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ON NEUTROSOPHIC θ-QUOTIENT FUNCTIONS

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Abstract

We continue the study on neutrosophic θ-closed set and neutrosophic θ-quotient, neutrosophic strongly θ-quotient andneutrosophic quasi θ-openfunctions are presented in neutrosophic topological spaces. Characterizations as well as properties are also discussed.

1. Introduction

Fuzzy set hypothesis is presented and considered as a mathematical tool for managing vulnerabilities where every component had a level of participation, membership function (mf) or truth (t), by Zadeh [15]. The degree of non-membership function (nmf), the falsehood (f), was introduced by Atanassov [2] in an intuitionistic fuzzy set. Coker [3] developed the notion

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of intuitionistic fuzzy topology. Neutrality (i), the degree of indeterminacy, as an independent notion, was introduced by Smarandache [11, 12, 13]. He likewise characterized the neutrosophic set (NS) on three parts $(t, f, i) =$ (truth, falsehood, indeterminacy). Salama et al. [11, 12] converted Neutrosophic crisp set into neutrosophic topological spaces (NTS). The introduction of NS opened a broadvariety of exploration in requisites of NTS decision making problems. A. A. Salama et al. in [10] introduced neutrosophic closed sets and continuous functions. R. Dhavaseelan et al. [4] presented generalized neutrosophic closed sets. Neutrosophic semi-open, and α-open are presented in [14]. In [8] the authors developed the notion of neutrosophic θ-closure operator and utilizing this, neutrosophic θ-closed set is elucidated. Neutrosophic θ-continuous, neutrosophic strongly θ-continuous including neutrosophic weakly continuous mappings are characterized concerning the operator defined as applications of neutrosophic θ-closure and neutrosophic θ-closed sets.

This paper is devoted to the continuation study on neutrosophic θ-closed [8] set and presented the concept of neutrosophic θ-quotient function, neutrosophic strongly θ-quotient functions are obtainable in NTS. Some properties are also discussed.

2. Preliminaries

Definition 2.1 [11, 12]**.** Let be a non-empty fixed set. A neutrosophic set (in a nutshell, NS) Λ is an object so as $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x)\rangle : x \in S_1\}$ wherein $\mu_\Lambda(x), \sigma_\Lambda(x)$ and $\gamma_\Lambda(x)$ which addresses the level of membership function (namely $\mu_\Lambda(x)$), the level of indeterminacy (viz $\sigma_\Lambda(x)$) along with the level of non-membership (namely $\gamma_\Lambda(x)$) separately of every component $x \in S_1$ to the set Λ.

Remark 2.2 [11, 12]**.** (i) An N-set $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x)\rangle : x \in S_1\}$ can be recognized to an arranged triple $\langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda\rangle$ in $]0^-, 1^+[$ on S_1.

(ii) We use the symbol $\Lambda = \langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda\rangle$ for the N-set $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x)\rangle : x \in S_1\}$.

Definition 2.3 [11, 12]. Let $S_1 \neq \emptyset$ and the N-sets Λ and Γ be named as $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x)\rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_\Gamma(x), \sigma_\Gamma(x), \Gamma_\Gamma(x)\rangle : x \in S_1\}$. Then I. $\Lambda \subseteq \Gamma$ iff $\mu_\Lambda(x) \leq \mu_\Gamma(x)$, $\sigma_\Lambda(x) \leq \sigma_\Gamma(x)$ and $\Gamma_\Lambda(x) \leq \Gamma_\Gamma(x)$ for all $x \in S_1$,

II. $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;

III. $\overline{\Lambda} = \{\langle x, \Gamma_\Lambda(x), \sigma_\Lambda(x), \mu_\Lambda(x)\rangle : x \in S_1\}$, [Complement of Λ]

IV. $\Lambda \cap \Gamma = \{\langle x, \mu_\Lambda(x) \wedge \mu_\Gamma(x), \sigma_\Lambda(x) \wedge \sigma_\Gamma(x), \Gamma_\Lambda(x) \vee \Gamma_\Gamma(x)\rangle : x \in S_1\}$,

V. $\Lambda \cup \Gamma = \{\langle x, \mu_\Lambda(x) \vee \mu_\Gamma(x), \sigma_\Lambda(x) \vee \sigma_\Gamma(x), \Gamma_\Lambda(x) \wedge \Gamma_\Gamma(x)\rangle : x \in S_1\}$,

VI. $[\ ]\ \Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), 1 - \mu_\Lambda(x)\rangle : x \in S_1\}$,

VII. $\langle\ \rangle\ \Lambda = \{\langle x, 1 - \Gamma_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x)\rangle : x \in S_1\}$.

Definition 2.4 [12, 13]. Let $\{\Lambda_i : i \in j\}$ be a discretionary family of N-sets in S_1. Therefore

I. $\cap \Lambda_i = \{\langle p, \mu_{\Lambda_i}(p), \wedge \sigma_{\Lambda_i}(p), \vee \Gamma_{\Lambda_i}(p)\rangle : p \in S_1\}$,

II. $\cup \Lambda_i = \{\langle p \vee \mu_{\Lambda_i}(p), \vee \sigma_{\Lambda_i}(p), \wedge \Gamma_{\Lambda_i}(p)\rangle : p \in S_1\}$.

The principal subject is to assemble the equipment for creating NTS, so we construct the neutrosophic sets $0_\aleph$ along with $1_\aleph$ in X as follows:

Definition 2.5 [12, 13]. $0_\aleph = \{\langle q, 0, 0, 1\rangle : q \in X\}$ and $1_\aleph = \{\langle q, 0, 0, 1\rangle : q \in X\}$.

Definition 2.6 [10]. A neutrosophic topology (briefly, $\aleph T$) $S_1 \neq \emptyset$ is a family ξ_1 of N-sets in S_1 obeying the axioms given underneath:

I. $0_\aleph, 1_\aleph \in \xi_1$,

II. $W_1 \cap W_2 \in T$ being $W_1, W_2 \in \xi_1$,

III. $\cup W_i \in \xi_1$ for arbitrary family $\{W_i \mid i \in \Lambda\} \subseteq \xi_1$.

For this circumstances, Ordered pair (S_1, ξ_1) or plainly S_1 is termed as and each NS in ξ_1 is termed as neutrosophic open set (in a nutshell, $\aleph$OS).

The complement of an $\aleph$-open set Λ in S_1 is known as neutrosophic closed set ($\aleph$CS, for short) in S_1.

Definition 2.7 [10, 11]**.** Consider a NS as $\Re$ in NTS S_1. Therefore $\aleph\operatorname{int}(\Re) = \bigcup\{W \mid W$ is an $\aleph$OS in S_1 and $W \subseteq \Re\}$ is called as neutrosophic interior (in a nutshell, $\aleph\operatorname{int}$) of $\Re$; $\aleph cl(\Re) = \bigcap\{W \mid W$ is an $\aleph$ CS in S_1 and $W \supseteq \Re\}$ is named as neutrosophic closure (in a nutshell, $\aleph cl$) of $\Re$.

Definition 2.8 [4]**.** Let X be a nonempty set. Whenever r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ at that time the $NSx_{r,t,s}$ is named as neutrosophic point (in a nutshell, NP) in X given by $x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$ for $x_p \in X$ is designated as the support of $x_{r,t,s}$, wherein demonstrates the mf, demonstrates the i and demonstrates then mf of $x_{r,t,s}$.

Definition 2.9 [14]**.** For an $\aleph$S D in an NTS (X, T). We have,

(i) $\aleph$eutrosophic semi open set ($\aleph$SOS) if $D \subseteq \aleph cl(\aleph\operatorname{int}(D))$.

(ii) $\aleph$eutrosophic α-open set ($\aleph\alpha$OS) if $D \subseteq \aleph\operatorname{int}(\aleph cl(\aleph\operatorname{int}(D)))$.

The complement of D is an $\aleph$SOS, $\aleph\alpha$OS, is called respectively as $\aleph$SCS and $\aleph\alpha$ CS.

Definition 2.10 [8]**.** A NP $x_{(\alpha,\beta,\gamma)}$ is named as neutrosophic θ-cluster point ($\aleph\alpha$-cluster point, shortly) of a NS K iff considering respective W in $N \in q$-nbd of $x_{(\alpha,\beta,\gamma)}$ with $Ncl(W)qK$. The collection of all $N\theta$-cluster points of K is labelled as neutrosophic θ-closure besides represented as $NCl_\theta(K)$.

An NS K will be Nθ-closed set (NθCS in precise) iff $K = NCl_\theta(K)$. The complement of a NθCS is N-open set (in precise, NθOS).

Definition 2.11 [7]**.** Let $\eta : (S_1, \tau) \to (T_1, \Gamma)$ be a surjective function. Then η is named as neutrosophic quotient function if η is neutrosophic continuous and $\eta^{-1}(M)$ is NOS in (T_1, Γ).

3. Neutrosophic θ-Quotient Function

Definition 3.1. Let $\eta : (S_1, \tau) \to (T_1, \Gamma)$ is termed as

(i) Neutrosophic θ-quotient function if η is neutrosophic θ-continuous and $\eta^{-1}(M)$ is NOS in S_1 implies M is a NOS in T_1.

(ii) Neutrosophic θ^*-quotient function if η is neutrosophic θ-irresolute and $\eta^{-1}(M)$ is NθOS in implies M is a NOS in T_1.

(iii) Neutrosophic strongly θ-open function if the image of every NθOS in S_1 is a NTOS in T_1.

(iv) Neutrosophic θ-open if the image of every NOS in S_1 is a NθOS in T_1.

Theorem 3.2. *If* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *is surjective neutrosophic* θ-*continuous and neutrosophic* θ-*open*, *then* η *is neutrosophic* θ-*quotient function*.

Proof. Let $\eta^{-1}(M)$ be NOS in S_1. Thereupon is a NTOS, as is neutrosophic θ-open. Accordingly, NθOS, as η is surjective along with $(\eta^{-1}(M)) = M$. So, η is neutrosophic θ-quotient function

Theorem 3.3. *If* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *is neutrosophic open surjective neutrosophic* θ-*irresolute and* $\kappa : (S_2, \Gamma) \to (T_2, \Psi)$ *be neutrosophic* θ-*quotient function*. *Then* $(\kappa \circ \eta)$ *is neutrosophic* θ-*quotient*.

Proof. Consider R be any NOS in (T_2, Ψ). Thereupon $\kappa^{-1}(R)$ is NθOS as is neutrosophic θ-quotient function along with this η is neutrosophic θ-irresolute, $\eta^{-1}(\kappa^{-1}(R))$ is NθOS. Henceforth $(\kappa \circ \eta)^{-1}(R)$ is a NθOS implies $(\kappa \circ \eta)$ is NθOS. So, $(\kappa \circ \eta)$ is a neutrosophic θ-continuous. Also, suppose $(\kappa \circ \eta)^{-1}(R)$ be NOS in S_1 for $T \subseteq R$, that is $\eta^{-1}(\kappa^{-1}(R))$ NθOS in S_1. As η is neutrosophic open, $\eta(\eta^{-1}(\kappa^{-1}(R)))$ is NOS in T_1. It follows that $\kappa^{-1}(R)$ is NOS in T_1, η is surjective. Since κ is neutrosophic θ-quotient function, R is NθOS. Accordingly, $(\kappa \circ \eta)$ is neutrosophic θ-quotient function.

Theorem 3.4. *Let* (S_1, τ) *and* (T_1, Γ) *be NTSs. If* $\eta : (S_1, \tau^\theta) \to (T_1, \Gamma^\theta)$ *is neutrosophic* θ*-quotient function then* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *is neutrosophic* θ*-quotient.*

Proof. Consider $W \in \Gamma$ so $W \in \tau^\theta$. As η is neutrosophic θ-quotient function, $\eta^{-1}(W) \in \tau^\theta$. So, it is claimed that when W is a NOS in (T_1, Γ) at that time $\eta^{-1}(W)$ is a NθOS in (S_1, τ). Accordingly, is neutrosophic θ-continuous function. Presume $\eta^{-1}(W)$ is NOS in (S_1, τ) thereupon $\eta^{-1}(W) \in \tau^\theta$. As η is neutrosophic quotient function, $W \in \tau^\theta$ and so W is NθOS in (T_1, Γ). Hence $\eta : (S_1, \tau) \to (T_1, \Gamma)$ is neutrosophic θ-quotient.

Theorem 3.5. *If* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *be onto neutrosophic strongly* θ*-open and a neutrosophic* θ*-irresolute function. Consider,* $\kappa : (S_2, \Gamma) \to (T_2, \Psi)$ *be neutrosophic* θ^**-quotient function. Then* $(\kappa \circ \eta)$ *is neutrosophic* θ^**-quotient.*

Proof. We prove that $(\kappa \circ \eta)$ is neutrosophic θ-irresolute. Consider R be any NθOS in (T_2, Ψ). Then $\kappa^{-1}(R)$ is NθOS as κ is neutrosophic θ^*-quotient function. Since, η is neutrosophic θ-irresolute, $\eta^{-1}(\kappa^{-1}(R)) = (\kappa \circ \eta)^{-1}(R)$ is NθOS. So, $(\kappa \circ \eta)$ is a neutrosophic θ-irresloute function. Suppose $(\kappa \circ \eta)^{-1}(R) = \eta^{-1}(\kappa^{-1}(R))$ be NθOS in S_1. Since, η is neutrosophic strongly θ-open, $\eta^{-1}(\eta^{-1}(\kappa^{-1}(R)))$ is NθOS in T_1. Since, η is an onto function $\eta(\eta^{-1}(\eta^{-1}(\kappa^{-1}(R)))) = \kappa^{-1}(R)$. So $\kappa^{-1}(R)$ is NθOS in T_1. This implies that R is NOS in (T_2, Ψ) as κ is a neutrosophic θ^*-quotient function. Therefore, $(\kappa \circ \eta)$ is neutrosophic θ^*-quotient.

Theorem 3.6. *Every neutrosophic quotient function is neutrosophic* θ*-quotient.*

Proof. Consider J be a NOS in (T_1, Γ). As, η is neutrosophic quotient function, $\eta^{-1}(J)$ is NOS in (S_1, τ) and so it is NθOS in (S_1, τ). So, η is neutrosophic θ-continuous function. Let $\eta^{-1}(J)$ is NOS in (S_1, τ) and so J is a NθOS in (T_1, Γ). Hence is neutrosophic θ-quotient function.

Theorem 3.7. *Every neutrosophic* θ^**-quotient function is neutrosophic strongly* θ*-quotient.*

Proof. Consider $\eta : (S_1, \tau) \to (T_1, \Gamma)$ be a neutrosophic θ^*-quotient function. Suppose R be NOS in T_1 at that time $\eta^{-1}(R)$ is NOS in S_1 as R is NθOS in T_1 along with η is neutrosophic θ-irrsolute. As η is neutrosophic θ^*-quotient function $\eta^{-1}(R)$ is NθOS in S_1 then R is NOS in T_1. So η is neutrosophic strongly θ-quotient.

Definition 3.8. Let (S_1, τ) and (T_1, Γ) be NTSs. A function $\eta : (S_1, \tau) \to (T_1, \Gamma)$ is named as neutrosophic quasi θ-open if the image of each NθOS in (S_1, τ) is NOS in (T_1, Γ).

Theorem 3.9. *Consider two NTSs as* (S_1, τ) *and* (T_1, Γ). *If* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *is neutrosophic quasi* θ*-open then is neutrosophic strongly* θ*-open.*

Proof. Consider J be a NθOS in (S_1, τ). So $J \in \tau^\theta$. Since $\eta : (S_1, \tau^\theta) \to (T_1, \Gamma^\theta)$ is neutrosophic quasi θ-open, $\eta(J)$ is NOS in (T_1, Γ^θ). So, $\eta(J)$ is a NθOS in (T_1, Γ). Hence it follows that $\eta : (S_1, \tau) \to (T_1, \Gamma)$ is neutrosophic strongly θ-open function.

Definition 3.10. Consider S_1 be a NTS and R is NS in S_1. Neutrosophic θ-interior of R is noted and defined as $N\operatorname{int}_\theta(R) = Ncl_\theta(R^c)^c$ and we have

(i) $(Ncl_\theta(R^c)) = (N\operatorname{int}_\theta(R))^c$

(ii) $(Ncl_\theta(R))^c = N\operatorname{int}_\theta(R^c)$

Theorem 3.11. *Taking J and K as NSs in NTS* (S_1, τ), *then*

(i) $N\operatorname{int}_\theta(1_\sim) = 1_\sim$

(ii) $N\,\mathrm{int}_\theta(J) \subseteq J$

(iii) $J \subseteq K \Rightarrow N\,\mathrm{int}_\theta(J) \subseteq N\,\mathrm{int}_\theta(K)$.

(iv) $N\,\mathrm{int}_\theta(J \cap K) = N\,\mathrm{int}_\theta(J) \cap N\,\mathrm{int}_\theta(K)$.

(v) $N\,\mathrm{int}_\theta(J) \cup N\,\mathrm{int}_\theta(K) \subseteq N\,\mathrm{int}_\theta(J \cap K)$.

Corollary 3.12. *For a NS R,* $N\,\mathrm{int}_\theta(R) \subseteq N\,\mathrm{int}(R)$.

Theorem 3.13. *If* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *is neutrosophic quasi* θ*-open iff each member W of* S_1, $\eta(N\,\mathrm{int}_\theta(W) \subset N\,\mathrm{int}(\eta(W))$.

Proof. Consider η be neutrosophic quasi θ-open. As $N\,\mathrm{int}(W) \subset W$ with $N\,\mathrm{int}_\theta(W)$ is NθOS. Accordingly, we get $\eta(N\,\mathrm{int}_\theta(W) \subset \eta(W)$. As, $\eta(N\,\mathrm{int}_\theta(W))$ is NOS, $\eta(N\,\mathrm{int}_\theta(W) \subset N\,\mathrm{int}(\eta(W))$. Conversely, presume that W is NθOS in S_1. Thereupon, $\eta(W) = \eta(N\,\mathrm{int}_\theta(W) \subset N\,\mathrm{int}(\eta(W))$. But, $N\,\mathrm{int}(\eta(W)) \subset \eta(W)$. Consequently, $\eta(W) = N\,\mathrm{int}(\eta(W))$ and hence η is neutrosophic quasi θ-open.

Theorem 3.14. *If* $\eta : (S_1, \tau) \to (T_1, \Gamma)$ *is neutrosophic quasi* θ*-open thereupon* $N\,\mathrm{int}_\theta(\eta^{-1}(W)) \subset \eta^{-1}(N\,\mathrm{int}(W))$ *for each member W of* T_1.

Proof. Consider W be member of T_1. At that time $N\,\mathrm{int}_\theta(\eta^{-1}(W))$ is NθOS in and η is neutrosophic quasi θ-open, so $\eta(N\,\mathrm{int}_\theta(\eta^{-1}(W)))$ $\subset N\,\mathrm{int}(\eta(\eta^{-1}(W)))$. Thus, $N\,\mathrm{int}_\theta(\eta^{-1}(W)) \subset \eta^{-1}(N\,\mathrm{int}(W))$.

4. Conclusions

We have presented concept of neutrosophic θ-quotient, neutrosophic strongly θ-quotient and neutrosophic quasi θ-open functions in NTS. Some results have been proved to show that how far topological structures are preserved by the new neutrosophic functions defined. Here we have presented the idea; still some more theoretical research is to be carried out to build up an overall frame work for dynamic and to characterize designs for complex organization considering and down to earth application.

References

[1] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, On some new notions and functions in neutrosophic topological spaces, Neutrosophic Sets and Systems 16(201) (2017), 16-19.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[3] D. Coker, An introduction to intuitionistic fuzzy topological space, Fuzzy Sets Syst. 88 (1997), 81-89.

[4] R. Dhavaseelan and S. Jafari, Generalized Neutrosophic closed sets, New Trends in Neutrosophic Theory and Applications II (2017), 261-273.

[5] R. Dhavaseelan and Md. Hanif PAGE, Neutrosophic Almost α-contra-continuous function, Neutrosophic Sets and Systems 29 (2019), 71-77.

[6] Mohamed El Fatini, Mohammed Louriki, Roger Pettersson and Zarife Zararsiz, Epidemic modeling: Diffusion approximation vs. stochastic differential equations allowing reflection, International Journal of Biomathematics 14(05) 2150036 (2021).

[7] T. Nandhini and M. Vigneshwaran, Neutrosophic Quotient Mappings, Advances and Applications in Mathematical Sciences 18(11) (2019), 1571-1583.

[8] Md. Hanif PAGE, R. Dhavaseelan and B. Gunasekar, Neutrosophic θ-Closure Operator, Neutrosophic Sets and Systems 38(1) (2020), 41-50. https://digitalrepository.unm.edu/nss_journal/vol38/iss1/4

[9] A. A. Salama and S. A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Mathematics 3(4) (2012), 31-35.

[10] A. A. Salama, F. Smarandache and K. Valeri, Neutrosophic closed set and neutrosophic continuous functions, Neutrosophic Sets and Systems 4 (2014), 4-8.

[11] F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA (2002), smarand@unm.edu.

[12] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, (1999).

[13] F. Smarandache, Neutrosophic set- a generalization of the intuitionistic fuzzy set International Journal of Pure and Applied Mathematics 24(3) (2005), 287-294.

[14] WadeiAl-Omeri and Smarandache, F. New Neutrosophic Sets via Neutrosophic Topological Spaces, In Neutrosophic Operational Research; Smarandache, F., Pramanik, S., Eds.; Pons Editions: Brussels, Belgium I (2017), 189-209.

[15] L. A. Zadeh, Fuzzy set, Inf. Control 8 (1965), 338-353.