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Solution of logistic differential equation in an uncertain environment using neutrosophic numbers

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Abstract

The modeling and forecasting of population dynamics, as well as growth in biological systems more generally, have required the construction of various growth models. This paper presents the logistic growth model, which is a modified version of the exponential growth model and offers a more flexible framework to capture fluctuations and potential deviations from deterministic predictions. This paper aims to advance the model of a logistic growth differential equation within a neutrosophic environment, with a focus on validating the proposed framework by applying it to predict the projected population of India from the year 2001 to 2050. We investigated the neutrosophic solutions and validated them with the exact solution with available projected data on the portal of UNITED NATION. The developed logistic growth differential equation in a neutrosophic environment is visually elucidated through graphical representation, enhancing the clarity and accessibility of the presented model.

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1. Introduction

The fields of economics, biomathematics, science, and engineering are some places where applied analysis plays a significant part in modeling real-world natural phenomena. In contrast, in the contemporary era, there

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is a widely acknowledged consensus that information pertaining to physical phenomena is inherently susceptible to varying degrees of uncertainty. This impreciseness can be overcome by utilizing the powerful fuzzy set theory tool. In expanding upon classical set theory, L.A. Zadeh's [1] creation of fuzzy set theory has significantly enriched our ability to mathematically represent and analyze phenomena that exhibit degrees of vagueness and uncertainty. This theory is helpful because it considers ambiguous environmental situations when determining membership values. In the context of fuzzy set theory, these conceptual ideas are often termed as "linguistic words," intimately tied to imprecision and vagueness within fuzzy set theory. These conceptual ideas are systematically assigned a potential range of numerical values, typically confined within the interval [0, 1], in order to quantify and express the varying degrees of membership associated with imprecision and vagueness within fuzzy set theory.

In addition, Atanassov [2] in the early 1980's came up with the novel concept of intuitionistic fuzzy sets (IFSs), another type of fuzzy set. The elements under consideration are evaluated by two distinct functions: the membership function, which assesses the degree to which an element belongs to a set, and the non-membership function, which gauges the extent to which an element does not belong to the set, offering a comprehensive evaluation within the framework of fuzzy set theory. The fuzzy sets are defined by the aspects that only look at the grade of membership and not the grade of non-membership of any given bit of information. In some situations, it is not enough to just look at the value of membership; the value of not being a member must also be considered. IFS is a natural extension of the fuzzy set. One more extension, which is known as the Neutrosophic set, is the one that has played the most important role in dealing with problems that occur in real life. Neutrosophic set theory is another idea in the field of uncertainty. It investigates the origin, explanation, and possibilities of neutral thoughts. Florentin Smarandache [3] came up with neutrosophic sets, which look at how truth, indeterminate, and false values fit together. In the realm of Neutrosophic logic, the degrees of membership pertaining to Truth value (T), Indeterminate value (I), and False value (F) are characterized within the unconventional non-standard interval $]^{-}0,1^{+}[$, expanding the expressive capabilities of the framework to encapsulate a broader spectrum of uncertainty and indeterminacy.

The application of Neutrosophic set theory with non-standard intervals proves to be highly effective in philosophical contexts.

Nevertheless, defining data within this non-standard range regarding scientific and technical problems poses a challenge due to the unconventional nature of the interval. To solve this problem, Wang et al. [4] made Single-Valued Neutrosophic Sets (SVNS) by choosing the standard form of the unit interval [0, 1]. Neutrosophic numbers are the numbers that fall within this range. Aal et al. [5], Deli and Subas [6], Ye [7], Chakraborty et al. [8], and others have explored and analyzed numerous other neutrosophic numbers, further enriching the field by offering diverse perspectives and applications within the framework of neutrosophic logic and set theory. Abdel-Basset et al. [9–14], who created Neutrosophic numbers and used them to solve problems such as the COVID-19 pandemic, problems with making decisions, supply chain models, and industrial and management problems, demonstrate the variety of applications that have been found for neutrosophic numbers. The study introduces by Abood et al. [15], a novel neutrosophic D-metric space derived from neutrosophic sets. In this way, substantial efforts have been invested in advancing Neutrosophic set theory by its application to real-world problems, underscoring its practical utility and potential for addressing complex issues across various domains.

Differential equations serve as foundational tools for constructing mathematical models across diverse disciplines, such as science, engineering, physics, medicine, etc. The presence of uncertainties or inconsistencies in variables, relevant factors, and initial conditions introduces challenges that can be resolved by converting them into fuzzy differential equations. The development of fuzzy differential equations has many conceptual and practical implications. Dubois et al. [16-18] initiated using fuzzy differential equations (FDE) by concentrating solely on membership values. The formulation of Fuzzy Differential Equations involved the incorporation of fuzzy numbers and functions, a significant contribution initially introduced by Chang et al. [19].

The solutions to these fuzzy differential equations necessitates a fundamental grasp of derivatives within a fuzzy environment, emphasizing the importance of refining our conceptual understanding in order to interpret and analyze the outcomes effectively. The exploration of differentials for fuzzy functions, initially investigated by Puri and Ralescu, plays a pivotal role in extending the mathematical tools available for understanding and solving fuzzy differential equations, contributing to the broader development of fuzzy calculus, which was looked at by Goetschel and Voxman [20], are just two examples of the key work that has been done in this area. The fuzzy derivative idea is used by Seikkala and

Kaleva [21-22] to solve first-order ordinary differential equations with initial conditions. Buckley and Feuring [23, 24] solved an ordinary differential equation with fuzzy initial conditions that were of the nth order. In 2005, Bede and Gal [25] suggested that generalization of the fuzzy-valued functions. Additionally, the methodology demonstrates the resolution of fuzzy differential equations through the application of this generalized differentiability, employing the lower-upper form of fuzzy integers to provide a comprehensive and effective solution strategy. In 2009, Stefanini et al. [26] employed the generalized Hukuhara derivative to illustrate the enhanced generality achievable for fuzzy interval-valued functions, highlighting the utility of this approach in broadening the scope of fuzzy analysis. In 2016, Paul et al. [27] elucidated the solution to the fuzzy quota harvesting model within a fuzzy environment, contributing to the evolving understanding and application of fuzzy models in ecological and resource management contexts.

Moreover, intuitionistic fuzzy differential equations are more general than fuzzy differential equations because they take belongingness and non-belongingness values into account. None of them thought about the indeterminacy factor as used in the neutrosophy set theory. The integration of philosophical logic with differential equations featuring uncertain and imprecise parameters offers a versatile approach that enhances the applicability and robustness of mathematical modeling in diverse scientific and engineering disciplines. Differential equation in a neutrosophic environment are designed to account for all three parameters: membership, indeterminacy, and not being a member. Neutrosophic Calculus [28] was made by Smarandache. In a neutrosophic theory, fundamental concepts such as limit, continuity, differentiability, essential functions like exponential and logarithmic, as well as the notions of differentials and integrals are incorporated, forming a comprehensive mathematical framework that embraces uncertainty and indeterminacy. In 2018, Sumathi and Priya [29] delved into the exploration of neutrosophic differential equations, contributing to the ongoing development and understanding of mathematical models that incorporate neutrosophic elements to address uncertainty and imprecision. Sumathi et al. and Parikh et al. [30-32] solved first-order differential equations with one independent variable by using different kinds of neutrosophic numbers. Fuzzy theory from the field of uncertain environments is still growing as a part of differential equations, and it gives a few ways to solve differential equations in uncertain situations. This review underscores the diverse array of numerical, semi-analytical, and analytical methods that have been devised and applied to attain more precise solutions for fuzzy differential equations, reflecting the ongoing efforts to enhance the accuracy and versatility of techniques in this domain. For example, Salahshour et al. [33] created the Laplace transform for a fuzzy differential equation, and Mondal et al. [34-35] used it to solve it. Similarly, in an uncertain environment, Sahni et al. [36–37] and Parikh et al. [38–39] also used Sumudu transform to solve a second-order fuzzy differential equation. In 2016, Tapaswini et al. [40] successfully addressed a fuzzy differential equation through an analytical approach, contributing to the expanding repertoire of solution methodologies in the field of fuzzy differential equations. In addition, Acharya et al. [41] worked on the application of modeling of glucose distribution in the bloodstream utilizing neutrosophic sets, and Biswas et al. [42] investigated the field of integral equations of second kind utilizing neutrosophic numbers. As more real-world situations are added to the theory of neutrosophic differential equations, more work needs to be done to help it grow.

In this article, we have successfully addressed the logistic growth differential equation amids uncertainties, employing the logistic growth model initially proposed by Pierre-Francois Verhulst in 1838 [43], which is articulated as follows:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right).$$

Here, within the logistic growth model, the variable P(t) denotes the population size at time t, while r signifies the intrinsic growth rate, and K represents the carrying capacity of the environment.

The findings are contrasted with the solution that can be reached by obtaining the values of the crisp set at various intervals of time. The work is structured as follows: in section 2, we describe certain mathematical preliminaries that will be beneficial for this investigation. We describe the strategies used to solve logistic growth differential equations using triangular numbers in section 3. In section 4, the generic population model-based fuzzified initial-value problem is presented, and the classical solution is employed to demonstrate their accuracy. A brief summary of the work accomplished is presented in section 5.

2. Mathematical Preliminaries

Definition 2.1 Fuzzy set (FS) [18]: In the formulation of fuzzy set F, where X_U denotes any universal set and $\xi_{\mathcal{F}}(x) \in [0,1], \quad \forall x \in X_U$ represents the membership value in the universal set for each element of X_U the fuzzy set F is formally defined as $\mathcal{F} = \{(x, \xi_{\mathcal{F}}(x)) \mid \forall x \in X_U\}$.

Definition 2.2 α-level of Fuzzy set [18]: The α-level of fuzzy set $\mathcal{F}$ denoted as $\mathcal{F}_\alpha$, where $x \in X_U$ is formally defined as $\mathcal{F}_\alpha = \{\xi_{\mathcal{F}}(x) \geq \alpha,\ \alpha \in [0,1]\}$, i.e. it contains membership values greater than or equal to α, providing a clear delineation of the elements exhibiting a certain level of membership in the fuzzy set F.

Definition 2.3 Intuitionistic fuzzy set (IFS) [18]: In the representation of an intuitionistic fuzzy set over the elements of the universal set X_U is represented by $\mathcal{F}_{IFS}$ and is formally defined as $\mathcal{F}_{IFS} = \{(x, \xi_{\mathcal{F}_{IFS}}(x), \zeta_{\mathcal{F}_{IFS}}(x)) \mid \forall x \in X_U\}$, where $\xi_{\mathcal{F}_{IFS}}(x)$ represents membership and $\zeta_{\mathcal{F}_{IFS}}(x)$ represents non-membership value of x in $\mathcal{F}_{IFS}$.

Definition 2.4 (α, β) - level of Intuitionistic Fuzzy set [18]: Establishing an (α,β) - level of Intuitionistic fuzzy set $\mathcal{F}_{IFS}$ involves the definition $\mathcal{F}_{IFS(\alpha,\beta)} = \{x : \xi_{\mathcal{F}_{IFS}}(x) \geq \alpha, \zeta_{\mathcal{F}_{IFS}}(x) \leq \beta,\ \forall x \in X_U,\ \alpha,\ \beta \in [0,1]\}$ under the constraint $0 \leq \alpha + \beta \leq 1$.

Definition 2.5 Neutrosophic set (NS) [3]: A neutrosophic set denoted as N and is defined as $N = \{(T_N(x),\ I_N(x),\ F_N(x)) : \forall x \in X_U\}$, where the values of $T_N(x),\ I_N(x),\ F_N(x)$ are drawn from the universal set X →]–0,1+[, are called as truth membership, indeterminacy membership, and false membership values respectively of the elements belonging to the Universal set with the constraint given as $-0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3+$.

Definition 2.6 Single-valued Neutrosophic set (SVNS) [3]: Consider the single-valued Neutrosophic set denoted as N and is defined as $N = \{(T_N(x),\ I_N(x),\ F_N(x)) : \forall x \in X_U\}$, where the values of $T_N(x),\ I_N(x),\ F_N(x)$ are considered from the universal set $X_U \rightarrow [0, 1]$, under the given standard constraint $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$.

Definition 2.7 Neutrosophic Number [3]: A neutrosophic set N defined over the universal single valued set of real numbers R is said to be a neutrosophic number if it exhibits the following properties:

- *SVNS* is normal: if $\exists x_0 \in R$, such that $T_N(x_0) = 1$ and the, indeterminacy and falsity value is zero.
- *SVNS* is convex set for the truth function $T_N(x)$,

i.e., $T_N(\mu x_1 + (1-\mu)x_2) \geq \min(T_N(x_1), T_N(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1]$.

- *SVNS* is a concave set for the indeterminacy function ($I_N(x)$) and false function ($F_N(x)$),

  i.e., $I_N(\mu x_1 + (1-\mu)x_2) \geq \max(I_N(x_1), I_N(x_2))$, and

  $F_N(\mu x_1 + (1-\mu)x_2) \geq \max(F_N(x_1), F_N(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1]$.

Definition 2.8 (α, β, γ)-level of Neutrosophic set [40]: Represented as G (α, β, γ), a neutrosophic set with (α, β, γ)-level over the universal set X_U, where α, β, γ range from 0 to 1 and is defined as G (α, β, γ) ={$T_N(x)$, $I_N(x)$, $F_N(x)$: $\forall x \in X_U$, $T_N(x) \geq \alpha$, $I_N(x) \leq \beta$, $F_N(x) \leq \gamma$}, under the given constraint $0 \leq \alpha + \beta + \gamma \leq 3$.

Definition 2.9 Triangular Neutrosophic Number [40]: Consider a single-valued neutrosophic set N characterized by Truth $T_N(x)$, Indeterminacy $I_N(x)$ and False $F_N(x)$ membership value over universal set X_U. In this context, the triangular neutrosophic number is defined as:

$$T_N(x) = \begin{cases} \left(\frac{x-a}{b-a}\right) & \text{for } a \leq x \leq b \\ 1, & \text{for } x = b \\ \left(\frac{c-x}{c-b}\right) & \text{for } b \leq x \leq c \\ 0 & \text{otherwise} \end{cases},$$

$$I_N(x) = \begin{cases} \left(\frac{b-x}{b-a}\right) & \text{for } a \leq x \leq b \\ 0, & \text{for } x = b \\ \left(\frac{x-c}{c-b}\right) & b \leq x \leq c \\ 1 & \text{otherwise} \end{cases},$$

and

$$F_N(x) = \begin{cases} \left(\frac{b-x}{b-a}\right) & \text{for } a \leq x \leq b \\ 0, & \text{for } x = b \\ \left(\frac{x-c}{c-b}\right) & \text{for } b \leq x \leq c \\ 1 & \text{otherwise} \end{cases},$$

In this definition, denoted as $N_T\langle(a,b,c)\rangle$, the triangular neutrosophic number is characterized by the condition $a < b < c$, where a, b, c belongs to the set of real numbers. Specifically, the truth membership function $T_N(x)$ exhibits a linear increase for x ∈ [a, b] and a linear decrease for x ∈ [b, c]. Conversely, the indeterminacy membership function $I_N(x)$ and false membership function $F_N(x)$ demonstrate an inverse behavior within the respective intervals [a, b] and [b, c].

Definition 2.10 (α, β, γ)-cut of a Triangular Neutrosophic Number [40]: A triangular neutrosophic set with (α, β, γ)-cut is denoted by A_TN(α, β, γ), where α, β, γ ∈ [0, 1], and is defined as a mathematical representation capturing the indeterminate, contradictory, and unknown aspects within a given set and is represented as,

$$A_{TN(\alpha,\beta,\gamma)} = \{T_N(x), I_N(x), F_N(x) : T_N(x) \geq \alpha,\ I_N(x) \leq \beta, F_N(x) \leq \gamma, x \in X\}.$$

Here $0 \leq \alpha + \beta + \gamma \leq 3$ and

$$A_{TN(\alpha,\beta,\gamma)} = [[(a+\alpha(b-a)),(c-\alpha(c-b))],[(b-\beta(b-a)),\\ (b+\beta(c-b))],[(b-\gamma(b-a)),(b+\gamma(c-b))]]$$

Definition 2.11 H-differentiability [27]: A function $f:(a,b) \to \mathbb{R}_F$ (where $\mathbb{R}_F$ is the set of real numbers in fuzzy environment) is called H-differentiable on $x_0 \in (a,b)$, if for $h > 0$ sufficiently small there exist the H-differences $f(x_0+h)\theta f(x_0)$, $f(x_0)\theta f(x_0-h)$ and an element $f'(x_0) \in \mathbb{R}_F$ such that

$$\lim_{h\to 0} \frac{f(x_0+h)\ominus f(x_0)}{h} = \lim_{h\to 0} \frac{f(x_0)\ominus f(x_0-h)}{h} = f'(x_0).$$

Definition 2.12 Strongly Generalized Differentiability [27]:

Let $f:(a,b) \to R_F$ and $x_0 \in (a,b)$, we assert that f is strongly generalized differentiable if there exists an element $f''(x_0) \in R_F$ such that,

(a) for all $h > 0$ small enough, there exists, a generalized difference quotient

$$f'(x_0+h) \ominus f'(x_0), f'(x_0) \ominus f'(x_0-h)$$

and the limits holds (in the metric D), then we have

$$\lim_{h\to 0} \frac{f'(x_0+h)\ominus f'(x_0)}{h} = \lim_{h\to 0} \frac{f'(x_0)\ominus f'(x_0-h)}{h} = f''(x_0) \text{ or} \quad (1)$$

(b) for all $h > 0$ small enough, there exists, a generalized difference quotient

$$f'(x_0) \ominus f'(x_0 + h),\ f'(x_0 - h) \ominus f'(x_0)$$

and the limits holds (in the metric D), then we have

$$\lim_{h\to 0} \frac{f'(x_0)\ominus f'(x_0+h)}{h} = \lim_{h\to 0} \frac{f'(x_0-h)\ominus f'(x_0)}{h} = f''(x_0) \text{ or} \tag{2}$$

(c) for all $h > 0$ small enough, there exists, a generalized difference quotient

$$f'(x_0 + h) \ominus f'(x_0), f'(x_0 - h) \ominus f'(x_0)$$

and the limits holds (in the metric D), then we have

$$\lim_{h\to 0} \frac{f'(x_0+h)\ominus f'(x_0)}{h} = \lim_{h\to 0} \frac{f'(x_0-h)\ominus f'(x_0)}{h} = f''(x_0) \text{ or} \tag{3}$$

(d) for all $h > 0$ small enough, there exists, a generalized difference quotient

$$f'(x_0) \ominus f'(x_0 + h), f'(x_0) \ominus f'(x_0 - h)$$

and the limits holds (in the metric D), then we have

$$\lim_{h\to 0} \frac{f'(x_0)\ominus f'(x_0+h)}{h} = \lim_{h\to 0} \frac{f'(x_0)\ominus f'(x_0-h)}{h} = f''(x_0) \tag{4}$$

3. Modelling of Logistic growth model in an uncertain environment

3.1 *Logistic differential equation (Classical environment)*

In a real-life phenomenon, there are various situations where both growth and decay models can be applied to describe the exponential events. Radioactive decay is the prototypical case of exponential development. Also to state, it is not always the case. When applied to the growth of a population, for instance, it is possible to model growth on an exponential scale; however, the equation $P = Ce^{rt}$ implies that population growth continues indefinitely. Due to a limiting factor r, population growth rates tend to decline over time. Population growth might be constrained by the lack of things to do in the region. The logistic differential equation may be the most appropriate model for describing such occurrences.

Consider the non-linear continuous function of the population as

$$\frac{dP}{dt} = f(P(t)) \tag{5}$$

Using Taylor series expansion of the function P(t), we get

$$f(P(t)) = r_0 + r_1 P + r_2 P^2 \dots \tag{6}$$

Reducing the given equation in quadratic form so we can ignore the higher order terms from the second order and equation (6) can be written as

$$f(P(t)) = r_0 + r_1 P + r_2 P^2 \tag{7}$$

Now we apply the initial and boundary conditions on equation (7).

i) $\frac{dP}{dt} = 0$ when $P = 0$ (No Population at an initial point). Using this condition in equation (7), we get

$$r_0 = 0 \tag{8}$$

ii) $\frac{dP}{dt} = 0$ when $P = P_{\max} = K(say)$ (Maximum Population size). Using this condition in equation (7) and $r_0 = 0$, we get

$$r_2 = -\frac{r_1}{K} \tag{9}$$

Putting the values of r_0 and r_2 in equation (7), we obtain

$$\frac{dP}{dt} = r_1 P\left(1 - \frac{P}{K}\right) \tag{10}$$

In equation (10), we consider $r_1 = r$in the differential equation of the Logistic growth model in classical environment.

3.2 *Solution of Logistic growth Differential Equation in a Neutrosophic fuzzy environment*

Consideration of the first-order differential equation (10) for the Logistic growth model within a neutrosophic fuzzy environment leads to the expression:

$$\frac{dP}{dt} = f(P(t)) = f(\underline{P}(t)_T, \overline{P}(t)_T, \underline{P}(t)_I, \overline{P}(t)_I, \underline{P}(t)_F, \overline{P}(t)_F) \tag{11}$$

where $[\underline{P}(t)_T, \overline{P}(t)_T]$ represents the lower bound and upper bound of truth membership respectively in a population. Likewise $[\underline{P}(t)_I, \overline{P}(t)_I]$ and $[\underline{P}(t)_F, \overline{P}(t)_F]$ denote the lower and upper bounds of indeterminacy and falsity membership for the population, respectively. By utilizing equations (10) and (11), we derive the differential equation for the logistic growth model, expressed as follows:

$$\frac{d\underline{P_T}}{dt} = r\underline{P_T}\left(1 - \frac{\underline{P_T}}{\underline{K_T}}\right) \tag{12}$$

$$\frac{d\overline{P_T}}{dt} = r\overline{P_T}\left(1 - \frac{\overline{P_T}}{\overline{K_T}}\right) \tag{13}$$

$$\frac{d\underline{P_I}}{dt} = r\underline{P_I}\left(1 - \frac{\underline{P_I}}{\underline{K_I}}\right) \tag{14}$$

$$\frac{d\overline{P_I}}{dt} = r\overline{P_I}\left(1 - \frac{\overline{P_I}}{\overline{K_I}}\right) \tag{15}$$

$$\frac{d\overline{P_F}}{dt} = r\overline{P_F}\left(1 - \frac{\overline{P_F}}{\overline{K_F}}\right) \tag{16}$$

$$\frac{d\overline{P_F}}{dt} = r\overline{P_F}\left(1 - \frac{\overline{P_F}}{\overline{K_F}}\right) \tag{17}$$

In the differential equations (12) to (17), we employ definition 2.10 to transform the initial value problem into the (α, β, γ)-cut representation of triangular neutrosophic fuzzy numbers. This conversion is expressed for both population and carrying capacity as follows:

$$P_{T\alpha}(t_0) = [a + \alpha(b-a), c - \alpha(c-b)], \tag{18}$$

$$P_{I\beta}(t_0) = [b - \beta(b-a), b + \beta(c-b)], \tag{19}$$

$$P_{F\gamma}(t_0) = [b - \gamma(b-a), b + \gamma(c-b)], \tag{20}$$

$$K_{T\alpha}(t) = [d + \alpha(e-d), f - \alpha(f-d)], \tag{21}$$

$$K_{I\beta}(t) = [e - \beta(e-d), e + \beta(f-d)], \tag{22}$$

$$K_{F\beta}(t) = [e - \beta(e-d), e + \beta(f-d)], \tag{23}$$

Using separation of variable method and neutrosophic fuzzified initial values i.e., equations (18) to (23), we get solutions as follows

$$\underline{P_{T\alpha}}(t) = \frac{\underline{K_{T\alpha}}}{(1+\underline{A}e^{-rt})}, \qquad \underline{A} = \frac{\underline{K_{T\alpha}} - \underline{P_{T\alpha}}(t_0)}{\underline{P_{T\alpha}}(t_0)} \tag{24}$$

$$\overline{P_{T\alpha}}(t) = \frac{\overline{K}_{T\alpha}}{(1+\overline{A}e^{-rt})}, \qquad \overline{A} = \frac{\overline{K}_{T\alpha} - \overline{P_{T\alpha}}(t_0)}{\overline{P}_{T\alpha}(t_0)} \tag{25}$$

$$\underline{P_{I\beta}}(t) = \frac{\underline{K_{I\beta}}}{(1+\underline{A}e^{-rt})}, \qquad \underline{A} = \frac{\underline{K_{I\beta}} - \underline{P_{I\beta}}(t_0)}{\underline{P_{I\beta}}(t_0)} \tag{26}$$

$$\overline{P_{I\beta}}(t) = \frac{\overline{K}_{I\beta}}{(1+\overline{A}e^{-rt})}, \qquad \overline{A} = \frac{\overline{K}_{I\beta} - \overline{P_{I\beta}}(t_0)}{\overline{P}_{I\beta}(t_0)} \tag{27}$$

$$\underline{P_{F\gamma}}(t) = \frac{\underline{K_{F\gamma}}}{(1+\underline{A}e^{-rt})}, \qquad \underline{A} = \frac{\underline{K_{F\gamma}} - \underline{P_{F\gamma}}(t_0)}{\underline{P_{F\gamma}}(t_0)} \tag{28}$$

$$\overline{P_{F\gamma}}(t) = \frac{\overline{K}_{F\gamma}}{(1+\overline{A}e^{-rt})}, \qquad \overline{A} = \frac{\overline{K}_{F\gamma} - \overline{P_{F\gamma}}(t_0)}{\overline{P}_{F\gamma}(t_0)} \tag{29}$$

From equations (24) to (29), we get $P(t) = [\underline{P_{T\alpha}}(t), \overline{P_{T\alpha}}(t), \underline{P_{I\beta}}(t), \overline{P_{I\beta}}(t), \underline{P_{F\gamma}}(t), \overline{P_{F\gamma}}(t)]$.

4. Illustration using Numerical Example

In this section, we consider example of the population and develop the theoretical approach for prediction of population. We find the solution of the logistic growth model in neutrosophic environment and then compared it to the crisp solution.

Problem Statement in classical manner:

To predict the future population of INDIA, we need to determine growth rate using the exponential growth model.

Using the actual population of INDIA in table 1 with t = 0, i.e. t_0 corresponding to the year 2001, we have P(0) = 107.9 . Again t = 1 i.e. t_1 corresponding to the year 2002, we have P(1) = 109.8. The population are considered in millions.

Solution: By Classical method

According to section 3.1, we consider logistic differential equation as follows,

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right).$$

By exponential growth model, we get the following equation

$$P(t) = P_0(t)e^{rt}.$$

Here we consider the initial population $P(0) = 107.9,$ and after one-year population $P(1) = 109.8,$ then we get exponential growth rate r = 0.0177.

Applying an analytical method to solve the logistic growth model, we get a solution in the following form, $P(t) = \frac{K}{(1 + Ae^{-rt})}, \quad A = \frac{K - P(t_0)}{P(t_0)}.$

Table 1

Projected population by Logistic model (in millions) from 2001 to 2050 [44].

Number of Years	Year	Actual population (in millions)	Projected population by Logistic model (in millions)	Absolute Percentage Error
1	2001	107.90	…	…
2	2002	109.83	109.8504	0.02045
3	2003	111.74	111.7805	0.04045
4	2004	113.74	113.6883	0.05167
5	2005	115.46	115.5725	0.11250
6	2006	117.24	117.4314	0.19144
7	2007	118.97	119.2637	0.29375
8	2008	120.67	121.0681	0.39811
9	2009	122.36	122.8433	0.48330
10	2010	124.06	124.5882	0.52820
11	2011	125.76	126.3018	0.54180
12	2012	127.45	127.9832	0.53317
13	2013	129.11	129.6315	0.52149
14	2014	130.72	131.2461	0.52606
15	2015	132.29	132.8262	0.53624
16	2016	133.86	134.3715	0.51152
17	2017	135.42	135.8815	0.46147
18	2018	136.90	137.3558	0.45575
19	2019	138.31	138.7941	0.48413

Contd...

20	2020	139.64	140.1964	0.55642
21	2021	140.76	141.5626	0.80257
22	2022	141.71	142.8926	1.18255
23	2023		144.1865	
24	2024		145.4444	
25	2025		146.6667	
26	2026		147.8534	
27	2027		149.0051	
28	2028		150.1220	
29	2029		151.2045	
30	2030		152.2532	
31	2031		153.2686	
32	2032		154.2511	
33	2033		155.2015	
34	2034		156.1202	
35	2035		157.0080	
36	2036		157.8654	
37	2037		158.6932	
38	2038		159.4920	
39	2039		160.2626	
40	2040		161.0056	
41	2041		161.7217	
42	2042		162.4117	
43	2043		163.0763	
44	2044		163.7162	
45	2045		164.3321	
46	2046		164.9248	
47	2047		165.4948	
48	2048		166.0431	
49	2049		166.5701	
50	2050		167.0766	

As seen from Table 1 that the logistic growth model well fits into the population model as the maximum percentage error occurred is less than 1 percent as seen in the year 2022. We have predicted that the population growth of INDIA till 2050.

4.1 *Problem Statement for Fuzzy environment*

Given that the data presented in the problem are simply estimates, we can assume that there is either a calculation error or an inadequacy. As a result, we will be looking into a fuzzy environment as a possible solution to this problem. In this situation, we will create the problem statement for the beginning population of INDIA and the carrying capacity, which will be expressed as triangular fuzzy numbers. To do so, we will make use of ambiguous phrases in neutrosophic environment.

Now, we construct the problem in the form of neutrosophic triangular numbers as per following way. For truth membership, consider that in the year 2001 the population of India was around 107.9 million people. We can determine the amount of the population by taking the average difference between 2.9 and 2.1 million. If this is the case, then we can estimate the population of India to be between 105 and 110 million, i.e. with the lowest being 105 million and the highest being 110 million. As the description of the problem indicates that there would be an annual increase in population, we are able to make the following assumption: after one year, there will be 109.8 million people in the population, i.e. in 2002. We may estimate that the population will be somewhere between 107 and 111 million after a year if there is an average difference of 2.8 and 1.2 million correspondingly. In 2002 the smallest population is 107 (in million), and the highest population is 111 (in million). In addition, the estimated carrying capacity based on the data is 178.65 (in million), which is provided in the data. We may describe this number as a triangular fuzzy number, and the population capacity that is the lowest is 150, while the population capacity that is the largest is 199. Now comes the time when we must compute the projected population from the year 2003 together all way up until 2050. Similar way we can assume for indeterminacy membership and falsity membership.

Using definition 2.9 we can construct the above description in the following form,

$$P(0) = [105, 107.9, 110]_T \leftrightarrow P(0) = [105 + 2.9\alpha, 111 - 2.1\alpha]_T$$

$$P(0) = [104, 107.9, 111]_I \leftrightarrow P(0) = [107.9 - 3.9\beta, 111 + 3.1\beta]_I$$

$$P(0) = [103, 107.9, 112]_F \leftrightarrow P(0) = [107.9 - 4.9\gamma, 107.9 + 4.1\gamma]_F$$

$$P(1) = [107, 109.8, 111]_T \leftrightarrow P(1) = [105 + 2.8\alpha, 111 - 1.2\alpha]_T$$

$$P(1) = [106, 109.8, 112]_I \leftrightarrow P(1) = [109.8 - 3.8\beta, 109.8 + 2.2\beta]_I$$

$$P(1) = [105, 109.8, 113]_F \leftrightarrow P(1) = [109.8 - 4.8\gamma, 109.8 + 3.2\gamma]_F$$

$$K(0)=[150,178.65,199]_T \leftrightarrow K(0)=[150+28.65\alpha,199-20.35\alpha]_T$$
$$K(0)=[139,178.65,209]_I \leftrightarrow K(0)=[178.65-38.65\beta,178.65+30.35\beta]_I$$
$$K(0)=[129,178.65,219]_F \leftrightarrow K(0)=[178.65-49.65\gamma,178.65+40.35\gamma]_F$$

Solution: According to section 3.2, we consider logistic differential equation in the neutrosophic environment (given in equations (12) to (17)).

Applying the exponential growth model within a neutrosophic environment, we derive growth rates represented as lower and upper bounds for truth, indeterminacy, and falsity membership, as illustrated in Table 2.

Table 2
Growth rate by Exponential model.

(α, β, γ) − *cut*	Lower bound of Truth value for growth rate $\underline{r}_T$	Upper bound of Truth value for growth rate $\overline{r}_T$	Lower bound of Indeterminacy value for growth rate $\underline{r}_I$	Upper bound of Indeterminacy value for growth rate $\overline{r}_I$	Lower bound of Falsity value for growth rate $\underline{r}_F$	Upper bound of Falsity value for growth rate $\overline{r}_F$
0.0	0.01887	0.00905	0.01746	0.01746	0.01746	0.01746
0.1	0.01872	0.00988	0.01761	0.01659	0.01763	0.01657
0.2	0.01858	0.01071	0.01777	0.01573	0.01780	0.01570
0.3	0.01844	0.01155	0.01792	0.01487	0.01797	0.01483
0.4	0.01829	0.01238	0.01808	0.01401	0.01815	0.01396
0.5	0.01815	0.01322	0.01824	0.01316	0.01832	0.01310
0.6	0.01801	0.01406	0.01840	0.01231	0.01850	0.01225
0.7	0.01787	0.01491	0.01856	0.01147	0.01868	0.01140
0.8	0.01773	0.01575	0.01872	0.01063	0.01886	0.01056
0.9	0.01759	0.01660	0.01888	0.00980	0.01905	0.00972
1.0	0.01746	0.01746	0.01904	0.00896	0.01923	0.00888

In Table 2, the growth rates are calculated for truth, indeterminancy and falsity values for various (α,β,γ)-cut. By utilizing the growth rates associated with each membership category and applying the logistic growth model, we obtain solutions in neutrosophic environment, and presenting varying perspectives based on different (α, β, γ)-cuts.

Tables 3 and 4 demonstrate the projected population of India from 2023 to 2050, showcasing the lower and upper bounds for truth, indeterminacy, and falsity membership at different (α, β, γ)-cuts, specifically at values of 0 and 1.

Table 3

Projected Population at (α, β, γ)-cut = 0.

Year	Actual population (in millions)	Lower bound of Projected Population by Logistic model (in millions) for truth membership	Upper bound of Projected population by Logistic model (in millions) for truth membership	Lower bound of Projected Population by Logistic model (in millions) for indeterminancy membership	Upper bound of Projected population by Logistic model (in millions) for indeterminancy membership	Lower bound of Projected Population by Logistic model (in millions) for false membership	Upper bound of Projected population by Logistic model (in millions) for false membership
2001	107.9	…	…	…	…	…	…
2002	109.83	106.4315	110.9989	109.7909	109.7909	109.7909	109.7909
2003	111.74	107.8358	111.9954	111.6743	111.6743	111.6743	111.6743
2004	113.74	109.2121	112.9895	113.5314	113.5314	113.5314	113.5314
2005	115.46	110.5600	113.9807	115.3665	115.3665	115.3665	115.3665
2006	117.24	111.8789	114.9691	117.1781	117.1781	117.1781	117.1781
2007	118.97	113.1683	115.9543	118.9650	118.9650	118.9650	118.9650
2008	120.67	114.4280	116.9362	120.7259	120.7259	120.7259	120.7259
2009	122.36	115.6577	117.9147	122.4598	122.4598	122.4598	122.4598
2010	124.06	116.8572	118.8894	124.1654	124.1654	124.1654	124.1654
2011	125.76	118.0264	119.8603	125.8420	125.8420	125.8420	125.8420
2012	127.45	119.1652	120.8272	127.4885	127.4885	127.4885	127.4885
2013	129.11	120.2737	121.7899	129.1043	129.1043	129.1043	129.1043
2014	130.72	121.3519	122.7482	130.6886	130.6886	130.6886	130.6886
2015	132.29	122.4000	123.7019	132.2408	132.2408	132.2408	132.2408
2016	133.86	123.4181	124.6510	133.7604	133.7604	133.7604	133.7604
2017	135.42	124.4065	125.5952	135.2470	135.2470	135.2470	135.2470

Contd...

2018	136.9	125.3655	126.5343	136.7001	136.7001	136.7001	136.7001
2019	138.31	126.2955	127.4683	138.1196	138.1196	138.1196	138.1196
2020	139.64	127.1967	128.3970	139.5052	139.5052	139.5052	139.5052
2021	140.76	128.0696	129.3202	140.8569	140.8569	140.8569	140.8569
2022	141.71	128.9146	130.2378	142.1744	142.1744	142.1744	142.1744
2023	142.86	129.7322	131.1497	143.4580	143.4580	143.4580	143.4580
2024	144.75	130.5229	132.0557	144.7076	144.7076	144.7076	144.7076
2025	145.46	131.2872	132.9556	145.9234	145.9234	145.9234	145.9234
2026	146.73	132.0256	133.8495	147.1055	147.1055	147.1055	147.1055
2027	147.95	132.7387	134.7371	148.2544	148.2544	148.2544	148.2544
2028	149.16	133.4271	135.6183	149.3702	149.3702	149.3702	149.3702
2029	150.34	134.0913	136.4931	150.4533	150.4533	150.4533	150.4533
2030	151.14	134.7320	137.3612	151.5041	151.5041	151.5041	151.5041
2031	152.62	135.3497	138.2227	152.5230	152.5230	152.5230	152.5230
2032	153.71	135.9449	139.0774	153.5105	153.5105	153.5105	153.5105
2033	154.76	136.5185	139.9252	154.4671	154.4671	154.4671	154.4671
2034	155.79	137.0708	140.7660	155.3933	155.3933	155.3933	155.3933
2035	156.78	137.6025	141.5998	156.2897	156.2897	156.2897	156.2897
2036	157.73	138.1143	142.4265	157.1569	157.1569	157.1569	157.1569
2037	158.64	138.6067	143.2459	157.9954	157.9954	157.9954	157.9954
2038	159.52	139.0803	144.0580	158.8058	158.8058	158.8058	158.8058
2039	160.36	139.5358	144.8628	159.5888	159.5888	159.5888	159.5888
2040	161.16	139.9736	145.6601	160.3450	160.3450	160.3450	160.3450
2041	161.93	140.3943	146.4500	161.0751	161.0751	161.0751	161.0751
2042	162.65	140.7985	147.2323	161.7796	161.7796	161.7796	161.7796
2043	163.34	141.1868	148.0070	162.4593	162.4593	162.4593	162.4593
2044	163.98	141.5597	148.7741	163.1149	163.1149	163.1149	163.1149
2045	164.58	141.9178	149.5335	163.7468	163.7468	163.7468	163.7468
2046	165.15	142.2614	150.2852	164.3559	164.3559	164.3559	164.3559
2047	165.67	142.5912	151.0292	164.9428	164.9428	164.9428	164.9428
2048	166.17	142.9077	151.7653	165.5080	165.5080	165.5080	165.5080
2049	166.62	143.2113	152.4936	166.0524	166.0524	166.0524	166.0524
2050	167.04	143.5025	153.2141	166.5764	166.5764	166.5764	166.5764

Table 3 presents the calculated exponential growth rates for truth, indeterminacy, and falsity membership pertaining to the projected population of India from 2003 to 2050, delineating the lower and upper bound values for each membership category. Furthermore, the data in Table 3 reveals that the lower and upper bounds of membership align precisely with the exact solution when β is set to 0 for indeterminacy membership and γ is set to 0 for falsity membership.

Table 4

Projected Population at (α, β, γ)-cut = 1.

Year	Actual population (in millions)	Lower bound of Projected Population by Logisticmodel (in millions) for truth membership	Upper bound of Projected population by Logistic model (in millions) for truth membership	Lower bound of Projected Population by Logistic model (in millions) for indeterminancy membership	Upper bound of Projected population by Logistic model (in millions) for indeterminancy membership	Lower bound of Projected Population by Logistic model (in millions) for false membership	Upper bound of Projected population by Logistic model (in millions) for false membership
2001	107.90	…	…	…	…	…	…
2002	109.83	109.7909	109.7909	105.9619	112.0666	104.9426	113.0871
2003	111.74	111.6743	111.6743	107.9314	113.1316	106.8907	114.1735
2004	113.74	113.5314	113.5314	109.7804	114.1948	108.6649	115.2590
2005	115.46	115.3665	115.3665	111.5509	115.2560	110.3279	116.3433
2006	117.24	117.1781	117.1781	113.2430	116.3149	111.8829	117.4263
2007	118.97	118.9650	118.9650	114.8571	117.3714	113.3335	118.5077
2008	120.67	120.7259	120.7259	116.3941	118.4253	114.6839	119.5874
2009	122.36	122.4598	122.4598	117.8553	119.4762	115.9385	120.6651
2010	124.06	124.1654	124.1654	119.2422	120.5241	117.1018	121.7406
2011	125.76	125.8420	125.8420	120.5565	121.5687	118.1787	122.8137
2012	127.45	127.4885	127.4885	121.8004	122.6098	119.1740	123.8842
2013	129.11	129.1043	129.1043	122.9759	123.6471	120.0926	124.9520
2014	130.72	130.6886	130.6886	124.0853	124.6807	120.9391	126.0167
2015	132.29	132.2408	132.2408	125.1313	125.7101	121.7183	127.0783
2016	133.86	133.7604	133.7604	126.1161	126.7352	122.4347	128.1365
2017	135.42	135.2470	135.2470	127.0425	127.7559	123.0926	129.1911

Contd...

2018	136.9	136.7001	136.7001	127.9130	128.7719	123.6962	130.2419
2019	138.31	138.1196	138.1196	128.7303	129.7831	124.2496	131.2888
2020	139.64	139.5052	139.5052	129.4967	130.7893	124.7564	132.3316
2021	140.76	140.8569	140.8569	130.2151	131.7904	125.2203	133.3701
2022	141.71	142.1744	142.1744	130.8877	132.7860	125.6446	134.4041
2023	142.86	143.4580	143.4580	131.5172	133.7762	126.0324	135.4334
2024	144.75	144.7076	144.7076	132.1057	134.7607	126.3867	136.4578
2025	145.46	145.9234	145.9234	132.6557	135.7394	126.7102	137.4773
2026	146.73	147.1055	147.1055	133.1694	136.7121	127.0054	138.4916
2027	147.95	148.2544	148.2544	133.6489	137.6786	127.2746	139.5006
2028	149.16	149.3702	149.3702	134.0962	138.6389	127.5202	140.5041
2029	150.34	150.4533	150.4533	134.5133	139.5928	127.7440	141.5020
2030	151.14	151.5041	151.5041	134.9020	140.5401	127.9479	142.4941
2031	152.62	152.5230	152.5230	135.2642	141.4808	128.1337	143.4803
2032	153.71	153.5105	153.5105	135.6015	142.4146	128.3029	144.4604
2033	154.76	154.4671	154.4671	135.9155	143.3415	128.4569	145.4344
2034	155.79	155.3933	155.3933	136.2077	144.2614	128.5971	146.4020
2035	156.78	156.2897	156.2897	136.4795	145.1742	128.7247	147.3631
2036	157.73	157.1569	157.1569	136.7323	146.0797	128.8408	148.3177
2037	158.64	157.9954	157.9954	136.9674	146.9778	128.9464	149.2655
2038	159.52	158.8058	158.8058	137.1859	147.8685	129.0424	150.2065
2039	160.36	159.5888	159.5888	137.3890	148.7516	129.1298	151.1407
2040	161.16	160.3450	160.3450	137.5776	149.6271	129.2092	152.0677
2041	161.93	161.0751	161.0751	137.7529	150.4948	129.2815	152.9877
2042	162.65	161.7796	161.7796	137.9157	151.3548	129.3471	153.9004
2043	163.34	162.4593	162.4593	138.0668	152.2069	129.4068	154.8058
2044	163.98	163.1149	163.1149	138.2071	153.0511	129.4611	155.7038
2045	164.58	163.7468	163.7468	138.3374	153.8872	129.5104	156.5943
2046	165.15	164.3559	164.3559	138.4583	154.7153	129.5552	157.4772
2047	165.67	164.9428	164.9428	138.5705	155.5353	129.5959	158.3524
2048	166.17	165.5080	165.5080	138.6746	156.3471	129.6329	159.2199
2049	166.62	166.0524	166.0524	138.7712	157.1507	129.6666	160.0797
2050	167.04	166.5764	166.5764	138.8608	157.9460	129.6971	160.9316

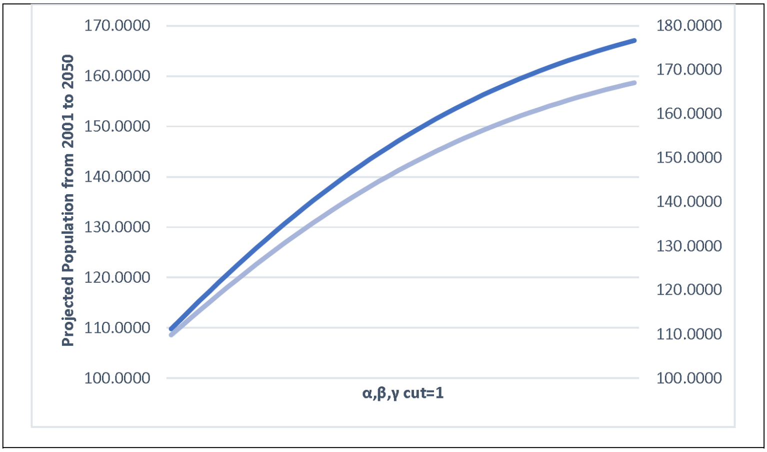

Figure 1

Solution of projected population at (α, β, γ)-cut = 1.

Furthermore, Table 4 illustrates that the numerical data therein signifies the lower and upper bounds of truth, indeterminacy, and false membership, respectively, at the (α, β, γ)-cut of 1 for the projected population spanning from 2003 to 2050. Moreover, the observation is evident in Table 4 that the lower and upper bounds of membership align precisely with the exact solution when α is set to 1 for truth membership. Furthermore, the solutions of truth membership are depicted by graphical form for . The Logistic growth model best fits into the population model as can been seen from the results and observed in Figure 1.

5. Conclusions

This paper presents the solution of logistic growth model with an uncertain environment. In addressing inherent uncertainties, we extended our investigation to the Neutrosophic fuzzified formulation of the logistic differential equation, incorporating initial conditions and carrying capacity, such as the effects of pandemics, natural disasters, and impacts on natural resources. It means that vagueness in data calculations are considered. Through the utilization of population growth data from INDIA [44], we demonstrated the practicality of the model for analysis and prediction of population for five decades. The derived solution is expressed in terms of truth, indeterminacy, and falsity membership grades, highlighting the effectiveness of the neutrosophic formation.

Produced results are systematically presented in the form of tables for various (α,β,γ)-cut values enhancing the clarity and accessibility of the results. As a direction for future research, we propose expanding this model to encompass diverse scenarios characterized by uncertain environments. In future time, effective management of such uncertainties will lead to more accurate data collection for modeling differential equations. Further , we are suggesting to explore other models that incorporate neutrosophic uncertainty and validate them using real-world data. By these efforts, we tried to connect the devations of this research to address uncertainties to bo encountered in practical applications.

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