INFORMATICA, 2023, Vol. 0, No. 0, 1–27

DOI: https://doi.org/10.15388/23-INFOR518

Linguistic Single-Valued Neutrosophic Multi-Criteria Group Decision Making Based on Personalized Individual Semantics and Consensus

Zhang-Peng TIAN[1], Fu-Xin XU[1], Ru-Xin NIE[1,*], Xiao-Kang WANG[2], Jian-Qiang WANG[3]

[1] *School of Economics and Management, China University of Mining and Technology, Xuzhou, 221116, PR China*
[2] *College of Management, Shenzhen University, Shenzhen 560180, PR China*
[3] *School of Business, Central South University, Changsha 410083, PR China*
e-mail: rx_nie@cumt.edu.cn

Received: December 2022; accepted: May 2023

Abstract. In practical linguistic multi-criteria group decision-making (MCGDM) problems, words may indicate different meanings for various decision makers (DMs), and a high level of group consensus indicates that most of the group members are satisfied with the final solution. This study aims at developing a novel framework that considers the personalized individual semantics (PISs) and group consensus of DMs to tackle linguistic single-valued neutrosophic MCGDM problems. First, a novel discrimination measure for linguistic single-valued neutrosophic numbers (LSVNNs) is proposed, based on which a discrimination-based optimization model is built to assign personalized numerical scales (PNSs). Second, an extended consensus-based optimization model is constructed to identify the weights of DMs considering the group consensus. Then, the overall evaluations of all the alternatives are obtained based on the LSVNN aggregation operator to identify the ranking of alternatives. Finally, the results of the illustrative example, sensitivity and comparative analysis are presented to verify the feasibility and effectiveness of the proposed method.

Key words: neutrosophic set, linguistic single-valued neutrosophic set, multi-criteria group decision making, personalized individual semantics, consensus reaching process.

1. Introduction

A multi-criteria group decision-making (MCGDM) problem is defined as a decision problem where several experts (judges, decision makers (DMs), etc.) provide their evaluations on a set of alternatives regarding multiple criteria and seek to achieve a common solution that is most acceptable by the group of experts as a whole (Kabak and Ervural, 2017; Wang *et al.*, 2021). One important issue in MCGDM is to depict the ratings of experts. As the socio-economic environment becomes increasingly complex, it becomes difficult for DMs to specify their preferences with crisp values. Zadeh (1965) pioneered fuzzy sets (FSs),

*Corresponding author.

which are characterized by membership functions and considered as an effective tool to capture uncertain information. In order to capture DMs' imprecise preferences, Atanassov (1986) introduced the concept of intuitionistic fuzzy sets (IFSs), which are considered as an extension of Zadeh's fuzzy sets (Zadeh, 1965). IFSs have been widely applied to address impreciseness and uncertainty information in MCGDM problems (Chen *et al.*, 2020; Garg and Kumar, 2019; Ohlan, 2022; Seikh and Mandal, 2022; Wan *et al.*, 2020; Zhao *et al.*, 2021). For an overview of IFSs and their extensions in MCGDM, one can refer to Xu and Zhao (2016).

Although FSs and IFSs have been extended to manage various MCGGDM problems, they failed to effectively handle a situation where the indeterminate and inconsistent information are involved. To manage such issues, Smarandache (1999) pioneered neutrosophic sets that consider the truth, indeterminacy and falsity memberships, simultaneously. Neutrosophic sets are regarded as a flexible tool in coping with information involving uncertainty, incompleteness and inconsistency (Peng and Dai, 2020). However, without specific description, neutrosophic sets are hard to be applied in actual situations. Wang *et al.* (2010) introduced single-valued neutrosophic sets (SVNSs), which are considered as an extension of neutrosophic sets. Moreover, various extensions of neutrosophic sets, such as interval neutrosophic sets (Liu *et al.*, 2022; Wang *et al.*, 2005), trapezoidal neutrosophic sets (Liang *et al.*, 2018b; Sarma *et al.*, 2019), type-2 neutrosophic sets (Gokasar *et al.*, 2022), multi-valued neutrosophic sets (MVNSs) (Ji *et al.*, 2018; Ye *et al.*, 2022), probability MVNSs (Liu and Cheng, 2019; Peng *et al.*, 2018), interval-valued fermatean neutrosophic sets (Broumi *et al.*, 2022) and complex fermatean neutrosophic sets (Broumi *et al.*, 2023) have been proposed and applied to address various neutrosophic multi-criteria decision making (MCDM) problems, such as investment decision, personnel selection, disaster management and designing a stable sustainable closed-loop supply chain network (Kalantari *et al.*, 2022). Among various forms of neutrosophic sets, SVNSs are considered as one of the most concise tools to capture DMs' evaluations (Peng and Dai, 2020).

Over the past years, lots of studies have been witnessed focusing on MCDM and MCGDM based on SVNSs and their extensions, which can be roughly grouped into two categories (Nguyen *et al.*, 2019; Peng and Dai, 2020). The first category is based on various neutrosophic aggregation operators, such as single-valued neutrosophic number (SVNN) generalized power averaging operator (Liu and Liu, 2018), SVNN weighted geometric averaging (WGA) operator (Refaat and El-Henawy, 2019), and SVNN ordered weighted harmonic averaging operator (Paulraj and Tamilarasi, 2022). The second category is based on kinds of neutrosophic measures (Hezam *et al.*, 2023; Karabasevic *et al.*, 2020; Kumar *et al.*, 2020; Peng and Dai, 2018; Sun *et al.*, 2019; Tian *et al.*, 2020; Zhang *et al.*, 2023). For example, Hezam *et al.* (2023) proposed a neutrosophic discrimination measure-based COPRAS framework, and applied it to evaluate the sustainable transport investment projects. Karabasevic *et al.* (2020) developed an extended TOPSIS method based on the SVNN Hamming distance measure, and applied it to e-commerce development strategies selection. Sun *et al.* (2019) developed a new distance measure for SVNNs, based on which an extended TODIM and ELECTRE III methods were proposed and applied in physician selection.

The above-mentioned approaches are effective when copying with MC(G)DM problems with SVNNs. However, specific numerical evaluations cannot always accurately reflect DMs' behaviour and opinions because of the limitation of their cognition. Actually, DMs usually prefer to elicit their evaluations with linguistic terms, such as "poor", "good" and "perfect" due to the prominent advantages of linguistic terms for characterizing ambiguous and inexact assessments (Zadeh, 1975). Recently, Li Y. *et al.* (2017) proposed linguistic single-valued neutrosophic sets (LSVNSs) which employ a triple-tuple linguistic structure to characterize the truth, indeterminacy and falsity memberships of LSVNSs. Since LSVNSs integrate the advantages of SVNSs and linguistic term sets (LTSs), various MC(G)DM methods on the basis of linguistic single-valued neutrosophic numbers (LSVNNs) have been proposed. For example, Fang and Ye (2017) developed a linguistic neutrosophic MCGDM method based on the LSVNN weighted arithmetic averaging (WAA) and WGA operators. Garg and Nancy (2018) proposed the LSVNN prioritized WAA and WGA operator-based MCGDM method. Moreover, several comprehensive MCGDM methods that integrate with the LSVNN power WAA and WGA operators and TOPSIS (Liang *et al.*, 2018a), and the EDAS (Li *et al.*, 2019) were developed. These linguistic neutrosophic MCGDM methods have been applied to the university human resource management evaluation and property management company selection, respectively.

Although great efforts have been made to improve and extend the application of linguistic neutrosophic MCGDM methods, there still exist some challenges. The existing methods seem to overlook the semantics of individual DMs and the consensual solution. In the existing methods, numerical values are identified through calculating the index values of linguistic terms. In this way, the numerical values cannot indicate experts' personalized individual semantics with respect to linguistic terms.

When tackling linguistic MCGDM problems, it is argued and accepted that words indicate different meanings for various DMs in computing with words (Mendel *et al.*, 2010). Considering the issue of personalized individual semantics (PISs) is necessary. For example, two referees are invited to express evaluations about a manuscript. Both of them provide comments with "good". However, linguistic term "good" may indicate different numerical semantics for them. Recently, different attempts have been made to copy with this issue, which can be roughly classified into three groups, including type-2 fuzzy set model (Mendel and Wu, 2010), multi-granular linguistic model (Morente-Molinera *et al.*, 2015), and consistency-driven models (Li *et al.*, 2017). Compared with the first two types of models, the consistency-driven model can effectively characterize the specific semantics of individuals, and becomes a popular tool to manage PISs in linguistic GDM. Thus, various consistency-driven models have been designed to assign personalized numerical scales (PNSs) based on linguistic preference relations (LPRs) (Zhang and Li, 2022), incomplete LPRs (Li *et al.*, 2022a), distribution LPRs (Tang *et al.*, 2020), and hesitant fuzzy LPRs with self-confidence (Zhang *et al.*, 2021). In these models, DMs' PISs can be explored according to their linguistic preferences in terms of a set of alternatives. The traditional PIS models are valid in the situations where the evaluations are expressed with LPRs or their extensions. However, they will fail to work when DMs' evaluations are in forms of linguistic MCGDM matrices.

Recently, Li *et al.* (2022a) designed a data-driven learning model to investigate the PISs of DMs in MCGDM. In this model, two objectives are achieved through maximizing the minimum overall deviation among alternatives between any two consecutive categories, and minimizing the overall deviation among alternatives in a category. Inspired by the idea of Li *et al.* (2022b), this study aims to propose an improved framework to handle PISs and GDM, where the evaluations are presented in forms of MCGDM matrices with LSVNNs. The main novelties and contributions are summarized as follows:

(1) A novel discrimination measure for SVNNs is proposed, based on which a discrimination-based optimization model is constructed to assign PNSs. The proposed framework is the first attempt to manage PISs in linguistic GDM, where DMs' assessments are presented with linguistic neutrosophic MCGDM matrices.

(2) An extended consensus-based optimization model is constructed to identify the weights of DMs considering group consensus. The proposed approach can cautiously assign DMs' weights to guarantee a level of agreement among members regarding the final solution, and reveal the differences among alternatives with the optimal discrimination degrees.

The rest of the paper is organized as follows. Section 2 introduces some concepts about 2-tuple linguistic model and numerical scale (NS) model, neutrosophic sets and LSVNSs. Section 3 presents the concept of distance and discrimination measures for LSVNNs and develops a discrimination-based optimization model to obtain PNSs. Section 4 presents a comprehensive linguistic neutrosophic MCGDM framework considering the PIS and group consensus. Section 5 presents an illustrative example, followed by the comparative analysis to valid the proposed framework. Finally, Section 6 concludes this study.

2. Preliminaries

2.1. *2-Tuple Linguistic and Numerical Scale Models*

Definition 1 (Herrera and Martínez, 2000). Let $S = \{s_0, s_1, \dots, s_\tau\}$ be an LTS and $\beta \in [0, \tau]$ indicate the result of a symbolic aggregation operation. The conversion function between 2-tuples and numerical values are defined as follows:

$$\Delta : [0, \tau] \to \bar{S} \text{ being } \Delta(\beta) = (s_\theta, \alpha) \quad \text{with} \begin{cases} s_\theta, & \theta = round(\beta), \\ \alpha = \beta - \theta, & \alpha \in [-0.5, 0.5). \end{cases} \tag{1}$$

The inverse function of Δ, $\Delta^{-1} : \bar{S} \to [0, \tau]$ is defined as $\Delta^{-1}(s_\theta, \alpha) = \theta + \alpha$. The corresponding negative operator is $Neg(\Delta^{-1}(s_\theta, \alpha)) = \Delta(\tau - \Delta^{-1}(s_\theta, \alpha))$.

Definition 2 (Dong *et al.*, 2009). Let $S = \{s_0, s_1, \dots, s_\tau\}$ be an LTS and R be a set of real numbers. A function $NS : S \to R$ is called an NS of S, and $NS(s_\theta)$ is the numerical index of s_θ. The numerical scale NS for (s_θ, α) is defined as follows:

$$NS(s_\theta, \alpha) = \begin{cases} NS(s_\theta) + \alpha \times \big(NS(s_{\theta+1}) - NS(s_\theta)\big), & \alpha \geqslant 0, \\ NS(s_\theta) + \alpha \times \big(NS(s_\theta) - NS(s_{\theta-1})\big), & \alpha < 0. \end{cases} \tag{2}$$

If $NS(s_\theta) < NS(s_{\theta+1})$ for $\theta = 0, 1, \dots, \tau - 1$, then the numerical scale NS on S is ordered. Furthermore, Li *et al.* (2017) added several conditions: $NS(s_0) = 0$, $NS(s_{\frac{\tau}{2}}) = 0.5$, $NS(s_\tau) = 1$ and $NS(s_\theta) \in \big[\frac{\theta-0.5}{\tau}, \frac{\theta+0.5}{\tau}\big)$ $(\theta = 1, 2, \dots, \tau - 1; \theta \neq \frac{\tau}{2})$ in order to generate the PNS.

The inverse operator of NS is defined as Dong *et al.* (2009):

$$NS^{-1} : R \to \bar{S} \quad \text{with}$$
$$NS^{-1}(r) = \begin{cases} \left(s_\theta, \dfrac{r - NS(s_\theta)}{NS(s_{\theta+1}) - NS(s_\theta)}\right), & NS(s_\theta) < r < \dfrac{NS(s_\theta) + NS(s_{\theta+1})}{2}, \\ \left(s_\theta, \dfrac{r - NS(s_\theta)}{NS(s_\theta) - NS(s_{\theta-1})}\right), & \dfrac{NS(s_{\theta-1}) + NS(s_\theta)}{2} < r \leqslant NS(s_\theta). \end{cases} \tag{3}$$

2.2. *Neutrosophic Sets*

Definition 3 (Smarandache, 1999). Let X be a space of points, where $x \in X$. Then, a neutrosophic set A in X is characterized by $A = \{\langle x, T_A(x), I_A(x), F_A(x) \mid x \in X\rangle\}$, where $T_A(x), I_A(x), F_A(x) \in]0^-, 1^+[$ represent the truth, indeterminacy and falsity-membership functions, respectively, such that $0^- \leqslant \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leqslant 3^+$.

Definition 4 (Wang *et al.*, 2010; Ye, 2013). An SVNS in A is characterized by $A = \{\langle x, T_A(x), I_A(x), F_A(x) \mid x \in X\rangle\}$, where $T_A(x), I_A(x), F_A(x) \in [0, 1]$, such that $0 \leqslant \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leqslant 3$.

2.3. *Linguistic Single-Valued Neutrosophic Sets*

Definition 5 (Li Y. *et al.*, 2017). Let $S = \{s_0, s_1, \dots, s_\tau\}$ be an LTS. Then, an LSVNS is defined as $A = \{\langle x, s_A^T(x), s_A^I(x), s_A^F(x) \mid x \in X\rangle\}$, where $s_A^T(x), s_A^I(x), s_A^F(x) \in S$ indicate the linguistic truth, indeterminacy and falsity degrees, respectively, such that $0 \leqslant s_A^T(x), s_A^I(x), s_A^F(x) \leqslant 3s_\tau$. Particularly, $\langle T_A(x), I_A(x), F_A(x)\rangle$ is described as an LSVNN, and each SVNN is $a = \langle s_a^T, s_a^I, s_a^F\rangle$.

Definition 6 (Fang and Ye, 2017). Let $r_j = \langle s_{r_j}^T, s_{r_j}^I, s_{r_j}^F\rangle$ $(j = 1, 2, \dots, n)$ be a set of LSVNNs and Ω be a set of all given values. Then, the LSVNN WAA operator is the mapping $LNWA : \Omega^n \to \Omega$, defined as follows:

$$\begin{aligned} LNWA(r_1, r_2, \dots, r_n) &= \sum_{j=1}^{n} w_j r_j = \langle s_r^T, s_r^I, s_r^F \rangle \\ &= \Big\langle NS^{-1}\Big(1 - \prod_{j=1}^{n}\big(1 - NS(s_{r_j}^T)\big)^{w_j}\Big), NS^{-1}\Big(\prod_{j=1}^{n}\big(NS(s_{r_j}^I)\big)^{w_j}\Big), \\ &\quad NS^{-1}\Big(\prod_{j=1}^{n}\big(NS(s_{r_j}^F)\big)^{w_j}\Big)\Big\rangle, \end{aligned} \tag{4}$$

where $w_j \in [0, 1]$ is the corresponding weight of r_j, satisfying $\sum_{j=1}^{n} w_j = 1$.

Definition 7 (Fang and Ye, 2017). Let $r = \langle s_r^T, s_r^I, s_r^F \rangle$ be an LSVNN over LTS S. Then, the score function $S(r)$ and accuracy function $H(r)$ of r are defined as follows:

$$S(r) = \frac{2 + NS(s_r^T) - NS(s_r^I) - NS(s_r^F)}{3} \quad \text{and} \quad H(r) = NS(s_r^T) - NS(s_r^F), \tag{5}$$

where $NS(s_\theta)$ is the ordered NS of linguistic term s_θ, as defined in Definition 2.

3. Optimization Model to Obtain PNSs in MCGDM with LSVNNs

This section presents the concepts of distance and discrimination measures of LSVNNs, based on which a programming model is constructed to derive the PNSs of each DM.

For convenience, assume that $M = \{1, 2, \dots, m\}$, $N = \{1, 2, \dots, n\}$ and $Q = \{1, 2, \dots, q\}$. Assume that $A = \{a_1, a_2, \dots, a_m\}$ $(m \geqslant 2)$ is a set of alternatives; $C = \{c_1, c_2, \dots, c_n\}$ $(n \geqslant 2)$ is a set of criteria and $w_j \in [0, 1]$ is the corresponding weight of criterion c_j, satisfying $\sum_{j=1}^{n} w_j = 1$; and $E = \{e_1, e_2, \dots, e_q\}$ $(q \geqslant 2)$ is a set of experts and each expert is assigned with a weight $\lambda_h \in [0, 1]$, satisfying $\sum_{h=1}^{q} \lambda_h = 1$. Suppose that $B^h = [b_{ij}^h]_{m \times n}$ $(h \in Q)$ are the decision matrices, where $b_{ij}^h = \langle s_{b_{ij}^h}^T, s_{b_{ij}^h}^I, s_{b_{ij}^h}^F \rangle$ is linguistic single-valued neutrosophic evaluation given by expert e_h.

Assume that the standardized matrices are denoted by $R^h = [r_{ij}^h]_{m \times n}$ $(h \in Q)$. The original decision matrices $B^h = [b_{ij}^h]_{m \times n}$ $(h \in Q)$ can then be normalized into $R^h = [r_{ij}^h]_{m \times n}$ $(h \in Q)$ based on the primary transformation rule of Li Y. *et al.* (2017), where

$$r_{ij}^h = \begin{cases} \big\langle s_{b_{ij}^h}^T, s_{b_{ij}^h}^I, s_{b_{ij}^h}^F \big\rangle, & \text{for benefit criterion } c_j, \\ \big\langle s_{b_{ij}^h}^F, s_{b_{ij}^h}^I, s_{b_{ij}^h}^T \big\rangle, & \text{for cost criterion } c_j. \end{cases} \tag{6}$$

3.1. *Distance and Discrimination Measures for LSVNNs*

Definition 8. Let $r_j = \langle s^T_{r_j}, s^I_{r_j}, s^F_{r_j} \rangle$ $(j = 1, 2)$ be any two LSVNNs. Then, the distance measure $d(r_1, r_2)$ between r_1 and r_2 is defined as follows:

$$d(r_1, r_2) = \left(\frac{1}{3}\left(\left|NS\left(s^T_{r_1}\right) - NS\left(s^T_{r_2}\right)\right|^{\rho} + \left|NS\left(s^I_{r_1}\right) - NS\left(s^I_{r_2}\right)\right|^{\rho} + \left|NS\left(s^F_{r_1}\right) - NS\left(s^F_{r_2}\right)\right|^{\rho}\right)\right)^{\frac{1}{\rho}}, \tag{7}$$

where $NS(s_\theta)$ is the ordered NS of linguistic term s_θ, as defined in Definition 2, and $\rho > 0$.

Remark 1. Particularly, $\rho = 1, 2$, Eq. (7) is degenerated into the Hamming and Euclidean distance measures, respectively. Due to the distinct applicability of the distance measure under different values of ρ, Eq. (7) can help DMs flexibly select suitable parameter ρ based on actual decision scenarios, thereby optimizing DMs' discrimination measures.

Theorem 1. *Let $r_j = \langle s^T_{r_j}, s^I_{r_j}, s^F_{r_j} \rangle$ $(j = 1, 2, 3)$ be any three LSVNNs. Then, the distance measure in Definition 8 satisfies the following properties*:

(1) $0 \leqslant d(r_1, r_2) \leqslant 1$;
(2) $d(r_1, r_2) = d(r_2, r_1)$;
(3) *If $s^T_{r_1} \leqslant s^T_{r_2} \leqslant s^T_{r_3}$, $s^I_{r_1} \geqslant s^I_{r_2} \geqslant s^I_{r_3}$ and $s^F_{r_1} \geqslant s^F_{r_2} \geqslant s^F_{r_3}$, then $d(r_1, r_2) \leqslant d(r_1, r_3)$ and $d(r_2, r_3) \leqslant d(r_1, r_3)$.*

Proof. It is obvious that Properties (1) and (2) hold. Thus, the proof of Property (3) is provided.

Since $NS(s_\theta)$ is ordered in terms of s_θ, it has $NS(s^T_{r_1}) \leqslant NS(s^T_{r_2}) \leqslant NS(s^T_{r_3})$, $NS(s^I_{r_1}) \geqslant NS(s^I_{r_2}) \geqslant NS(s^I_{r_3})$ and $NS(s^F_{r_1}) \geqslant NS(s^F_{r_2}) \geqslant NS(s^F_{r_3})$. Thus, $S(r_1) \leqslant S(r_2) \leqslant S(r_3)$ based on Eq. (5) and the following inequalities can be obtained: $|NS(s^T_{r_1}) - NS(s^T_{r_2})|^\rho \leqslant |NS(s^T_{r_1}) - NS(s^T_{r_3})|^\rho$, $|NS(s^I_{r_1}) - NS(s^I_{r_2})|^\rho \leqslant |NS(s^I_{r_1}) - NS(s^I_{r_3})|^\rho$ and $|NS(s^F_{r_1}) - NS(s^F_{r_2})|^\rho \leqslant |NS(s^F_{r_1}) - NS(s^F_{r_3})|^\rho$. Therefore,

$$\begin{aligned}
&\left|NS\left(s^T_{r_1}\right) - NS\left(s^T_{r_2}\right)\right|^{\rho} + \left|NS\left(s^I_{r_1}\right) - NS\left(s^I_{r_2}\right)\right|^{\rho} + \left|NS\left(s^F_{r_1}\right) - NS\left(s^F_{r_2}\right)\right|^{\rho} \\
&\quad \leqslant \left|NS\left(s^T_{r_1}\right) - NS\left(s^T_{r_3}\right)\right|^{\rho} + \left|NS\left(s^I_{r_1}\right) - NS\left(s^I_{r_3}\right)\right|^{\rho} + \left|NS\left(s^F_{r_1}\right) - NS\left(s^F_{r_3}\right)\right|^{\rho} \\
&\quad \Rightarrow d(r_1, r_2) \leqslant d(r_1, r_3).
\end{aligned}$$

Thus, we have $d(r_1, r_2) \leqslant d(r_1, r_3)$. Similarly, it can be demonstrated that $d(r_2, r_3) \leqslant d(r_1, r_3)$. This completes the proof of Property (3). □

In GDM, a panel of experts are invited to provide their evaluations about a set of alternatives. It is required that an expert should be qualified with the ability to differentiate between cases which are similar but not identical (Herowati *et al.*, 2017). Motivated by

Table 1
LSVNN evaluations of Example 1.

	c_1	c_2	c_3	c_4
a_1	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_3, s_1\rangle$	$\langle s_6, s_1, s_1\rangle$	$\langle s_4, s_2, s_2\rangle$
a_2	$\langle s_4, s_3, s_1\rangle$	$\langle s_7, s_3, s_2\rangle$	$\langle s_6, s_3, s_2\rangle$	$\langle s_6, s_2, s_3\rangle$
a_3	$\langle s_5, s_3, s_2\rangle$	$\langle s_4, s_2, s_3\rangle$	$\langle s_4, s_3, s_1\rangle$	$\langle s_5, s_2, s_2\rangle$
a_4	$\langle s_5, s_3, s_3\rangle$	$\langle s_7, s_2, s_1\rangle$	$\langle s_6, s_2, s_3\rangle$	$\langle s_6, s_2, s_3\rangle$

the idea of maximizing deviation method (Wang, 1997), the total deviation among all alternatives is considered an effective tool to measure the discrimination of an expert (Tian *et al.*, 2019).

Definition 9. Let $R^h = [r_{ij}^h]_{m\times n}$ is a linguistic single-valued neutrosophic evaluation matrix, given by DM e_h. Then, the discrimination measure $Dis(e_h)$ of e_h is defined as follows:

$$Dis(e_h) = \frac{1}{m(m-1)} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1,k\neq i}^{m} w_j d\big(r_{ij}^h, r_{kj}^h\big), \tag{8}$$

where $Dis(e_h) \in [0, 1]$, $w_j \in [0, 1]$ is the weight of criterion c_j, and $d(r_{ij}^h, r_{kj}^h)$ is the distance between LSVNNs r_{ij}^h and r_{kj}^h, as per Eq. (7).

Remark 2. Although the discrimination measures defined in this study and Tian *et al.* (2019) are both used to measure experts' discrimination degrees among alternatives, there are differences in application. The discrimination measure in Tian *et al.* (2019) is defined for measuring experts' discrimination degrees among alternatives with evaluations in forms of interval type-2 fuzzy numbers. However, the discrimination measure defined in this study is suitable for experts who elicit qualitative ratings with LSVNSs.

Example 1. Assume that $S = \{s_0, s_1, \dots, s_8\}$ is an LTS and $R^1 = [r_{ij}^1]_{4\times 4}$ is a linguistic single-valued neutrosophic evaluation matrix, given by DM e_1 based on S, as shown in Table 1. The corresponding criterion weights are $w_j = 0.25$ $(j = 1, 2, 3, 4)$. Assume that the NSs on S are predetermined as $NS(s_\theta) = \frac{\theta}{8}$ $(\theta = 0, 1, \dots, 8)$. Then, according to Eq. (8), the discrimination degree is $Dis(e_1) = 0.1128$.

3.2. *Discrimination-Based Optimization Model to Obtain PNSs*

As mentioned above, an expert is expected to be skilled and have the ability to discriminate the differences between cases. When experts are required to express linguistic ratings, their PISs of linguistic terms are embedded in the evaluations, which implicitly indicate the subtle differences among alternatives distinguished by experts. Therefore, an optimization model by maximizing the discrimination degree can be considered as a good solution to derive the PISs of DMs.

$$\text{Max}\,Dis(e_h) = \frac{1}{m(m-1)} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1,k\neq i}^{m} w_j d\big(r_{ij}^h, r_{kj}^h\big)$$
$$\text{s.t.} \begin{cases} NS^h(s_0) = 0, \\ NS^h(s_\theta) \in \Big(\dfrac{\theta-1}{\tau}, \dfrac{\theta+1}{\tau}\Big], \quad \theta = 1, 2, \dots, \tau-1;\ \theta \neq \dfrac{\tau}{2}, \\ NS^h(s_{\frac{\tau}{2}}) = 0.5, \\ NS^h(s_\tau) = 1, \end{cases} \tag{9}$$

where $d(r_{ij}^h, r_{kj}^h)$ is the distance measure of LSVNNs, as per Eq. (7).

By resolving Model (9), a set of possible PNSs of linguistic terms for DM e_h can be derived, i.e. $APS^h = \{(NS^h(s_0), NS^h(s_1), \dots, NS^h(s_\tau)); \dots\}$. The obtained NSs can guarantee the maximum discrimination degree with respect to alternatives.

Example 2 (*Continuation of Example 1*). Assume that $R^1 = [r_{ij}^1]_{4\times4}$ and w are the same as Example 1. Then, the PNSs of linguistic terms for DM e_1 can be obtained as follows:

Without loss of generality, ρ is set as $\rho = 1$. The discrimination-based optimization model can be established based on Model (9).

$$\text{Max}\,Dis(e_1) = \frac{1}{3\times4\times4} \sum_{j=1}^{4} \sum_{i=1}^{4} \sum_{k=1,k\neq i}^{4} d\big(r_{ij}^1, r_{kj}^1\big)$$
$$\text{s.t.} \begin{cases} NS^1(s_0) = 0, \\ NS^1(s_1) \in \Big(\dfrac{0}{8}, \dfrac{2}{8}\Big], \\ NS^1(s_2) \in \Big(\dfrac{1}{8}, \dfrac{3}{8}\Big], \\ NS^1(s_3) \in \Big(\dfrac{2}{8}, \dfrac{4}{8}\Big], \\ NS^1(s_4) = 0.5, \\ NS^1(s_5) \in \Big(\dfrac{4}{8}, \dfrac{6}{8}\Big], \\ NS^1(s_6) \in \Big(\dfrac{5}{8}, \dfrac{7}{8}\Big], \\ NS^1(s_7) \in \Big(\dfrac{6}{8}, 1\Big], \\ NS^1(s_8) = 1. \end{cases} \tag{10}$$

By resolving Model (10), the PNSs of linguistic terms for DM e_1 can be identified, i.e. $NS^1(s_0) = 0$, $NS^1(s_1) = 0.05$, $NS^1(s_2) = 0.125$, $NS^1(s_3) = 0.45$, $NS^1(s_4) = 0.5$, $NS^1(s_5) = 0.55$, $NS^1(s_6) = 0.875$, $NS^1(s_7) = 0.95$, and $NS^1(s_8) = 1$. Moreover, according to Eq. (8), the discrimination degree is $Dis'(e_1) = 0.1816$, which is higher than that derived from Example 1. Specifically, compared to the method of without considering PISs in Example 1, the approach of considering PISs increases the discrimination

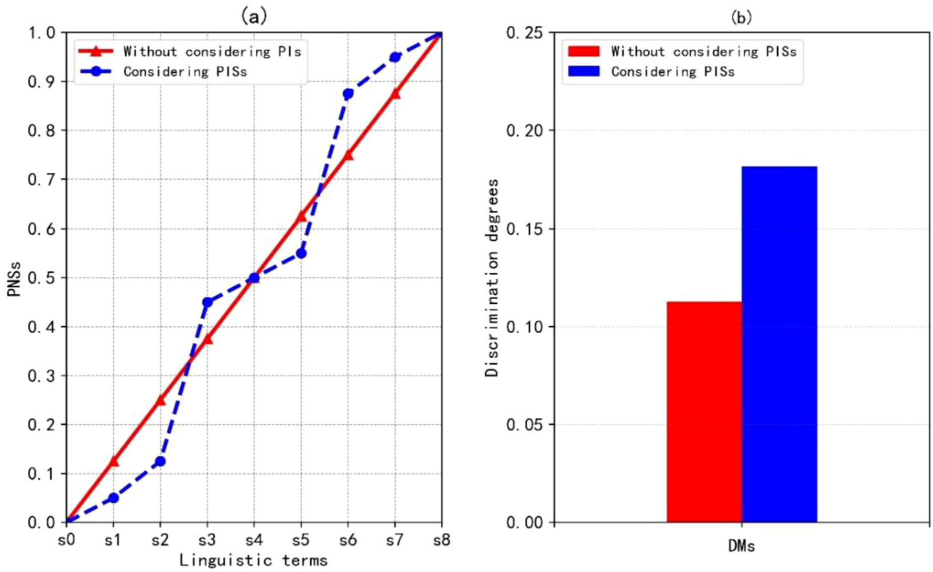

Fig. 1. Results derived from without considering and considering PISs.

degree of DM e_1 by approximately 61%. Figure 1 vividly reveals the difference between the method of without considering and considering PISs.

4. Linguistic Single-Valued Neutrosophic MCGDM Considering PIS and Consensus

This section develops a comprehensive linguistic neutrosophic MCGDM framework that considers the PIS and group consensus.

4.1. *Consensus Measure*

Definition 10. Let $R^h = [r_{ij}^h]_{m\times n}(h \in Q)$ be a set of individual linguistic neutrosophic evaluation matrices, $R^c = [r_{ij}^c]_{m\times n}$ be the collective evaluation matrix, and $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_q)^T$ be the weight vector of DMs. Then, the distance $d(R^h, R^c)$ between R^h and R^c is defined as follows:

$$d\big(R^h, R^c\big) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} d\big(r_{ij}^h, r_{ij}^c\big), \tag{11}$$

where $d(r_{ij}^h, r_{ij}^c)$ is the distance between LNNs based on Eq. (7) and $r_{ij}^c = LNWA(r_{ij}^1, r_{ij}^2, \dots, r_{ij}^q)$ is the collective evaluation of all DMs, which is obtained based on Eq. (4).

Definition 11. For a MCGDM problem, let $R^h = [r_{ij}^h]_{m\times n}$ $(h \in Q)$, $R^c = [r_{ij}^c]_{m\times n}$ and $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_q)^T$ be as the same as in Definition 10. Then, the consensus measure of such a decision group is defined as:

$$GCL = 1 - \underset{h\in Q}{\mathrm{Max}}\{d(R^h, R^c)\}, \tag{12}$$

where $d(R^h, R^c)$ is the distance between R^h and R^c based on Eq. (11). The group consensus $GCL \in [0, 1]$ indicates the agreement level among group members regarding the final solution. Higher value of GCL indicates more reliable result of the MCGDM (Wan *et al.*, 2017).

4.2. *Determination the Weights of DMs*

In order to merge the group consensus into the MCGDM, an optimization model is established to derive the weights of DMs by maximizing the group consensus as follows:

$$\begin{aligned} &\mathrm{Max}\, GCL = 1 - \underset{h\in Q}{\mathrm{Max}}\{d(R^h, R^c)\}, \\ &\text{s.t.} \begin{cases} \sum_{h=1}^{q} \lambda_h = 1, \\ \lambda_h \in [0, 1], \quad h \in Q. \end{cases} \end{aligned} \tag{13}$$

Set $\eta = mn\,\mathrm{Max}_{h\in Q}\{d(R^h, R^c)\}$ and it has $d(R^h, R^c) \leqslant \frac{\eta}{mn}$ for all $h \in Q$. Thus, Model (13) can be equally converted into the following model.

$$\begin{aligned} &\mathrm{Min}\, \eta \\ &\text{s.t.} \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n} d(r_{ij}^h, r_{ij}^c) \leqslant \eta, \quad h \in Q, \\ r_{ij}^c = \sum_{h=1}^{q} \lambda_h r_{ij}^h, \\ \sum_{h=1}^{q} \lambda_h = 1, \\ \lambda_h \in [0, 1], \quad h \in Q, \end{cases} \end{aligned} \tag{14}$$

where $\lambda_h (h \in Q)$ are the decision variables, $d(r_{ij}^h, r_{ij}^c)$ is the distance between LSVNNs based on Eq. (7), and $r_{ij}^c = LNWA(r_{ij}^1, r_{ij}^2, \dots, r_{ij}^q)$ is the collective evaluation of all DMs, which is obtained based on Eq. (4).

By resolving Model (14), the weight vector of DMs $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_q)^T$ can be obtained. Moreover, the group consensus GCL can be calculated based on Eq. (12). According to the consensus-driven optimization model, DMs who make great contribution to achieve the high level of group consensus will be assigned large weight. In this way,

Table 2
PNSs of linguistic terms for DMs in Fang and Ye (2017).

	$NS(s_0)$	$NS(s_1)$	$NS(s_2)$	$NS(s_3)$	$NS(s_4)$	$NS(s_5)$	$NS(s_6)$	$NS(s_7)$	$NS(s_8)$
APS^1	0	0.05	0.375	0.45	0.5	0.563	0625	0.95	1
APS^2	0	0.05	0.125	0.45	0.5	0.55	0.875	0.95	1
APS^3	0	0.05	0.375	0.45	0.5	0.55	0.625	0.95	1
APS^c	0	0.05	0.311	0.45	0.5	0.553	0.689	0.95	1

the proposed weighting framework can be considered as an indirect reward mechanism to encourage group members to obtain the consensual solutions.

Example 3. Take the numerical example with LSVNN decision matrices $R^h = [r_{ij}^h]_{4\times 3}$ $(h = 1, 2, 3)$ and criterion weights $w = (0.35, 0.25, 0.4)^T$ in Fang and Ye (2017) as an instance. Then, the PNSs of linguistic terms and weights for each DM can be obtained as follows:

First, the PNSs of linguistic terms for each DM can be derived based on Model (9). The obtained results are presented in Table 2.

Second, the weights of each DM can be identified based on Model (14)

$$\text{Min}\ \eta$$
$$\text{s.t.}\ \begin{cases} \sum_{i=1}^{4}\sum_{j=1}^{4} d\left(r_{ij}^h, r_{ij}^c\right) \leqslant \eta, \quad h = 1, 2, 3, \\ r_{ij}^c = \lambda_1 r_{ij}^1 + \lambda_2 r_{ij}^2 + \lambda_3 r_{ij}^3, \\ \lambda_1 + \lambda_2 + \lambda_3 = 1, \\ \lambda_h \in [0, 1], \quad h = 1, 2, 3, \end{cases} \tag{15}$$

where $d(r_{ij}^h, r_{ij}^c)$ is the distance between LSVNNs based on Eq. (7).

By resolving the above model with the software MATLAB or LINGO, the weight vector of DMs is $\lambda = (0.254, 0.258, 0.488)^T$. The collective PNSs APS^c for group members can be calculated based on the simple weighted average of APS^h $(h = 1, 2, 3)$, as shown in Table 2.

4.3. *Proposed Framework for MCGDM with LSVNNs*

This subsection presents a framework for managing MCGDM with LSVNNs considering the PIS and consensus, which is described in Fig. 2. The detailed steps are as follows:

Step 1: Normalize the decision matrices of group members.

Each member provides evaluation information with LSVNNs and the individual decision matrices are normalized based on Eq. (6), i.e. $R^h = [r_{ij}^h]_{m\times n}$ $(h \in Q)$.

Step 2: Identify PNSs of linguistic terms for each DM.

The PNSs APS^h $(h \in Q)$ of linguistic terms can be derived based on Model (9).

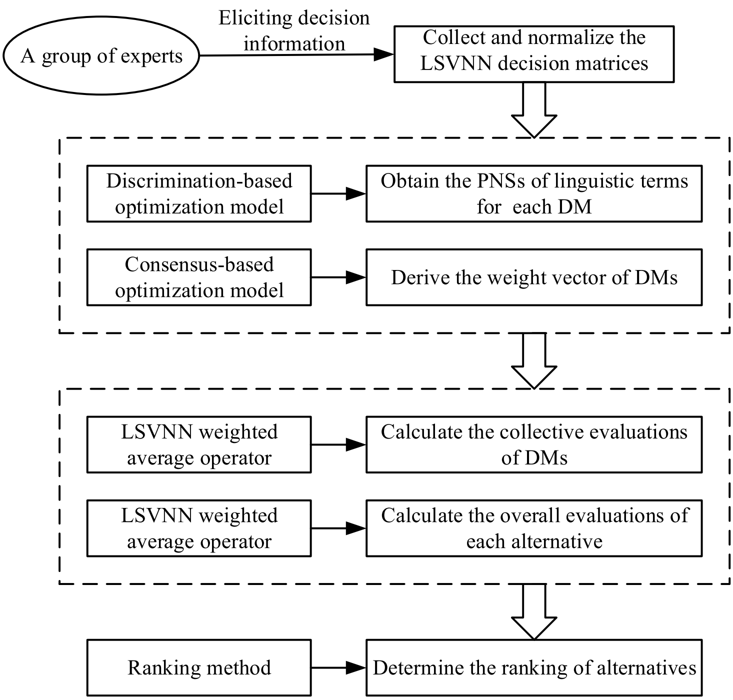

Fig. 2. Flowchart of the proposed approach.

Step 3: Determine the weight vector and obtain the collective evaluations of DMs.

The weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_q)^T$ can be derived based on Model (14), and the collective evaluation matrix $R^c = [r_{ij}^c]_{m \times n}$ can be calculated based on Eq. (4).

Step 4: Derive the overall evaluations of each alternative.

The overall evaluations r_i^c $(i \in M)$ of each alternative can be calculated based on Eq. (4).

Step 5: Determine the ranking of alternatives.

The ranking of alternatives can be identified based on the score values $S(r_i^c)$ $(i \in M)$ and accuracy values $H(r_i^c)$ $(i \in M)$.

5. Illustrative Example

This section presents an illustrative example adapted from Garg and Nancy (2018) to demonstrate the application of the proposed approach.

5.1. *Illustration of the Proposed MCGDM Method*

A panel involving five experts are invited to express their evaluations and select the best internet service provider(s). After conducting the preliminary investigation, four internet

Table 3
LSVNN evaluations given by DM e_1.

	c_1	c_2	c_3	c_4
a_1	$\langle s_6, s_1, s_1\rangle$	$\langle s_5, s_3, s_3\rangle$	$\langle s_4, s_3, s_1\rangle$	$\langle s_6, s_3, s_1\rangle$
a_2	$\langle s_5, s_3, s_3\rangle$	$\langle s_5, s_4, s_1\rangle$	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_2, s_2\rangle$
a_3	$\langle s_5, s_3, s_2\rangle$	$\langle s_4, s_3, s_2\rangle$	$\langle s_3, s_4, s_2\rangle$	$\langle s_6, s_2, s_2\rangle$
a_4	$\langle s_5, s_3, s_3\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_4, s_3, s_2\rangle$	$\langle s_7, s_1, s_2\rangle$

Table 4
LSVNN evaluations given by DM e_2.

	c_1	c_2	c_3	c_4
a_1	$\langle s_4, s_4, s_1\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_6, s_1, s_1\rangle$
a_2	$\langle s_5, s_2, s_2\rangle$	$\langle s_4, s_3, s_3\rangle$	$\langle s_3, s_4, s_2\rangle$	$\langle s_5, s_3, s_2\rangle$
a_3	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_4, s_2\rangle$	$\langle s_6, s_2, s_4\rangle$	$\langle s_6, s_2, s_4\rangle$
a_4	$\langle s_5, s_2, s_2\rangle$	$\langle s_7, s_1, s_2\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_6, s_5, s_2\rangle$

Table 5
LSVNN evaluations given by DM e_3.

	c_1	c_2	c_3	c_4
a_1	$\langle s_5, s_3, s_3\rangle$	$\langle s_5, s_3, s_1\rangle$	$\langle s_4, s_3, s_1\rangle$	$\langle s_6, s_3, s_1\rangle$
a_2	$\langle s_4, s_2, s_7\rangle$	$\langle s_5, s_4, s_1\rangle$	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_2, s_2\rangle$
a_3	$\langle s_5, s_3, s_3\rangle$	$\langle s_4, s_3, s_2\rangle$	$\langle s_3, s_4, s_2\rangle$	$\langle s_6, s_2, s_2\rangle$
a_4	$\langle s_4, s_2, s_5\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_4, s_3, s_2\rangle$	$\langle s_7, s_1, s_2\rangle$

service providers, namely, Bharti Airtel (a_1), Relience Communications (a_2), Vodafone India (a_3) and Mahanagar Telecom Nigam (a_4) are considered as alternatives. The criteria are Customer Service (c_1), Bandwidth (c_2), Package Deal (c_3) and Total Cost (c_4). The group of experts, represented as e_h ($h = 1, 2, 3, 4, 5$) provide their ratings about alternatives a_i ($i = 1, 2, 3, 4$) in terms of each criterion c_j ($j = 1, 2, 3, 4$) with LSVNNs based on the LTS $S = \{s_0 = \text{extremely poor},\ s_1 = \text{very poor},\ s_2 = \text{poor},\ s_3 = \text{slightly poor},\ s_4 = \text{fair},\ s_5 = \text{slightly good},\ s_6 = \text{good},\ s_7 = \text{very good},\ s_8 = \text{extremely good}\}$. Assume that the decision matrices of experts are denoted by $B^h = [b^h_{ij}]_{m\times n}$ ($h = 1, 2, 3, 4, 5$) and the criterion weight vector is $w = (0.35, 0.3, 0.2, 0.15)^T$.

The proposed approach is employed to deal with the above linguistic neutrosophic MCGDM problem. Let the parameter $\rho = 1$.

Step 1: Normalize the decision matrices of group members.

Each member provides evaluation information with LSVNNs and the individual decision matrices are normalized based on Eq. (6), as presented in Tables 3–7.

Step 2: Identify PNSs of linguistic terms for each DM.

The PNSs APS^h ($h = 1, 2, 3, 4, 5$) of linguistic terms on LTS S can be identified based on the discrimination-driven optimization model. Taking R^1 as an example, a program-

Table 6
LSVNN evaluations given by DM e_4.

	c_1	c_2	c_3	c_4
a_1	$\langle s_4, s_1, s_3\rangle$	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_4, s_2\rangle$	$\langle s_6, s_1, s_3\rangle$
a_2	$\langle s_7, s_2, s_4\rangle$	$\langle s_7, s_4, s_2\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_6, s_4, s_3\rangle$
a_3	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_4, s_3\rangle$	$\langle s_6, s_1, s_2\rangle$	$\langle s_6, s_3, s_1\rangle$
a_4	$\langle s_4, s_2, s_3\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_6, s_5, s_1\rangle$	$\langle s_7, s_4, s_2\rangle$

Table 7
LSVNN evaluations given by DM e_5.

	c_1	c_2	c_3	c_4
a_1	$\langle s_5, s_1, s_2\rangle$	$\langle s_6, s_2, s_3\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_6, s_1, s_1\rangle$
a_2	$\langle s_7, s_1, s_1\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_5, s_2, s_2\rangle$	$\langle s_7, s_1, s_2\rangle$
a_3	$\langle s_5, s_2, s_3\rangle$	$\langle s_5, s_1, s_1\rangle$	$\langle s_5, s_2, s_2\rangle$	$\langle s_5, s_3, s_1\rangle$
a_4	$\langle s_4, s_3, s_1\rangle$	$\langle s_7, s_2, s_1\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_6, s_1, s_1\rangle$

ming model is established based on Model (9) as follows:

$$\begin{aligned}\text{Max}\, Dis(e_1) = \frac{1}{12}\bigg(&0.35\sum_{i=1}^{4}\sum_{k=1,k\neq i}^{4} d\big(r_{i1}^1, r_{k1}^1\big) + 0.3\sum_{i=1}^{4}\sum_{k=1,k\neq i}^{4} d\big(r_{i2}^1, r_{k2}^1\big)\\ &+ 0.2\sum_{i=1}^{4}\sum_{k=1,k\neq i}^{4} d\big(r_{i3}^1, r_{k3}^1\big) + 0.15\sum_{i=1}^{4}\sum_{k=1,k\neq i}^{4} d\big(r_{i4}^1, r_{k4}^1\big)\bigg)\end{aligned}$$

$$\text{s.t.}\begin{cases} NS^1(s_0) = 0,\\ NS^1(s_1) \in \left(\frac{0}{8}, \frac{2}{8}\right],\\ NS^1(s_2) \in \left(\frac{1}{8}, \frac{3}{8}\right],\\ NS^1(s_3) \in \left(\frac{2}{8}, \frac{4}{8}\right],\\ NS^1(s_4) = 0.5,\\ NS^1(s_5) \in \left(\frac{4}{8}, \frac{6}{8}\right],\\ NS^1(s_6) \in \left(\frac{5}{8}, \frac{7}{8}\right],\\ NS^1(s_7) \in \left(\frac{6}{8}, 1\right],\\ NS^1(s_8) = 1.\end{cases}$$

By resolving the above model with the software MATLAB or LINGO, the PNS APS^1 for DM e_1 can be obtained. Similarly, APS^h $(h = 2, 3, 4, 5)$ can be derived and the results are presented in Table 8. There are subtle differences about the semantics of linguistic terms among DMs.

Table 8
PNSs of linguistic terms.

	$NS(s_0)$	$NS(s_1)$	$NS(s_2)$	$NS(s_3)$	$NS(s_4)$	$NS(s_5)$	$NS(s_6)$	$NS(s_7)$	$NS(s_8)$
APS^1	0	0.05	0.201	0.45	0.5	0.55	0.875	0.95	1
APS^2	0	0.05	0.125	0.45	0.5	0.651	0.875	0.95	1
APS^3	0	0.05	0.125	0.25	0.5	0.75	0.875	0.95	1
APS^4	0	0.05	0.375	0.45	0.5	0.55	0.875	0.95	1
APS^5	0	0.05	0.375	0.45	0.5	0.55	0.751	0.95	1
APS^c	0	0.05	0.262	0.406	0.5	0.605	0.854	0.95	1

Table 9
Overall evaluations of alternatives in terms of each criterion.

	c_1	c_2
a_1	$\langle(s_5, 0.379), (s_1, 0.2), (s_2, -0.41)\rangle$	$\langle(s_6, -0.424), (s_3, -0.276), (s_1, 0.418)\rangle$
a_2	$\langle(s_6, -0, 121), (s_2, -0.327), (s_2, 0.423)\rangle$	$\langle(s_6, -0.249), (s_3, -0.015), (s_2, -0.367)\rangle$
a_3	$\langle(s_6, -0.409), (s_2, 0.382), (s_1, 0.491)\rangle$	$\langle(s_5, 0.001), (s_2, 0.46), (s_2, -0.411)\rangle$
a_4	$\langle(s_4, 0.305), (s_2, 0.133), (s_2, 0.255)\rangle$	$\langle(s_6, -0.033), (s_2, -0.184), (s_2, -0.318)\rangle$
	c_3	c_4
a_1	$\langle(s_5, 0.375), (s_2, -0.069), (s_2, -0.484)\rangle$	$\langle(s_6, 0.058), (s_1, 0.339), (s_1, 0.266)\rangle$
a_2	$\langle(s_5, 0.202), (s_2, 0.19), (s_2, -0.084)\rangle$	$\langle(s_6, -0.18, (s_2, -0.095), (s_3, -0.193)\rangle$
a_3	$\langle(s_6, -0.028), (s_2, -0.159), (s_2, -0.169)\rangle$	$\langle(s_6, 0.196), (s_3, -0.421), (s_1, 0.32)\rangle$
a_4	$\langle(s_6, -0.291), (s_2, -0.041), (s_1, 0.313)\rangle$	$\langle(s_6, 0.439), (s_2, -0.302), (s_2, -0.448)\rangle$

Step 3: Determine the weight vector and obtain the collective evaluations of DMs.

A consensus-based optimization model can be established based on Model (14) as follows:

$$\text{Min}\ \eta$$
$$\text{s.t.}\ \begin{cases} \sum_{i=1}^{4}\sum_{j=1}^{4} d\left(r_{ij}^{h}, r_{ij}^{c}\right) \leqslant \eta, \quad h = 1, 2, 3, 4, 5, \\ r_{ij}^{c} = \lambda_1 r_{ij}^{1} + \lambda_2 r_{ij}^{2} + \lambda_3 r_{ij}^{3} + \lambda_4 r_{ij}^{4} + \lambda_5 r_{ij}^{5}, \\ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1, \\ \lambda_h \in [0, 1], \quad h = 1, 2, 3, 4, 5. \end{cases}$$

By resolving the above model with the software MATLAB or LINGO, the weight vector of DMs is $\lambda = (0.244, 0.113, 0.219, 0.252, 0.172)^T$. The collective PNSs APS^c for group members can be calculated based on the simple weighted average of APS^h ($h = 1, 2, 3, 4, 5$), as shown in Table 8. The collective evaluation matrix $R^c = [r_{ij}^c]_{4\times 4}$ is calculated based on Eq. (4). The result is presented in Table 9. Moreover, the group consensus GCL can be calculated based on Eq. (12), namely, $GCL = 0.858$.

Step 4: Derive the overall evaluations of each alternative.

The overall evaluations r_i^c ($i = 1, 2, 3, 4$) are calculated based on Eq. (4). The results are shown in Table 10.

Table 10
Overall values of each alternative.

	r_i^c	$S(r_i^c)$	$H(r_i^c)$	Rankings
a_1	$\langle(s_6, -0.435), (s_2, -0.399), (s_1, 0.465)\rangle$	0.807	0.597	1
a_2	$\langle(s_6, -0.274), (s_2, 0.057), (s_2, 0.017)\rangle$	0.751	0.522	4
a_3	$\langle(s_6, -0.354), (s_2, 0.291), (s_2, -0.453)\rangle$	0.765	0.600	3
a_4	$\langle(s_6, -0.337), (s_2, -0.086), (s_2, -0.295)\rangle$	0.776	0.571	2

Step 5: Determine the ranking of alternatives.

The score and accuracy values $S(r_i^c)$ $(i = 1, 2, 3, 4)$ and $H(r_i^c)$ $(i = 1, 2, 3, 4)$ are calculated based on Eq. (5), as presented in Table 10. Based on the comparison method for LSVNNs, the ranking of alternatives is $a_1 \succ a_4 \succ a_3 \succ a_2$, and Bharti Airtel ($a_1$) can be considered as the best internet service provider.

5.2. *Sensitivity Analysis*

In order to investigate the influence of parameter ρ on the final result, different values of ρ are assigned, namely, $\rho = 1$, $\rho = 2$ and $\rho \to \infty$. The PNSs of linguistic terms and discrimination degrees of each DM with different values of ρ are presented in Figs. 3–5, respectively. From Figs. 3(a)–5(a), there are slight differences about the PNSs of linguistic terms when ρ is assigned different values. From Figs. 3(b)–5(b), although the discrimination levels are various when ρ is assigned different values, the orders of discrimination levels of DMs remain unchanged, except $\rho = 2$. Moreover, from Table 11, the orders of alternatives almost have no change, except $\rho = 2$. The worst alternative is Relience Communications (a_2) in the three cases. In summary, different values of ρ can generate various distance measures of LSVNNs. However, it seems to have little influence on the final ranking of alternatives. Thus, one can choose the Hamming distance measure of LSVNNs with $\rho = 1$ to assign the PNSs and drive the ranking of alternatives, which is simple and straightforward.

5.3. *Comparative Analysis*

A comparative analysis is conducted between the existing MCGDM and the proposed approaches with LSVNNs. Two common MCGDM methods are employed in this comparison, and they are the LSVNN WAA and WGA operator-based method (M1 for short) (Fang and Ye, 2017) and the LSVNN prioritized WAA and WGA operator-based method (M2 for short) (Garg and Nancy, 2018).

The proposed approach is employed to solve the MCGDM problems in Fang and Ye (2017) and Garg and Nancy (2018), where the criterion weights keep the same as M1 and M2, respectively. The ranking results obtained by different methods are presented in Table 12. The sorting results obtained from the first group of comparative analysis are consistent, all of which are $a_4 \succ a_2 \succ a_3 \succ a_1$. However, there are differences in the ranking results obtained from the second group of comparative analysis, which reflected in the order of alternatives a_1, a_2 and a_4. The possible reason for this difference is that the

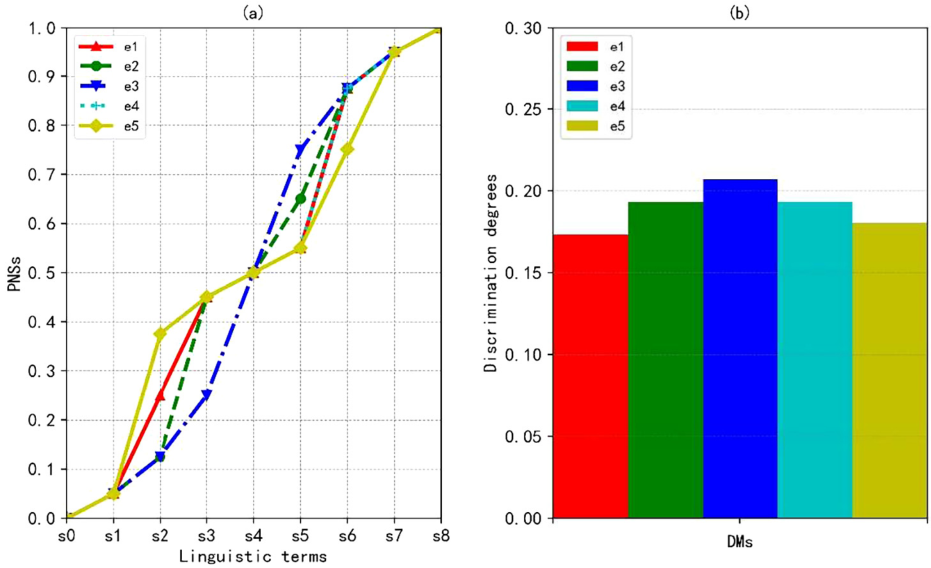

Fig. 3. PNSs of linguistic terms and discrimination degrees with $\rho = 1$.

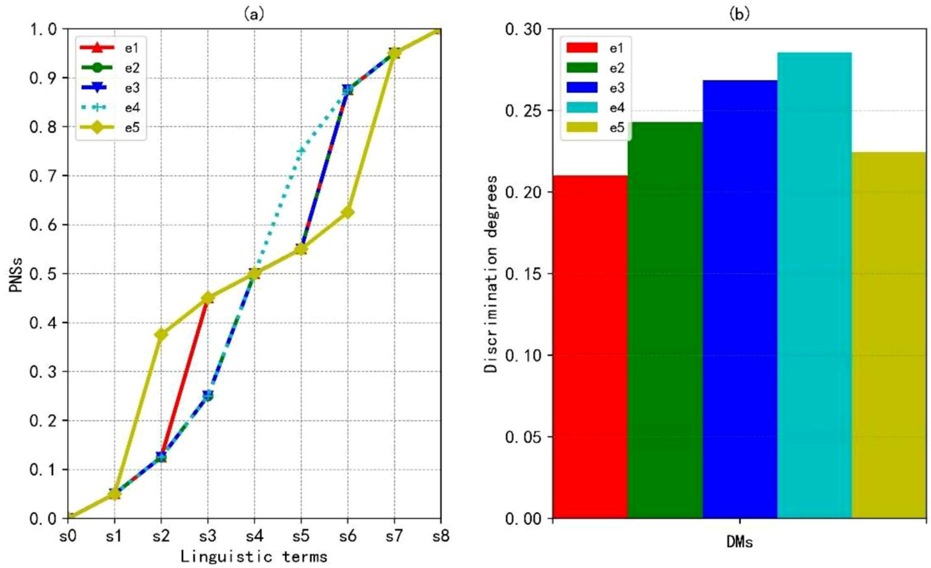

Fig. 4. PNSs of linguistic terms and discrimination degrees with $\rho = 2$.

M1 needs to identify the orders of DMs and criteria before determining their weights. The aggregated results are highly related to the predefined orders. By contrast, the proposed approach identifies the weights of DMs based on a consensus-based optimization model, omitting the extra pre-procedure of determining the orders of DMs and criteria. Moreover,

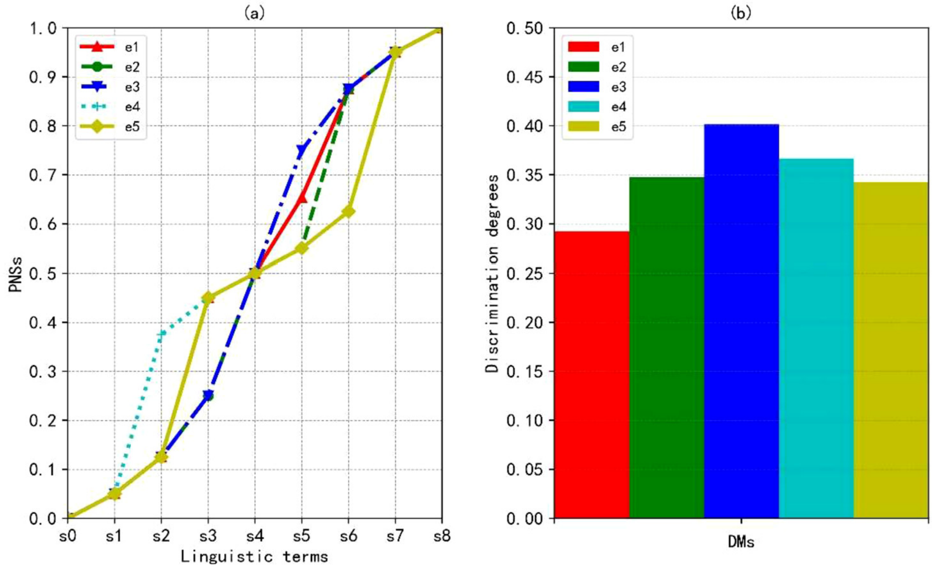

Fig. 5. PNSs of linguistic terms and discrimination degrees with $\rho \to \infty$.

Table 11
Rankings of alternatives with different values of ρ.

ρ	$S(r_i^c)$	Rankings
$\rho = 1$	$S(r_1^c) = 0.807$, $S(r_2^c) = 0.751$, $S(r_3^c) = 0.765$, $S(r_4^c) = 0.776$	$a_1 \succ a_4 \succ a_3 \succ a_2$
$\rho = 2$	$S(r_1^c) = 0.795$, $S(r_2^c) = 0.757$, $S(r_3^c) = 0.763$, $S(r_4^c) = 0.797$	$a_4 \succ a_1 \succ a_3 \succ a_2$
$\rho \to \infty$	$S(r_1^c) = 0.802$, $S(r_2^c) = 0.762$, $S(r_3^c) = 0.775$, $S(r_4^c) = 0.781$	$a_1 \succ a_4 \succ a_3 \succ a_2$

Table 12
Rankings of alternatives yielded by different methods.

Methods	Rankings	Discrimination degrees
M1 (Fang and Ye, 2017)	$a_4 \succ a_2 \succ a_3 \succ a_1$	$Dis(e_1) = 0.069$, $Dis(e_2) = 0.085$, and $Dis(e_3) = 0.121$
The proposed approach	$a_4 \succ a_2 \succ a_3 \succ a_1$	$Dis(e_1) = 0.159$, $Dis(e_2) = 0.123$, and $Dis(e_3) = 0.166$
M2 (Garg and Nancy, 2018)	$a_2 \succ a_4 \succ a_1 \succ a_3$	$Dis(e_1) = 0.159$, $Dis(e_2) = 0.214$, and $Dis(e_3) = 0.283$
The proposed approach	$a_1 \succ a_2 \succ a_4 \succ a_3$	$Dis(e_1) = 0.231$, $Dis(e_2) = 0.319$, and $Dis(e_3) = 0.386$

the alternatives are ranked based on the proposed PNSs-based score and accuracy functions. The different weight determination methods, aggregation rules and ranking methods may yield various results. The differences between M1, M2 and the proposed approach are summarized in Table 13.

In order to highlight the characteristics of considering PISs, the discrimination degrees of decision matrices are calculated. Since the PISs of DMs are overlooked in M1 and M2, the fixed NSs for LTS S are set, namely, $FNS(s_\theta) = \frac{\theta}{8}$ ($\theta = 0, 1, \ldots, 8$). Then, the discrimination degrees of decision matrices R^h ($h = 1, 2, 3$) in Fang and Ye (2017)

Table 13
Comparisons between the existing and the proposed methods.

Methods	Aggregation operators	Ways of addressing LSVNNs	PISs considered	Group consensus considered
M1 (Fang and Ye, 2017)	LSVNN WAA and WGA operators	Consider indices of linguistic terms	No	No
M2 (Garg and Nancy, 2018)	LSVNN prioritized WAA and WGA operators	Consider indices of linguistic terms	No	No
Method in Li Y. *et al.* (2017)	LSVNN geometric Heronian mean and prioritized geometric Heronian mean operators	Consider indices of linguistic terms	No	No
Method in Liang *et al.* (2018a)	LSVNN power WAA and WGA operators	Consider indices of linguistic terms	No	No
Method in Li *et al.* (2019)	LSVNN power WAA and WGA operators	Consider indices of linguistic terms	No	No
The proposed approach	LSVNN WAA operator	NS-based 2-tuple linguistic model	Yes	Yes

and Garg and Nancy (2018) are calculated by using Eq. (8) based on the fixed and personalized NSs. The results are presented in Figs. 6 and 7. It shows that when expressing linguistic rating with LSVNNs, different DMs may present various linguistic semantics, as shown in Fig. 6(a) and Fig. 7(a). Therefore, it is necessary to take the PISs of each DM into account. Moreover, the proposed approach presents higher discrimination degrees than M1 and M2 that both fail to consider PISs of DMs, as shown in Fig. 6(b) and Fig. 7(b). Although the discrimination degrees obtained from the comparative analysis of each method are implicit, their significance can be achieved by describing the percentage of the improvement value brought by the proposed method compared to the results obtained by existing methods, as shown in Fig. 8. For example, in the first group of comparative analysis, the discrimination degree of e_1 calculated using the proposed method is greater than that using the M1 approach, and the discrimination degree is significantly improved by about 130.43% compared to the M1 method. In this way, the differences of alternatives in MCGDM problems with LSVNNs can be robustly distinguished by employing the proposed approach.

Furthermore, comparisons are conducted between the existing LSVNN MCGDM methods and the proposed approach. The comparison results are summarized in Table 13. The results reflect that the existing methods aggregate LSVNNs based on the indices of linguistic variables of them. In this way, various virtual linguistic terms will be output and they may fail to be mapped to any original linguistic terms, reducing the readability. By contrast, the proposed approach considers the PISs of DMs and employs an NS-based 2-tuple linguistic model to address LSVNNs. The proposed approach can effectively avoid generating virtual linguistic terms. Meanwhile, the group consensus considered in the proposed approach can yield a final solution that is highly accepted by the group. In summary, the suggested method is better compared to other approaches.

Based on the discussion in the illustrative example, sensitivity and comparative analysis, the prominent features of the developed framework are summarized as follows:

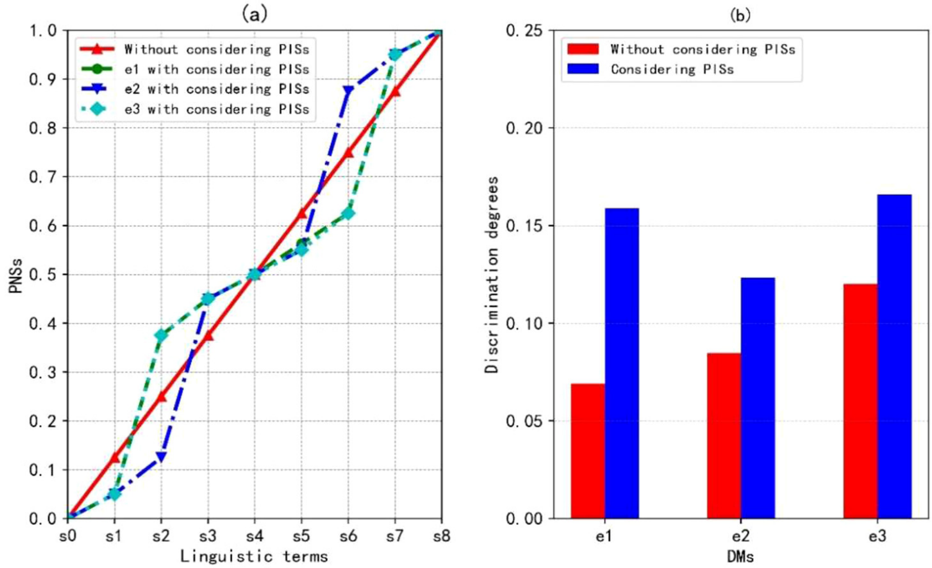

Fig. 6. Comparisons between M1 and the proposed approach.

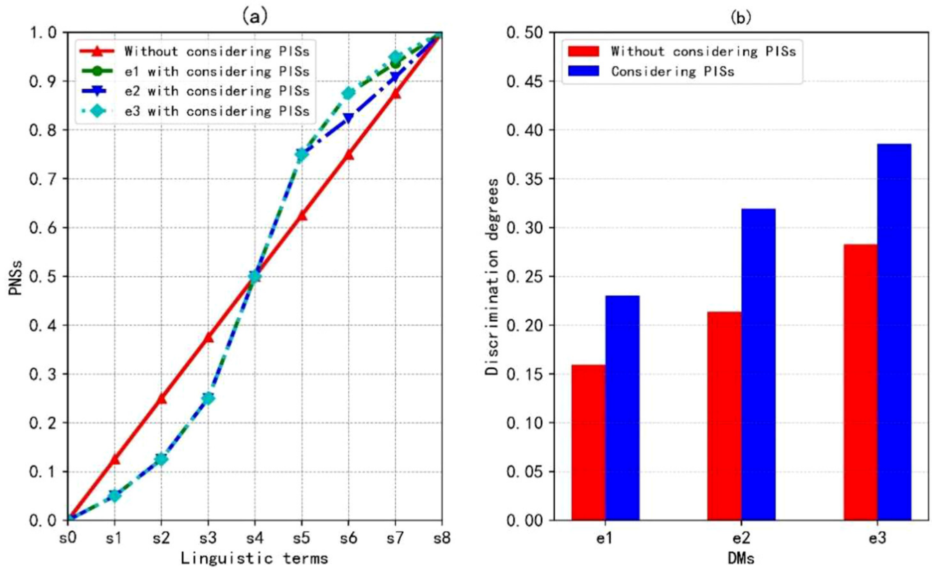

Fig. 7. Comparisons between M2 and the proposed approach.

(1) An effective solution for addressing PISs. The proposed PIS model can provide an effective solution to assign PNSs of linguistic terms for DMs, characterizing their personalized semantic preferences regarding linguistic MCGDM with LSVNNs.
(2) A cautious method to assign the weights of DMs considering group consensus. The developed consensus-driven optimization model is utilized to identify the weights of

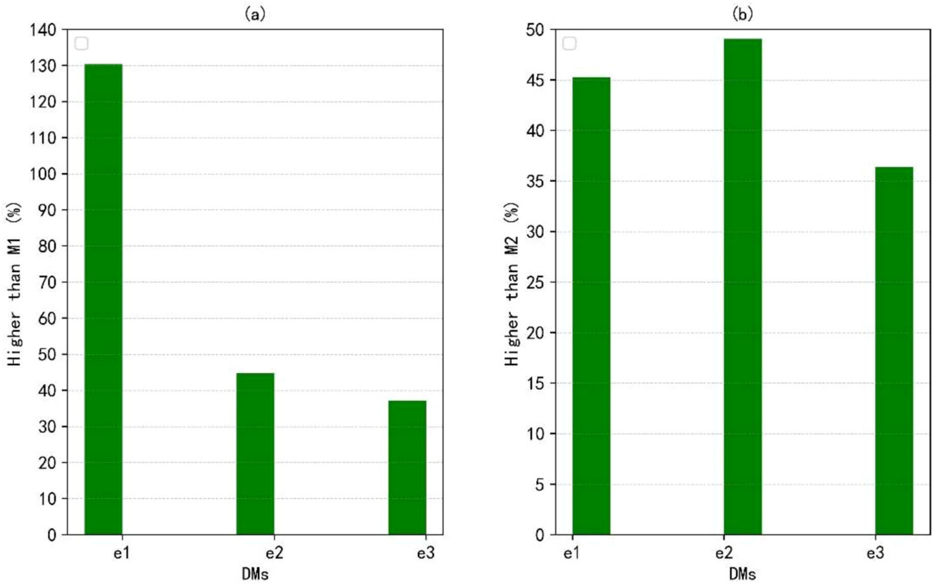

Fig. 8. Discrimination degrees derived from considering and without considering PISs.

DMs, guaranteeing a high level of agreement among members in terms of the final solution.

(3) A robust method to determine the differences among alternatives. The proposed approach can not only consider the PISs of DMs, but also provide a robust method to reveal the differences among alternatives with the optimal discrimination degrees.

However, although the proposed approach equips outstanding characteristics in dealing with linguistic MCGDM problems with LSVNNs, DMs may have to derive the PNSs and weights of DMs by resolving some mathematic programming models. Compared to the existing methods without considering PISs, the proposed approach is intricate and time-consuming.

6. Conclusions

LSVNNs are valuable for describing qualitative ratings involving uncertain, incomplete, and inconsistent information. When eliciting linguistic evaluations, words may be assigned different meanings for various people, that is, DMs have PISs with regard to linguistic terms. Considering PISs of DMs can lead to a realistic and effective methodology for addressing linguistic neutrosophic MCGDM problems. This study firstly develops a discrimination-based optimization model to assign PNSs of linguistic terms on LTS for DMs, and effectively describe their personalized semantic preferences regarding linguistic MCGDM with LSVNNs. Then, an optimization model on the basis of group consensus is constructed to identify the weights of DMs, which guarantees a high level of agreement among members in terms of the final solution. Subsequently, an LSVNN WAA aggregation operator and PNSs-based score and accuracy functions are utilized to determine the ranking of alternatives. Finally, by comparing with existing methods, the results demonstrate that the proposed approach which developed PIS can effectively derive PNSs of linguistic terms on LTS for DMs and lead to higher discrimination degrees than those without considering PISs.

In the future study, it would be an interesting topic to investigate the PIS-based approach for addressing incomplete MCGDM problems with LSVNNs. Moreover, complex MCGDM involving large-scale members and considering their social relationships has attracted much attention (Liao *et al.*, 2021). It would be an interesting extension of the proposed framework for tackling social network large-scale MCGDM problems.

Appendix

The notation used in this study is summarized in Table 14.

Table 14
Notation in this study.

Indicators	Meanings
$S = \{s_0, s_1, \dots, s_\tau\}$	Set of linguistic terms
$NS(s_\theta)$	Numerical index of linguistic term s_θ
$A = \{\langle x, s_A^T(x), s_A^I(x), s_A^F(x) \mid x \in X\rangle\}$	LSVNS with linguistic truth degree $s_A^T(x)$, indeterminacy degree $s_A^I(x)$ and falsity degree $s_A^F(x)$
$r_j = \langle s_{r_j}^T, s_{r_j}^I, s_{r_j}^F\rangle$ $(j = 1, 2, \dots, n)$	Collection of LSVNNs
$LNWA(r_1, r_2, \dots, r_n)$	LSVNN WAA operator
$S(r)$	Score function of LSVNN r
$H(r)$	Accuracy function of LSVNN r
$A = \{a_1, a_2, \dots, a_m\}$	Set of alternatives
$C = \{c_1, c_2, \dots, c_n\}$	Set of criteria
w_j	Weight of criterion c_j
$E = \{e_1, e_2, \dots, e_q\}$	Set of experts
λ_h	Weight of expert e_h
$B^h = [b_{ij}^h]_{m\times n}$	Original decision matrix of expert e_h
$b_{ij}^h = \langle s_{b_{ij}^h}^T, s_{b_{ij}^h}^I, s_{b_{ij}^h}^F\rangle$	LSVNN evaluation given by expert e_h
$R^h = [r_{ij}^h]_{m\times n}$	Standardized decision matrix of expert e_h
$d(r_1, r_2)$	Distance measure between LSVNNs r_1 and r_2
ρ	Parameter of distance measure $d(r_1, r_2)$
$Dis(e_h)$	Discrimination measure of expert e_h
$APS^h = \{(NS(s_0), NS(s_1), \dots, NS(s_\tau)); \dots\}$	Set of possible PNSs of linguistic terms for expert e_h
GCL	Group consensus measure

Acknowledgements

The authors are very grateful to the editors and the anonymous referees for their valuable comments and suggestions.

Funding

This work was supported by the National Natural Science Foundation of China (No. 72001204), the China Postdoctoral Science Foundation (No. 2020M681782), the Postgraduate Research & Practice Innovation Program of Jiangsu Province, and the Graduate Innovation Program of China University of Mining and Technology (No. 2023WLJCR-CZL134).

References

Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.

Broumi, S., Sundareswaran, R., Shanmugapriya, M., Nordo, G., Nordo, G., Bakali, A., Smarandache, F. (2022). Interval-valued fermatean neutrosophic graphs. *Decision Making: Applications in Management and Engineering*, 5(2), 176–200.

Broumi, S., Mohanaselvi, S., Witczak, T., Talea, M., Bakali, A., Smarandache, F. (2023). Complex fermatean neutrosophic graph and application to decision making. *Decision Making: Applications in Management and Engineering*, 6(1), 474–501. https://doi.org/10.31181/dmame24022023b.

Chen, Z., Zhang, X., Pedrycz, W., Wang, X., Skibniewski, M.J. (2020). Bid evaluation in civil construction under uncertainty: a two-stage LSP-ELECTRE III-based approach. *Engineering Applications of Artificial Intelligence*, 94, 103835.

Dong, Y., Xu, Y., Yu, S. (2009). Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model. *IEEE Transactions on Fuzzy Systems*, 17(6), 1366–1378.

Fang, Z., Ye, J. (2017). Multiple attribute group decision-making method based on linguistic neutrosophic numbers. *Symmetry*, 9(7), 111.

Garg, H., Nancy (2018). Linguistic single-valued neutrosophic prioritized aggregation operators and their applications to multiple-attribute group decision-making. *Journal of Ambient Intelligence and Humanized Computing*, 9(6), 1975–1997.

Garg, H., Kumar, K. (2019). Linguistic interval-valued Atanassov intuitionistic fuzzy sets and their applications to group decision-making problems. *IEEE Transactions on Fuzzy Systems*, 27(2), 2302–2311.

Gokasar, I., Deveci, M., Kalan, O. (2022). CO_2 emission based prioritization of bridge maintenance projects using neutrosophic fuzzy sets based decision making approach. *Research in Transportation Economics*, 91, 101029.

Herowati, E., Ciptomulyono, U., Parung, J., Suparno (2017). Expertise-based ranking of experts: an assessment level approach. *Fuzzy Sets and Systems*, 315, 44–56.

Herrera, F., Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 8(6), 746–752.

Hezam, I.M., Mishra, A.R., Krishankumar, R., Ravichandran, K.S., Kar, S., Pamucar, D.S. (2023). A single-valued neutrosophic decision framework for the assessment of sustainable transport investment projects based on discrimination measure. *Management Decision*, 61(2), 443–471.

Ji, P., Zhang, H.Y., Wang, J.Q. (2018). A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. *Neural Computing and Applications*, 29(1), 221–234.

Kabak, Ö., Ervural, B. (2017). Multiple attribute group decision making: a generic conceptual framework and a classification scheme. *Knowledge-Based Systems*, 123, 13–30.

Kalantari, S., Kazemipoor, H., Sobhani, F.M., Molana, S.M.H. (2022). A neutrosophical model for optimal sustainable closed-loop supply chain network with considering inflation and carbon emission policies. *Decision Making: Applications in Management and Engineering*, 5(2), 46–77.

Karabasevic, D., Stanujkic, D., Zavadskas, E.K., Stanimirovic, P., Ulutas, A. (2020). A novel extension of the TOPSIS method adapted for the use of single-valued neutrosophic sets and hamming distance for e-commerce development strategies selection. *Symmetry*, 12(8), 1263.

Kumar, A., Gandhi, C.P., Zhou, Y., Tang, H., Xiang, J. (2020). Fault diagnosis of rolling element bearing based on symmetric cross entropy of neutrosophic sets. *Measurement*, 152, 107318.

Li, C., Dong, Y., Herrera, F., Herrera-Viedma, E., Martínez, L. (2017). Personalized individual semantics in computing with words for supporting linguistic group decision making. An application on consensus reaching. *Information Fusion*, 33, 29–40.

Li, C., Dong, Y., Chiclana, F., Herrera-Viedma, E. (2022a). Consistency-driven methodology to manage incomplete linguistic preference relation: a perspective based on personalized individual semantics. *IEEE Transactions on Cybernetics*, 52(7), 6170–6180.

Li, C., Dong, Y., Liang, H., Pedrycz, W., Herrera, F. (2022b). Data-driven method to learning personalized individual semantics to support linguistic multi-attribute decision making. *Omega*, 111, 102642.

Li, Y., Zhang, H.Y., Wang, J. (2017). Linguistic neutrosophic sets and its application to multi-criteria decision-making problems. *International Journal for Uncertainty Quantification*, 7(2), 135–154.

Li, Y., Wang, J., Wang, T. (2019). A linguistic neutrosophic multi-criteria group decision-making approach with EDAS method. *Arabian Journal for Science and Engineering*, 44(3), 2737–2749.

Liang, R., Jiang, Z., Wang, J. (2018a). A linguistic neutrosophic multi-criteria group decision-making method to university human resource management. *Symmetry*, 10(9), 364.

Liang, R., Wang, J., Li, L. (2018b). Multi-criteria group decision-making method based on interdependent inputs of single-valued trapezoidal neutrosophic information. *Neural Computing and Applications*, 30(1), 241–260.

Liao, H., Li, X., Tang, M. (2021). How to process local and global consensus? A large-scale group decision making model based on social network analysis with probabilistic linguistic information. *Information Sciences*, 579, 368–387.

Liu, P., Liu, X. (2018). The neutrosophic number generalized weighted power averaging operator and its application in multiple attribute group decision making. *International Journal of Machine Learning & Cybernetics*, 9(2), 347–358.

Liu, P., Cheng, S. (2019). An extension of ARAS methodology for multi-criteria group decision-making problems within probability multi-valued neutrosophic sets. *International Journal of Fuzzy Systems*, 21(8), 2472–2489.

Liu, P., Khan, Q., Mahmood, T. (2022). Application of interval neutrosophic power hamy mean operators in MAGDM. *Informatica*, 30(2), 293–325.

Mendel, J., Wu, D. (2010). *Perceptual Computing: Aiding People in Making Subjective Judgments*. Wiley and Sons.

Mendel, J.M., Zadeh, L.A., Trilas, E., Yager, R., Lawry, J., Hagras, H., Guadarrama, S. (2010). What computing with words means to me. *IEEE Computational Intelligence Magazine*, 5(1), 20–26.

Morente-Molinera, J.A., Pérez, I.J., Ureña, M.R., Herrera-Viedma, E. (2015). On multi-granular fuzzy linguistic modeling in group decision making problems: a systematic review and future trends. *Knowledge-Based Systems*, 74, 49–60.

Nguyen, G.N., Ashour, A.S., Dey, N. (2019). A survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses. *International Journal of Machine Learning and Cybernetics*, 10(1), 1–13.

Ohlan, A. (2022). Novel entropy and distance measures for interval-valued intuitionistic fuzzy sets with application in multi-criteria group decision-making. *International Journal of General Systems*, 51(4), 413–440.

Paulraj, S., Tamilarasi, G. (2022). Generalized ordered weighted harmonic averaging operator with trapezoidal neutrosophic numbers for solving MADM problems. *Journal of Ambient Intelligence and Humanized Computing*, 13(8), 4089–4102.

Peng, H., Zhang, H., Wang, J. (2018). Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems. *Neural Computing and Applications*, 30(2), 563–583.

Peng, X., Dai, J. (2018). Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. *Neural Computing and Applications*, 29(10), 939–954.

Peng, X., Dai, J. (2020). A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017. *Artificial Intelligence Review*, 53(1), 199–255.

Refaat, R., El-Henawy, I.M. (2019). Innovative method to evaluate quality management system audit results' using single value neutrosophic number. *Cognitive Systems Research*, 57, 197–206.

Sarma, D., Das, A., Bera, U.K., Hezam, I.M. (2019). Redistribution for cost minimization in disaster management under uncertainty with trapezoidal neutrosophic number. *Computers in Industry*, 109, 226–238.

Seikh, M.R., Mandal, U. (2022). Multiple attribute group decision making based on quasirung orthopair fuzzy sets: application to electric vehicle charging station site selection problem. *Engineering Applications of Artificial Intelligence*, 115, 105299.

Smarandache, F. (1999). *A Unifying Field in Logics: Neutrsophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability*. American Research Press, Rehoboth.

Sun, R., Hu, J., Chen, X. (2019). Novel single-valued neutrosophic decision-making approaches based on prospect theory and their applications in physician selection. *Soft Computing*, 23(1), 211–225.

Tang, X., Peng, Z., Zhang, Q., Pedrycz, W., Yang, S. (2020). Consistency and consensus-driven models to personalize individual semantics of linguistic terms for supporting group decision making with distribution linguistic preference relations. *Knowledge-Based Systems*, 185, 105078.

Tian, Z.P., Nie, R.X., Wang, X.K., Wang, J.Q. (2020). Single-valued neutrosophic ELECTRE II for multi-criteria group decision-making with unknown weight information. *Computational & Applied Mathematics*, 39, 224.

Wan, S., Wang, F., Dong, J. (2017). Additive consistent interval-valued Atanassov intuitionistic fuzzy preference relation and likelihood comparison algorithm based group decision making. *European Journal of Operational Research*, 263(2), 571–582.

Tian, Z.P., Nie, R.X., Wang, J.Q. (2019). Social network analysis-based consensus-supporting framework for large-scale group decision-making with incomplete interval type-2 fuzzy information. *Information Sciences*, 502, 446–471.

Wan, S., Xu, G., Dong, J. (2020). An Atanassov intuitionistic fuzzy programming method for group decision making with interval-valued Atanassov intuitionistic fuzzy preference relations. *Applied Soft Computing*, 95, 106556.

Wang, H.B., Smarandache, F., Zhang, Y.Q., Sunderraman, R. (2005). *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*. Hexis, Phoenix, Ariz, USA.

Wang, H.B., Smarandache, F., Zhang, Y.Q., Sunderraman, R. (2010). Single valued neutrosophic sets. *The Scientific Informative Review*, 16(1), 10–14.

Wang, X., Xu, Z., Su, S., Zhou, W. (2021). A comprehensive bibliometric analysis of uncertain group decision making from 1980 to 2019. *Information Sciences*, 547, 328–353.

Wang, Y.M. (1997). Using the method of maximizing deviation to make decision for multiindices. *Journal of Systems Engineering & Electronics*, 8(3), 21–26.

Xu, Z., Zhao, N. (2016). Information fusion for intuitionistic fuzzy decision making: an overview. *Information Fusion*, 28, 10–23.

Ye, J. (2013). Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *International Journal of General Systems*, 42(4), 386–394.

Ye, J., Song, J., Du, S. (2022). Correlation coefficients of consistency neutrosophic sets regarding neutrosophic multi-valued sets and their multi-attribute decision-making method. *International Journal of Fuzzy Systems*, 24(2), 925–932.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.

Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, 8(3), 199–249.

Zhang, H., Li, C., Liu, Y., Dong, Y. (2021). Modelling personalized individual semantics and consensus in comparative linguistic expression preference relations with self-confidence: an optimization-based approach. *IEEE Transactions on Fuzzy Systems*, 29(3), 627–640.

Zhang, K., Xie, Y., Noorkhah, S.A., Imeni, M., Das, S.K. (2023). Neutrosophic management evaluation of insurance companies by a hybrid TODIM-BSC method: a case study in private insurance companies. *Management Decision*, 61(2), 363–381.

Zhang, Z., Li, Z. (2022). Personalized individual semantics-based consistency control and consensus reaching in linguistic group decision making. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 52(9), 5623–5635.

Zhao, M., Wei, G., Wei, C., Wu, J., Guo, Y. (2021). Extended TODIM based on cumulative prospect theory for picture fuzzy multiple attribute group decision making. *Informatica*, 32(4), 865–886.

Z.-P. Tian received the PhD degree in management science and engineering from the Central South University, Changsha, China, in 2019. He is currently an associate professor with the School of Economics and Management, China University of Mining and Technology. His current research interests include social network, decision-making theory and application, risk management and control, and information management.

F.-X. Xu studied at Shandong University of Finance and Economics from 2018 to 2022 and obtained Bachelor of Management in 2022. At present, he is pursuing a master's degree in management science and engineering at China University of Mining and Technology. His current research interests include conflict analysis and group decision.

R.-X. Nie received the PhD degree in management science and engineering from the Central South University, Changsha, China, in 2022. She is currently a lecturer with the School of Economics and Management, China University of Mining and Technology. Her current research interests include customer preference analysis, decision-making theory and application, and quality and innovation management.

X.-K. Wang received the PhD degree in management science and engineering from the Central South University, Changsha, China, in 2022. He is currently a lecturer with the School of Management, Shenzhen University. His current research interests include decision-making theory and application, social commerce, and machine learning.

J.-Q. Wang received the PhD degree in management science and engineering from Central South University, Changsha, China, in 2005. He is currently a professor at the Business School, Central South University. His current research interests include decision-making theory and application, risk management and control, and information management.