrm{Cl}_j(\mathrm{Int}_i(A))$;*
(6) *(τ_i, τ_j)-β-open if $A \subset \mathrm{Cl}_j(\mathrm{Int}_i(\mathrm{Cl}_j(A)))$;*
(7) *(τ_i, τ_j)-δ-open if $\mathrm{Int}_i(\mathrm{Cl}_j(A)) \subset \mathrm{Cl}_j(\mathrm{Int}_i(A))$.*

On each definition above, $i, j = 1, 2$ and $i \neq j$.

The complement of an (i, j)-semiopen (resp. (i, j)-preopen, (i, j)-b-open, (i, j)-β-open, (i, j)-regular open) set is called an (i, j)-semiclosed (resp. (i, j)-preclosed, (i, j)-b-closed, (i, j)-β-closed, (i, j)-regular closed) set.
The family of all (i, j)-regular open (resp. (i, j)-preopen, (i, j)-semiopen, (i, j)-b-open, (i, j)-β-open, (i, j)-regular closed, (i, j)-preclosed, (i, j)-semiclosed, (i, j)-b-closed, (i, j)-β-closed) subsets of (X, τ_1, τ_2) is denoted by (i, j)-$RO(X)$ (resp. (i, j)-$PO(X)$, (i, j)-$SO(X)$, (i, j)-$BO(X)$, (i, j)-$\beta O(X)$, (i, j)-$RC(X)$, (i, j)-$PC(X)$, (i, j)-$SC(X)$, (i, j)-$BC(X)$, (i, j)-$\beta C(X)$).

Theorem 3.3. *Let A and B be neutrosophic subsets of (X, τ_1, τ_2). Then*

(1) *A is (τ_1, τ_2)-semiopen if and only if $\mathrm{Cl}_2(A) = \mathrm{Cl}_2(\mathrm{Int}_1(A))$.*
(2) *A is (τ_2, τ_1)-semiopen if and only if $\mathrm{Cl}_1(A) = \mathrm{Cl}_1(\mathrm{Int}_2(A))$.*
(3) *If $A \in \tau_1$ and B is (τ_1, τ_2)-preopen, then $A \cap B$ is (τ_1, τ_2)-preopen.*
(4) *If $A \in \tau_2$ and B is (τ_2, τ_1)-preopen, then $A \cap B$ is (τ_2, τ_1)-preopen.*

Proof. The proof is clear. □

Theorem 3.4. *Let A and B be any two neutrosophic subsets of a mixed neutrosophic topological space (X, τ_1, τ_2). Then*

(1) *If A is a (τ_1, τ_2)-semiopen or B is a (τ_1, τ_2)-semiopen set, then $\mathrm{Int}_1(\mathrm{Cl}_2(A \cap B)) = \mathrm{Int}_1(\mathrm{Cl}_2(A)) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))$.*

(2) *If A is a (τ_2, τ_1)-semiopen or B is a (τ_2, τ_1)-semiopen set, then* $\mathrm{Int}_2(\mathrm{Cl}_1(A \cap B)) = \mathrm{Int}_2(\mathrm{Cl}_1(A)) \cap \mathrm{Int}_2(\mathrm{Cl}_1(B))$.

Proof. (1). Clearly, $\mathrm{Int}_1(\mathrm{Cl}_2(A \cap B)) \subset \mathrm{Int}_1(\mathrm{Cl}_2(A)) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))$. If A is a (τ_1, τ_2)-semiopen set, then $\mathrm{Cl}_2(A) = \mathrm{Cl}_2(\mathrm{Int}_1(A))$. Then $\mathrm{Int}_1(\mathrm{Cl}_2(A)) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B)) = \mathrm{Int}_1(\mathrm{Cl}_2(A) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))) = \mathrm{Int}_1(\mathrm{Cl}_2 (\mathrm{Int}_1(A)) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))) \subset \mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))) = \mathrm{Int}_1(\mathrm{Cl}_2 (\mathrm{Int}_1(A) \cap \mathrm{Cl}_2(B)))) \subset \mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A \cap B))))) \subset \mathrm{Int}_1(\mathrm{Cl}_2 (\mathrm{Int}_1(\mathrm{Cl}_2(A \cap B)))) = \mathrm{Int}_1(\mathrm{Cl}_2(A \cap B))$.
(2). The proof is similar. □

Theorem 3.5. *Let A and B be any two neutrosophic subsets of a mixed neutrosophic topological space (X, τ_1, τ_2). Then*

(1) *If B is a (τ_1, τ_2)-α-open set if and only if there exists $B \in \tau_1$ such that $A \subset B \subset \mathrm{Int}_1(\mathrm{Cl}_2(A))$.*
(2) *If A is a (τ_1, τ_2)-α-open set and $A \subset B \subset \mathrm{Int}_1(\mathrm{Cl}_2(A))$, then A is (τ_1, τ_2)-α-open set.*
(3) *If B is a (τ_2, τ_1)-α-open set if and only if there exists $B \in \tau_2$ such that $A \subset B \subset \mathrm{Int}_2(\mathrm{Cl}_1(A))$.*
(4) *If A is a (τ_2, τ_1)-α-open set and $A \subset B \subset \mathrm{Int}_2(\mathrm{Cl}_1(A))$, then A is (τ_2, τ_1)-α-open set.*

Proof. (1). If $\mathrm{Int}_1(A) = B$, then $B \subset A \subset \mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A))) = \mathrm{Int}_1(\mathrm{Cl}_2(A))$. Conversely, Let $B \in \tau_1$ and$B \subset A \subset \mathrm{Int}_1(\mathrm{Cl}_2(A))$. Then $\mathrm{Int}_1(B) = B \subset \mathrm{Int}_1(A))$. Hence $A \subset \mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(B))) \subset \mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A)))$. Thus, B is a (τ_1, τ_2)-α-open set.
The other proofs are similar. □

Theorem 3.6. *Let A and B be any two neutrosophic subsets of a mixed neutrosophic topological space (X, τ_1, τ_2). Then*

(1) *If A is a (τ_1, τ_2)-α-open set and B is a (τ_1, τ_2)-β-open set, then $A \cap B$ is a (τ_1, τ_2)-β-open set.*
(2) *If A is a (τ_2, τ_1)-α-open set and B is a (τ_2, τ_1)-β-open set, then $A \cap B$ is a (τ_2, τ_1)-β-open set.*
(3) *If A is a (τ_1, τ_2)-α-open set and B is a (τ_1, τ_2)-semiopen set, then $A \cap B$ is a (τ_1, τ_2)-semiopen set.*
(4) *If A is a (τ_2, τ_1)-α-open set and B is a (τ_2, τ_1)-semiopen set, then $A \cap B$ is a (τ_2, τ_1)-semiopen set.*

Proof. (1). We have $A \cap B \subset \mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A))) \cap \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(B))) \subset \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A))) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))) = \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A)) \cap \mathrm{Int}_1(\mathrm{Cl}_2 (B)))) \subset \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A) \cap \mathrm{Int}_1(\mathrm{Cl}_2(B))))) = \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Int}_1 (A) \cap \mathrm{Cl}_2(B))))) \subset \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(\mathrm{Int}_1(A) \cap B))))) \subset \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2 (\mathrm{Int}_1(\mathrm{Cl}_2(A \cap B)))) \subset \mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(A \cap B)))$. Hence (τ_1, τ_2)-β-open set.
The other proofs are similar. □

Theorem 3.7. *If A is a neutrosophic subset of a mixed neutrosophic topological space (X, τ_1, τ_2). Then*

(1) *A is* (τ_1, τ_2)*-semiclosed if and only if* $\mathrm{Int}_2(\mathrm{Cl}_1(A)) \subset A$.
(2) *A is* (τ_2, τ_1)*-semiclosed if and only if* $\mathrm{Int}_1(\mathrm{Cl}_2(A)) \subset A$.
(3) *A is* (τ_1, τ_2)*-preclosed if and only if* $\mathrm{Cl}_1(\mathrm{Int}_2(A)) \subset A$.
(4) *A is* (τ_2, τ_1)*-preclosed if and only if* $\mathrm{Cl}_1(\mathrm{Int}_2(A)) \subset A$.
(5) *A is* (τ_1, τ_2)*-*α*-closed if and only if* $\mathrm{Cl}_2(\mathrm{Int}_1(\mathrm{Cl}_2(A))) \subset A$.
(6) *A is* (τ_2, τ_1)*-*α*-closed if and only if* $\mathrm{Cl}_1(\mathrm{Int}_2(\mathrm{Cl}_1(A))) \subset A$.
(7) *A is* (τ_1, τ_2)*-*β*-closed if and only if* $\mathrm{Int}_2(\mathrm{Cl}_1(\mathrm{Int}_2(A))) \subset A$.
(8) *A is* (τ_2, τ_1)*-*β*-closed if and only if* $\mathrm{Cl}_1(\mathrm{Int}_2(\mathrm{Cl}_1(A))) \subset A$.

Proof. The proofs follow from the respective definitions. □

Lemma 3.8. *If A is a neutrosophic subset of a mixed neutrosophic topological space* (X, τ_1, τ_2)*. Then*
(1) $\mathrm{Cl}_i(\mathrm{Int}_j(A)) = \mathrm{Cl}_i(\mathrm{Int}_j(\mathrm{Cl}_i(\mathrm{Int}_j(A))))$;
(2) $\mathrm{Int}_i(\mathrm{Cl}_j(A)) = \mathrm{Int}_i(\mathrm{Cl}_j(\mathrm{Int}_i(\mathrm{Cl}_j(A))))$.

Proof. (1). Clearly, the following holds $\mathrm{Int}_j(A) \subset \mathrm{Cl}_i(\mathrm{Int}_j(A))$. Then $\mathrm{Int}_j(\mathrm{Int}_j(A)) = \mathrm{Int}_j(A) \subset \mathrm{Int}_j(\mathrm{Cl}_i(\mathrm{Int}_j(A)))$ and consequently $\mathrm{Cl}_i(\mathrm{Int}_j(A)) \subset \mathrm{Cl}_i(\mathrm{Int}_j(\mathrm{Cl}_i(\mathrm{Int}_j(A))))$. Conversely, one has that $\mathrm{Int}_j(\mathrm{Cl}_i(\mathrm{Int}_j(A))) \subset \mathrm{Cl}_i(\mathrm{Int}_j(A))$ and hence we have the inclusion $\mathrm{Cl}_i(\mathrm{Int}_j(\mathrm{Cl}_i(\mathrm{Int}_j(A)))) \subset \mathrm{Cl}_i(\mathrm{Cl}_i(\mathrm{Int}_j(A))) = \mathrm{Cl}_i(\mathrm{Int}_j(A))$, and the proof is complete.
(2). Dual to (1). □

Proposition 3.9. (1) *Every* (τ_i, τ_j)*-*α*-open set is* (τ_i, τ_j)*-semiopen.*
(2) *Every* (τ_i, τ_j)*-semiopen set is* (τ_i, τ_j)*-b-open.*

Proof. The proof follows from the definitions. □

Corollary 3.10. (1) *Every* (τ_i, τ_j)*-semiopen set is* (τ_i, τ_j)*-*δ*-open.*
(2) *Every* (τ_i, τ_j)*-semiopen set is* (τ_i, τ_j)*-semipreopen.*

Remark 3.11. *It is clear that* (τ_i, τ_j)*-semiopenness and* (τ_i, τ_j)*-preopenness are independent notions.*

Theorem 3.12. *If* $\{A_\alpha\}_{\alpha\in\Delta}$ *is the collection of* (τ_i, τ_j)*-semiopen sets of* (X, τ_1, τ_2)*, then* $\bigcup_{\alpha\in\Delta} A_\alpha$ *is also a* (τ_i, τ_j)*-semiopen set.*

Proof. Since each A_α is (τ_i, τ_j)-semiopen and $A_\alpha \subset \bigcup_{\alpha\in\Delta} A_\alpha$, implies that $\bigcup_{\alpha\in\Delta} A_\alpha \subset \mathrm{Cl}_j(\mathrm{Int}_i(\bigcup_{\alpha\in\Delta} A_\alpha))$. Hence $\bigcup_{\alpha\in\Delta} A_\alpha$ is also a (τ_i, τ_j)-semiopen set in (X, τ_1, τ_2). □

Proposition 3.13. *A subset A of X is* (τ_i, τ_j)*-semiopen if and only if* $\mathrm{Cl}_j(A) = \mathrm{Cl}_j(\mathrm{Int}_i(A))$.

Proof. Let $A \in (\tau_i, \tau_j)\text{-}SO(X)$. Then we have $A \subset \mathrm{Cl}_j(\mathrm{Int}_i(A))$. Then $\mathrm{Cl}_j(A) \subset \mathrm{Cl}_j(\mathrm{Int}_i(A))$ and hence $\mathrm{Cl}_j(A) = \mathrm{Cl}_j(\mathrm{Int}_i(A))$. The converse is obvious. □

Corollary 3.14. *If A is a nonempty* (τ_i, τ_j)*-semiopen set, then* $\mathrm{Int}_i(A) \neq \emptyset$.

Proof. Since A is (τ_i, τ_j)-semiopen, by Proposition 3.13, we have $\text{Cl}_j(A) = \text{Cl}_j(\text{Int}_i(A))$. Suppose $\text{Int}_i(A) = \emptyset$. Then we have $\text{Cl}_j(A) = \emptyset$ and hence $A = \emptyset$. This is contrary to the hypothesis. Therefore, $\text{Int}_i(A) \neq \emptyset$. □

Proposition 3.15. *A subset A is (τ_i, τ_j)-semiopen if and only if there exists $U \in \tau_i$ such that $U \subset A \subset \text{Cl}_j(U)$.*

Proof. Let $A \in (\tau_i, \tau_j)$-$SO(X)$. Then we have $A \subset \text{Cl}_j(\text{Int}(A))$. Take $\text{Int}_i(A) = U$. Then $U \subset A \subset \text{Cl}_j(U)$. Conversely, let U be a τ_i-open set such that $U \subset A \subset \text{Cl}_j(U)$. Since $U \subset A$, $U \subset \text{Int}_i(A)$ and hence $\text{Cl}_j(U) \subset \text{Cl}_j(\text{Int}_i(A))$. Thus, we obtain $A \subset \text{Cl}_j(\text{Int}_i(A))$. □

Proposition 3.16. *If A is a (τ_i, τ_j)-semiopen set in a mixed neutrosophic topological space (X, τ_1, τ_2) and $A \subset B \subset \text{Cl}_j(A)$, then B is a (τ_i, τ_j)-semiopen set in (X, τ_1, τ_2).*

Proof. Since A is (τ_i, τ_j)-semiopen, there exists a τ_i-open set U such that $U \subset A \subset \text{Cl}_j(U)$. Then we have $U \subset A \subset B \subset \text{Cl}_j(A) \subset \text{Cl}_j(\text{Cl}_j(U)) = \text{Cl}_j(U)$ and hence $U \subset B \subset \text{Cl}_j(U)$. By Proposition 3.15, we obtain $B \in (\tau_i, \tau_j)$-$SO(X)$. □

Theorem 3.17. *A subset A of X is (τ_i, τ_j)-semiopen if and only if it is both (τ_i, τ_j)-δ-open and (τ_i, τ_j)-β-preopen.*

Proof. Let A be a (τ_i, τ_j)-semiopen set, then $A \subset \text{Cl}_j(\text{Int}_i(A)) \subset \text{Cl}_j(\text{Int}_i(\text{Cl}_j(A)))$. This shows that A is (τ_i, τ_j)-β-open. Moreover, $\text{Int}_i(\text{Cl}_j(A)) \subset \text{Cl}_j(A) \subset \text{Cl}_j(\text{Int}_i(A))$. Therefore, A is (τ_i, τ_j)-δ-open. Conversely, let A be (τ_i, τ_j)-δ-open and (τ_i, τ_j)-β-open set, then we have $\text{Int}_i(\text{Cl}_j(A)) \subset \text{Cl}_j(\text{Int}_i(A))$. Thus we obtain that $\text{Cl}_j(\text{Int}_i(\text{Cl}_j(A))) \subset \text{Cl}_j(\text{Int}_i(A))$. Since A is (τ_i, τ_j)-β-open, we have $A \subset \text{Cl}_j(\text{Int}_i(\text{Cl}_j(A))) \subset \text{Cl}_j(\text{Int}_i(A))$ and $A \subset \text{Cl}_j(\text{Int}_i(A))$. Hence A is a (τ_i, τ_j)-semiopen set. □

Theorem 3.18. *A subset A of X is (τ_i, τ_j)-semiclosed if and only if there exists a τ_j-closed set F such that $\text{Int}_i(F) \subset A \subset F$.*

Proof. Suppose that A is (τ_i, τ_j)-semiclosed. Then $\text{Int}_i(\text{Cl}_j(A)) \subset A$. Let $F = \text{Cl}_j(A)$, then F is τ_j-closed set such that $\text{Int}_i(F) \subset A \subset F$. Conversely, let F be a τ_j-closed set such that $\text{Int}_i(F) \subset A \subset F$. But $F \supset \text{Cl}_j(A)$, so $\text{Int}_i(F) \supset \text{Int}_i(\text{Cl}_j(A))$. Hence $\text{Int}_i(\text{Cl}_j(A)) \subset A$. Therefore, A is (τ_i, τ_j)-semiclosed. □

Proposition 3.19. *A subset A of X is (τ_i, τ_j)-β-closed and (τ_i, τ_j)-δ-open, then it is (τ_i, τ_j)-semiclosed.*

Proof. The proof follows from the definitions. □

Theorem 3.20. *Arbitrary intersection of (τ_i, τ_j)-semiclosed sets is always (τ_i, τ_j)-semiclosed.*

Proof. Follows from Theorem 3.12. □

Definition 3.21. *Let A be subset of a mixed neutrosophic topological space (X, τ_1, τ_2). Then*

(1) *the (τ_i, τ_j)-semiclosure of A is defined as intersection of all (τ_i, τ_j)-semiclosed sets containing A. That is, $(\tau_i, \tau_j)\text{-}s\,\mathrm{Cl}(A) = \bigcap\{F : F$ is (τ_i, τ_j)-semiclosed and $A \subset F\}$.*

(2) *the (τ_i, τ_j)-semiinterior of A is defined as union of all (τ_i, τ_j)-semiopen sets contained in A. That is, $(\tau_i, \tau_j)\text{-}s\,\mathrm{Int}(A) = \bigcup\{U :$ U is (τ_i, τ_j)-semiopen and $U \subset A\}$.*

Theorem 3.22. *For a subset A of X, the following hold:*

(1) $(\tau_i, \tau_j)\text{-}s\,\mathrm{Cl}(A) = A \cup \mathrm{Int}_i(\mathrm{Cl}_j(A))$.

(2) $(\tau_i, \tau_j)\text{-}s\,\mathrm{Int}(A) = A \cap \mathrm{Cl}_i(\mathrm{Int}_j(A))$.

Proof. The proof follows from the definitions. □

4. Extremally disconnected mixed neutrosophic topological spaces

Definition 4.1. *A mixed neutrosophic topological space (X, τ_1, τ_2) is said to be*

(1) *(τ_i, τ_j)-extremally disconnected if τ_j-closure of every τ_i-open set is τ_i-open in X,*

(2) *pairwise extremally disconnetcted if (X, τ_1, τ_2) is (τ_1, τ_2)-extremally disconnected and (τ_2, τ_1)-extremally disconnected.*

Theorem 4.2. *A mixed neutrosophic topological space (X, τ_1, τ_2) is pairwise extremally disconnected if and only if for each τ_i-open set A and each τ_j-open set B such that $A \cap B = \emptyset$, $\tau_j\text{-Cl}(A) \cap \tau_i\text{-Cl}(B) = \emptyset$.*

Proof. Assume that (X, τ_1, τ_2) is pairwise extremally disconnected. Let A and B, respectively, be τ_1-open and τ_2-open sets such that $A \cap B = \emptyset$. Then $\tau_j\text{-Cl}(A) \in \tau_i$ and hence $\tau_j\text{-Cl}(A) \cap \tau_i\text{-Cl}(B) = \emptyset$. Conversely, let U be a τ_i-open set in X. Then $X \backslash \tau_j\text{-Cl}(U)$ is τ_j-open in X. Now, we have

$U \cap (X \backslash \tau_j\text{-Cl}(U)) = \emptyset$

$\Rightarrow \tau_j\text{-Cl}(U) \cap \tau_i\text{-Cl}(X \backslash \tau_j\text{-Cl}(U))$

$\Rightarrow \tau_i\text{-Cl}(X \backslash \tau_j\text{-Cl}(U)) \subset X \backslash \tau_j\text{-Cl}(U)$

$\Rightarrow \tau_i\text{-Cl}(X \backslash \tau_j\text{-Cl}(U)) = X \backslash \tau_j\text{-Cl}(U)$

$\Rightarrow (X \backslash \tau_j\text{-Cl}(U))$ is τ_i-closed

$\Rightarrow \tau_j\text{-Cl}(U)$ is τ_i-open.

Thus (X, τ_1, τ_2) is (τ_i, τ_j)-extremally disconnected. Similarly, (X, τ_1, τ_2) is (τ_j, τ_i)-extremally disconnected. Hence (X, τ_1, τ_2) is pairwise extremally disconnected. □

Theorem 4.3. *The following are equivalent for a mixed neutrosophic topological space (X, τ_1, τ_2):*

(1) *(X, τ_1, τ_2) is pairwise extremally disconnected.*

(2) *For each (τ_j, τ_i)-semiopen set A in X, $\tau_j\text{-Cl}(A)$ is τ_i-open set.*

(3) *For each* (τ_i, τ_j)*-semiopen set* A *in* X*,* (τ_j, τ_i)-$s\,\mathrm{Cl}(A)$ *is* τ_i*-open set.*
(4) *For each* (τ_i, τ_j)*-semiopen set* A *and each* (τ_j, τ_i)*-semiopen set* B *with* $A \cap B = \emptyset$*,* τ_j-$\mathrm{Cl}(A) \cap \tau_i$-$\mathrm{Cl}(B) = \emptyset$*.*
(5) *For each* (τ_j, τ_i)*-semiopen set* A *in* X*,* τ_j-$\mathrm{Cl}(A) = (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A)$*.*
(6) *For each* (τ_i, τ_j)*-semiopen set* A *in* X*,* (τ_j, τ_i)-$s\,\mathrm{Cl}(A)$ *is* τ_j*-closed set.*
(7) *For each* (τ_i, τ_j)*-semiclosed set* A *in* X*,* τ_j-$\mathrm{Int}(A) = (\tau_j, \tau_i)$-$s\,\mathrm{Int}(A)$*.*
(8) *For each* (τ_i, τ_j)*-semiclosed set* A *in* X*,* (τ_j, τ_i)-$s\,\mathrm{Int}(A)$ *is* τ_j*-open set.*

Proof. (1) $\Rightarrow$ (2): Clear.
(1) $\Rightarrow$ (5): Since (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \subset \tau_j$-$\mathrm{Cl}(A)$ for any set A of X, it is sufficient to show that (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \supset \tau_j$-$\mathrm{Cl}(A)$ for any (τ_i, τ_j)-semiopen set A of X. Let $x \notin (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A)$. Then there exists a (τ_j, τ_i)-semiopen set W with $x \in W$ such that $W \cap A = \emptyset$. Thus τ_j-$\mathrm{Int}(W)$ and τ_i-$\mathrm{Int}(A)$ are, respectively, τ_j-open and τ_i-open such that τ_j-$\mathrm{Int}(X) \cap \tau_i$-$\mathrm{Int}(A) = \emptyset$. By Theorem 4.2, τ_i-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}(W)) \cap \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A)) = \emptyset$ and hence $x \notin \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A)) = \tau_j$-$\mathrm{Cl}(A)$. Hence τ_j-$\mathrm{Cl}(A) \subset (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A)$.
(5) $\Rightarrow$ (6): Obvious.
(6) $\Rightarrow$ (5): For any set A in X, $A \subset (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A) \subset \tau_j$-$\mathrm{Cl}(A)$. Then τ_j-$\mathrm{Cl}(A) = \tau_j$-$\mathrm{Cl}((\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A))$. Since A is (τ_i, τ_j)-semiopen, by (6), (τ_j, τ_i)-$s\,\mathrm{Cl}(A)$ is τ_j-closed. Hence, τ_j-$\mathrm{Cl}(A) = (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A)$.
(6) $\Leftrightarrow$ (8): Clear.
(7) $\Rightarrow$ (8): Obvious.
(8) $\Rightarrow$ (7): For any subset A of X, τ_j-$\mathrm{Int}(A) \subset (\tau_j, \tau_i)$-$s\,\mathrm{Int}(A) \subset A$ and hence τ_j-$\mathrm{Int}(A) = \tau_j$-$\mathrm{Int}((\tau_j, \tau_i)$-$s\,\mathrm{Int}(A))$. Since A is (τ_i, τ_j)-semiclosed, by (8), (τ_j, τ_i)-$s\,\mathrm{Int}(A)$ is τ_j-open. Hence τ_j-$\mathrm{Int}(A) = (\tau_j, \tau_i)$-$s\,\mathrm{Int}(A)$.
(1) $\Rightarrow$ (4): Let A be a (τ_i, τ_j)-open set and B a (τ_j, τ_i)-semiopen set such that $A \cap B = \emptyset$. Then τ_i-$\mathrm{Int}(A) \cap \tau_j$-$\mathrm{Int}(B) = \emptyset$ and thus by Theorem 4.2, τ_j-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}(A)) \cap \tau_i$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}(B)) = \emptyset$. Hence τ_j-$\mathrm{Cl}(A) \cap \tau_i$-$\mathrm{Cl}(B) = \emptyset$.
(4) $\Rightarrow$ (2): Let A be a (τ_i, τ_j)-semiopen subset of X. Then $X \backslash \tau_j$-$\mathrm{Cl}(A)$ is (τ_j, τ_i)-semiopen and $A \cap (X \backslash \tau_j$-$\mathrm{Cl}(A))$. Thus, by (4), τ_j-$\mathrm{Cl}(A) \cap \tau_i$-$\mathrm{Cl}(X \backslash \tau_j$-$\mathrm{Cl}(A)) = \emptyset$ which implies τ_j-$\mathrm{Cl}(A) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. Hence, τ_j-$\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$ and consequently τ_j-$\mathrm{Cl}(A)$ is τ_i-open in X.
(5) $\Rightarrow$ (4): Let A be a (τ_i, τ_j)-semiopen set and B be a (τ_j, τ_i)-semiopen set such that $A \cap B = \emptyset$. Then (τ_j, τ_i)-$s\,\mathrm{Cl}(A)$ is (τ_i, τ_j)-semiopen and (τ_i, τ_j)-$s\,\mathrm{Cl}(B)$ is (τ_j, τ_i)-semiopen in X and hence (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \cap (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(B) = \emptyset$. By (5), τ_j-$\mathrm{Cl}(A) \cap \tau_i$-$\mathrm{Cl}(B) = \emptyset$.
(1) $\Rightarrow$ (3): Clear.
(3) $\Rightarrow$ (1): Let A be a τ_i-open set in (X, τ_1, τ_2). It is sufficient to prove that τ_j-$\mathrm{Cl}(A) = (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A)$. Obviously, (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \subset \tau_j$-$\mathrm{Cl}(A)$.

Let $x \notin (\tau_j,\tau_i)$-$s\,\mathrm{Cl}(A)$. Then there exists a (τ_j,τ_i)-semiopen set U with $x \in U$ such that $A \cap U = \emptyset$. Hence (τ_i,τ_j)-$s\,\mathrm{Cl}(U) \subset (\tau_i,\tau_j)$-$s\,\mathrm{Cl}(X\backslash A) = X\backslash A$ and thus (τ_i,τ_j)-$s\,\mathrm{Cl}(U) \cap A = \emptyset$. Since (τ_i,τ_j)-$s\,\mathrm{Cl}(U)$ is a τ_j-open set with $x \in (\tau_i,\tau_j)$-$s\,\mathrm{Cl}(U)$, $x \notin \tau_j$-$\mathrm{Cl}(A)$. Hence τ_j-$\mathrm{Cl}(A) \subset (\tau_j,\tau_i)$-$\mathrm{Cl}(A)$. □

Definition 4.4. *A point x in a mixed neutrosophic toplogical space (X,τ_1,τ_2) is said to be (τ_i,τ_j)-θ-cluster point of a set A if for every τ_i-open, say, U containing x, τ_j-$\mathrm{Cl}(U) \cap A \neq \emptyset$. The set of all (τ_i,τ_j)-θ-closure of A and will be denoted by (τ_i,τ_j)-$\mathrm{Cl}_\theta(A)$. A set A is called (τ_i,τ_j)-θ-closed if $A = (\tau_i,\tau_j)$-$\mathrm{Cl}_\theta(A)$.*

Lemma 4.5. *For any (τ_j,τ_i)-preopen set A in a mixed neutrosophic topological space (X,τ_1,τ_2), τ_i-$\mathrm{Cl}(A) = (\tau_i,\tau_j)$-$\mathrm{Cl}_\theta(A)$.*

Proof. It is obvious that τ_i-$\mathrm{Cl}(A) \subset (\tau_i,\tau_j)$-$\mathrm{Cl}_\theta(A)$, for any subset A of (X,τ_1,τ_2). Thus, it remains to be shown that (τ_i,τ_j)-$\mathrm{Cl}_\theta(A) \subset \tau_i$-$\mathrm{Cl}(A)$. If $x \notin \tau_i$-$\mathrm{Cl}(A)$, then there exists a τ_i-open set U containing x such that $U \cap A = \emptyset$ and thus $U \cap \tau_i$-$\mathrm{Cl}(A) = \emptyset$. But $U \cap \tau_j$-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A)) = \emptyset$ which implies τ_j-$\mathrm{Cl}(U) \cap \tau_j$-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A)) = \emptyset$ and so τ_j-$\mathrm{Cl}(U) \cap A = \emptyset$ since A is (τ_j,τ_i)-preopen. Hence $x \notin (\tau_j,\tau_i)$-$\mathrm{Cl}_\theta(A)$ and consequently (τ_j,τ_i)-$\mathrm{Cl}_\theta(A) \subset \tau_i$-$\mathrm{Cl}(A)$. □

Theorem 4.6. *The following are equivalent for a mixed neutrosophic topological space (X,τ_1,τ_2):*

1. *(X,τ_1,τ_2) is pairwise extremally disconnected.*
2. *The τ_j-closure of every (τ_i,τ_j)-β-open set of X is τ_i-open set.*
3. *The (τ_j,τ_i)-θ-closure of every (τ_i,τ_j)-preopen set of X is τ_i-open set.*
4. *The τ_j-closure of every (τ_i,τ_j)-preopen set of X is τ_i-open set.*

Proof. (1) ⇒ (2): Let A be a (τ_i,τ_j)-β-open set. Then τ_j-$\mathrm{Cl}(A) = \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A)))$. Since (X,τ_1,τ_2) is pairwise extremally disconnected. τ_j-$\mathrm{Cl}(A)$ is a τ_i-open set.
(2) ⇒ (4): Follows from the fact that every (τ_i,τ_j)-preopen set is (τ_i,τ_j)-β-open.
(4) ⇒ (1): Clear.
(3) ⇔ (4): Follows from Lemma 4.5. □

Theorem 4.7. *A mixed neutrosophic topological space (X,τ_1,τ_2) is pairwise extremally disconnected if and only if every (τ_i,τ_j)-semiopen set is a (τ_i,τ_j)-preopen set.*

Proof. Let A be a (τ_i,τ_j)-semiopen set. Then $A \subset \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A))$. Since X is pairwise extremally disconnected, τ_j-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A))$ is a τ_i-open set and then $A \subset \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A)) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A))) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. Hence A is a (τ_i,τ_j)-preopen set. Conversely, let A be a τ_i-open set. Since τ_j-$\mathrm{Cl}(A) = \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(A))$, we have τ_j-$\mathrm{Cl}(A) = \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A)))$. Then τ_j-$\mathrm{Cl}(A)$ is (τ_j,τ_i)-regular closed and hence

A is (τ_i, τ_j)-semiopen. By hypothesis, A is (τ_i, τ_j)-propen so that τ_j-$\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. Then τ_j-$\mathrm{Cl}(A)$ is τ_i-open in X and hence X is pairwise extremally disconnected. □

Lemma 4.8. *For a subset A of a mixed neutrosophic topological space (X, τ_1, τ_2),*

(1) τ_j-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A)) \subset (\tau_i, \tau_j)$-$s\,\mathrm{Cl}(A)$,
(2) τ_j-$\mathrm{Int}((\tau_i, \tau_j)$-$s\,\mathrm{Cl}(A)) = \tau_j$-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A))$.

Proof. (1) Since (τ_i, τ_j)-$s\,\mathrm{Cl}(A)$ is (τ_i, τ_j)-semiclosed, there exists a τ_i-closed set U in X such that τ_j-$\mathrm{Int}(U) \subset (\tau_i, \tau_j)$-$s\,\mathrm{Cl}(A) \subset U$. Then τ_j-$\mathrm{Int}(U) \subset (\tau_i, \tau_j)$-$s\,\mathrm{Cl}(A) \subset \tau_i$-$\mathrm{Cl}(A) \subset U$, and consequently τ_j-$\mathrm{Int}(U) \subset \tau_j$-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A)) \subset \tau_j$-$\mathrm{Int}(U)$. Hence, τ_j-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A)) \subset (\tau_i, \tau_j)$-$s\,\mathrm{Cl}(A)$.
(2) Follows easily from (1). □

Theorem 4.9. *Let A be a subset of a mixed neutrosophic topological space (X, τ_1, τ_2). Then A is (τ_i, τ_j)-regular open if and only if A is τ_i-open and τ_j-closed.*

Proof. Let A be a (τ_i, τ_j)-regular open set of a bitoplogical space (X, τ_1, τ_2). Then τ_i-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A)) = A$. Now, $X \backslash \tau_j$-$\mathrm{Cl}(A)$ and A are, respectively, τ_j-open and τ_i-open such that $(X \backslash \tau_j$-$\mathrm{Cl}(A)) \cap A = \emptyset$. Since (X, τ_1, τ_2) is pairwise extremally disconnected, by Theorem 4.2, τ_i-$\mathrm{Cl}(X \backslash \tau_j$-$\mathrm{Cl}(A)) \cap \tau_j$-$\mathrm{Cl}(A) = \emptyset$. Then τ_i-$\mathrm{Cl}(X \backslash \tau_j$-$\mathrm{Cl}(A)) = X \backslash \tau_j$-$\mathrm{Cl}(A)$ and $X \backslash \tau_j$-$\mathrm{Cl}(A)$ is τ_i-closed. Hence, τ_j-$\mathrm{Cl}(A)$ is τ_i-open, so that τ_j-$\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A)) = A$ is τ_i-open and τ_j-closed. The converse is clear. □

Lemma 4.10. *Let A be a subset of a mixed neutrosophic topological space (X, τ_1, τ_2). Then we have*

(1) *A is (τ_i, τ_j)-preopen if and only if (τ_j, τ_i)-$s\,\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$.*
(2) *A is (τ_i, τ_j)-preopen if and only if (τ_j, τ_i)-$s\,\mathrm{Cl}(A)$ is (τ_i, τ_j)-regular open.*
(3) *A is (τ_i, τ_j)-regular open if and only if A is (τ_i, τ_j)-preopen and (τ_j, τ_i)-semiclosed.*

Proof. (1). Let A b e a (τ_i, τ_j)-preopen set. Then (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \subset (\tau_j, \tau_i)$-$s\,\mathrm{Cl}(\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A)))$. Since τ_i-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$ is (τ_j, τ_i)-semiclosed, (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. Hence, by Lemma 4.8 (1), (τ_j, τ_i)-$s\,\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. The converse is obvious.
(2). Let (τ_j, τ_i)-$s\,\mathrm{Cl}(A)$ be a (τ_i, τ_j)-regular open set. Then we have (τ_j, τ_i)-$s\,\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(\tau_j, \tau_i)$-$s\,\mathrm{Cl}(A))$ and hence (τ_j, τ_i)-$s\,\mathrm{Cl}(A) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(\tau_j$-$\mathrm{Cl}(A))) = \tau_i$- $\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. By Lemma 4.8 (1), we have (τ_j, τ_i)-$s\,\mathrm{Cl}(A) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}(A))$. Hence, A is a (τ_i, τ_j)-preopen set from (1). The converse follows from (1).
(3). Let A be a (τ_i, τ_j)-preopen and a (τ_j, τ_i)-semiclosed set. Then by

(2), A is (τ_i,τ_j)-regular open in X. Conversely, let A be a (τ_i,τ_j)-regular open set. Then $A = \tau_i\text{-Int}(\tau_j\text{-Cl}(A))$ and thus $\tau_i\text{-Int}(\tau_j\text{-Cl}(A)) = (\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) = A$. Hence A is (τ_i,τ_j)-preopen and (τ_j,τ_i)-semiclosed. □

Theorem 4.11. *In a mixed neutrosophic topological space* (X,τ_1,τ_2)*, the following are equivalent:*

(1) *X is pairwise extremally disconnected.*
(2) $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) = (\tau_j,\tau_i)\text{-Cl}_\theta(A)$ *for every* (τ_i,τ_j)*-preopen (or* (τ_i,τ_j)*-semiopen) set A in X.*
(3) $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) = \tau_j\text{-Cl}(A)$ *for every* (τ_i,τ_j)*-β-open set A in X.*

Proof. (1) ⇒ (2): Since $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) \subset (\tau_j,\tau_i)\text{-Cl}_\theta(A)$ for any subset A of X, it is sufficient to show that $(\tau_j,\tau_i)\text{-Cl}_\theta(A) \subset (\tau_j,\tau_i)\text{-}s\,\text{Cl}(A)$ for any (τ_i,τ_j)-preopen or (τ_i,τ_j)-semiopen set A of X. Let $x \notin (\tau_j,\tau_i)\text{-}s\,\text{Cl}(A)$. Then there exists a (τ_j,τ_i)-semiopen set U with $x \in U$ such that $U \cap A = \emptyset$ and thus there exists a τ_j-open set V such that $V \subset U \subset \tau_j\text{-Cl}(V)$ with $V \cap A = \emptyset$ which implies $V \cap \tau_j\text{-Cl}(A) = \emptyset$. This means $V \cap \tau_i\text{-Int}(\tau_j\text{-Cl}(A)) = \emptyset$ and hence $\tau_i\text{-Cl}(V) \cap \tau_i\text{-Int}(\tau_j\text{-Cl}(A)) = \emptyset$. Now, if A is (τ_i,τ_j)-preopen, then $A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}(A))$ and hence $\tau_i\text{-Cl}(V)\cap A = \emptyset$. If A is (τ_i,τ_j)-semiopen, since X is pairwise extremally disconnected, $\tau_i\text{-Cl}(V)$ is τ_j-open and thus $\tau_i\text{-Cl}(V) \cap \tau_j\text{-Cl}(\tau_i\text{-Int}(\tau_j\text{-Cl}(A))) = \emptyset$ which implies $\tau_i\text{-Cl}(V)\cap A = \emptyset$. Hence, in any case, $x \notin (\tau_j,\tau_i)\text{-Cl}_\theta(A)$.
(2) ⇒ (1): First let A be a (τ_i,τ_j)-preopen set in X. By Lemmas 4.10 and 4.5, we have $\tau_i\text{-Int}(\tau_j\text{-Cl}(A)) = (\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) = (\tau_j,\tau_i)\text{-Cl}_\theta(A) = \tau_j\text{-Cl}(A)$. Then $\tau_j\text{-Cl}(A)$ is τ_i-open and hence by Theorem 4.6, X is pairwise extremally disconnected. Next, let A be a (τ_i,τ_j)-semiopen set in X. Then $(\tau_j,\tau_i)\text{-Cl}(A) \subset \tau_j\text{-Cl}(A) \subset (\tau_j,\tau_i)\text{-Cl}_\theta(A) = (\tau_j,\tau_i)\text{-}s\,\text{Cl}(A)$ and thus $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) = \tau_j\text{-Cl}(A)$. Hence, X is pairwise extremally disconnected from Theorem 4.6.
(1) ⇒ (3): Let A be a (τ_i,τ_j)-β-open set in X. Since X is pairwise extremally disconnected, by Theorem 4.6, $\tau_j\text{-Cl}(A)$ is τ_i-open in X. Hence, by Lemma 4.10, $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(A) = \tau_j\text{-Cl}(A)$.
(3) ⇒(1): Let U and V, respectively, be τ_i-open and τ_j-open sets such that $U \cap V = \emptyset$. Then $U \subset X\backslash V$ which implies $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(U) \subset (\tau_j,\tau_i)\text{-}s\,\text{Cl}(X\backslash V) = X\backslash V$ and hence $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(U) \cap V = \emptyset$. Since $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(U)$ is (τ_i,τ_j)-semiopen in X, $(\tau_j,\tau_i)\text{-}s\,\text{Cl}(U)\cap(\tau_i,\tau_j)\text{-}s\,\text{Cl}(V) = \emptyset$. Then by (3) $\tau_j\text{-Cl}(U) \cap \tau_i\text{-Cl}(V) = \emptyset$ and hence by Theorem 4.2, X is pairwise extremally disconnected. □

Theorem 4.12. *In a mixed neutrosophic topological space* (X,τ_1,τ_2)*, the following are equivalent:*

(1) *X is pairwise extremally disconnected.*
(2) *For each* (τ_i,τ_j)*-β-open set A in X and each* (τ_j,τ_i)*-semiopen set B in X such that* $A \cap B = \emptyset$*,* $\tau_i\text{-Cl}(A) \cap \tau_j\text{-Cl}(B) = \emptyset$

(3) *For each* (τ_i, τ_j)*-preopen set* A *in* X *and each* (τ_j, τ_i)*-semiopen set* B *in* X *such that* $A \cap B = \emptyset$*,* τ_i-Cl$(A) \cap \tau - j$-Cl$(B) = \emptyset$*.*

Proof. (1)⇒ (2): Let A be a (τ_i, τ_j)-β-open set and B a (τ_j, τ_i)-semiopen set such that $A \cap B = \emptyset$. Then $A \cap \tau_j$-Int$(B) = \emptyset$ and hence τ_j-Cl$(A) \cap \tau_j$-Int$(B) = \emptyset$. By Theorem 4.6, τ_j-Cl(A) is a τ_i-open set in X and hence τ_j-Cl$(A) \cap \tau_i$-Cl$(\tau_j$-Int$(B)) = \emptyset$. Since B is (τ_j, τ_i)-semiopen in X, τ_i-Cl$(B) = \tau_i$-Cl$(\tau_j$-Int$(B))$. Thus τ_j-Cl$(A) \cap \tau_i$-Cl$(B) = \emptyset$.
(2) ⇒ (3): Straightforward.
(3) ⇒ (1): Let A be a τ_i-open set and B a τ_j-open set such that $A \cap B = \emptyset$. Since every τ_i-open set is a (τ_i, τ_j)-semiopen set and every τ_j-open set is a (τ_i, τ_j)-semiopen set and every τ_j-open set is a (τ_j, τ_i)-preopen set, τ_j-Cl$(A) \cap \tau_i$-Cl$(B) = \emptyset$. Hence by Theorem 4.2, X is pairwise extremally disconnected. □

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College of Vestsjaelland South, Herrestraede 11, and, Mathematical and Physical Science Foundation,, 4200 Slagelse, Denmark.
Email address: jafaripersia@gmail.com

Dipartimento di Scienze Matematiche e Informatiche, Scienze Fisiche e Scienze della Terra Dell' Uuniversità delgli Studi di Messina, Viale Ferdinando Stango d'Alcontres, 31 - 98166 Messina, Italy.
Email address: giorgio.nordo@unime.it

Department of Mathematics, Rajah Serfoji Govt. College, Thanjavur-613005, Tamilnadu, India.
Email address: nrajesh_topology@yahoo.co.in