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Article *in* AIMS Mathematics · January 2023

DOI: 10.3934/math.2023380

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AIMS Mathematics, 8(3): 7584–7605.
DOI: 10.3934/math.2023380
Received: 26 October 2022
Revised: 21 December 2022
Accepted: 23 December 2022
Published: 17 January 2023

http://www.aimspress.com/journal/Math

Research article

Multi-objective optimization model for uncertain crop production under neutrosophic fuzzy environment: A case study

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Abstract: In real world uncertainty exist in almost every problem. Decision-makers are often unable to describe the situation accurately or predict the outcome of potential solutions due to uncertainty. To resolve these complicated situations, which include uncertainty, we use expert descriptive knowledge which can be expressed as fuzzy data. Pakistan, a country with a key geographic and strategic position in South Asia, relies heavily on irrigation for its economy, which involves careful consideration of the limits. A variety of factors can affect yield, including the weather and water availability. Crop productivity from reservoirs and other sources is affected by climate change. The project aims to optimize Kharif and Rabbi crop output in canal-irrigated areas. The optimization model is designed to maximize net profit and crop output during cropping seasons. Canal-connected farmed areas are variables in the crop planning model. Seasonal crop area, crop cultivated area, crop water requirement, canal capacity, reservoir evaporation, minimum and maximum storage, and overflow limits affect the two goals. The uncertainties associated with the entire production planning are incorporated by considering suitable membership functions and solved using the Multi-Objective Neutrosophic Fuzzy Linear Programming Model (MONFLP). For the validity and effectiveness of the technique, the model is tested for the wheat and rice production in Pakistan. The study puts forth the advantages of neutrosophic fuzzy algorithm which has been proposed, and the analyses derived can be stated to deal with yield uncertainty in the neutrosophic environments more effectively by considering the parameters which are prone to abrupt changes characterized by unpredictability.

Keywords: multi-objective linear programming; neutrosophic optimization; crop production; water reservoir active storage; unpredictability
Mathematics Subject Classification: 90C29, 90C70, 90C90

1. Introduction

Modeling of complex dynamic systems can pose some challenges given their main attributes, so there needs to be a requirement of the development of methods regarding qualitative analysis to handle the dynamics and behavior of such systems besides the constructing of efficient control algorithms toward efficient operation, classification, recognition, identification, optimization and simulation. Several types of uncertainty representation can be addressed such as interval, fuzzy, granular as well as combined uncertain sets. The concept of the fuzzy set given by Zadeh [1] to deal with ambiguity and vagueness is valid for situations where true grades of membership exist. However, all physical and logical models may not rely completely on the valuations of membership. The generalization of fuzzy sets was introduced as intuitionistic fuzzy set [2], being more effective in dealing with ambiguity. In contrast to the traditional the fuzzy set theory, it assigns membership and non-membership to generic elements. Fuzzy optimization is an improved method that handles the uncertainty and imprecision associated with any optimization problem by involving parameters, arithmetic operations and relations governed from fuzzy sets. Bellman and Zadeh [3] were the first individuals to combine the ideas of programming and fuzziness. Tanaka et al. [4] and Zimmerman [5] extended the work from single objective to several objective functions. Angelov [6] established intuitionistic optimization technique that depends on the remodeling of single objective minimizations problem in fuzzy environment. The single objective is converted into two objectives that are to maximize the membership degree and minimize the non-membership degree. Using intuitionistic fuzzy sets is one way to tackle uncertainty since it yields the guarantee of less violation of risks emerging from vagueness during decision-making processes. While assigning and designating the priorities in multi-objective situations, flexibility can be generated corresponding to each objective and evaluation [7]. Complex optimization problems pose multiple unknown parameters that occur due to uncontrollable and unavoidable factors. Climate, weather and water storage are some of these elements which are bound by such uncontrollable factors. Imprecise data that are reliant of different parameters can be well represented by fuzzy numbers and membership grades. More complexity occurs when there is uncertainty involved in that process regarding the membership degree with parameters being uncertain as well. Thus, the degrees of membership function are important in decision-making under uncertainty and vagueness [8]. The neutrosophic optimization technique involves degree of truth, falsity and indeterminacy memberships. Degrees of truth and falsity memberships are not complemented of each other. They are, in fact, independent of degree of indeterminacy.

Founded by Florentin Smarandache in 1998, the neutrosophic theory constitutes a further generalization of fuzzy sets, triangular dense fuzzy sets [9,10], picture fuzzy sets [11,12] and spherical fuzzy sets [13] among others. Accordingly, the study [14] developed robust neutrosophic programming model to deal with multi-objective intuitionistic fuzzy optimization problem where the approach is based on intuitionistic fuzzy numbers and ranking functions for these numbers. The model proposed in the study is effective in such a way that it includes neutral thoughts during decision-making. The study, with its practical implications, provides the different types of membership functions depicted for the marginal evaluation of each objective concurrently. Another relevant study [15] has the aim of showing how neutrosophic optimization technique can be employed for the solution of a nonlinear structural problem. The problem involves the requirement of investigating the consequence of non-linearity of the truth, indeterminacy and falsity membership function in view of multi-objective optimization problem under consideration. The study considers a non-linear three bar truss design problem. A further study [16] involves the development of an algorithm to evaluate the multi-level and multi-objective fractional programming problems by making use of the notion of a neutrosophic fuzzy set. The authors construct a neutrosophic fuzzy goal model to minimize the group tolerance of a satisfactory degree and attain the optimal degree for truth, falsity and indeterminacy of each kind of the prearranged membership functions goals to the most possible through minimizing their corresponding deviational variables, with the ultimate goal of obtaining the optimal solutions. The study puts forward the key benefit of the neutrosophic fuzzy goal programming algorithms proposed, which can be stated that if the attained optimal solution is refused by the relevant stakeholders repeatedly, the problem can be reevaluated by defining suitable and more appropriate membership functions until the desired result is obtained.

Uncertainty and counter intuition in geological interpretations generate an often-unquantified risk for related industries and activities. The challenge related to quantifying such interpretation for uncertainty has been addressed using various methods including the empirical quantification of uncertainties as derived from comparison of interpretations of different complex data. Pakistan, in that regard, holds a significant position in South Asia in terms of its geographic and geo-political conjuncture. Moreover, the country is highly vulnerable to the climatic changes that cause unpredictable weather patterns. Droughts and floods have frequently been observed in the last two decades resulting in a huge loss in the country's GDP. Pakistan is classified as an agricultural to semi-agricultural country, to put it differently, an agrarian country. Agriculture is the major supplier to food security and acts as a major contributor in Pakistan's economy [17]. More than 60% of the population living in rural areas of Pakistan is dependent on agriculture of the country [18,19]. Approximately 22 million hectors (Mha) of land are cultivated out of the total land and is a major user of water resources. Water used with the ratio of 17:5 Mha of a total 22 Mha is irrigated and rain is fed, respectively. Pakistan's irrigation system relies primarily on fresh surface water and it possesses the world's largest network of canals and reservoirs. With three major reservoirs, 19 barrages, 12 link canals, 46 main canals and thousands of hydraulic structures, Pakistan has the world's largest continuous irrigation system [20]. The most imported source of water for irrigation in Pakistan is the Indus basin which rises from Gilgit in Pakistan (see Figure 1). Flowing through the North in a southerly direction along the entire length of Pakistan, it falls into the Arabian Sea near Pakistan's port city of Karachi. It is also aided by the four other large rivers like Jehlum, Chenab and Sutluj as well as several some small rivers like Kabul, Swat, Haro, Kunhar and Chitral [21].

In terms of agricultural production, Kharif and Rabbi make up the two types of seasonal crops in Pakistan. The growing of Kharif starts from May and ends in October. Rice, sugar cane, cotton, maize, and millet belong to the class of Kharif crops, whereas Rabbi Season starts from November and ends in April. Wheat, gram, tobacco, rapeseed, barley and mustard belong to the category of Rabi crops [22]. The season of these crops is influenced by different climatic variables such as temperature and rainfall. Excess of any climatic variables has unfavorable impacts on the agricultural crops [23]. Climate change severely impacts water and land, which can cause a change in output as much as 60%. It can also stimulate the timing of agricultural seasons, water stress, the magnitude and duration of heat [24].

Several uncertainties must be considered when analyzing the effects of climate change. Most impact assessments rely on general circulation models (GCM) forecasts [25]. In an unpredictable situation, exact reservoir capacity and reservoir operation policies are among the climate change adaptation solutions. Hence, uncertainty analysis is regarded as an essential component of reservoir yield analysis [26]. The multi-objective optimization problem (MOP) is widely used in water management, agriculture, industry, engineering, economics, mining and many other fields where it is necessary to optimize several conflicting objectives at the same time. Multi-objective optimization models are extensively used in agriculture under the influence of climate change [27–33].

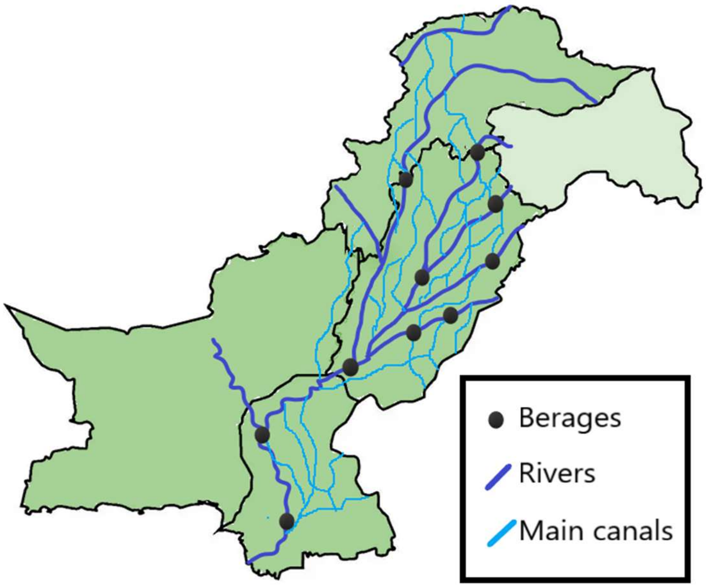

Figure 1. Irrigation system.

A two-stage stochastic fuzzy-interval credibility constraint programming method (ISFICP) has been developed that can be applied to the allocation of water resources under uncertain and complex situations. There may be trade-offs between system benefits and risk of violation in this strategy and the many complications of water resource management. To address the problem of farmland and ecological water allocation in irrigation regions under uncertainty, Pan et al. [34] proposed an interval multi-objective fuzzy-interval credibility-constrained nonlinear prograg (IMFICNP) model. The water demand of ecological vegetation is upgraded from a site-specific sample to a spatial decision-making unit (DMU), which offers a full range of spatial data for constraint inputs and the water requirement of ecological vegetation is divided into three categories using remote sensing (RS) and geoinformation system (GIS) tools, including forest land, grassland and shrubland. Jin et al. [35] developed an improved teaching-learning-based optimization algorithm for the purpose of capturing more robust scheduling schemes. The authors aimed to obtain promising solutions with score values of the uncertain completion times on each machine having been compared and optimized. The definition of distinct levels of fluctuations or uncertainties on processing times was performed in testing Kim's benchmark instances and the performance of computational results was analyzed in the related study. Moreover, Ren et al. [36] proposed an improved interval multi-objective programming system for dealing with numerous objectives and uncertainties in irrigation water resource allocation. The model was tested in a case study in China's Shaanxi Province's Jinghuiqu basin. Maximizing economic benefit and lowering energy use are two of the planning goals. Various planting structure optimization strategies were found in multiple typical hydrological years. They found that a lack of water resources negatively influenced the development of irrigation systems in that study. The Fuhe River basin was used to test a multi-objective water resource allocation (GWAS) model, which considered the goals of socioeconomic water consumption, power generation and river biological flow. Yue et al. [37,38] designed the models to cope with the water shortage caused by diverse natural conditions and ineffective irrigation water management Type-2 fuzzy mixed-integer bi-level and full fuzzy-interval credibility-constrained nonlinear programming approach. Sahoo et al. [39] discussed the Genetic algorithm and particle swarm optimization in combined form to solve mixed integer nonlinear reliability optimization issues in series, series-parallel, and bridge systems.

Indeterminacy allows flexibility in the decision-making process since the function and decision variable obtain optimum indeterminate outcomes. Improved and advanced methods are required as fuzzy framework has shown an ever-growing expansion. Fuzziness, ambiguity, uncertainty and vagueness connected with real-life data and the desire to get the optimal solution under these factors are what determine the formulation of fuzzy optimization. Neutrosophic set and logic appear to be a comparatively new idea introduced by Smarandache [40] and further flourished by Wang et al. [41]. Since indeterminacy always appears in our routine activities, the Neutrosophic theory and its generalization can analyze various situations smoothly [42,43]. The goal functions are transformed into neutrosophic constraints when using the neutrosophic technique for optimization. Neutrosophic is also used in many fields related to real-life problems such as transportation, tourism management, supply-demand chain, production planning, health sector, and matrix game solving, in a stock portfolio, among others [44–60].

The complexity of environmental and socio-economic factors has increased agricultural production uncertainty, especially under present climate change, highlighting the need for robust multi-objective optimization models. Crop production is subject to a variety of uncertainties, including crop yield, crop quality, input parameters, and output objectives. Multi-objective optimization models based on unified frameworks must be devised to address these uncertainties. A multi-objective optimization model for uncertain crop production in a neutrosophic fuzzy environment is proposed to integrate various uncertainty handling strategies to address the various crop production uncertainties. The proposed model will use neutrosophic fuzzy logic to account for uncertainty in input parameters, fuzzy logic to account for uncertainty in output objectives, and optimization techniques to optimize the multi-objective problem. The proposed model will provide an effective method for optimizing the multi-objective problem in the presence of several sources of uncertainty like climate change and active reservoirs, thereby addressing the issue of uncertain crop production. Furthermore, the proposed model can be used to assess the effectiveness of various crop management practices in a variety of situations.

2. Preliminaries

3.1. Linear programming

There are many challenges involved in linear programming, including maximizing or minimizing an objective function with constraints. The constraints are linear equality/inequality. It consists of the following parts:

Decision variables set X;
Objective Function: a linear function $f(x)$ to be maximized (minimized);

3.2. Multi-objective linear programming

Both the objective function and constraints can be different in multi-objective linear programming [5]. Multi-objective linear programming can be shown in mathematics as:
Minimize /maximize $f_s(x) = a_{s1}.x_1 + a_{s2}.x_2 + \ldots + a_{sn}.x_n$ for all s.
Such that

$$a_{t1}.x_1 + a_{t1}.x_1 + \ldots + a_{t1}.x_1 \gtreqless b_t.$$

In general, there will not exist a single point $(x) = (x_1, x_2, \ldots, x_n)$ which optimizes each objective function individually. Multi-objective linear programming problems (MOLP) can be used to mimic a variety of real-world issues. A multiple objective linear programme is a linear programme with numerous objective functions.

3.3. Fuzzy linear programming

Fuzzy linear programming [5] represented as:

$$\text{Max} \quad \tilde{Z}.x,$$

subject to $\sum_{j=1}^{n} \tilde{X}_{ij}.x_j \geq \tilde{Y}_i,\ i = 1,2,\ldots m$

$$x_j \geq 0,\ j = 1,2,\ldots n.$$

Where X_{ij} are decision variables in the constraints and x_j are non-negative fuzzy number. Mathematical modelling, manufacturing, environmental management, supply chain management, and transportation management are all applications of fuzzy linear programming. Fuzziness is considered in fuzzy linear programming's objective function and constraint equations.

3.4. Neutrosophic set

Neutrosophic set [8] is a generalized concept in which each component $\boldsymbol{x} \in \boldsymbol{X}$to a set $\widetilde{\boldsymbol{A}}^{\boldsymbol{N}}$ has a membership degree $\boldsymbol{T}_{\widetilde{\boldsymbol{A}}^N}(\boldsymbol{x})$, non membership degree $\boldsymbol{F}_{\widetilde{\boldsymbol{A}}^N}(\boldsymbol{x})$ as well as a degree of indeterminacy $\boldsymbol{I}_{\widetilde{\boldsymbol{A}}^N}(\boldsymbol{x})$, where $\boldsymbol{T}_{\widetilde{\boldsymbol{A}}^N}(\boldsymbol{x}), \boldsymbol{I}_{\widetilde{\boldsymbol{A}}^N}(\boldsymbol{x})$ and $\boldsymbol{F}_{\widetilde{\boldsymbol{A}}^N}(\boldsymbol{x})$ are real standard or nonstandard subsets of $]\boldsymbol{0}^-, \boldsymbol{1}^+[$.

3. Multi-objective crop production model

The two related functions are constructed in this section of the study. Water shortage, being the main issue for crop production and availability of water during a specific period in Pakistan, determines the productivity of the sectors in the country concerned with agriculture. Within this framework, modern agriculture is characterized by several parameters including conflicting optimization criteria (total cost, net benefit, production, and so forth).

3.1. The related parameters

The set of parameters used in this study are presented in Table 1.

Table 1. The related parameters.

Symbol	Parameter
sc	Seeds cost (PKR/ha)
ca	Cultivated area (Hector)
cst	Cost of chemical for seeds treatment (PKR/ha)
drp	Dry ploughing cost (PKR/ha)
wp	Wet ploughing cost (PKR/ha)
dep	Deep ploughing (PKR/ha)
pc	Ploughing/cultivator (PKR/ha)
plk	Planking (PKR/ha)
$wp*$	Wet planking (PKR/ha)
ll	Laser leveling (PKR/ha)
bm	Bund making (PKR/ha)
ur	Urea (PKR/ha)
dap	DAP (PKR/ha)
zs	Zinc sulphate (PKR/ha)
fta	Fertilizer transport & application (PKR/ha)
mn	Manual (25%) (PKR/ha)
wc	Weedicides (75%) (PKR/ha)
pp	Plant protection (PKR/ha)
qpr	Quintal average production per hectare
cpq	Crop price of one quintal
up	Uprooting transplanting & transport (PKR/ha)
cwc	Canal water cost (PKR/ha)
pt	Private tube well (PKR/ha)
li	Labor for irrigation and cleaning (PKR/ha)
lc	Labor cost (PKR/ha)
ait	Agriculture income tax (PKR/ha)
$mngc$	Management charges (PKR)
rlc	Cost of rent land (PKR/ha)
hc	Harvesting cost (PKR/ha)
thc	Threshing cost (PKR/ha)
$\widetilde{K_k}$	Profit obtained from Kharif crops (PKR/ha)
$\widetilde{R_r}$	Profit obtained from Rabbi crops (PKR/ha)

3.2. Decision variables

$\mathcal{K}_{k,c}$: Canal c connected cultivated area for Kharif crop k.
$\mathcal{R}_{r,c}$: Canal c connected cultivated area Rabbi crop r.

3.3. The first objective concerned with maximizing net profits

The first objective is to maximize net profit obtained by cultivating various crops in canal-connected areas

$$Maximize\ \ Z_1 = \sum_{c=1}^{w}\sum_{k=1}^{m}\widetilde{K_k}\,\mathcal{K}_{k,c} + \sum_{c=1}^{w}\sum_{r=k+1}^{n}\widetilde{R_r}\,\mathcal{R}_{r,c}.$$

Where $k =$ Kharif Crop index, r = Rabbi Crop index, $\mathcal{K}_{k,c}$=Kharif k canal-connected cultivated area, $\mathcal{R}_{r,c} =$ Rabbi Crop r canal-connected cultivated area. Here $\widetilde{K_k}$, $\widetilde{R_r}$ is the profit obtained from Kharif and Rabbi crops, respectively. Profit is calculated by subtracting the total cost from the gross income. Calculation of the total cost from Kharif and Rabbi Crops is performed as follows:

(1) Land Preparation Cost Kharif Crops: $LPC_k = drp + wp + wp *$
(2) Land Preparation Cost Rabbi Crops: $LPC_r = dep + pc + plk + ll$
(3) Seed and Sowing Cost Kharif Crops: $SOC_k = sc + up,$
(4) Seed and Sowing Cost Rabbi Crops: $SOC_r = sc + cst + pc + plk + bm + tc$
(5) Weeding Cost Kharif Crops: $WC_k = mn + wc + pp$
(6) Fertilizer Cost: $FC_k = FC_r = ur + dap + zs + fta$
(7) Protection Measure Cost: $\mathrm{P}MC_k = PMC_r = cp$
(8) Irrigation Cost Kharif Crops: $IC_k = cwc + pt + li$
(9) Irrigation Cost Rabbi Crops: $IC_k = cwc + pt$
(10) Harvesting and Threshing Cost: $HTC_k = HTC_r = hc + thc$
(11) Cost of Miscellaneous Activities: $MC_r = MC_k = rlc + ait + mngc + lc$
(12) Total Cost Kharif Crops: $TC_k = SC_k + LPC_k + SOC_k + WC_k + FC_k + PMC_k + HTC_k + MC_k$
(13) Total Cost Rabbi Crops: $TC_r = SC_r + LPC_r + SOC_r + FC_r + PMC_r + HTC_r + MC_r$
(14) Gross Income: $GI_r = GI_k = qpr \times cpq$
(15) $\widetilde{K_g} = GI_k - TC_k$ and $\widetilde{R_g} = GI_r - TC_r$.

3.4. The second objective concerned with maximizing yield of crops

Secondly, the multi-objective optimization model maximizes the yield of various crops grown in the connected regions to the dam irrigation system during the Rabi and Kharif seasons. An area is directly related to the amount of crop productivity.

$$Maximize\ Z_2 = \sum_{c=1}^{w}\sum_{k=1}^{m}Y_g\,\mathcal{K}_{k,c} + \sum_{c=1}^{w}\sum_{r=k+1}^{k}Y_g\,\mathcal{R}_{r,c}.$$

Where Y_g denotes the average yield of crop g in tones/ha.

3.5. The constraints concerned with crop yield

3.5.1. Seasonal crop area constraints

During each growing season, the area allocated for cultivation near the dam should be smaller than or equal to the total land area of all crops.

$$\text{Kharif Crop: } \sum_{g=1}^{k} \mathcal{K}_{g,c} \leq \mathcal{Z}\mathcal{K}_c \quad c = 1, \dots, 10.$$

$$Rabbi\ Crop: \sum_{g=k+1}^{n} \mathcal{R}_{g,c} \leq \mathcal{Z}\mathcal{R}_c \quad c = 1, \dots, 10.$$

Where $\mathcal{Z}\mathcal{R}_c$ and $\mathcal{Z}\mathcal{K}_c$ represent the maximum areas of the canals during the Rabbi and Kharif seasons, respectively.

3.5.2. Social constraint

In the dam-operating region, staple crops must be grown to satisfy social needs. Therefore, the social constraint must be used.

$$\text{Kharif Crop: } \mathcal{K}_{k,c} \geq \mathcal{K}_{g,c}^{min} \quad g = 1, \dots, k; c = 1, \dots, w.$$

$$Rabbi\ Crop: \mathcal{R}_{r,c} \geq \mathcal{R}_{g,c}^{min} \quad g = k+1, \dots, n; c = 1, \dots, w.$$

Where $\mathcal{K}_{\mathrm{g,c}}^{\mathrm{min}}$ and $\mathcal{R}_{\mathrm{g,c}}^{\mathrm{min}}$ are the minimal area to be planted for crop g with canal c in both seasons.

3.5.3. Crop water requirement constraint

Modified Penman method is used to estimate the amount of water required for different crops in the area connected to the canal in each season. Water loss during the entire process is also considered while estimating the total irrigation requirement. The constraints for kharif and rabbi seasons are given as follows:

$$\text{Kharif Time: } \rho\xi\mathcal{R}_{t,c} \geq \sum_{g=1}^{k} W\,\mathcal{R}_{g,t}\mathcal{K}_{g,c} \quad t = 1, \dots, 4; c = 1, \dots, w.$$

$$Rabbi\ Time: \rho\xi\mathcal{R}_{t,c} \geq \sum_{g=k+1}^{n} W\,\mathcal{R}_{g,t}\mathcal{R}_{g,c} \quad t = 5, \dots, 8; c = 1, \dots, w.$$

Where $\mathcal{R}_{t,c}$ is the water discharge from the canal, $W\mathcal{R}_{g,t}$ is the water required for irrigation in the time t for the cultivation area $\mathcal{K}_{g,c}$ and $\mathcal{R}_{g,c}$ of the crop. The time regarding the Rabi and Kharif crops is considered, respectively. ρ and ξ are the application and transmission efficiencies.

3.5.4. Canal capacity constraint

Water discharged for irrigation from the dam in the time t should be less than or equal to the maximum capacity of the canal. We have:

$$\mathcal{R}_{t,c} \leq C_c \quad t = 1, \dots, 12: c = 1, \dots, w.$$

Where C_c is the maximum capacity of the canal c.

3.5.5. Reservoir evaporation constraint

The water loss caused by evaporation $(\grave{\mathrm{E}}_{n,t})$ from reservoir n in the period t can be calculated by counting the final and initial storage in the given period. The reservoir evaporation constraint is presented as:

$$\grave{\mathrm{E}}_{n,t} = a_{n,t} \times \frac{(\mathrm{Y}_{n,t} + \mathrm{Y}_{n,t+1})}{2} + b_{n,t} \quad n = 1, \dots, 5; t = 1, \dots, 12.$$

Where $a_{n,t}$ and $b_{n,t}$ are the regression coefficients. $Ɣ_{n,t}$ and $Ɣ_{n,t+1}$ are reservoir n water levels during the time t.

3.5.6. Minimum and maximum storage constraint

Physically for the safety of any reservoir, water storage level at any time should be kept under the maximum storage capacity and to compete with the demand it should be above the minimum storage level. The storage constraint is conveyed as follows:

$$Ɣ_{n,min} \leq Ɣ_{n,t} \leq Ɣ_{n,max} \quad n = 1,\ldots,5; \quad t = 1,\ldots,12$$

where $Ɣ_{n,min}$ and $Ɣ_{n,max}$ are precisely the extreme storage capacities of the reservoir n.

3.5.7. Overflow constraint

Natural phenomena like excessive rains, floods and melting of glaciers turn into heavy water inflow to the reservoirs in that case the water beyond maximum storage capacity need to release immediately. The overflow constraint is stated as follows:

$$O_{n,t} = Ɣ_{n,t+1} - Ɣ_{n,max} \quad n = 1,\ldots,5; \quad t = 1,\ldots,12$$

$$O_{n,t} \geq 0 \quad n = 1,\ldots,5; t = 1,\ldots,12$$

where $O_{n,t}$ is the amount of water released from the water reservoir n in time t. The crop production planning objectives are initially solved as single objective functions subjected to the given set of constraints by using the linear programming (LP) technique. The optimal values of the objective functions obtained are then used to establish Multi-Objective Neutrosophic Fuzzy Linear Programming Model (MONFLP) approach to maximize net profit and crop production in uncertain environment.

4. Multi-objective neutrosophic fuzzy linear programming model

The programming model previously noted is based on the formulation of membership and non-membership function but there are several situations where these two components are not enough to incorporate all the necessary information about the problem. It is obvious that there will be some sorts on neutrality or indeterminacy within the information about the data under consideration. To deal with such scenario the intuitionistic fuzzy subset is upgraded in terms of the neutrosophic sets. As defined earlier in a neutrosophic set, each $z \in X$ to a set $\tilde{A}^{SN}$ is characterized by $T_{\tilde{A}^{SN}}(z), I_{\tilde{A}^{SN}}(z), F_{\tilde{A}^{SN}}(z)$, where $T_{\tilde{A}^{SN}}(z)$, $I_{\tilde{A}^{SN}}(z)$, $F_{\tilde{A}^{SN}}(z)$ belongs to $[0,1]$ termed as degree of truth, indeterminacy and falsity, respectively. These components must satisfy

$$0 \leq T_{\tilde{A}^{SN}}(z) + I_{\tilde{A}^{SN}}(z) + F_{\tilde{A}^{SN}}(z) \leq 3.$$

Thus, a single-valued neutrosophic set $\tilde{A}^{SN}$ is expressed as

$$\tilde{A}^{SN} = \{(x, T_{\tilde{A}^{SN}}(z), I_{\tilde{A}^{SN}}(z), F_{\tilde{A}^{SN}}(z)) : z \in X\}.$$

Recall the optimization problem with the following steps:

$$\text{Maximize } Z_1(\mathcal{K}_{g,c}, \mathcal{R}_{g,c});$$

$$\text{Maximize } Z_2(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}),$$

which are subject to the following related constraints:

(1) Seasonal crop area constraint;
(2) Crop area constraint;
(3) Crop cultivated area constraint;
(4) Crop water requirement constraint;
(5) Canal capacity constraint;
(6) Reservoir evaporation constraint;
(7) Constraint of mass balance;
(8) Minimum and maximum storage constraint;
(9) Overflow constraint.

To conduct optimization in a neutrosophic environment, we define the decision set $\widetilde{D}$, of neurotrophic objectives and constraints as follows:

$$\widetilde{D} = đ_{z_1} \cap đ_{z_2} \bigcap_{\ell=1}^{9} đ_{\ell}^{\mathcal{C}},$$

where $đ_{z_1}$, $đ_{z_2}$ and $đ_{\ell}^{\mathcal{C}}$ represent the decision set for objectives Z_1, Z_2 and the ℓ^{th} constraint set respectively. Over the decision set $\widetilde{D}$ the truth, indeterminacy and falsity of neutrosophic set can be evaluated as

$$T_{\widetilde{D}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) = \min\{T_{đ_{z_1}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), T_{đ_{z_2}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), T_{đ_1^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), \dots, T_{đ_9^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c})\}$$

$$I_{\widetilde{D}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) = \min\{I_{đ_{z_1}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), I_{đ_{z_2}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), I_{đ_1^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), \dots, I_{đ_9^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c})\}$$

$$F_{\widetilde{D}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) = \max\{F_{đ_{z_1}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), F_{đ_{z_2}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), F_{đ_1^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}), \dots, F_{đ_9^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c})\}$$

At this point, we remodel the optimization problem by defining aspiration levels α, β, γ for truth, indeterminacy and falsity, respectively. The new optimization problem is narrated as:

$$\text{Maximize } \alpha, \text{ Maximize } \beta, \text{ Minimize } \gamma,$$

such that

(1) $T_{đ_{z_1}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \propto$, $\quad T_{đ_{z_2}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \propto$
(2) $T_{đ_1^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \propto, \dots, T_{đ_9^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \propto$
(3) $I_{đ_{z_1}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \beta$, $\quad I_{đ_{z_2}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \beta$
(4) $I_{đ_1^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \beta, \dots, I_{đ_9^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \beta$
(5) $F_{đ_{z_1}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \gamma$, $\quad F_{đ_{z_2}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \gamma$
(6) $F_{đ_1^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \gamma, \dots, F_{đ_9^{\mathcal{C}}}(\mathcal{K}_{g,c}, \mathcal{R}_{g,c}) \geq \gamma$
(7) $\alpha \geq \beta$ and $\alpha \geq \gamma$
(8) $\alpha + \beta + \gamma \leq 3$
(9) $\alpha, \beta, \gamma \in [0,1]$
(10) $\sum_{c=1}^{w} \sum_{g=1}^{k} \mathcal{K}_{g,c} \leq \mathcal{Z}\mathcal{Z}_{\mathcal{K}}$
(11) $\sum_{c=1}^{w} \sum_{g=k+1}^{n} \mathcal{R}_{g,c} \leq \mathcal{Z}\mathcal{Z}_{\mathcal{R}}$
(12) $\sum_{g=1}^{k} \mathcal{K}_{g,c} \leq \mathcal{Z}\mathcal{Z}\mathcal{K}_c \quad c = 1, \dots, 10$

(13) $\sum_{g=k+1}^{n} \mathcal{R}_{g,c} \leq \mathcal{Z}\mathcal{Z}\mathcal{R}_{c} \quad c = 1,\ldots,10$
(14) $\mathcal{K}_{g,c} \geq \mathcal{K}_{g,c}^{min} \quad g = 1,\ldots,k; c = 1,\ldots,w$
(15) $\mathcal{R}_{g,c} \geq \mathcal{R}_{g,c}^{min} \quad g = k+1,\ldots,n; c = 1,\ldots,w$
(16) $\rho\xi\mathcal{R}_{t,c} \geq \sum_{g=1}^{k} W\,\mathcal{R}_{g,t}\mathcal{K}_{g,c} \quad t = 1,\ldots,4; c = 1,\ldots,w$
(17) $\rho\xi\mathcal{R}_{t,c} \geq \sum_{g=k+1}^{n} W\,\mathcal{R}_{g,t}\mathcal{R}_{g,c} \quad t = 5,\ldots,8; c = 1,\ldots,w$
(18) $\mathcal{R}_{t,c} \leq C_{c} \quad t = 1,\ldots,12{:}\ c = 1,\ldots,w$
(19) $\grave{E}_{n,t} = a_{n,t} \times \frac{(\text{Ɣ}_{n,t}+\text{Ɣ}_{n,t+1})}{2} + b_{n,t} \quad n = 1,\ldots,5; t = 1,\ldots,12$
(20) $\text{Ɣ}_{n,min} \leq \text{Ɣ}_{n,t} \leq \text{Ɣ}_{n,max} \quad n = 1,\ldots,5; \quad t = 1,\ldots,12$
(21) $O_{n,t} = \text{Ɣ}_{n,t+1} - \text{Ɣ}_{n,max} \quad n = 1,\ldots,5; \quad t = 1,\ldots,12$
(22) $O_{n,t} \geq 0 \quad n = 1,\ldots,5; \quad t = 1,\ldots,12.$

Computational algorithm based on multi-objective neutrosophic linear programming

The computation algorithm for multi-objective neutrosophic linear programming is based on the following six steps:

Step 1: Solve the objective function $Z_1(\mathcal{K}_{g,c}, \mathcal{R}_{g,c})$ as a single objective subject to the given set of constraints and decision variables.

Step 2: Using the solution computed in step 1, find value of objective function $Z_2(\mathcal{K}_{g,c}, \mathcal{R}_{g,c})$.

Step 3: Solve the objective function $Z_2(\mathcal{K}_{g,c}, \mathcal{R}_{g,c})$ as a single objective subject to the given set of constraints and decision variables. Use the value of decision variables and compute the value of the first objective function. Suppose $(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1})$ and $(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2})$ be values of decision variables computed by considering Z_1 and Z_2 as a single objective function subject to the given set of constraints, respectively. Using these values, the pay-off matrix will be

$$\begin{bmatrix} Z_1^*(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1}) & Z_2(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1}) \\ Z_1(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2}) & Z_2^*(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2}) \end{bmatrix}.$$

Step 4: Compute the upper bound $\mathfrak{U}^T$ and lower bound $\mathfrak{L}^T$ for truth function of each objective function as:

$$\mathfrak{U}_{Z_1}^T = \max\{Z_1^*(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1}), Z_1(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2})\}$$

$$\mathfrak{L}_{Z_1}^T = \min\{Z_1^*(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1}), Z_1(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2})\}$$

$$\mathfrak{U}_{Z_2}^T = \max\{Z_2^*(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1}), Z_2(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2})\}$$

$$\mathfrak{L}_{Z_2}^T = \min\{Z_2^*(\mathcal{K}_{g,c}^{*1}, \mathcal{R}_{g,c}^{*1}), Z_2(\mathcal{K}_{g,c}^{*2}, \mathcal{R}_{g,c}^{*2})\}.$$

Upper $\mathfrak{U}^F$ and lower $\mathfrak{L}^F$ bounds for falsity membership of objectives are

$$\mathfrak{U}_{Z_1}^F = \mathfrak{U}_{Z_1}^T \quad \text{and} \quad \mathfrak{L}_{Z_1}^F = \mathfrak{L}_{Z_1}^T + t(\mathfrak{U}_{Z_1}^T - \mathfrak{L}_{Z_1}^T)$$

$$\mathfrak{U}_{Z_2}^F = \mathfrak{U}_{Z_2}^T \quad \text{and} \quad \mathfrak{L}_{Z_2}^F = \mathfrak{L}_{Z_2}^T + t(\mathfrak{U}_{Z_2}^T - \mathfrak{L}_{Z_2}^T)$$

and upper $\mathfrak{U}^I$ and lowers $\mathfrak{L}^I$ for indeterminacy membership of objectives are

$$\mathfrak{U}_{Z_1}^I = \mathfrak{L}_{Z_1}^T + s(\mathfrak{U}_{Z_1}^T - \mathfrak{L}_{Z_1}^T) \quad \text{and} \quad \mathfrak{L}_{Z_1}^I = \mathfrak{L}_{Z_1}^T$$

$$\mathfrak{U}_{Z_2}^I = \mathfrak{L}_{Z_2}^T + s(\mathfrak{U}_{Z_2}^T - \mathfrak{L}_{Z_2}^T) \quad \text{and} \quad \mathfrak{L}_{Z_2}^I = \mathfrak{L}_{Z_2}^T$$

where $t, s \in (0,1)$.

Step 5: Define membership functions for truth, indeterminacy and falsity of each objective function

$$T_{Z_1}(Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) = \begin{cases} 1 & Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{L}_{Z_1}^T \\ \dfrac{\mathfrak{U}_{Z_1}^T - Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})}{\mathfrak{U}_{Z_1}^T - \mathfrak{L}_{Z_1}^T} & \mathfrak{L}_{Z_1}^T \le Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{U}_{Z_1}^T \\ 0 & Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \ge \mathfrak{U}_{Z_1}^T \end{cases}$$

$$F_{Z_1}(Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) = \begin{cases} 0 & Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{L}_{Z_1}^F \\ \dfrac{Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) - \mathfrak{L}_{Z_1}^F}{\mathfrak{U}_{Z_1}^F - \mathfrak{L}_{Z_1}^F} & \mathfrak{L}_{Z_1}^F \le Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{U}_{Z_1}^F \\ 1 & Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \ge \mathfrak{U}_{Z_1}^F \end{cases}$$

$$I_{Z_1}(Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) = \begin{cases} 1 & Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{L}_{Z_1}^I \\ \dfrac{\mathfrak{U}_{Z_1}^I - Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})}{\mathfrak{U}_{Z_1}^I - \mathfrak{L}_{Z_1}^I} & \mathfrak{L}_{Z_1}^I \le Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{U}_{Z_1}^I \\ 0 & Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) \ge \mathfrak{U}_{Z_1}^I \end{cases}$$

$$T_{Z_2}(Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) = \begin{cases} 1 & Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{L}_{Z_2}^T \\ \dfrac{\mathfrak{U}_{Z_2}^T - Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})}{\mathfrak{U}_{Z_2}^T - \mathfrak{L}_{Z_2}^T} & \mathfrak{L}_{Z_2}^T \le Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{U}_{Z_2}^T \\ 0 & Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \ge \mathfrak{U}_{Z_2}^T \end{cases}$$

$$F_{Z_2}(Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) = \begin{cases} 0 & Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{L}_{Z_2}^F \\ \dfrac{Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) - \mathfrak{L}_{Z_2}^F}{\mathfrak{U}_{Z_2}^F - \mathfrak{L}_{Z_2}^F} & \mathfrak{L}_{Z_2}^F \le Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{U}_{Z_2}^F \\ 1 & Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \ge \mathfrak{U}_{Z_2}^F \end{cases}$$

$$I_{Z_2}(Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) = \begin{cases} 1 & Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{L}_{Z_2}^I \\ \dfrac{\mathfrak{U}_{Z_2}^I - Z_1(\mathcal{K}_{g,c},\mathcal{R}_{g,c})}{\mathfrak{U}_{Z_2}^I - \mathfrak{L}_{Z_2}^I} & \mathfrak{L}_{Z_2}^I \le Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) \le \mathfrak{U}_{Z_2}^I. \\ 0 & Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c})) \ge \mathfrak{U}_{Z_2}^I \end{cases}$$

Step 6: At this step, it is necessary to solve the following neutrosophic linear programming problem Maximize $\alpha + \beta - \gamma$ such that

$$Z_1\left(\mathcal{K}_{g,c},\mathcal{R}_{g,c}\right) + \left(\mathfrak{U}_{Z_1}^T - \mathfrak{L}_{Z_1}^T\right).\zeta \le \mathfrak{U}_{Z_1}^T, Z_1\left(\mathcal{K}_{g,c},\mathcal{R}_{g,c}\right) - \left(\mathfrak{U}_{Z_1}^F - \mathfrak{L}_{Z_1}^F\right).\eta \le \mathfrak{L}_{Z_1}^F$$

$$Z_1\left(\mathcal{K}_{g,c},\mathcal{R}_{g,c}\right) + \left(\mathfrak{U}_{Z_1}^I - \mathfrak{L}_{Z_1}^I\right).\xi \le \mathfrak{U}_{Z_1}^I,\ Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) + (\mathfrak{U}_{Z_2}^T - \mathfrak{L}_{Z_2}^T).\zeta \le \mathfrak{U}_{Z_2}^T$$

$$Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) - (\mathfrak{U}_{Z_2}^F - \mathfrak{L}_{Z_2}^F).\eta \le \mathfrak{L}_{Z_2}^F,\ Z_2(\mathcal{K}_{g,c},\mathcal{R}_{g,c}) + (\mathfrak{U}_{Z_2}^I - \mathfrak{L}_{Z_2}^I).\xi \le \mathfrak{U}_{Z_2}^I$$

$$\alpha \ge \beta \text{ and } \alpha \ge \gamma,\ \alpha + \beta + \gamma \le 3, \alpha,\ \beta,\ \gamma \in [0,1]$$

$$\textstyle\sum_{c=1}^{w}\sum_{g=1}^{k}\mathcal{K}_{g,c} \le \mathcal{ZZ}_{\mathcal{K}}, \quad \sum_{c=1}^{w}\sum_{g=k+1}^{n}\mathcal{R}_{g,c} \le \mathcal{ZZ}_{\mathcal{R}}$$

$$\textstyle\sum_{g=1}^{k}\mathcal{K}_{g,c} \le \mathcal{ZZK}_c, \quad \sum_{g=k+1}^{n}\mathcal{R}_{g,c} \le \mathcal{ZZR}_c \quad c = 1,\dots,10$$

$$\mathcal{K}_{g,c} \geq \mathcal{K}_{g,c}^{min} \quad g = 1,\ldots,k; c = 1,\ldots,w,$$

$$\mathcal{R}_{g,c} \geq \mathcal{R}_{g,c}^{min} \quad g = k+1,\ldots,n; c = 1,\ldots,w,$$

$$\rho\xi\mathcal{R}_{t,c} \geq \sum_{g=1}^{k} W\,\mathcal{R}_{g,t}\mathcal{K}_{g,c} \quad t = 1,\ldots,4;$$

$$\rho\xi\mathcal{R}_{t,c} \geq \sum_{g=k+1}^{n} W\,\mathcal{R}_{g,t}\mathcal{R}_{g,c} \quad t = 5,\ldots,8; c = 1,\ldots,w,$$

$$\mathcal{R}_{t,c} \leq \mathrm{C}_c \quad t = 1,\ldots,12: c = 1,\ldots,w,$$

$$\grave{E}_{n,t} = a_{n,t} \times \frac{(\gamma_{n,t} + \gamma n, t+1)}{2} + b_{n,t} \quad n = 1,\ldots,5; t = 1,\ldots,12$$

$$\gamma_{n,min} \leq \gamma_{n,t} \leq \gamma_{n,max}, \quad O_{n,t} = \gamma_{n,t+1} - \gamma_{n,max}, O_{n,t} \geq 0 \quad n = 1,\ldots,5; t = 1,\ldots,12.$$

The accuracy of multi-objective neurotrophic linear programming model is higher than the accuracy of other related models while dealing with imprecise date. These steps are presented in the Figure 2.

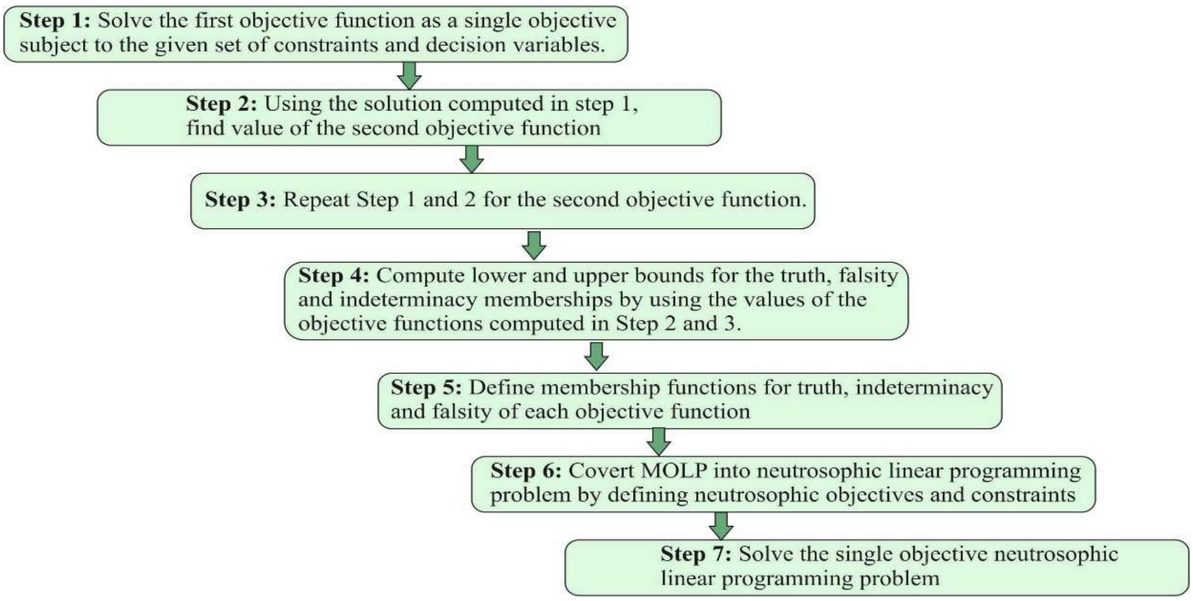

Figure 2. Computational procedural steps.

5. Case study: Multi-objective neutrosophic model for crop production

Pakistan has two distinguish cropping seasons and despite of modern irrigation technologies still a large fragment of agriculture is highly depending upon water flow from dams and canals. Climatic changes affect the water supply and ultimately the crop production in the adjacent areas. There is a need to construct a Multi-objective Neutrosophic Model for Rabbi and Kharif crop production using water reservoirs active storage under yield uncertainty. The primary data was taken from the agriculture marketing information services to calculate and maximize net profit from Rabbi and Kharif crops and crop production. For the validation of the constructed model, the two main crops play a significant role in the agriculture of Pakistan: wheat and rice from Rabbi and Kharif crops, respectively. The cultivation areas are in Figures 3 and 4.

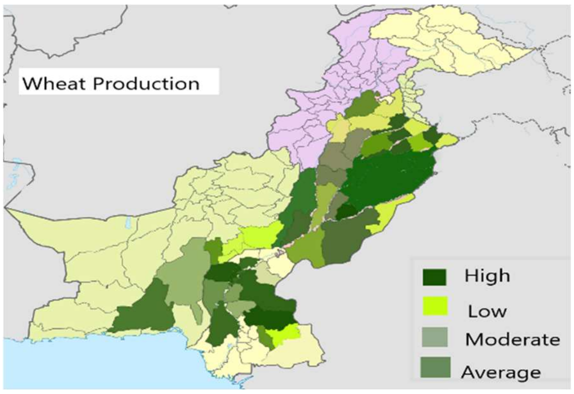

Figure 3. Wheat productions areas.

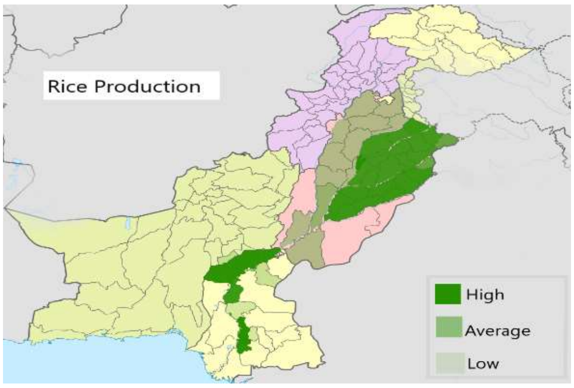

Figure 4. Rice productions areas.

First, the profit from Rabbi and Kharif crops using active storage water reservoirs is separately calculated since profit is the subtraction of gross income and the total cost used for crop production. Various factors are involved in cost production.

6. Marginal insights and interpretations

The first step of crop production is the land preparation involving deep ploughing, cultivator, planking, laser levelling, wet ploughing and wet planking are included. Rabbi's seed and sowing costs include seed treatment, ploughing, planking, bund making and tractor with the drill as depicted in Table 2.

Table 2. Land preparation cost (in PKR).

Crops	Deep ploughing	Ploughing /cultivator	Planking	Laser leveling	Wet ploughing	Wet planking	Total cost/Acre
Wheat	235	1033	516	360	--	--	2144
Rice	--	2533	--	--	2041	510	5084

In contrast, for Kharif seeding, uprooting, transplanting and transporting, manure weedicides and seed treatment are shown in Table 3.

Table 3. Seed and sowing cost/ Acre (in PKR).

Costs	Wheat	Rice
Seed	2250	--
Seed Treatment	80	--
Ploughing /cultivator	1033	--
Planking	516	--
Bund Making	270	--
Tractor with Drill	516	--
Seedling		1200
Uprooting transplanting		3412
Manure (25%)		550
Weedicide (75%)		1100
Seed Treatment		1300

Fertilizer cost comprises urea, DAP, transportation, fertilizer application, zinc sulphate and plant protection as presented in Table 4.

Table 4. Fertilizer cost (in PKR).

Crops	Urea	DAP	Transportation	Fertilizer Application	Zinc Sulphate	Plant Protection (Herbicides /Weedicides)	Total cost /Acre
Wheat	2460	3350	100	265	-	1200	7375
Rice	1520	3250	100	200	650	1200	6920

Furthermore, irrigation costs are based on canal water rate, private tube-well, labor for irrigation and cleansing, as followed by Table 5.

Table 5. Irrigation cost (in PKR).

Crops	Canal Water Rate	Private Tube well	Labor for irrigation and cleansing	Total cost /Acre
Wheat	56	2000	1050	3106
Rice	9572	7000	3150	5720

Moreover, harvesting and threshing cost is composed of harvesting and threshing cost for both. Additionally, miscellaneous costs involved land rent, agriculture income tax and management charges are presented in Table 6.

Table 6. Harvesting, threshing and miscellaneous cost (in PKR).

Crop	Harvesting and threshing cost			Miscellaneous cost			
	Harvesting	Threshing	Total cost /Acre	Land Rent	Agriculture Income Tax	Management charges	Total cost /Acre
Wheat	3600	3600	7200	12500	50	960	13510
Rice	5500	3775	9275	15000	50	900	15950

Overall, the total cost and gross income for the Rice (Kharif) and Wheat (Rabbi) is calculated as given in Table 7.

Table 7. Total cost and gross income (in PKR).

Crops	Total Cost (pkr/hect)	Gross Income (Pkr/hect)
Rice	64238	327050
Wheat	45653	447620

While using linear optimization, intuitionistic optimization and neutrosophic optimization the profit for cultivation wheat and rice and average production needed for second objective function in the case of Rice (Kharif) and Wheat (Rabbi) crop is given by Table 8. In this study, the models were solved using MATLAB software version 2018a on a device with the specification Core(TM) i3-8130U CPU and 4GB RAM.

Table 8. Profit (in PKR/hector) and average production (Tons/ Hector) needed.

Crops		Profit (Ton/Hect)	Average cultivation
Intuitionistic	Rice	380060	7.04
	Wheat	200401	7.04
Neutrosophic	Rice	418060	8.52
	Wheat	285460	8.52
Linear	Rice	356796	7.01
	Wheat	210056	7.01

Whereas graphical representation of linear optimization, intuitionistic optimization and neutrosophic optimization the profit is given by Figure 5 and average production needed, is given by Figure 6.

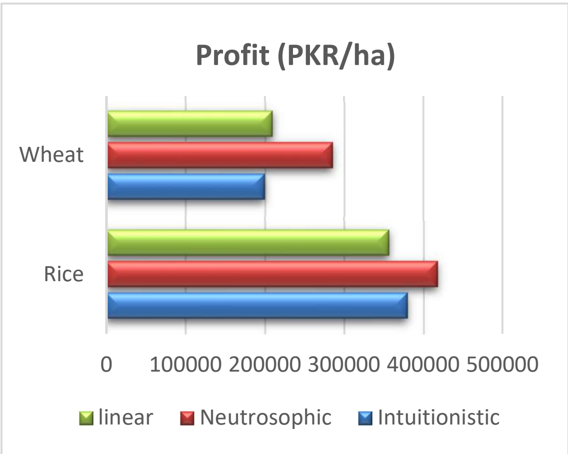

Figure 5. Profit (PKR/ha).

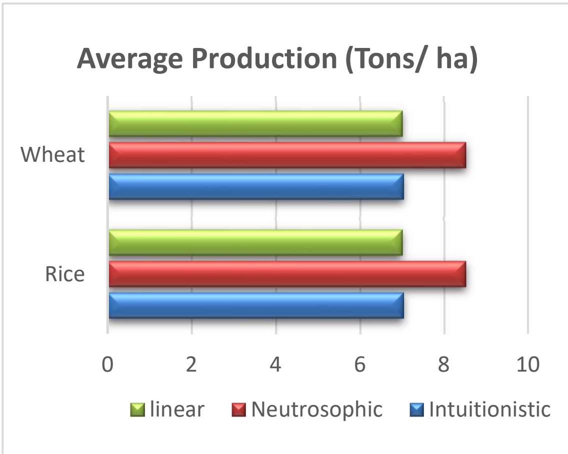

Figure 6. Average Production needed (Tons/ ha).

Comparing the results in Table 8 and Figure 5, the profit computed by employing neutrosophic technique is higher than the profit computed by linear and intuitionistic optimization model. Thus, the proposed model is more accurate and efficient in maximizing profit and gross production.

6. Conclusions

It can be concluded that modelling of complex dynamic systems points some challenges considering their main attributes. Hence, the need of the development of methods regarding qualitative analysis to address the dynamics and behaviors of these systems along with the constructing of efficient control algorithms toward efficient operation, classification, recognition, identification, optimization and simulation becomes conspicuous. Several types of uncertainty representation can be noted such as interval, fuzzy, granular as well as combined uncertain sets. Furthermore, uncertainty is apparently prevalent as a challenge under the unpredictable and uncontrollable climatic conditions, so provision of accurate and prompt solutions are important undertakings to fulfill the required different missions. Neutrosophic optimization techniques have the objective of providing how neutrosophic optimization technique can be employed to solve a structural problem which requires the investigation of the effect of conflicting constraints. Accordingly, our study has aimed to investigate the profitability and marketing of Rabbi and Kharif crops in Pakistan. Firstly, we proposed a multi-objective model in which standard parameters values are taken from agriculture marketing information services. To ensure the applicability of research findings, a selection of rice and wheat crops was undertaken. The analysis in our study adopts a neutrosophic technique that considers the degrees of truth, falsity and indeterminacy. The maximum net profit was obtained by using water reservoirs active storage at 285460 PKR. Ton/ ha. Our study has put forth the benefits of neutrosophic fuzzy algorithm which has been proposed and the analyses derived can be stated to deal with yield uncertainty in the neutrosophic environments more effectively considering the different parameters in nonlinear and dynamic settings in which abrupt changes may occur any time. It is worth noting that decision-making problems manifests the challenge of being imprecise, vague, or ambiguous. Yield production is highly crucial considering the ever-changing dynamics of our current time which affects yield uncertainty. Since uncertainty is considered by a mathematical model, the neutrosophic environment is more appropriate and applicable since it deals with uncertainty more efficiently and effectively. While Uncertainty in crop production is difficult to capture in the model as it is based on assumptions rather than empirical data. Climate change will increase the frequency and intensity of extreme weather events, such as droughts and floods, which can lead to reduce crop yields. This will reduce the amount of food produced, making it more difficult for countries to feed their populations. Increased temperatures and more frequent extreme weather events can also lead to increased crop losses due to increased pests and diseases, and heat and drought stress. Climate change can also cause changes in the suitability of different crop varieties, as some may become better adapted to the changing climate while others may struggle. This could have a significant effect on crop production, as some crops may no longer be suitable for certain regions. Climate change can also lead to water scarcity, as increased temperatures can cause more water to evaporate and less precipitation. This can cause water stress for crops, leading to decreased yields. Climate change can also lead to soil degradation, as increased temperatures can cause soil to dry out, leading to nutrient loss and reduced soil fertility. This can make it more difficult for crops to grow.

The following suggestions as future directions can be outlined to improve Rabbi and Kharif crops' agriculture profitability and marketing using active water reservoirs' storage: first, the government should provide subsidies on pesticides, fertilizer, and other nutrients. There is also a need for appropriate crops farming guidance for farmers. Thus, it would be a viable option if the government could assign active experts and extend departments for relevant guidance to be able to conceptualize, implement, maintain and control the applicable strategies and decision-making processes in a systematic, expeditious and accurate way. In future, using the proposed methodology crop production models can be desinged for the rain fed areas to meet the food requirements of the growing population.

Acknowledgments

Thanks to American University of the Middle East for their support for this research.

Conflict of interest

The authors declare that they have no conflict of interest regarding publication of this research work.

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