On the Split-Complex Neutrosophic Numbers and Their Algebraic Properties

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On the Split-Complex Neutrosophic Numbers and Their Algebraic Properties

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Abstract

The objective of this paper is to define for the first time the concept of split-complex numbers as a new generalization of classical split-complex numbers by using neutrosophic numbers. Also, we study the elementary properties of this new numerical class such as equations, conjugates, and vector spaces formed by it. On the other hand, some special AH-subspaces will be presented and handled with many corresponding examples.

Keywords: split-complex number; neutrosophic number; neutrosophic split-complex number

1. Introduction

The split complex numbers are defined as a similar algebraic structure to the complex numbers. The set of split-complex numbers is defined as follows: $S(J) = \{a + bJ; a, b \in R, J^2 = 1\}$. [1-2]

Split-complex numbers S(J) have a commutative ring structure, and they have many applications in physics and relativity [3].

Neutrosophy is a new kind of generalized logic presented by Smarandache [11] to deal with indeterminacy in all fields of science and real-life problems. As a result of Smarandache's work neutrosophic numbers were defined and studied widely by many researchers [4-10], were we find neutrosophic algebraic structures such as modules, spaces, matrices, and rings. The ring of neutrosophic real numbers is defined as follows: $R(I) = \{a + bI; a, b \in R, I^2 = I\}$. The element I is called the indeterminacy with a logical property $I^2 = I$. Neutrosophic numbers are considered as a generalization of classical real numbers with many deep applications in many scientific fields, see [12-17].

The previous works have motivated us to combine split-complex numbers with neutrosophic numbers to get a novel generalization of real numbers, where we define the concept of neutrosophic split-complex numbers, and we present many elementary properties of these numbers in terms of theorems. On the other hand, we clarify the validity of this work by illustrating some examples to explain the algebraic properties of these numbers.

2. Main discussion

Definition: Let $x = x_1 + x_2 I$, $y = y_1 + y_2 I$ be two neutrosophic real numbers, where $I^2 = I$.
We call $X = x + yJ$; $J^2 = 1$ a split-complex neutrosophic number.
Remark:
1). The split-complex neutrosophic number X can be written as follows:
$X = (x_1 + x_2 I) + (y_1 + y_2 I)J = x_1 + x_2 I + y_1 J + y_2 IJ$; $I^2 = I, J^2 = 1, IJ = JI$.
2). $(IJ)^2 = I^2 J^2 = I^2 .1 = J^2$.
3). We denote the set of all neutrosophic split-complex numbers by $NS = \{x_1 + x_2 I + y_1 J + y_2 IJ;\ x_i, y_i \in R\}$
Definition: the addition operation on NS is defined as follows:
$(+)$: $NS \times NS \rightarrow NS$ such that:
$(x_1 + x_2 I + y_1 J + y_2 IJ) + (z_1 + z_2 I + t_1 J + t_2 IJ) = (x_1 + z_1) + (x_2 + z_2)I + (y_1 + t_1)J + (y_2 + t_2)IJ$

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The multiplication operation on NS is defined as follows:
$(.): NS \times NS \to NS$ such that:
$[(x_1 + x_2I + y_1J + y_2IJ).(z_1 + z_2I + t_1J + t_2IJ)]$
$= x_1z_1 + x_1z_2I + x_1t_1J + x_1t_2IJ + x_2z_1I + x_2z_2I^2 + x_2t_1IJ + x_2t_2I^2J + y_1z_1J + y_1z_2JI + y_1t_1J^2 + y_1t_2IJ^2 + y_2z_1IJ + y_2z_2I^2J + y_2t_1IJ^2 + y_2t_2(IJ)^2$
Since $I^2 = I, J^2 = 1$, we get:
$= (x_1z_1 + y_1t_1) + (x_1z_2 + x_2z_1 + x_2z_2 + y_2t_2 + y_1t_2 + y_2t_1)I + (x_1t_1 + y_1z_1)J + (x_1t_2 + x_2t_1 + x_2t_2 + y_1z_2 + y_2z_1 + y_2z_2)IJ$
Example : consider the following two split-complex neutrosophic numbers:

$$X = 1 + I + 2J - IJ\ , Y = 2 - I + J + 3IJ$$

We have:

$$X + Y = 3 + 3J + 2IJ\ , X.Y = 2 - I + J + 3IJ + 2I - I + IJ + 3IJ + 4J - 2IJ + 2 + 6I - 2IJ + IJ - I - 3I$$

$$X.Y = 4 + 2I + 5J + 4IJ$$

Definition: Let $X = (x_0 + x_1I) + (x_2 + x_3I)J$be a neutrosophic split-complex number, we define its conjugate as follows:

$$\bar{X} = (x_0 + x_1I) - (x_2 + x_3I)J = x_0 + x_1I - x_2J - x_3IJ$$

Definition: Let $X = (x_0 + x_1I) + (x_2 + x_3I)J$ be a neutrosophic split-complex number, we define the s-norm as follows:

$\|X\|^2 = X.\bar{X}$

$\|X\| = \sqrt{|X.\bar{X}|}$ is called the norm of X.

Theorem:

Let $X = (x_0 + x_1I) + (x_2 + x_3I)J$ be a neutrosophic split-complex number, then

$$\|X\|^2 = (x_0 + x_1I)^2 - (x_2 + x_3I)^2$$

Proof.

$\|X\|^2 = [(x_0 + x_1I) + (x_2 + x_3I)J].[(x_0 + x_1I) + (x_2 + x_3I)J]$

$= (x_0 + x_1I)^2 - (x_2 + x_3I)^2J^2 = (x_0 + x_1I)^2 - (x_2 + x_3I)^2$

Remarks:

1). The s-norm of a neutrosophic split-complex number is a neutrosophic real number, i.e. $\|X\| \in R(I)$.

2). According to [ ], the absolute value of a real neutrosophic number $a + bI$ can be computed as follows:

$|a + bI| = |a| + (|a + b| - |a|)I \geq 0.$

Also, if $a + bI \geq 0$, i.e. $a \geq 0, a + b \geq 0$, then $\sqrt{a + bI} = \sqrt{a} + \left(\sqrt{a + b} - \sqrt{a}\right)I$

The formula of norms:

Consider $X = (x_0 + x_1I) + (x_2 + x_3I)J \in NS$, then:

$\|X\|^2 = (x_0 + x_1I)^2 - (x_2 + x_3I)^2 = ({x_0}^2 - {x_2}^2) + I({x_1}^2 + 2x_0x_1 - {x_3}^2 - 2x_2x_3) = a + bI$, where

$a = ({x_0}^2 - {x_2}^2)\ , b = ({x_1}^2 + 2x_0x_1 - {x_3}^2 - 2x_2x_3)$

First of all, we have: $|a + bI| = |{x_0}^2 - {x_2}^2| + I[|(x_0 + x_1)^2 - (x_2 + x_3)^2| - |{x_0}^2 - {x_2}^2|]$

This means that:

$\|X\| = \sqrt{|X.\bar{X}|} = \sqrt{|a + bI|} = \sqrt{|a| + (|a + b| - |a|)I} = \sqrt{|a|} + \left(\sqrt{|a + b|} - \sqrt{|a|}\right)I$

$= \sqrt{|{x_0}^2 - {x_2}^2|} + \left(\sqrt{|(x_0 + x_1)^2 - (x_2 + x_3)^2|} - \sqrt{|{x_0}^2 - {x_2}^2|}\right)I$

Example:

Take $X = (1 + 2I) + (2 - I)J; x_0 = 1, x_1 = 2, x_2 = 2, x_3 = -1$

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$\bar{X} = (1+2I) - (2-I)J$, $\|X\|^2 = (1+2I)^2 - (2-I)^2 = 1+8I-(4-4I+I) = 1+8I-4+3I = -3+11I$

$\|X\| = \sqrt{|-3+11I|} = \sqrt{|-3| + I(|8| - |-3|)} = \sqrt{3+5I} = \sqrt{3} + (\sqrt{8}-\sqrt{3})I$

On other hand, we have:

$\sqrt{|x_0{}^2 - x_2{}^2|} = \sqrt{3}\,, \sqrt{|(x_0+x_1)^2-(x_2+x_3)^2|} = \sqrt{9-1} = \sqrt{8}$

Thus, by using the previous, we get:

$\|X\| = \sqrt{3} + (\sqrt{8}-\sqrt{3})I$

Remark:

$(N_S, +, .)$ is a ring that contains $R(I)$.

The condition of invertibility :

Let $X = (x_0+x_1I) + (x_2+x_3I)J$, then $X$is invertible if and only if $x_0+x_2J, (x_0+x_1)+(x_2+x_3)J$ are invertible split-complex numbers.

Proof.

A neutrosophic number $a+bI$ is invertible if and only if $a, a+b$ are invertible [ ].

On the other hand, X can be written as follows:

$X = (x_0+x_2J) + I(x_1+x_3J)$, thus X is invertible if and only if $x_0+x_2J, (x_0+x_1)+(x_2+x_3)J$ are invertible.

The inverse formula:

Let $X = (x_0+x_1I) + (x_2+x_3I)J$ be an invertible neutrosophic split-complex number, then:

$$X^{-1} = \frac{1}{X} = \frac{x_0 - x_2J}{x_0{}^2 - x_2{}^2} + I\left[\frac{x_0+x_2-(x_2+x_3)J}{(x_0+x_2)^2-(x_2+x_3)^2} - \frac{x_0-x_2J}{x_0{}^2-x_2{}^2}\right]$$

Proof.

We write $(x_0+x_2I) + (x_1+x_3I)J$, by using the formula of the inverse of a neutrosophic number, we get:

$$X^{-1} = (x_0+x_2J)^{-1} + I[(x_0+x_2J+x_1+x_3J)^{-1} - (x_0+x_2J)^{-1}]$$
$$= \frac{x_0-x_2J}{x_0{}^2-x_2{}^2} + I\left[\frac{x_0+x_1-(x_2+x_3)J}{(x_0+x_1)^2-(x_2+x_3)^2} - \frac{x_0-x_2J}{x_0{}^2-x_2{}^2}\right]$$

Example:

Take $X = (1+I) + (2+3I)J = 1+2J+I+3IJ$, where:

$x_0 = 1, x_1 = 1, x_2 = 2, x_3 = 3$, we have:

$$X^{-1} = \frac{1-2J}{-3} + I\left[\frac{2-5J}{-21} - \frac{1-2J}{-3}\right] = \frac{-1}{3} + \frac{2}{3}J + I\left[\frac{-2}{21} + \frac{5}{21}J + \frac{-1}{3} - \frac{2}{3}J\right] = \frac{-1}{3} + \frac{2}{3}J + \frac{5}{21}I - \frac{9}{21}IJ$$

Let's compute

$$X.X^{-1} = (1+2J+I+3IJ)\left(\frac{-1}{3} + \frac{2}{3}J + \frac{5}{21}I - \frac{9}{21}IJ\right)$$
$$= \frac{-1}{3} + \frac{2}{3}J + \frac{5}{21}I - \frac{9}{21}IJ - \frac{2}{3}J + \frac{4}{3} + \frac{10}{21}IJ - \frac{18}{21}I - \frac{1}{3}I + \frac{2}{3}IJ + \frac{5}{21}I - \frac{9}{21}IJ - IJ + 2I$$
$$+ \frac{15}{21}IJ - \frac{27}{21}I = 1$$

3. Zero divisors

Let X be a neutrosophic non zero split-complex number, we say that X is a zero divisor if there exists $Y \in NS - \{0\}$ such that $X.Y = 0$.

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Theorem:

Let $X = (x_0 + x_1 I) + (x_2 + x_3 I)J$ be a neutrosophic split-complex number, then X is a zero divisor if and only if one of the following is true:

1). $x_0 = x_2$.

2). $x_0 = -x_2$.

3). $x_0 + x_1 = x_2 + x_3$

4). $x_0 + x_1 = -x_2 - x_3$.

Proof.

$X = (x_0 + x_2 J) + I(x_1 + x_3 J)$, by using the condition of being a zero divisor in a neutrosophic ring, we get that:

$x_0 + x_2 J$ or $(x_0 + x_2 J) + (x_1 + x_3 J)$ is a zero divisor.

If $x_0 + x_2 J$ is a zero divisor then it is not invertible, then ${x_0}^2 - {x_2}^2 = 0$, this implies that $x_0 = x_2$ or $x_0 = -x_2$.

On the other hand, if $(x_0 + x_2 J) + (x_1 + x_3 J)$ is a zero divisor, then $(x_0 + x_1)^2 - (x_2 + x_3)^2 = 0$, thus $x_0 + x_1 = x_2 + x_3$ or $x_0 + x_1 = -x_2 - x_3$.

4. Idempotency

Definition.

A neutrosophic split-complex number X is called idempotent if and only if:

$X^2 = X$.

Theorem:

Let $X = (x_0 + x_2 J) + I(x_1 + x_3 J)$ be a neutrosophic split-complex number, then X is idempotent if and only if:

1). $x_0 - x_2 \in \{0,1\}$.

2). $x_0 + x_2 \in \{0,1\}$.

3). $x_0 + x_1 - (x_2 + x_3) \in \{0,1\}$.

4). $x_0 + x_1 + (x_2 + x_3) \in \{0,1\}$.

Proof.

$X = (x_0 + x_2 J) + I(x_1 + x_3 J)$ is idempotent if and only if $x_0 + x_2 J$, $x_0 + x_1 + (x_2 + x_3)J$ are idempotent split-complex numbers.

Assume that $x_0 + x_2 J$ is idempotent, then:

$(x_0 + x_2 J)^2 = x_0 + x_2 J$, thus: $\begin{cases} {x_0}^2 + {x_2}^2 = x_0 \\ 2x_0 x_2 = x_2 \end{cases}$.

So that $\begin{cases} (x_0 + x_2)^2 = x_0 + x_2 \\ (x_0 - x_2)^2 = x_0 - x_2 \end{cases}$, hence $x_0 - x_2, x_0 + x_2 \in \{0,1\}$.

By the same method, we can see that $x_0 + x_1 + x_2 + x_3, x_0 + x_1 - x_2 - x_3 \in \{0,1\}$.

Example.

$X = \frac{1}{2} + \frac{1}{2}J$, $X^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{2}J = \frac{1}{2} + \frac{1}{2}J = X$, so that X is idempotent.

5. Neutrosophic split-complex quadratic equation

Theorem.

Let $AX^2 + BX + C = 0$ be a quadratic equations such that $A, B, C \in NS$ with

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$A=(a_0+a_1I)+(a_2+a_3I)J\ ,B=(b_0+b_1I)+(b_2+b_3I)J,C=(c_0+c_1I)+(c_2+c_3I)J$

$X=(x_0+x_1J)+(x_2+x_3I)J$, then it is equivalent to the following two split-complex equations:

$$\begin{cases} (a_0+a_2J)(x_0+x_2J)^2+(b_0+b_2J)(x_0+x_2J)+(c_0+c_2J)=0 \\ [(a_0+a_1)+(a_2+a_3)J][(x_0+x_1)+(x_2+x_3)J]^2+[(b_0+b_1)+(b_2+b_3)J][(x_0+x_1)+(x_2+x_3)J] \\ +(c_0+c_1)+(c_2+c_3)=0 \end{cases}$$

Proof.

We write
$A=(a_0+a_2J)+(a_1+a_3J)I\ ,B=(b_0+b_2J)+(b_1+b_3J)I,C=(c_0+c_2I)+(c_1+c_3J)I,$

$X=(x_0+x_2J)+(x_1+x_3J)I$

According to [ ], a neutrosophic quadratic equation:

$$(a_0+a_2J)(x_0+x_2J)^2+(b_0+b_2J)(x_0+x_2J)+(c_0+c_2J)=0$$

Is equivalent to:

$$\begin{cases} a_0{x_0}^2+b_0x_0+c_0=0 \\ (a_0+a_1)(x_0+x_1)^2+(b_0+b_1)(x_0+x_1)+(c_0+c_1)=0 \end{cases}$$

By using this criteria, we get the following two equations:

$$\begin{cases} (a_0+a_2J)(x_0+x_2J)^2+(b_0+b_2J)(x_0+x_2J)+(c_0+c_2J)=0 \\ [(a_0+a_1)+(a_2+a_3)J][(x_0+x_1)+(x_2+x_3)J]^2+[(b_0+b_1)+(b_2+b_3)J][(x_0+x_1)+(x_2+x_3)J] \\ +(c_0+c_1)+(c_2+c_3)=0 \end{cases}$$

Definition.

A square matrix $T\ is$ called a neutrosophic split-complex matrix if it has neutrosophic split-complex entries.

Example.

$T=\begin{pmatrix} 1+I-J & 1+IJ \\ -2I & 1+3IJ \end{pmatrix}$ is a 2×2 neutrosophic split-complex matrix.

Remark.

A neutrosophic split-complex matrix is invertible if and only if its determinant is invertible.

For example: $T=\begin{pmatrix} 1+I-J & 1+IJ \\ -2I & 1+3IJ \end{pmatrix}$.

$detT=(1+I-J)(1+3IJ)-(-2I)(1+IJ)=1+3IJ+I+3IJ-J-3I+2I+2IJ=1-J+8IJ$

6. Neutrosophic split-complex vector space

Definition.

Let V be a vector space over the real field, we define the corresponding neutrosophic split-complex vector space as follows:

$$V_{NS}=V+VI+VJ+VIJ=\{x+yI+zJ+tIJ;\ x,y,z,t\in V\}$$

Definition:

We define addition operation on V_{NS} is defined as follows:
$(+)$: $V_{NS}\times V_{NS}\to V_{NS}$ such that:
For $X=x_1+y_1I+z_1J+t_1IJ\ ,Y=x_2+y_2I+z_2J+t_2IJ\in V_{NS}$
$X+Y=(x_1+y_1I+z_1J+t_1IJ)+(x_2+y_2I+z_2J+t_2IJ)$
$=(x_1+x_2)+(y_1+y_2)I+(z_1+z_2)J+(t_1+t_2)IJ$
We define multiplication operation on V_{NS} as follows:
$(.)$: $V_{NS}\times V_{NS}\to V_{NS}$ such that:
For $X=x_1+y_1I+z_1J+t_1IJ\in V_{NS}\ ,A=a_1+b_1I+c_1J+d_1IJ\in NS$

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$$A.X = (x_1 + y_1 I + z_1 J + t_1 IJ).(a_1 + b_1 I + c_1 J + d_1 IJ)$$
$$= a_1x_1 + a_1y_1I + a_1z_1J + a_1t_1IJ + b_1x_1I + b_1y_1I + b_1z_1IJ + b_1t_1IJ + c_1x_1J + c_1y_1JI + c_1z_1 + c_1t_1I + d_1x_1IJ + d_1y_1IJ + d_1z_1I + d_1t_1I$$
$$= (a_1x_1 + c_1z_1) + (a_1y_1 + b_1x_1 + b_1y_1 + c_1t_1 + d_1z_1 + d_1t_1)I + (a_1z_1 + c_1x_1)J + (a_1t_1 + b_1z_1 + b_1t_1 + d_1x_1 + d_1y_1)IJ$$

Theorem.

$(V_{NS}, +, .)$Is a module over ring NS.

Proof.

It is that $(V_{NS}, +, .)$Is an abelian group.

On other hand, for $A, B \in NS$, $X, Y \in V_{NS}$, we have:

$1.X = X, (A + B).X = AX + BX, (AB)X = A.(B.X), A(X + Y) = AX + AY.$

Example.

Consider the space $V = R^2 = \{(x, y); x, y \in R\}$ over R.

The corresponding neutrosophic split-complex vector space is:

$V_{NS} = \{(x_0, y_0) + (x_1, y_1)I + (x_2, y_2)J + (x_3, y_3)IJ; x_i, y_j \in R\}$

For example $X = (1,1) + (1,2)I + (0,1)J + (1,0)IJ = (1 + I + IJ, 1 + 2I + J) \in V_{NS}$

$A = 1 - J \in NS$, thus:

$A.X = 1.(1,1) + (1,2)I + (0,1)J + (1,0)IJ - (1,2)IJ - (0,1) - (1,0)I$

$= (1,0) + I(0,2) + J(-1,0) + IJ(0,-2) = (1 - J, 2I - 2IJ) \in V_{NS}$

Definition.

Let w_1, w_2, w_3, w_4 be subspace of V, then we define:

$w_{NS} = w_1 + w_2I + w_3J + w_4IJ = \{x + yI + zJ + tIJ; x \in w_1, y \in w_2, z \in w_3, t \in w_4\}$

w_{NS} is called an AH-subspace.

If $w_1 = w_2 = w_3 = w_4$, then w_{NS} is called AHS-subspace.

Theorem.

Let W_{NS} be an AHS-subspace of V_{NS}, then W_{NS} is a submodule of V_{NS}.

Proof.

It is clear that $(W_{NS}, +)$is a subgroup of $(V_{NS}, +)$.

$\forall A = a + bI + cJ + dIJ \in NS, X = x + yI + zJ + tIJ \in W_{NS}$, we have:

$A.X = ax + ayI + azJ + atIJ + bxI + byI + bzIJ + btIJ + cxJ + cyIJ + cz + ctI + dxIJ + dyIJ + dzI + dtI$

$= (ax + cz) + (ay + bx + by + ct + dz + dt)I + (az + cx)J + (at + bz + bt + cy + dx + dy)IJ \in W_{NS}$

That is because:

$$\begin{cases} ax + cz \in w_1, ay + +bx + by + ct + dz + dt \in w_2 \\ az + cx \in w_3, at + bz + bt + cy + dx + dy \in w_4 \end{cases}$$

Example.

Let $V = R^2$, $w = \langle(0,1)\rangle = \{(0, x); x \in R\}$ is a subspace of V.

Let V_{NS}be the corresponding neutrosophic split-complex vector space.

$W_{NS} = w + wI + wJ + wIJ = \{(0, x) + (0, y)I + (0, z)J + (0, t)IJ\} = \{(0, x + yI + zJ + tIJ); x, y, z, t \in R\}$.

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W_{NS} is an AHS-subspace of V_{NS}, and it is a sub module in the ordinary algebraic meaning.

Definition.

Let V_{NS} be a neutrosophic split-complex vector space.

Let f_1, f_2, f_3, f_4 be linear transformations, where:

$f_1, f_2, f_3, f_4 : V \to T$, T is a vector space over R.

We define the AH-linear transformation as follows:

$f: V_{NS} \to T_{NS}$ such that:

$f(x + yI + zJ + tIJ) = f_1(x) + f_2(y)I + f_3(z)J + f_4(t)IJ$

If $f_1 = f_2 = f_3 = f_4$, we call f an AHS-linear transformation.

Example.

Let $V_{NS} = {R_{NS}}^2, T_{NS} = {R_{NS}}^2$, we have:

$f_1: V \to T$ such that $f_1(x, y) = (x, y, y)$.

$f_2: V \to T$ such that $f_2(x, y) = (y, 2x, 2y)$.

$f_3: V \to T$ such that $f_3(x, y) = (x + y, x - y, -3x)$.

$f_4: V \to T$ such that $f_4(x, y) = (2x + y, 3x - y, 4x + 5y)$.

Are four classical linear transformations. The corresponding AH-linear transformation is:

$f: V_{NS} \to T_{NS}$ such that:

$f\big((x_0, y_0) + (x_1, y_1)I + (x_2, y_2)J + (x_3, y_3)IJ\big) = f_1(x_0, y_0) + f_2(x_1, y_1)I + f_3(x_2, y_2)J + f_4(x_3, y_3)IJ$

$= (x_0, y_0, y_0) + (y_1, 2x_1, 2y_1)I + (x_2 + y_2, x_2 - y_2, -3x_2)J + (2x_3 + y_3, 3x_3 - y_3, 4x_3 + 5y_3)IJ$

The function $g: V_{NS} \to T_{NS}$ such that:

$g\big((x_0, y_0) + (x_1, y_1)I + (x_2, y_2)J + (x_3, y_3)IJ\big) = f_1(x_0, y_0) + f_1(x_1, y_1)I + f_1(x_2, y_2)J + f_1(x_3, y_3)IJ$

$= (x_0, y_0, y_0) + (x_1, y_1, y_1)I + (x_2, y_2, y_2)J + (x_3, y_3, y_3)IJ$

Theorem.

Let V, T be two vector space over R, $f_1: V \to T$is a linear transformation.

LetV_{NS}, T_{NS} be the corresponding neutrosophic split-complex vector spaces. $f: V_{NS} \to T_{NS}$ be the corresponding AHS-linear transformation, then $f$is a module homomorphism.

Proof.

Let $X = x_0 + x_1 I + x_2 J + x_3 IJ, Y = y_0 + y_1 I + y_2 J + y_3 IJ \in V_{NS}$, and $A = a + bI + cJ + dIJ \in NS$.

$f(X + Y) = f_1(x_0, y_0) + f_2(x_1, y_1)I + f_3(x_2, y_2)J + f_4(x_3, y_3)IJ$

$= [f_1(x_0) + f_2(x_1)I + f_3(x_2)J + f_4(x_3)IJ] + [f_1(y_0) + f_2(y_1)I + f_3(y_2)J + f_4(y_3)IJ] = f(X) + f(Y)$

$$A.X = ax_0 + ax_1 I + ax_2 J + ax_3 IJ + bx_0 I + bx_1 I + bx_2 IJ + bx_3 IJ + cx_0 J + cx_1 IJ + cx_2 + cx_3 I + dx_0 IJ + dx_1 IJ + dx_2 I + dx_3 I$$

$$= (ax_0 + cx_2) + (ax_1 + bx_0 + bx_1 + cx_3 + dx_0 + dx_1)I + (ax_2 + cx_0)J + (ax_3 + bx_2 + bx_3 + cx_1 + dx_2 + dx_3)IJ \in W_{NS}$$

$$f(A.X) = f_1(ax_0 + cx_2) + If_1(ax_1 + bx_0 + bx_1 + cx_3 + dx_0 + dx_1) + Jf_1(ax_2 + cx_0) + IJf_1(ax_3 + bx_2 + bx_3 + cx_1 + dx_2 + dx_3)$$

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$$= af_1(x_0) + cf_1(x_2) + I[af_1(x_1) + bf_1(x_0) + bf_1(x_1) + cf_1(x_3) + df_1(x_2) + df_1(x_1)] + J[af_1(x_2) + cf_1(x_0)] + IJ[af_1(x_3) + bf_1(x_2) + bf_1(x_3) + cf_1(x_1) + df_1(x_2) + df_1(x_3)] = A.f(X)$$

So that, the proof is complete.

7. Conclusion

In this paper, we have defined the neutrosophic split-complex numbers as a combination between classical well-known split-complex numbers and neutrosophic numbers. Many related concepts such as neutrosophic split-complex equations, neutrosophic split-complex vector spaces, and neutrosophic split-complex AH-spaces were presented and discussed in terms of theorems. Also, we have illustrated many examples to explain the validity of this work. As a future research direction, we aim to find many applications of neutrosophic split-complex numbers especially in algebraic structures and physics.

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