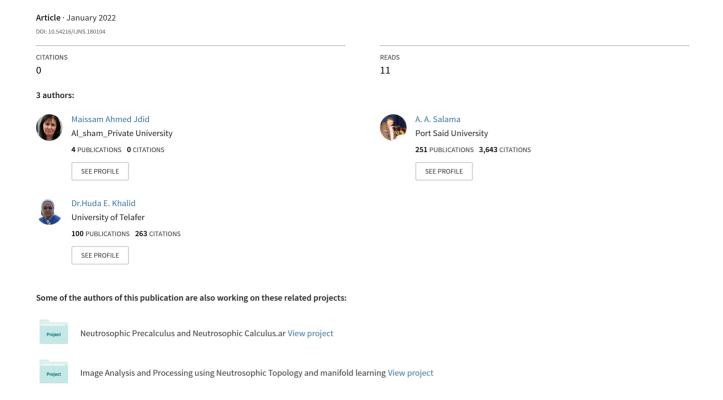# Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming

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Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming

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Abstract

Mathematical programming has the ability to express competency concepts in a well-defined mathematical model for a particular situation or system, and the ability to derive computational methods to solve this mathematical model, it is also a mathematical tool that allows us to model, analyze and solve a wide range of problems concerned with allocating rare resources of labor, materials, machinery, and capitals. Consequently, using them in the best attainable way in order to minimize costs or maximize profits. In such issues, the linear programming is one of the most widely used types of mathematical programming because it is a method that helps to make good decisions and decide the best program for independent activities, taking into account the available sources. It does not take in consideration the continuous and rapid changes and the state of instability in data. So, this manuscript studies one of the methods to solve linear models, which is the simplex method using the neutrosophic theory that covers all the data in analysing, whether specific or not, determined or not, having consistency or not, as well as it deeming all occurring changes. However, the optimal solution is related to the variables in the objective function, which in turn are affected by the fixed quantities that express the available possibilities. This article presents a study to solve the linear model using the simplex method in which the variables and their coefficients are indeterminate values, and we will explain the effected of the indeterminate values on the optimal solution of the mathematical model. The product mixture problem has been presented as case study to demonstrate the efficiency of the proposed method.

Keywords: Simplex Algorithm; Operations Research; Mathematical Programming; Linear Programming; Neutrosophic Logic; Products Mixture Model.

1. Introduction

The search for the best maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost) that is optimal solutions to various problems has been the focus of human attention at all era. The great scientific development witnessed by our contemporary world was the reason for the emergence of the so-called operations research, its originating in military efforts before World War II. Operational research (OR) encompasses the development and the use of a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as simulation, mathematical optimization, queuing theory and other stochastic-process models. It is one of the modern sciences whose applications have achieved wide success in various fields of life and it was a helper for companies of all kinds of problems that deal with conditions that are characterized by continuous and very rapid change, which imposes on it a set of challenges and a continuous search for guarantees of survival and continuity through the continuous quest for the best and the increasing need to improve its performance levels and the adoption of various scientific methods that allow achieving different performance standards. The characteristic of this science is the development of mathematical models, tools and techniques that allow modelling, analysing, and solving the issues posed by our contemporary and complex lives and looking for their optimal solutions. The major sub-disciplines in modern operational research are Computing and information technologies, financial engineering, manufacturing, service sciences and supply chain management, policy modelling and public sector work, revenue management, simulation, stochastic models, and transportation problems.

One of the most important methods that help management in making decisions is the linear programming manner, which is a mathematical tool that allows us to model, analyze, and solve a broad spectrum of problems that resulted from the great scientific development witnessed in our contemporary world, it concerned with allocating rare resources of labour, materials, machinery and capital and using them in the best possible way so as to minimize costs or maximize profits [1,2,3,4,5].

All previous traditional studies, the optimal solution for the problems were specific values resulting from specific or determined data provided by the conducted field study subject to the existed conditions, where the available resources are subject to instantaneous change. However, the reality of the situation indicates that the position of companies is not stable, the future is unpredictable, and these specific values of profits and available resources are subject to instantaneous change. Consequently, to this companies' urgent needs, it was necessary to originate a new theory that takes into account these fluctuations that productive companies go through and provides results that enjoy indeterminate and give companies a margin of freedom during applying linear programming models. This article gives linear model which is the direct simplex method using neutrosophic logic, the logic that is the new vision of modelling and is designed to effectively address the uncertainties inherent in the real world founded by the Romanian mathematician Florentine Smarandache [7, 9, 10, 11, 13].

F. Smarandache presented neutrosophic logic in 1995 as a generalization of fuzzy logic that setup by Lotfi A. Zadeh [6] in 1965. And it is also a generalization for intuitionistic fuzzy logic that founded by K. Atanasove [8] in 1882. In addition to that, Ahmed A. Salama presented the theory of neutrosophic classical categories as a generalization of the theory of classical categories [12,20], also, he developed, introduced and formulated new

concepts in the various disciplinary of mathematics, statistics, computer science by neutrosophic theory [17,18,19,22,28].

In the recent past decades, the neutrosophic theory has broad and significant expanded in both its theoretical and applied aspects, where it has been applied in dozens of application fields such as but not limited to measurement, graph theory, statistics, probability...etc. [8,14,15, 16, 21, 23, 25, 26, 27].

2. Discussion:

Linear programming is the method that helps in selecting decisions and choosing the best program for independent activities, taking into account the available resources. Linear programming is used to solve problems in which the goal is specific, such as securing a maximum profit, securing a minimum cost, saving time or effort ... etc. The linear programming problems consisting of a linear objective function subject to some constraints (inequalities or equations), is characterized by the existence of a large number of acceptable solutions to it, the goal is to find an optimal solution among the set of those accepted solutions [1,2,3].

It is well known that to get an optimal solution for any linear programming problem using the direct simplex algorithm should be processed to be in standard form, the simplex method for solving an LP problem requires the problem to be expressed in the standard form. But not all LP problems appear in the standard form. In many cases, some of the constraints are expressed as inequalities rather than equations; at least it is most often true in the case of water resources problems. In some problems, all the decision variables may not be even nonnegative. Hence, the first step in solving an LP problem is to convert it to the standard form. The procedure to convert a general program to the standard form is outlined below:

1- Convert all inequalities to equalities.
2- Convert all decision variables unrestricted in sign to strictly non-negative.
3- Make all the right-hand side constants of the constraints nonnegative.

3. Mathematical Model of Linear Programming Having Neutrosophic Values in Its Coefficients

Using simplex method, find the optimal solution for the following linear programming problem (1):

$$\max Z = c_{1N}x_1 + c_{2N}x_2 + \cdots + c_{nN}x_n$$

$$subject\ to \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_{1N} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_{2N} \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n \le b_{3N} \\ \qquad\qquad\qquad . \\ \qquad\qquad\qquad . \\ \qquad\qquad\qquad . \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_{mN} \end{cases}$$

With the non-negativity conditions $x_1, x_2, \dots, x_n \ge 0$.

It is worthy to mention that the coefficients subscribed by the index N are of neutrosophic values.

The objective function coefficients $c_{1N}, c_{2N}, \dots, c_{nN}$ have neutrosophic meaning are intervals of possible values :

That is, $c_{jN} = [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2, \dots, n$. Also, we have the values of the right-hand side of the inequality constraints $b_{1N}, b_{2N}, \dots, b_{mN}$ are regarded as neutrosophic interval values:

$b_{iN} = [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint $i = 1,2, \dots, m$.

As previously mentioned to solve any linear problem by simplex method, we should put it in its standard form, so, it should be handled using the slack, surplus, and artificial variables which have their usual meaning here,

- slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. If the model is in standard form, the slack variables will always have a zero coefficient in the objective function and +1 coefficient in corresponding constraint. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.

- A surplus variable represents the amount by which solution values exceed a resource. These variables are also called 'Negative Slack Variables'. Surplus variables like slack variables carry a zero coefficient in the objective function. it is added to greater than or equal to (>) type constraints in order to get an equality constraint.

- Artificial variables are added to those constraints with equality (=) and greater than or equal to (>) sign. An Artificial variable is added to the constraints to get an initial solution to an LP problem. Artificial variables have no meaning in a physical sense and are not only used as a tool for generating an initial solution to an LP problem.

After getting a well-defined for problem (1), solve it using simplex algorithm:

$$\max Z = c_{1N}x_1 + c_{2N}x_2 + \cdots + c_{nN}x_n + 0y_1 + 0y_2 + \cdots + 0y_m$$

$$subject\ to \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 = b_{1N} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 = b_{2N} \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + y_3 = b_{3N} \\ \qquad\qquad\qquad . \\ \qquad\qquad\qquad . \\ \qquad\qquad\qquad . \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m = b_{mN} \end{cases}$$

With the non-negativity conditions $x_1, x_2, \dots, x_n, y_1, y_2, y_3, \dots, y_m \geq 0$. The below tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function.

The initial Simplex Tableau

Basic Variables	Coefficient of								Right Side b_{iN}
	x_1	x_2	.....	x_n	y_1	y_2	....	y_m	
y_1	a_{11}	a_{12}	......	a_{1n}	1	0	.....	0	b_{1N}
y_2	a_{21}	a_{22}	......	a_{2n}	0	1	.....	0	b_{2N}
.	.	.		.	.	.			.
.	.	.		.	.	.			.
.	.	.		.	.	.			.
y_m	a_{m1}	a_{m2}		a_{mn}	0	0	......	1	b_{mN}
Z	c_{1N}	c_{2N}	......	c_{nN}	0	0	....	0	$Z - q_o$

The optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model. To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. If a tableau is not optimal, the next step is to identify the pivot variable to base a new tableau on, as described below:

The added variables $\{y_1, y_2, ..., y_m\}$ represent the first basic variables, and the variables $\{x_1, x_2, ..., x_n\}$ are non-basic variables. Since the problem is for the maximum objective function, the maximum positive coefficient in the objective function has been selected, i.e. $Max\{c_{1N}, c_{2N}, ..., c_{nN}\} = c_{sN}$. Suppose the coefficient c_{sN} of the term x_s is the maximum positive number. The column verses the number c_{sN} is the required column that set the variable x_s as the enter variable , it is still required to determine the row that crossed to the selected column to determine the pivot element:

$$\theta = min\left[\frac{b_{iN}}{a_{is}}\right] = \frac{b_{tN}}{a_{ts}} > 0; \quad a_{is} > 0, b_{iN} > 0$$

The pivot element is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot element can be identified by looking at the bottom row of the tableau and the indicator. Assuming that the solution is not optimal, pick the smallest negative value in the bottom row. One of the values lying in the column of this value will be the pivot variable. To find the indicator, divide the beta values (b_{iN}) of the linear constraints by their corresponding values from the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row will become the pivot element.

To optimize the pivot element, it will need to be transformed into a unit value (value of 1). To transform the value, multiply the row containing the pivot variable by the reciprocal of the pivot value.

In order to keep the tableau equivalent, the other variables not contained in the pivot column or pivot row must be calculated by using the new pivot values. For each new value, multiply the negative of the value in the old pivot column by the value in the new pivot row that corresponds to the value being calculated. Then add this to the old value from the old tableau to produce the new value for the new tableau. This step can be condensed into the equation on the following:

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value).

The following mathematical symbols and operations indicate the above explain:

$$\frac{a_{t1}}{a_{ts}}, \frac{a_{t2}}{a_{ts}}, \frac{a_{t3}}{a_{ts}}, \dots, \frac{a_{ts-1}}{a_{ts}}, 1, \frac{a_{ts+1}}{a_{ts}}, \dots, 0, 0, \frac{b_{tN}}{a_{ts}}.$$

$$a'_{ij} = a_{ij} - a_{tj}\frac{a_{is}}{a_{ts}} = \frac{a_{ij}a_{ts} - a_{tj}a_{is}}{a_{ts}}.$$

$$b'_{iN} = b_{iN} - b_{tN}\frac{a_{is}}{a_{ts}} = \frac{b_{iN}a_{ts} - b_{tN}a_{is}}{a_{ts}}.$$

$$c'_{jN} = c_{jN} - c_{sN}\frac{a_{tj}}{a_{ts}} = \frac{c_{jN}a_{ts} - c_{sN}a_{tj}}{a_{ts}}.$$

The $(b'_{1N}, b'_{2N}, \dots, b'_{mN}, 0, 0, \dots, 0)$ represents the non-negative basic solution which gives a value for $Z = q_{oN}$, the following tableau has been constructed:

The Iteration Tableau of Simplex Method

Basic Variables	Coefficient of												Right Side b'_{iN}
	x_1	x_2	...	x_m	x_{m+1}	...	x_n	y_1	y_2	...	y_m		
x_1	1	0	...	0	a'_{1m+1}	...	a'_{1n}	α_{11}	α_{12}	...	α_{11}		b'_{1N}
x_2	0	1	...	0	a'_{2m+1}	...	a'_{2n}	α_{21}	α_{22}	...	α_{21}		b'_{2N}
.	.	.		.	.		.	.	.	...	.		.
.	.	.		.	.		.	.	.	...	.		.
.	.	.		.	.		.	.	.	...	.		.
x_m	0	0		1	a'_{mm+1}	...	a'_{mn}	α_{m1}	α_{m2}	...	α_{mN}		b'_{mN}
Z	0	0	...	0	$c'_{m+1 N}$	...	c'_{nN}	q_{1N}	q_{2N}	...	q_{mN}		$Z - q_{oN}$

Where α_{ij} and q_{iN} are the coefficients of the added variables in the objective function and in the constraints after execute the previous iterative operations. The optimal solution is:

$$x_1^* = b'_{1N}, x_2^* = b'_{2N}, \dots, x_m^* = b'_{mN} \ ,$$

The optimal maximum solution of the objective function is:

$$Z^* = c_{1N}b'_1 + c_{2N}b'_2 + \cdots + c_{mN}b'_m.$$

3.1 Important Notes: [2,3]

1- A basic solution is admissible if all variables of the basic solution are nonnegative.

2- Each basic solution of linear programming problem for which all variables are nonnegative, is called an admissible basic solution. This admissible basic solution corresponds to an extreme point (corner solution).

3- There are two types of unboundedness:

- The objective function is unbounded (as is the feasible region).

- The feasible region is unbounded, but the objective function is not.

   " The LP is unbounded if there is a negative coefficient in the row versus of the objective function, and all the remaining elements in the column are negative or zero ".

4- Stop if all coefficients of the row versus to the objective function are negative or zeros. The current solution is optimal and unique.

4. Practical Example:

A. The Classical Context of the Problem [3]

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	19
F_2	2	1	13
F_3	0	3	15
F_4	3	0	18
Profit Returned per unit	7	5	

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible. We symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2, after building the appropriate mathematical model and solving it, we conclude that $x_1 = 5, x_2 = 3$, and hence the maximum profit $Z^* = 50$ of monetary unit.

B. The Neutrosophic Context of the Problem

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Products ⟍ Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible. Symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , the problem will be reformulated from the neutrosophic perspective as follow:

$$\max Z = [5,8] \, x_1 + [3,6] \, x_2$$

$$subject\ to \begin{cases} 2x_1 + 3x_2 \leq [14,20] \\ 2x_1 + x_2 \leq [10,16] \\ 3x_2 \leq [12,18] \\ 3x_1 \leq [15,21] \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0$$

The above program need to reformulated to an equivalent form by adding slack variables:

$$\max Z = [5,8] \, x_1 + [3,6] \, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4$$

$$subject\ to \begin{cases} 2x_1 + 3x_2 + y_1 = [14,20] \\ 2x_1 + x_2 + y_2 = [10,16] \\ 3x_2 + y_3 = [12,18] \\ 3x_1 + y_4 = [15,21] \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0, y_1 \geq 0, y_2 \geq 0, \ y_3 \geq 0, \ y_4 \geq 0$$

Non-basic var. ⟍ Basic var.	x_1	x_1	y_1	y_2	y_3	y_4	R. H. S. b_i
y_1	2	3	1	0	0	0	[14,20]

y_2	2	1	0	1	0	0	[10,16]
y_3	0	3	0	0	1	0	[12,18]
y_4	3	0	0	0	0	1	[15,21]
Objective fun.	[5,8]	[3,6]	0	0	0	0	$Z - 0$

It is clear that max([5,8], [3,6]) = [5,8] versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables. Now to demonstrate which basic variables should be ejected, the following calculation has been done:

$$\theta = min \left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3} \right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 which is the pivot element, divide the elements of the row versus to y_4 yields:

$$\frac{3}{3}, \frac{0}{3}, \frac{0}{3}, \frac{0}{3}, \frac{0}{3}, \frac{1}{3}, \frac{[15,21]}{3} = [5,7]$$

Non-basic var. \\ Basic var.	x_1	x_2	y_1	y_2	y_3	y_4	R. H. S. b_i
y_1	0	3	1	0	0	$\frac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\frac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
Object. Fun.	0	[3,6]	0	0	0	$[\frac{-8}{3}, \frac{-5}{3}]$	$Z - [25,56]$

It still there is a non-negative value in the row of the objective function (i.e. [3,6]) which is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. Now, the question is: which basic variable should be ejected? Track the following calculation to answer this question, $\theta = min \left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3} \right] = \frac{[4,6]}{3} = [\frac{4}{3}, 2]$ which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3 , the required calculations yield the following tableau:

Non-basic var. \\ basic var.	x_1	x_2	y_1	y_2	y_3	y_4	R. H. S. b_i

x_2	0	1	$\frac{1}{3}$	0	0	$\frac{-2}{9}$	$[\frac{4}{3},2]$
y_2	0	0	$\frac{1}{3}$	1	0	$\frac{-4}{9}$	$[\frac{-4}{3},2]$
y_3	0	0	-1	0	1	$\frac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
Object. Fun.	0	0	[-2,-1]	0	0	$[\frac{-6}{9},-1]$	$Z-[29,68]$

It is clear from the row of the objective function that all the elements are either zero or neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* = [5,7], x_2^* = \left[\frac{4}{3},2\right], y_2^* = \left[\frac{-4}{3},2\right], y_3^* = [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function, yields:

$$\max Z = [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$ which is identical to the result in the previous tableau.

To more checking, the optimal solution has to be satisfied the constraints:

4. Summarization

The below table summarizing the previous study by indicating the benefit of using neutrosophic theory:

Classical logic								
issue data						results		
c_1	c_2	b_1	b_2	b_3	b_4	x_1	x_2	$max\ Z$
7	5	19	13	15	18	5	3	50
Neutrosophic logic								
issue data						results		
c_{1N}	c_{2N}	b_{1N}	b_{2N}	b_{3N}	b_{4N}	x_{1N}	x_{2N}	$max\ Z_N$
[5,8]	[3,6]	[14,20]	[10,16]	[12,18]	[15,21]	[5,7]	$[\frac{4}{3},2]$	[29,68]

the reader could easily noticed that, when the neutrosophic strategy has been applied in solving linear programming problem, indeterminate vales have been obtained, these indeterminate values have well simulated the reality, and takes into account most of the changes that may occur in the working environment of the system represented by the linear mathematical model, while the values we obtain when solving according to traditional classical logic are specific values and do not take into account the change that It can occur in the working environment of the system represented by the mathematical model. Accordingly, the neutrosophic logic provides us with a more general and

comprehensive study than the well-known classical study and does not neglect any data just because they are not explicitly specified, and therefore the presence of indeterminacy actually affects the results and these unspecified values cannot Ignoring it and keeping it away from the framework of the study in order to obtain as accurate results as possible, that is, working within the classical logic is no longer sufficient at the present time, because the development of science and the instability in the work environment of the facilities put before us a large number of cases that need quick treatment and accurate in order to avoid losses that may be incurred by the facilities, which cannot be handled using the classical logic and here comes the role of the neutrosophic logic, which provides us with a more comprehensive, It helps us to interpret the study data and helps us obtain the more logical results.

We look forward in the near future studying other topics of linear programming and its applications in practical life using neutrosophic logic, accompanying programs, sensitivity analysis...etc.

Conflicts of Interest: "The authors declare no conflict of interest."

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