The Connectivity indices concept of Neutrosophic Graph and Their Application of Computer Network, Highway System and Transport Network Flow

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ABSTRACT

It was suggested that a neutrosophic set be used as a strategy to deal with the neutrally ambiguous facts. The three membership functions T_r, I_n, and F_i are used to describe this set. These functions represent an object's degree of truth, indeterminacy, and false membership. The most intriguing aspect of the neutrosophic set is that it is assumed to have a symmetric form since the truth membership T_r is symmetric to the opposite false membership F_i with respect to the indeterminacy membership I_n, which precisely serves as an axis of symmetry. The neutrosophic connectivity index (CI_N) is crucial in solving real-world issues, particularly those involving traffic network flow. Neutrosophic graphs can be used to designate two features of knowledge with the membership, intermediate membership as well as non-membership degrees in hesitations. Given the standing of CI_N in real-life problems and in understanding neutrosophic graphs, we aim to come out with some CI_N in the environs of neutrosophic graphs. We are acquainted with two natures of CI_Ns, namely CI_N and mean CI_N, in the framework of neutrosophic graphs. Nevertheless, certain types of nodes are called neutrosophic graph connectivity enhancing nodes $(NCEN)$, neutrosophic connectivity reducing nodes $(NCRN)$, and neutrosophic neutral nodes. The node is enabled for neutrosophic diagrams. Let's make applications of CI_N in two varieties of networks, traffic network flow and examples to demonstrate the applicability of the proposed work.

1 Introduction

Zadeh[54] introduced the idea of a fuzzy set by assigning degrees of membership between 0 and 1 to the elements of the set. In 1975, Rosenfeld[43] examined the fuzzy graph. Thereafter, the identical idea was separately offered over the same time period by Yeh and Bang[52]. While Yeh and Bang provided applications for the concept of a fuzzy connected graph and[40],[36],[45]-[47],[53],[24]and[31], Rosenfeld identified a few basic features. Mathew and Sunitha introduced arc types[11]to fuzzy terminal nodes[12] and geodesics[10]. Atanassov[8] introduced the intuitionistic fuzzy set in 1986, which is a development of the fuzzy set. The intuitionistic fuzzy graph described and elaborated by Parvathi and Karunambigai[41] and[20],[19],[38],[39] is a generalisation of the fuzzy graph. Karunambigai and Buvaneswari[29] introduced slurs in IFG, like strong and weakest arcs, strong path, α, β-strong, and γ-weak arcs. Karunambigai and Kalaivani[28] viewed $IFGs$ as a matrix representation. The cyclic CI_9N and the mean cyclic CI_N of fuzzy sets are two connectivity measurements that Binu et al., introduced[13]. The ideas of CI_N, average CI_N, and connectivity node types were introduced by Poulik and Ghorai to the bipolar fuzzy graph environment with applications[42]. CI_N and ACI_N were presented by Mathew and Mordeson[14], who also explored their characteristics and practical uses. With a focus on illegal immigration networks, Binu et al.,[15] examined the Wiener index idea and the connection between the Wiener index and the connectedness index. Abdu Gumaei et al.,[1] presented IFG connectivity indices and their applications. Many researches using CI in intuitionisitic fuzzy graphs(See ([51],([2]-[7])).[49] Tulat Naeem et al., established the notion of wiener index of intuitionistic fuzzy graphs with an application to transport network flow and[48]. A generalisation of the fuzzy set and the intuitionistic fuzzy set, the neutrosophic set was proposed by Smarandache[46]. It has the ability to interpret information that is hazy, unclear, and inconsistent. The idea of single-valued neutrosophical set ($SVNS$), a subclass of the neutrosophical set in which each membership of truth, indeterminacy, and falsehood accepts values between [0, 1], was then put forward by Wang et al.,[50]. Strong, complete, and regular monovalent bipolar neutrosophic graphs were characterised and new findings on monovalent bipolar neutrosophic graphs were presented by Broumi et al.,[16] while Hassan et al., established a number of distinct types of bipolar neutrosophic graphs[23] and[30],[34].[37]Merkepci and Ahmad introduced the notion on the conditions of imperfect neutrosophic duplets and imperfect neutrosophic triplets.[18] Celik and Hatip presented the concepet on the refined AH-Isometry and its applications in refined neutrosophic surfaces galoitica[17]; then Celik and Olgun defined some basic properties of the classification of neutrosophic complex inner product spaces.[35] Masoud Ghods and Zahra Rostami examined the wiener index and applications in the Neutrosophic graphs and compared this index with the connectivity index, which is one of the most important degree-based indicators. Mathematically, it seems neutrosophic logic is more generalized than intuitionistic fuzzy logic. neutrosophic logic can be applied to any field, to provide the solution for indeterminacy problem. Many of the real-world data have a problem of inconsistency, indeterminacy, and incompleteness.

Fuzzy sets provide a solution for uncertainties, and intuitionistic fuzzy sets handle incomplete information, but both concepts have failed to handle indeterminate information. Neutrosophic sets provide a solution for both incomplete and indeterminate information. It has mainly three degrees of membership, namely, indeterminacy, and falsity.

Related Work With Different Component			
Reference	Year	Techniques used	Solved Problem
22	2020	Intuitionistic fuzzy soft graphs	Gain and loss of vertices pair
1	2021	Intuitionistic Fuzzy Graphs	Connectivity Indices
21	2023	Intuitionistic Fuzzy Graphs	Wiener index
33	2022	bipolar fuzzy incidence graph	Cyclic connectivity index
13	2020	Fuzzy graphs	Cyclic connectivity index
34	2020	Neutrosophic trees	Connectivity index
35	2021	Neutrosophic graphs	Wiener index

Table 1. Literature Review

In the above Table 1 works CI was obtained for fuzzy and intuitionistic fuzzy graphs but in real time average connectivity index and also be interpreted in neutrosophic graph.

1.1 Motivation

The authors discovered that, to the best of their knowledge, no study has reported on the connectedness indices of neutrosophic graphs and their applications in transport network flow after becoming informed and motivated by the aforementioned works. The following explanations provide a rundown of this work's main contributions:

1. It introduces the concepts of neutrosophic graphs. The notion of neutrosophic graphs is the focus of the first approach in the literature, which is made in this study.
2. It is investigated the significance of this new class of graphs and how to differentiate it from the other existing classes.
3. Additionally, the neutrosophic graph's connection indices and average connectivity indices are established.

1.2 Novelty

1. To define neutrosophic graph.
2. To provide a new definition of neutrosophic connectivity index.
3. To define a neutrosophic connectivity index with edge and vertex.
4. comparing the numerical results for the average connection index with the neutrosophic connectivity index.

This may help us to make better decision. The investigation of neutrosophic graphs and a few related ideas is the focus of this study. Preliminary specifications for this job are outlined in Section 2. The CI_N ideas and CI_N bounds for neutrosophic sets are developed in Section 3. The CI_N of neutrosophic sub graphs with deleted vertices and edges is shown in Section 4, and Section 5 discusses the ACI_N and its attributes. In Section 6, applications of $CI_N s$ are covered. real time applications for Sections 7 and 8 complete this study providing a conclusion.

2 Preliminaries

This section presents definitions and examples relating to neutrosophic graphs, arcs, in neutrosophic and neutrosophic cycles pertinent to the current work.

Definition 1. [30] *A pair $G = (N, M)$ is called a neutrosophic graph if,*

1. $\check{\check{V}} = \{u_{p_1}, u_{p_2}, u_{p_3}, \ldots\ldots, u_{p_n}\}$ *with* $\check{\check{V}} \xrightarrow{T_r^N} [0,1], \check{\check{V}} \xrightarrow{I_n^N} [0,1]$ *and* $\check{\check{V}} \xrightarrow{F_i^N} [0,1]$ *representing the truth-membership function, indeterminacy membership function and falsity membership function,* $0 \leq T_r^N(u_{p_i}) + I_n^N(u_{p_i}) + F_i^N(u_{p_i}) \leq 3$ *for each* $u_{p_i} \in \check{\check{V}}$.

2. $\check{\check{E}} \subseteq \check{\check{V}} \times \check{\check{V}}$ *with* $\check{\check{E}} \xrightarrow{T_r^M} [0,1], \check{\check{E}} \xrightarrow{I_n^M} [0,1]$, *and* $\check{\check{E}} \xrightarrow{F_i^M} [0,1]$ *being as follows:*

$$T_r^M(p_i, u_{p_j}) \leq min\{T_r^N(u_{p_i}), T_r^N(u_{p_j})\}$$
$$I_n^M(u_{p_i}, u_{p_j}) \leq min\{I_n^N(u_{p_i}), I_n^N(u_{p_j})\}$$
$$F_i^M(u_{p_i}, u_{p_j}) \geq max\{F_i^N(u_{p_i}), F_i^N(u_{p_j})\}$$

and $0 \leq T_r^M(u_{p_i}, u_{p_j}) + I_n^M(u_{p_i}, u_{p_j}) + F_i^M(u_{p_i}, u_{p_j}) \leq 3$ *for all edge* $(u_{p_i}, u_{p_j}) \in E$.

Definition 2. [30] *A neutrosophic graph G is complete if*

$$T_r^M(u_{p_i}, u_{p_j}) = min\{T_r^N(u_{p_i}), T_r^N(u_{p_j})\}$$
$$I_n^M(u_{p_i}, u_{p_j}) = min\{I_n^N(u_{p_i}), I_n^N(u_{p_j})\}$$
$$F_i^M(u_{p_i}, u_{p_j}) = max\{F_i^N(u_{p_i}), F_i^N(u_{p_j})\}$$

for each $(u_{p_i}, u_{p_j}) \in E$.

Path has a significant and well-known part in neutrosophic graphs. We may define the path idea in neutrosophic graphs using the following definition.

Definition 3. [16] *A neutrosophic graph with different vertices* $u_{P_1}, u_{P_2}, u_{P_3}, \ldots\ldots, u_{P_n}$ *said to have a path* $\check{\check{V}}$*,if it met one of the conditions below.*

1. $T_r^M(u_{p_i}, u_{p_j}) > 0,\ I_n^M(u_{p_i}, u_{p_j}) > 0,\ F_i^M(u_{p_i}, u_{p_j}) = 0$
2. $T_r^M(u_{p_i}, u_{p_j}) = 0,\ I_n^M(u_{p_i}, u_{p_j}) = 0,\ F_i^M(u_{p_i}, u_{p_j}) > 0$
3. $T_r^M(u_{p_i}, u_{p_j}) > 0,\ I_n^M(u_{p_i}, u_{p_j}) > 0,\ F_i^M(u_{p_i}, u_{p_j}) < 0$.

The neutrosophic graphical representations play a significant role to analyse the strength of the paths of vertices in a two dimensional space. The limitations of the strength of the paths have been defined component wise as well as total strength wise in the following statements discussed in the Definition 4:

Definition 4. [16] *Assume taht a neutrosophic graph G contain a path* $\check{\check{V}} = u_{p_1}, u_{p_2}, u_{p_2}, \ldots\ldots, u_{p_n}$. *Then* $\check{\check{P}}$ *is defined by*

1. T_r*-strenthgh if* $S_{T_r} = min\{T_r^M(u_{p_i}, u_{p_j})\}$
2. I_n*-strenthgh if* $S_{I_n} = min\{I_n^M(u_{p_i}, u_{p_j})\}$

3. F_i-strenthgh if $S_{F_i} = max\{F_i^M(u_{p_i}, u_{p_j})\}$

4. The $S_{\check{\check{P}}} = (S_{T_r}, S_{I_n}, S_{F_i})$ is said to be a $\check{\check{P}}$ strength if both S_{T_r}, S_{I_n} and S_{F_i} to the same edge occur.

Definition 5. [23] *The T_r-strength of connecting vertices u_{p_i} and u_{p_j} is define by $CONN_{T_r(\c{G})}(u_{p_i}, u_{p_j}) = max\{S_{T_r}\}$, I_n-strength of connecting vertices u_{p_i} and u_{p_j} is define by $CONN_{I_n^N(\c{G})}(u_{p_i}, u_{p_j}) = max\{S_{I_n}\}$ and F_i-strength of connecting vertices u_{p_i} and u_{p_j} is define by $CONN_{F_i(\c{G})}(u_{p_i}, u_{p_j}) = min\{S_{F_i}\}$ for all possible paths between u_{p_i} and u_{p_j}, where $CONN_{T_r(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$, $CONN_{I_n(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$ and $CONN_{F_i(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$ denotes the T_r, I_n and F_i-strength of connected with u_{p_i} and u_{p_j} achieved by eliminating the (u_{p_i}, u_{p_j}) edge from $\c{G}$.*

Definition 6. [30] *An edge (u_{p_i}, u_{p_j}) in a neutrosophic graph*

1. *strongest, if $T_r^M(u_{p_i}, u_{p_j}) \geq CONN_{T_r(\c{G})}(u_{p_i}, u_{p_j})$, $I_n^M(u_{p_i}, u_{p_j}) \geq CONN_{I_n(\c{G})}(u_{p_i}, u_{p_j})$ and $F_i^M(u_{p_i}, u_{p_j}) \leq CONN_{F_i(\c{G})}(u_{p_i}, u_{p_j})$ for each $u_{p_i}, u_{p_j} \in V$.*

2. *Weakest, if $T_r^M(u_{p_i}, u_{p_j}) < CONN_{T_r(\c{G})}(u_{p_i}, u_{p_j})$, $I_n^M(u_{p_i}, u_{p_j}) < CONN_{I_n(\c{G})}(u_{p_i}, u_{p_j})$ and $F_i^M(u_{p_i}, u_{p_j}) > CONN_{F_i(\c{G})}(u_{p_i}, u_{p_j})$ for each $u_{p_i}, u_{p_j} \in V$.*

Definition 7. [34] *Let $\c{G} = (\d{N}, \c{M})$ be a neutrosophic graph. A path $P : u_{p_i} - u_{p_j}$ in $\c{G}$ is said to be a strong path if P consists of only strong edges.*

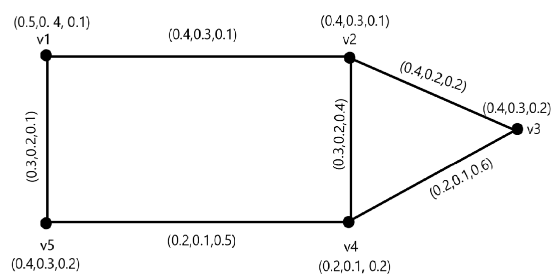

Figure 1. Neutrosophic graph with strong and weakest arcs.

Example 1. *In Fig. 1, $T_r^M(v_{p_1}, v_{p_2}) = 0.7 = CONN_{T_r(\c{G})}(v_{p_1}, v_{p_2})$, $I_n^M(v_{p_1}, v_{p_2}) = 0.6 = CONN_{I_n(\c{G})}(v_{p_1}, v_{p_2})$ and $F_i^M(v_{p_1}, v_{p_2}) = 0.3 = CONN_{F_i(\c{G})}(v_{p_1}, v_{p_2})$, which is implies that (v_{p_1}, v_{p_2}) is a strong arc. Similarly, $(v_{p_2}, v_{p_3}), (v_{p_1}, v_{p_5}), (v_{p_2}, v_{p_4})$, are strong arc and $(v_{p_3}, v_{p_4}), (v_{p_4}, v_{p_5})$ are weakest arcs.*
In this regard, $P = v_{p_1} v_{p_2} v_{p_3}$ is a strong path.

Definition 8. [21] *An arc (u_{p_i}, u_{p_j}) in a neutrosophic graph $(\c{G}) = (\d{N}, \c{M})$ is*

1. *If $T_r^M(u_{p_i}, u_{p_j}) > CONN_{T_r(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$,*
   $I_n^M(u_{p_i}, u_{p_j}) > CONN_{I_n(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$
   and $F_i^M(u_{p_i}, u_{p_j}) < CONN_{F_i(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$ is called α-strong.

2. *If $T_r^M(u_{p_i}, u_{p_j}) = CONN_{T_r(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$,*
   $I_n^M(u_{p_i}, u_{p_j}) = CONN_{I_n(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$
   and $F_i^M(u_{p_i}, u_{p_j}) = CONN_{F_i(\c{G})-(u_{p_i},u_{p_j})}(u_{p_i}, u_{p_j})$ is called β-strong.

3. If $T_r^M(u_{p_i},u_{p_j}) < CONN_{T_r(\underset{\cdot}{G})-(u_{p_i},u_{p_j})}(u_{p_i},u_{p_j})$,
$I_n^M(u_{p_i},u_{p_j}) < CONN_{I_n(\underset{\cdot}{G})-(u_{p_i},u_{p_j})}(u_{p_i},u_{p_j})$
and $F_i^M(u_{p_i},u_{p_j}) > CONN_{F_i(\underset{\cdot}{G})-(u_{p_i},u_{p_j})}(u_{p_i},u_{p_j})$ is called γ-weak.

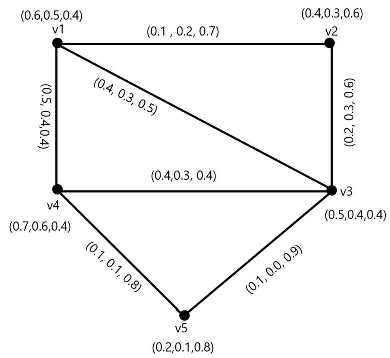

Figure 2. A Neutrosophic graph with α,β-strong, and γ-weakest arcs.

Example 2. *Fig 2, shows, the arcs* $(v_{p_1},v_{p_4}),(v_{p_2},v_{p_3}),(v_{p_4},v_{p_5}),(v_{p_1},v_{p_3})$ *are* α*-strong,* (v_{p_3},v_{p_4}) *is* β*-strong, and* $(v_{p_1},v_{p_2}),(v_{p_3},v_{p_5})$ *are* γ*-weak.*

Definition 9. [34] *A path in a neutrosophic graph containing only* $\alpha\&\beta$*-strong arc are called* $\alpha\&\beta$*-strong.*

Definition 10. [34]

1. If $\underset{\cdot}{G}^* = (N^*, M*)$ is a cycle, then $\underset{\cdot}{G} = (\underset{\cdot}{N}, \underset{\cdot}{M})$ is said to be a cycle
2. If $\underset{\cdot}{G}^* = (N^*, M*)$ is cycle, and $\nexists$ a pair $(x,y) \in M^*$ be such that $T_r^M(t,x) = min\{T_r^M(a,b) \mid (a,b) \in M^*\}, I_n^M(t,x) = min\{I_n^M(a,b) \mid (a,b) \in M^*\}$, and $F_i^M(t,x) = max\{F_i^M(a,b) \mid (a,b) \in M^*\}$, then $\underset{\cdot}{G}$ is said to be a neutrosophic cycle.

Example 3. *In Fig 3. Take* $T_r^N(u_p), I_n^N(u_p), F_i^N(u_p) = (.3,.3,.4), \forall u \in N^*$*.Then,* $min\{T_r^M(u_p,v_p)\} = 0.3, min\{I_n^M(u_p,v_p)\} = 0.3$ *and* $max\{F_i^M(u_p,v_p)\} = 0.4$.

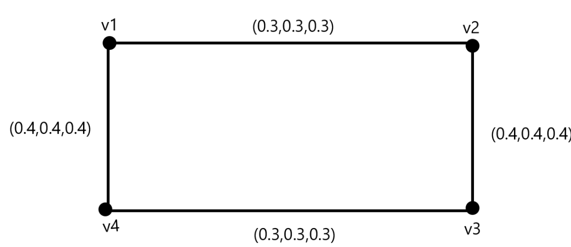

Figure 3. A neutrosophic graph cycle.

Our study's primary goal is to increase the precision and accuracy of topological indices research, specifically in the perspective of connection indices. Neutrosophic graphs provide more information than intuitionistic fuzzy graphs. In a certain condition of haziness and ambiguity, intuitionistic are termed by having membership grade and non-membership grade, but neutrosophic graphs are considered by the 3 grades, namely true, indeterminacy, and false membership grades. As we are using three membership grades, neutrosophic graphs do not have as much loss of information as relating to intutionistic graphs. For this reason, we would want to suggest different CIN models for neutrosophic graphs and learn how to use them.

3 Neutrosophic Connectivity Index Graph

Naturally, when we discuss a network similar to a transport network, we consider its connectedness. The network's connectedness indicates how stable and dynamic it is. Therefore, we may claim that this connectedness metric is the most fundamental and essential. The connection metric is already present in neutrosophic graphs. However, neutrosophic graph is an extension of intuitionistic fuzzy graph, performs better when intuitionistic fuzzy graphs are not permeable. The authors have thus suggested this notion of connectedness from intuitionistic fuzzy graphs to neutrosophic graphs for the above mentioned purpose.The authors have given some findings about the connectivity of neutrosophic graphs.

Definition 11. *The CI_N of a neutrosophic graph $G = (N, M)$ is demarcated as*

$$\begin{aligned}
CI_N(G) &= \sum_{u_p,v_p \in \check{V}(G)} (T_r^N(u_p), I_n^N(u_p), F_i^N(u_p))(T_r^N(v_p), I_n^N(v_p), F_i^N(v_p)) \times CONN_G(u_p, v_p) \\
&= \sum_{u_p,v_p \in \check{V}(G)} (T_r^N(u_p), I_n^N(u_p), F_i^N(u_p))(T_r^N(v_p), I_n^N(v_p), F_i^N(v_p)) \\
&\quad (CONN_{T_r(G)}(u_p, v_p), CONN_{I_n(G)}(u_p, v_p), CONN_{F_i(G)}(u_p, v_p)) \\
&= \sum_{u_p,v_p \in \check{V}(G)} (T_r^N(u_p) T_r^N(v_p) CONN_{T_r(G)}(u_p, v_p)) \\
&\quad + (I_n^N(u_p) I_n^N(v_p) CONN_{I_n(G)}(u_p, v_p)) + (F_i^N(u_p) F_i^N(v_p) CONN_{F_i(G)}(u_p, v_p)) \\
&= \sum_{u_p,v_p \in \check{V}(G)} T_r^N(u_p) T_r^N(v_p) CONN_{T_r(G)}(u_p, v_p) \\
&\quad + \sum_{u_p,v_p \in \check{V}(G)} I_n^N(u_p) I_n^N(v_p) CONN_{I_n(G)}(u_p, v_p) \\
&\quad + \sum_{u_p,v_p \in \check{V}(G)} F_i^N(u_p) F_i^N(v_p) CONN_{F_i(G)}(u_p, v_p) \qquad (3.1) \\
&= T_r CI_N(G) + I_n CI_N(G) + F_i CI_N(G) \qquad (3.2)
\end{aligned}$$

where $T_r CI_N(G), I_n CI_N(G) \& F_i CI_N(G)$ are $T_r, I_n \& F_i$-connectivity index of G, and $CONN_{T_r(G)}(u_p, v_p), CONN_{I_n(G)}(u_p, v_p) \& CONN_{F_i(G)}(u_p, v_p)$ are $T_r, I_n \& F_i$-strength $u_p - v_p$.

Example 4. *In Fig 1,*

$$\begin{aligned}
T_r CI_N &= \sum_{u_p,v_p \in \check{V}(G)} T_r^N(u_p) T_r^N(v_p) CONN_{T_r(G)}(u_p, v_p) \\
&= (.5)(.4)(.2) + (.5)(.4)(.4) + (.5)(.2)(.3) + (.5)(.4)(.3) + (.4)(.4)(.4) \\
&\qquad + (.4)(.2)(.3) + (.4)(.4)(.3) + (.4)(.2)(.3) + (.4)(.4)(.3) + (.2)(.4)(.3) \\
&= 0.442
\end{aligned}$$

$$I_nCI_N = \sum_{u_p,v_p \in \check{V}(G)} I_n^N(u_p) I_n^N(v_p) CONN_{I_n(G)}(u_p,v_p)$$

$$= (.4)(.3)(.1)+(.4)(.3)(.2)+(.4)(.1)(.2)+(.4)(.3)(.2)+(.3)(.2)(.2) +(.3)(.1)(.2)+(.3)(.3)(.2)+(.3)(.1)(.2)+(.3)(.3)(.2)+(.1)(.3)(.2)$$

$$= 0.140$$

$$F_iCI_N = \sum_{u_p,v_p \in \check{V}(G)} F_i^N(u_p) F_i^N(v_p) CONN_{F_i(G)}(u_p,v_p)$$

$$= (.1)(.1)(.5)+(.1)(.2)(.2)+(.1)(.2)(.4)+(.1)(.2)(.1)+(.1)(.2)(.2) +(.1)(.2)(.4)+(.1)(.2)(.1)+(.2)(.2)(.4)+(.2)(.2)(.2)+(.2)(.2)(.4)$$

$$= 0.073$$

$$CI_N = T_rCI_N + I_nCI_N + F_iCI_N$$

$$CI_N = 0.442+0.140+0.073$$

$$CI_N = 0.635$$

It may be observe that $T_rCI_N(G) > I_nCI_N(G) > F_iCI_N(G)$,which show that the level of $F_iCI_N(G)$ is lower than the level of $I_nCI_N(G)$ is lower than the level of $T_rCI_N(G)$.

Proposition 1. *If $G = (N,M)$ is a complete neutrosophic graph with $N^* = \{v_{p_1}, v_{p_2}, ..., v_{P_\kappa}\}$ be such that $t_1 \le t_2 \le ... \le t_n, r_1 \le r_2 \le ... \le r_\kappa \& s_1 \ge s_2 \ge ... \ge s_\kappa$, where $t_{p_i} = T_r^N(v_{pi})$,*
$r_{p_i} = I_n^N(v_{pi})$, and $s_i = F_i^N(v_{pi})$,

$$CI_N(G) = \sum_{p_i=1}^{\kappa-1} t_i^2 \sum_{j=i+1}^{\kappa} t_j + \sum_{i=1}^{n-1} r_i^2 \sum_{j=i+1}^{n} r_j + \sum_{i=1}^{\kappa-1} s_i^2 \sum_{j=i+1}^{\kappa} s_j. \tag{3.3}$$

Proof. Assume that v_{p_1} is the vertex with the lowest truth-membership value t_1.
A complete neutrosophic graphis $CONN_{T_r(G)}(u_p,v_p) = T_r^M(u_p,v_p) \forall u_p,v_p \in N^*$, so, $T_r^M(v_{p_1},v_{pi}) = t_1; 2 \le v_{pi} \le \kappa$ and hence, $T_r^N(v_{p_1})T_r^N(v_{pi})CONN_{T_r(G)}(v_{p_1},v_{pi}) = t_1.t_{P_i}.t_1 = t_1^2 t_{P_i}; 2 \le p_i \le \kappa$. we have

$$\sum_{p_i=2}^{\kappa} T_r^N(v_{p_1})T_r^N(v_{p_i})CONN_{T_r(G)}(v_{p_1},v_{p_i}) = \sum_{p_i=2}^{\kappa} t_1^2 t_{p_i}, \tag{3.4}$$

for v_{p_2}, is

$$\sum_{p_i=3}^{\kappa} T_r^N(v_{p_2})T_r^N(v_{p_i})CONN_{T_r(G)}(v_{p_2},v_{p_i}) = \sum_{p_i=3}^{\kappa} t_2^2 t_{p_i}, \tag{3.5}$$

for v_{p_3} is

$$\sum_{p_i=4}^{\kappa} T_r^N(v_{p_3})T_r^N(v_{pi})CONN_{T_r(G)}(v_{p_3},v_{pi}) = \sum_{i=4}^{\kappa} t_3^2 t_i, \tag{3.6}$$

and for $v_{p_{\kappa-1}}$ is

$$\sum_{p_i=\kappa}^{\kappa} T_r^N(v_{p_{\kappa-1}})T_r^N(v_{p_i})CONN_{T_r(G)}(v_{\kappa-1},v_{p_i}) = \sum_{p_i=\kappa}^{\kappa} t_{\kappa-1}^2 t_{p_i}. \tag{3.7}$$

The result of combining the equations above is

$$T_rCI^N_\kappa(\c{G}) = \sum_{p_i=2}^{\kappa} t_1^2 t_{p_i} + \sum_{p_i=3}^{\kappa} t_2^2 t_{p_i} + \sum_{p_i=4}^{\kappa} t_3^2 t_{p_i} + ... + \sum_{p_i=\kappa}^{\kappa} t_{\kappa-1}^2 t_{p_i} \tag{3.8}$$

$$= \sum_{p_i=1}^{\kappa-1} t_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} t_{p_j}. \tag{3.9}$$

Suppose v_{p_1} is the vertex with least indermatiance-membership value r_1.

Then, for a complete neutrosophic graph, $CONN_{I_n(\c{G})}(u_p, v_p) = I_n^M(u_p, v_p) \forall u_p, v_p \in N^*$,

So, $I_n^M(v_{p_1}, v_{pi}) = r_1; 2 \leq i \leq n$ and hence,$I_n^N(v_{p_1}) I_n^N(v_{pi}) CONN_{I_n^N(\c{G})}(v_{p_1}, v_{pi}) = r_1.r_i.r_1 = r_1^2 r_i; 2 \leq p_i \leq \kappa$. Taking summation over P_i, we have

$$\sum_{p_i=2}^{\kappa} I_n^N(v_{p_1}) I_n^N(v_{pi}) CONN_{I_n^N(\c{G})}(v_{p_1}, v_{pi}) = \sum_{p_i=2}^{\kappa} r_1^2 r_{p_i}, \tag{3.10}$$

for v_{p_2}, is

$$\sum_{p_i=3}^{\kappa} I_n^N(v_{p_2}) I_n^N(v_{pi}) CONN_{I_n(\c{G})}(v_{p_2}, v_{pi}) = \sum_{p_i=3}^{\kappa} r_2^2 r_{p_i}, \tag{3.11}$$

for v_{p_3} is

$$\sum_{p_i=4}^{\kappa} I_n^N(v_{p_3}) I_n^N(v_{pi}) CONN_{I_n(\c{G})}(v_{p_3}, v_{pi}) = \sum_{p_i=4}^{\kappa} r_3^2 r_{p_i}, \tag{3.12}$$

and for $v_{\kappa-1}$ is

$$\sum_{p_i=\kappa}^{\kappa} I_n^N(v_{n-1}) I_n^N(v_{pi}) CONN_{I_n(\c{G})}(v_{m-1}, v_{pi}) = \sum_{p_i=\kappa}^{\kappa} r_{n-1}^2 r_{p_i}. \tag{3.13}$$

The result of combining the equations above is

$$I_nCI_N(\c{G}) = \sum_{p_i=2}^{\kappa} r_1^2 r_{p_i} + \sum_{p_i=3}^{\kappa} r_2^2 r_i + \sum_{p_i=4}^{\kappa} r_3^2 r_{p_i} + ... + \sum_{p_i=\kappa}^{\kappa} r_{n-1}^2 r_{p_i} \tag{3.14}$$

$$= \sum_{i=1}^{\kappa-1} r_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} r_{p_j}. \tag{3.15}$$

and

Suppose v_{p_1} is the vertex with least falsity-membership value s_1. Then, for a complete neutrosophic graph, $CONN_{F_i(\c{G})}(u_p, v_p) = F_i^M(u_p, v_p) \forall u_p, v_p \in N^*$, So, $F_i^M(v_{p_1}, v_{p_i}) = s_1; 2 \leq p_i \leq \kappa$ and hence,

$F_i^N(v_{p_1}) F_i^N(v_{p_i}) CONN_{F_i(\c{G})}(v_{p_1}, v_{p_i}) = s_1.s_{p_i}.s_1 = s_1^2 s_{p_i}; 2 \leq p_i \leq \kappa$.

$$\sum_{p_i=2}^{\kappa} F_i^N(v_{p_1}) F_i^N(v_{pi}) CONN_{F_i(\c{G})}(v_{p_1}, v_{p_i}) = \sum_{p_i=2}^{\kappa} s_1^2 s_{p_i}, \tag{3.16}$$

for v_{p_2}, is

$$\sum_{p_i=3}^{\kappa} F_i^N(v_{p_2}) F_i^N(v_{pi}) CONN_{F_i(\c{G})}(v_{p_2}, v_{p_i}) = \sum_{p_i=3}^{\kappa} s_2^2 s_{p_i}, \tag{3.17}$$

for v_{p_3} is

$$\sum_{p_i=4}^{\kappa} F_i^N(v_{p_3}) F_i^N(v_{pi}) CONN_{F_i(\c{G})}(v_{p_3}, v_{pi}) = \sum_{p_i=4}^{\kappa} s_3^2 s_{p_i}, \tag{3.18}$$

and for $v_{\kappa-1}$ is

$$\sum_{p_i=\kappa}^{\kappa} F_i^N(v_{\kappa-1})F_i^N(v_{pi})CONN_{F_i(\c{G})}(v_{\kappa-1},v_{p_i}) = \sum_{p_i=\kappa}^{\kappa} s_{\kappa-1}^2 s_{p_i}. \tag{3.19}$$

By adding all the above equations, we get

$$F_iCI_n^N(\c{G}) = \sum_{p_i=2}^{\kappa} s_1^2 s_{p_i} + \sum_{p_i=3}^{\kappa} s_2^2 s_{p_i} + \sum_{p_i=4}^{\kappa} s_3^2 s_{p_i} + \ldots.. + \sum_{p_i=\kappa}^{\kappa} s_{\kappa-1}^2 s_i \tag{3.20}$$

$$= \sum_{i=1}^{\kappa-1} s_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} s_{p_j}. \tag{3.21}$$

Finally, Sum of all $CI_N s$,we Ģet

$$CI_N(\c{G}) = T_rCI_N(\c{G}) + I_nCI_N(\c{G}) + F_iCI_N(\c{G}) \tag{3.22}$$

$$= \sum_{p_i=1}^{\kappa-1} t_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} t_{p_j} + \sum_{i=1}^{\kappa-1} r_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} r_{p_j} + \sum_{p_i=1}^{\kappa-1} s_{p_i}^2 \sum_{p_j=pi+1}^{\kappa} s_{p_j}. \tag{3.23}$$

□

Example 5. *Fig 4 makes it clear that k_3 is an entirely neutrosophic graph. So,*

$$T_rCI_N(\c{G}) = \sum_{p_i=1}^{3} T_r^N(v_{pi})T_r^N(v_{pj})CONN_{T_r(\c{G})}(v_{pi},v_{pj})$$

$$= (0.5)(0.6)(0.5) + (0.6)(0.6)(0.6) + (0.5)(0.6)(0.5)$$

$$= 0.516$$

$$I_nCI_N(\c{G}) = \sum_{p_i=1}^{3} I_n^N(v_{pi})I_n^N(v_{pj})CONN_{I_n(\c{G})}(v_{pi},v_{pj})$$

$$= (0.4)(0.5)(0.4) + (0.5)(0.5)(0.5) + (0.4)(0.5)(0.4)$$

$$= 0.285$$

$$F_iCI_N(\c{G}) = \sum_{p_i=1}^{3} F_i^N(v_{pi})F_i^N(v_{pj})CONN_{F_i(\c{G})}(v_{pi},v_{pj})$$

$$= (.5)(.4)(.5) + (.4)(.3)(.4) + (.5)(.3)(.5)$$

$$= 0.211.$$

$$CI_N(\c{G}) = T_rCI_N(\c{G}) + I_nCI_N(\c{G}) + F_iCI_N(\c{G})$$

$$= 0.516 + 0.285 + 0.211$$

$$= 1.012.$$

Now, we use above theorem

$$\sum_{p_i=1}^{\kappa-1} t_{p_i}^2 \sum_{pj=pi+1}^{\kappa} t_{p_j} = \sum_{p_i=1}^{2} t_{p_i}^2 \sum_{p_j=p_i+1}^{3} t_{p_j}$$

$$= t_1^2(t_2+t_3)+t_2^2 t_3$$

$$= (0.5)^2(0.6+0.6)+(0.6)^2(0.6)$$

$$= 0.516$$

$$\sum_{p_i=1}^{\kappa-1} r_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} r_{p_j} = \sum_{p_i=1}^{2} r_{p_i}^2 \sum_{p_j=p_i+1}^{3} r_{p_j}$$

$$= r_1^2(r_2+r_3)+r_2^2 r_3$$

$$= (0.4)^2(0.5+0.5)+(0.5)^2(0.5)$$

$$= 0.285$$

$$\sum_{p_i=1}^{\kappa-1} s_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} s_{p_j} = \sum_{p_i=1}^{2} s_{p_i}^2 \sum_{p_j=p_i+1}^{3} s_{p_j}$$

$$= s_1^2(s_2+s_3)+s_2^2 s_3$$

$$= (.5)^2(.3+.4)+(.3)^2(.4) = .211$$

Adding these three summations, we get

$$\sum_{p_i=1}^{\kappa-1} t_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} t_{p_j} + \sum_{p_i=1}^{\kappa-1} r_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} r_{p_j} + \sum_{p_i=1}^{\kappa-1} s_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} s_{p_j} = 0.516+0.285+0.211.$$

$$= 1.012$$

Hence, it is verified that

$$CI_N(\underset{\cdot}{G}) = \textstyle\sum_{p_i=1}^{\kappa-1} t_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} t_{p_j} + \sum_{p_i=1}^{\kappa-1} r_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} r_{p_j} + \sum_{p_i=1}^{\kappa-1} s_{p_i}^2 \sum_{p_j=p_i+1}^{\kappa} s_j.$$

4 Connectivity Index with edge and Vertex Deleted Neutrosophic graphs

A vertex or an edge deletion may or may not have an impact on the CI_N. It is based on how the edge and vertex that must be omitted behave.

Example 6. *In Fig 5, take* $CI_N = 2.923$ $\underset{\cdot}{G} = (N,M)$. *Then* $(v_{p_1}, v_{p_4}), (v_{p_2}, v_{p_3}), (v_{p_4}, v_{p_5})$ *are* α*-strong arcs,*$(v_{p_1}, v_{p_3}), (v_{p_3}, v_{p_4})$ *are* β*-strong arcs and* $(v_{p_1}, v_{p_2}), (v_{p_3}, v_{p_5})$ *are* γ*-strong arcs. Then,*

$$T_r CI_N(\underset{\cdot}{G}) = \sum_{p_i=1}^{10} T_r^N(v_{pi}) T_r^N(v_{pj}) CONN_{T_r(\underset{\cdot}{G})}(v_{pi}, v_{pj})$$

$$= (0.6)(0.4)(0.2)+(0.6)(0.5)(0.4)+(0.6)(0.7)(0.5)+(0.6)(0.2)(0.1)$$

$$+(0.4)(0.5)(0.2)+(0.4)(0.7)(0.2)+(0.4)(0.2)(0.1)+(0.5)(0.7)(0.4)$$

$$+(0.5)(0.2)(0.1)+(0.7)(0.2)(0.1)$$

$$= 0.658$$

$$
\begin{aligned}
I_nCI_N(G) &= \sum_{p_i=1}^{10} I_n^N(v_{pi})I_n^N(v_{pj})CONN_{I_n(G)}(v_{pi},v_{pj})\\
&= (0.5)(0.3)(0.3)+(0.5)(0.4)(0.3)+(0.5)(0.6)(0.4)+(0.5)(0.1)(0.1)\\
&+(0.3)(0.4)(0.3)+(0.3)(0.6)(0.3)+(0.3)(0.1)(0.1)+(0.4)(0.6)(0.3)\\
&+(0.4)(0.1)(0.1)+(0.6)(0.1)(0.1)\\
&= 0.405\\
F_iCI_N(G) &= \sum_{p_i=1}^{10} F_i^N(v_{pi})F_i^N(v_{pj})CONN_{F_i(G)}(v_{pi},v_{pj})\\
&= (.4)(.7)(.5)+(.4)(.4)(.5)+(.4)(.4)(.4)+(.4)(.8)(.8)+(.7)(.4)(.5)\\
&+(.7)(.4)(.5)+(.7)(.8)(.8)+(.4)(.4)(.5)+(.4)(.8)(.8)+(.4)(.8)(.8)\\
&= 1.86
\end{aligned}
$$

$CI_N(G) = T_rCI_N(G) + I_nCI_N(G) + F_iCI_N(G)$ = 0.658+0.405+1.86 = 2.923. So, we have $CI_N(G-(v_{p_1},v_{p_4})) = T_rCI_N(G-(v_{p_1},v_{p_4})) + I_nCI_N(G-(v_{p_1},v_{p_4})) + F_iCI_N(G-(v_{p_1},v_{p_4})) = 0.448+0.285+1.796 = 2.529$. Thus, $CI_N(G-(v_{p_1},v_{p_4})) < CI_N(G)$, this implies CI_N of G have reduced by neglecting α-strong edge (v_{p_1},v_{p_4}). The neutrosohic graph, $G-(v_{p_1},v_{p_2})$ like Fig 6(a).

If we remove the β-strong edge (v_{p_1},v_{p_3}), then every pair of vertices strength of connectivity is constant, ie.,

$$
\begin{aligned}
&CONN_{T_r(G)}(v_{pi},v_{pj}) = CONN_{T_r(G)-(v_{p_1},v_{p_3})}(v_{pi},v_{pj}),\\
&CONN_{I_n(G)}(v_{pi},v_{pj}) = CONN_{I_n(G)-(v_{p_1},v_{p_3})}(v_{pi},v_{pj})\\
&CONN_{F_i(G)}(v_{pi},v_{pj}) = CONN_{F_i(G)-(v_{p_1},v_{p_3})}(v_{pi},v_{pj}),
\end{aligned}
$$

so $CI_N(G-(v_{p_1},v_{p_3})) = CI_N(G)$. The graph of $G-(v_{p_1},v_{p_3})$ according to Fig.6(b).

Similarly, when we delete the γ-arc (v_{p_1},v_{p_2}), then both the CI_N and the strength of connectivity between each pair of vertices remain constant. The graph of $G-(v_{p_1},v_{p_2})$ appears in Fig 6(c).

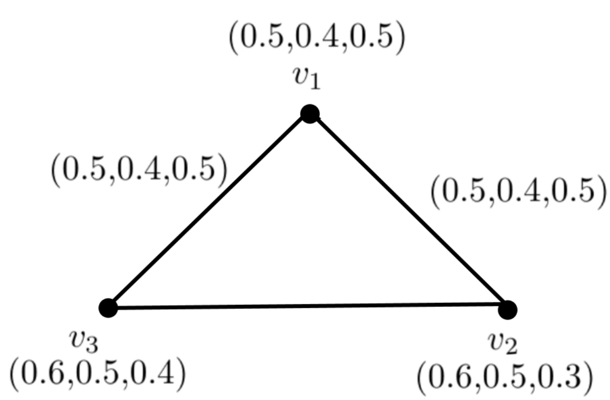

Figure 4. A complete neutrosophic graphv with $CI_N = 1.012$.

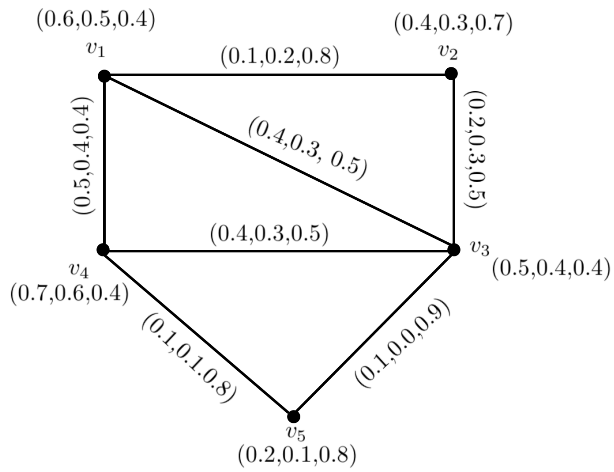

Figure 5. A neutrosophic graph $CI_N = 2.529$.

$$T_rCI_N(Ģ-(v_{p_1},v_{p_4})) = \sum_{i=1}^{10} T_r^N(v_{pi})T_r^N(v_{pj})CONN_{T_r(Ģ)-(v_{p_1},v_{p_4})}(v_{pi},v_{pj}) = 0.448 \tag{4.1}$$

$$I_nCI_N(Ģ-(v_{p_1},v_{p_4})) = \sum_{i=1}^{10} I_n^N(v_{pi})I_n^N(v_{pj})CONN_{I_n(Ģ)-(v_{p_1},v_{p_4})}(v_{pi},v_{pj}) = 0.285 \tag{4.2}$$

$$F_iCI_N(Ģ-(v_{p_1},v_{p_4})) = \sum_{i=1}^{10} F_i^N(v_{pi})F_r^N(v_{pj})CONN_{F_i(Ģ)-(v_{p_1},v_{p_4})}(v_{pi},v_{pj}) = 1.790 \tag{4.3}$$

Theorem 1. *Let H be the neutrosophic sub graph of a neutrosophic graph $Ģ = (N,M)$ produced by taking away an edge $u_pv_p \in M_Ģ$ from $Ģ$. Then, $CI_n^N(Ģ) > CI_N(H)$ or $(CI_N)_n^N(Ģ) < CI_N(H)$ iff u_pv_p is a bridge.*

Proof. Consider u_pv_q as a bridge. As stated in Definition 6, there exit u_p and v_p in a way that reduces the strength of their connection. So, We determine that $(CI_N)_n^N(Ģ) > CI_N(H)$ or $(CI_N)_n^N(Ģ) < CI_N(H)$. Conversely, let that $CI_N(Ģ) > CI_N(H)$ or $CI_N(Ģ) < CI_N(H)$ and give the following options some thought.

Case(a). Consider, u_pv_p is a γ-arc. Then, $CONN_{T_r(Ģ)-u_pv_p}(u_p,v_p) = CONN_{T_r(Ģ)}(u_p,v_p)$, $CONN_{I_n(Ģ)-u_pv_p}(u_p,v_p) = CONN_{I_n(Ģ)}(u_p,v_p)$, and $CONN_{F_i(Ģ)-u_pv_p}(u_p,v_p)$ $= CONN_{F_i(Ģ)}(u_p,v_p)$. So, we have $(T_rCI_N)_n^N(Ģ) = T_rCI_N(H), (I_nCI_N)_n(Ģ) = I_nCI_N(H)$ and $(F_iCI_N)_n^N(Ģ) = F_iCI_N(H)$ and therefore, $(CI_N)_n^N(Ģ) = CI_N(H)$.

Case(b). Let u_pv_p as γ-strong arc. Then, $T_r^M(u_p,v_p) = CONN_{T_r(Ģ)-u_pv_p}(u_p,v_p)$, $I_n^M(u_p,v_p) = CONN_{I_n(Ģ)-u_pv_p}(u_p,v_p)$ and $F_i^M(u_p,v_p) = CONN_{F_i(Ģ)-u_pv_p}(u_p,v_p)$. This implies that there is another $u_p - v_p$ path different from u_pv_p edge. Therefore, the removal of the arc u_pv_p will have no effect on the strength of connectedness between u_p and v_p. So, $CI_N(Ģ) = CI_N(H)$.

Case(c). Now, let u_pv_p be α-strong edge. Then, $T_r^M(u_p,v_p) > CONN_{T_r(Ģ)-uv}(u_p,v_p)$, $I_n^M(u_p,v_p) > CONN_{I_n(Ģ)-uv}(u_p,v_p)$ and $F_i^M(u_p,v_p) < CONN_{F_i(Ģ)-u_pv_p}(u_p,v_p)$. Thus, the strongest path is u_pv_p edge with strength equal to $(T_r^M(u_p,v_p), I_n^M(u_p,v_p)$, $F_i^M(u_p,v_p))$. Then, clearly $CI_N(Ģ) > CI_N(H)$, or $CI_N(Ģ) < CI_N(H), T_rCI_N(Ģ) - T_rCI_N(H) > I_nCI_N(Ģ) - I_nCI_N(H) > F_iCI_N(Ģ) -$

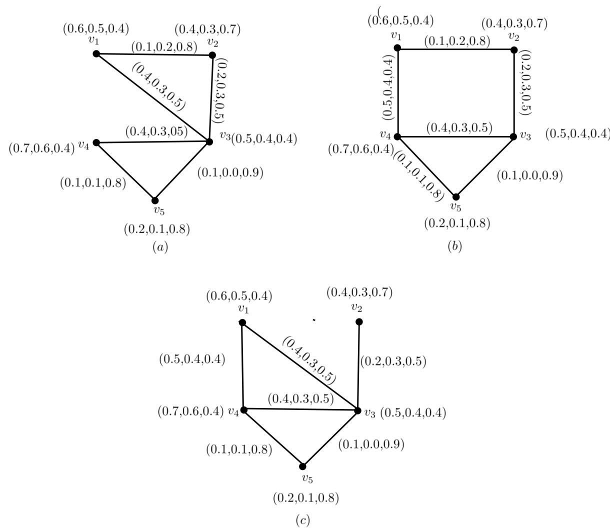

Figure 6. $Ģ-(v_{p_1},v_{p_4}), Ģ-(v_{p_1},v_{p_3})$ and $Ģ-(v_{p_1},v_{p_2})$,
(a) $Ģ-(v_{p_1},v_{p_4})$. (b) $Ģ-(v_{p_1},v_{p_3})$, (c) $Ģ-(v_{p_1},v_{p_2})$.

$F_iCI_N(H)$ or $T_rCI_N(Ģ)-T_rCI_N(H) < I_nCI_N(Ģ)-I_nCI_N(H) < F_iCI_N(Ģ)-F_iCI_N(H)$ since α-strong edges are neutrosophic bridges. This implies that u_pv_p is a bridge.

□

Remark 1. *Let H be the neutrosophic sub graphof a neutrosophic graph$Ģ=(N,M)$ formulated by removing an edge $u_pv_p \in M_{Ģ}$ from $Ģ$. Then, $CI_N(G) = CI_N(H) \Leftrightarrow u_pv_p$ is either β-strong or γ-edge.*

Remark 2. *Consider the case when u_pv_p is an edge of a neutrosophic graph$Ģ=(N,M)$. Then, $CI_N(G) \neq CI_N(G-u_pv_p)$ if and only if a unique neutrosophic bridge of G is u_pv_p.*

Theorem 2. *Let $G_1=(N_1,M_1)$ and $G_2=(N_2,M_2)$ be the two isomorphic neutrosophic graphs. Then, $CI_N(G_1)=CI_N(G_2)$.*

Proof. Suppose that $Ģ_1=(N_1,M_1)$ and $Ģ_2=(N_2,M_2)$ are isomorphic neutrosophic graphs. Then, $\exists$ is a mapping $h: N_1 \to N_2$ such that h is bijective and $T_{N_1}(u_{p_i}) = T_{N_2}(h(u_{p_i})), I_{N_1}(u_{p_i}) = I_{N_2}(h(u_{p_i}))$ and $F_{N_1}(u_{p_i}) = F_{N_2}(h(u_{p_i}))$ for all $u_{p_i} \in N^*$ as well as $T_{M_1}(u_{p_i},u_{p_j}) = T_{M_2}(h(u_{p_i}),(u_{p_j})), I_{M_1}(u_{p_i},u_{p_j}) = I_{M_2}(h(u_{p_i}),(u_{p_j}))$ and $F_{M_1}(u_{p_i},u_{p_j}) = F_{M_2}(h(u_{p_i}),(u_{p_j}))$ for all $u_{p_i} \in M^*$. As $Ģ_1$ and $Ģ_2$ isomorphic, then the strength of any strongest $u_{p_i}-u_{p_j}$ is equal to $h(u_{p_i})-h(u_{p_j})$ in $Ģ_2$. Thus, $u_p, v_p \in N^*$

$$CONN_{T_r(Ģ_1)}(u_{p_i},u_{p_j}) = CONN_{T_r(Ģ_2)}(h(u_{p_i}),h(u_{p_j})), \tag{4.4}$$

$$CONN_{I_n(Ģ_1)}(u_{p_i},u_{p_j}) = CONN_{I_n(Ģ_2)}(h(u_{p_i}),h(u_{p_j})), \tag{4.5}$$

$$CONN_{F_i(Ģ_1)}(u_{p_i},u_{p_j}) = CONN_{F_i(Ģ_2)}(h(u_{p_i}),h(u_{p_j})) \tag{4.6}$$

So, we have

$$T_rCI_N(\c{G}_1) = \sum_{u_{p_i},u_{p_j}\in V(\c{G}_1)} T_{N_1}(u_{p_i})T_{N_1}(u_{p_j})CONN_{T(\c{G}_1)}(u_{p_i},u_{p_j}) \tag{4.7}$$

$$= \sum_{h(u_{p_i}),h(u_{p_j})\in V(\c{G}_2)} T_{N_2}(h(u_{p_i}))T_{N_2}(h(u_{p_j}))CONN_{T(\c{G}_2)}(h(u_{p_i}),h(u_{p_j})) \tag{4.8}$$

$$= T_rCI_N(\c{G}_2) \tag{4.9}$$

$$I_nCI_N(\c{G}_1) = \sum_{u_{p_i},u_{p_j}\in V(\c{G}_1)} I_{N_1}(u_{p_i})I_{N_1}(u_{p_j})CONN_{I(\c{G}_1)}(u_{p_i},u_{p_j}) \tag{4.10}$$

$$= \sum_{h(u_{p_i}),h(u_{p_j})\in V(\c{G}_2)} I_{N_2}(h(u_{p_i}))I_{N_2}(h(u_{p_j}))CONN_{I(\c{G}_2)}(h(u_{p_i}),h(u_{p_j})) \tag{4.11}$$

$$= I_nCI_N(\c{G}_2) \tag{4.12}$$

$$F_iCI_N(\c{G}_1) = \sum_{u_{p_i},u_{p_j}\in V(\c{G}_1)} F_{N_1}(u_{p_i})F_{N_1}(u_{p_j})CONN_{F(\c{G}_1)}(u_{p_i},u_{p_j}) \tag{4.13}$$

$$= \sum_{h(u_{p_i}),h(u_{p_j})\in V(\c{G}_2)} F_{N_2}(h(u_{p_i}))F_{N_2}(h(u_{p_j}))CONN_{F(\c{G}_2)}(h(u_{p_i}),h(u_{p_j})) \tag{4.14}$$

$$= F_iCI_N(\c{G}_2). \tag{4.15}$$

Thus,
$T_rCI_N(\c{G}_1)+I_nCI_N(\c{G}_1)+F_iCI_N(\c{G}_1) = T_rCI_N(\c{G}_2)+I_nCI_N(\c{G}_2)+F_iCI_N(\c{G}_2$.
This implies that $CI_N(\c{G}_1) = CI_N(\c{G}_2)$. □

5 Neutrosophic Average Connectivity Index graph

The literature on intuitionistic fuzzy graphs contains the idea of average connection index. Therefore, the authors have presented this idea for neutrosophic graphs. The average flow of a network ensures its stability.

Definition 12. *The average T_r-connectivity index of $\c{G}$ is defined by*

$$AT_rCI_N(\c{G}) = \frac{1}{\binom{\kappa}{2}} \sum_{u_p,v_p\in N^*} T_r^N(u_p)T_r^N(v_p)CONN_{T_r(\c{G})}(u_p,v_p) \tag{5.1}$$

the average I_n-connectivity index of $\c{G}$ is defined by

$$AI_nCI_N(\c{G}) = \frac{1}{\binom{\kappa}{2}} \sum_{u_p,v_p\in N^*} I_n^N(u_p)I_n^N(v_p)CONN_{I_n(\c{G})}(u_p,v_p) \tag{5.2}$$

the average F_i-connectivity index of $\c{G}$ is defined by

$$AF_iCI_N(\c{G}) = \frac{1}{\binom{\kappa}{2}} \sum_{u_p,v_p\in N^*} F_i^N(u_p)F_i^N(v_p)CONN_{F_i(\c{G})}(u_p,v_p) \tag{5.3}$$

where $CONN_{T_r(\c{G})}(u_p,v_p)$ is the T_r-strength of connectedness,$CONN_{I_n^N(\c{G})}(u_p,v_p)$ is the I_n-strength of connectedness and $CONN_{F_i(\c{G})}(u_p,v_p)$ is the F_i-strength $u_p - v_p$.

Definition 13. *Let $\c{G} = (\c{N}, \c{M})$ be a neutrosophic graph. Then, the average connectivity index of $\c{G}$ is defined to be the sum of average T_r-connectivity index, I_n-connectivity index and F_i-connectivity index of $\c{G}$,*

$$
\begin{aligned}
ACI_N(\c{G}) = & \frac{1}{\binom{\kappa}{2}} \sum_{u_p, v_p \in N^*} T_r^N(u_p) T_r^N(v_p) CONN_{T_r(\c{G})}(u_p, v_p) \\
& + \frac{1}{\binom{\kappa}{2}} \sum_{u_p, v_p \in N^*} I_n^N(u_p) I_n^N(v_p) CONN_{I_n(\c{G})}(u_p, v_p) \\
& + \frac{1}{\binom{\kappa}{2}} \sum_{u_p, v_p \in N^*} F_i^N(u_p) F_i^N(v_p) CONN_{F_i(\c{G})}(u_p, v_p) \qquad (5.4) \\
& = AT_rCI_N(\c{G}) + AI_nCI_N(\c{G}) + AF_iCI_N(\c{G}) \qquad (5.5)
\end{aligned}
$$

where $CONN_{T_r(\c{G})}(u_p, v_p)$ is the T_r-strength of connectedness,$CONN_{I_n(\c{G})}(u_p, v_p)$ is the I_n-strength of connectedness and $CONN_{F_i(\c{G})}(u_p, v_p)$ is the F_i-strength $u_p - v_p$.

Example 7. *In Fig 7, let $\c{G} = (\c{N}, \c{M})$ be the neutrosophic graph with $(T_r^N(v_p), I_n^N(v_p), F_i^N(v_p)) = (.9, .9, .2) \forall v_p \in N^*$. Then, we have $(T_rCI_N)_n^N(\c{G}) = 1.215, (I_nCI_N)_n^N(\c{G}) = 1.134, (F_iCI_N)_n^N(\c{G}) = 0.056$ and $CI_N = 2.405$.*

From Example 7, $(CI_N)_n^N(\c{G}) = 2.405$ and the number of pair in Ģ is $\binom{4}{6} = 6$. By average in Ģ the $(T_rCI_N)_n^N(\c{G}), (I_nCI_N)_n^N(\c{G}), (F_iCI_N)_n^N(\c{G})$ and $(CI_N)_n^N(\c{G})$, we get

$$
\begin{aligned}
(AT_rCI_N)_n^N(\c{G}) &= \frac{1}{6}(T_rCI_N)_n^N(\c{G}) = \frac{1}{6}(1.215) = 0.2025, \\
(AI_nCI_N)_n^N(\c{G}) &= \frac{1}{6}(I_nCI_N)_n^N(\c{G}) = \frac{1}{6}(1.134) = 0.189, \\
(AF_iCI_N)_n^N(\c{G}) &= \frac{1}{6}(F_iCI_N)_n^N(\c{G}) = \frac{1}{6}(0.056) = 0.0093 \\
(ACI_N)_n^N(\c{G}) &= 0.2025 + 0.189 + 0.0093 = 0.4008.
\end{aligned}
$$

Now,consider $\c{G} - v_{p_4}$ and we have

$T_rCI_N(\c{G} - v_{p_4}) = 0.567, I_nCI_N(\c{G} - v_{p_4}) = 0.648, F_iCI_N(\c{G} - v_{p_4}) = 0.036$ and $CI_N(\c{G} - v_{p_4}) = 0.567 + 0.648 + 0.036 = 1.251$.

On average in Ģ them, we have

$$
\begin{aligned}
AT_rCI_N(\c{G} - v_{p_4}) &= \frac{0.567}{3} = 0.188, \\
AI_nCI_N(\c{G} - v_{p_4}) &= \frac{0.648}{3} = 0.216, \\
AF_iCI_N(\c{G} - v_{p_4}) &= \frac{0.036}{3} = 0.012 \\
ACI_N(\c{G} - v_{p_4}) &= 0.188 + 0.216 + 0.012 = 0.416.
\end{aligned}
$$

By eliminating vertices, Ģ's total connectedness is enhanced v_{p_4} from Ģ.

Definition 14. *Let $\c{G} = (N, M)$ be a neutrosohic graph and $u_p, v_p \in N^*$. Then, u_p is called a neutrosophic connectivity reducing node $(NCRN)$ of $\c{G}$ if $ACI_N(\c{G} - u_p) < ACI_N(\c{G})$.*

u_p is termed as a neutrosophic connectivity enhancing node $(NCEN)$ of $\c{G}$ if $ACI_N(\c{G} - u_p) > ACI_N(\c{G}).u_p$ is said to be a neutrosophic neutral node of $\c{G}$ if $ACI_N(\c{G} - u_p) = ACI_N(\c{G})$.

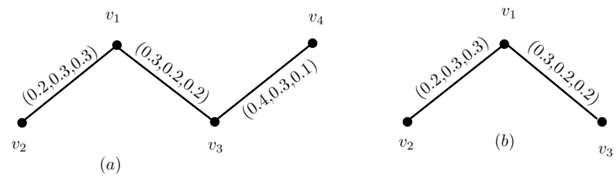

Figure 7. A neutrosophic graph $\c{G} - v_{p_4}$ (a) A neutrosophic graph with $CI_N = 2.405$. (b)$\c{G} - v_{p_4}$.

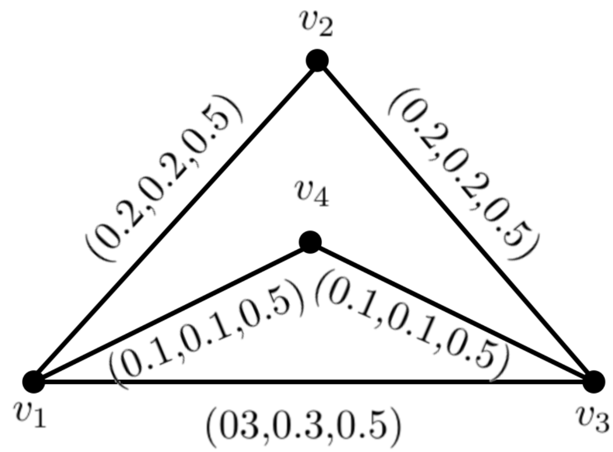

Figure 8. A neutrosophic graph with NCRN,NCEN and neutrosophic neutal nodes.

Example 8. *Consider the neutrosophic graphs shown in Fig 8, We have taken here* $(T_r^N(u_p), I_n^N(u_p), F_i^N(u_p)_= (0.5, 0.5, 0.5), \forall all u_p \in N^*. ACI_N(\c{G}) = 0.2083, ACI_N(\c{G} - v_{p_1}) = 0.191, ACI_N(\c{G} - v_{p_2}) = 0.225, ACI_N(\c{G} - v_{p_3}) = 0.2083, ACI_N(\c{G} - v_{p_4}) = 0.2416.$
$ACI_N(\c{G} - v_{p_1}) < ACI_N(\c{G}), ACI_N(\c{G} - v_{p_3}) = ACI_N(\c{G})$, *and* $ACI_N(\c{G} - v_{p_2}), ACI_N(\c{G} - v_{p_4}) > ACI_N(\c{G})$. *Thus* v_{p_1} *is a NCRN,*v_{p_3} *is a neutral node, and* v_{p_2}, v_{p_4} *are NCEN.*

Theorem 3. *Let* $\c{G} = (\d{N}, \c{M})$ *be a neutrosophic graph and* $u_p \in N^*$ *having* $|N^*| \geq 3$. *Suppose* $r = CI_N(\c{G})/CI_N(\c{G} - u_p).u$ *is a NCEN iff* $r < \kappa/(\kappa - 2)$. u_p *is a NCRN iff* $r > \kappa/(\kappa - 2).u_p$ *is a neutrosophic neutral node iff* $r = \kappa/(\kappa - 2)$.

Proof. Suppose u_p is a neutrosophic neutral node. Then, $ACI_N(\c{G} - u_p) = ACI_N(\c{G})$.
The ACI_N is

$$\frac{1}{\binom{\kappa}{2}} CI_N(\c{G}) = \frac{1}{\binom{\kappa-1}{2}} CI_N(\c{G} - u_p). \tag{5.6}$$

From here, we get

$$\frac{(CI_N)_m^N(\c{G})}{CI_N(\c{G} - u_p)} = \frac{\binom{\kappa}{2}}{\binom{\kappa-1}{2}} \tag{5.7}$$

$$= \frac{\kappa(\kappa-1)/2}{(\kappa-1)(\kappa-2)/2} \tag{5.8}$$

$$= \frac{\kappa}{\kappa - 2}. \tag{5.9}$$

□

Theorem 4. *Suppose* $\c{G} = (\c{N}, \c{M})$ *be a neutrosophic graph with* $|N^*| \geq 3$. *If* $w_p \in N^*$ *is an end vertex of* $\c{G}$, *let* $l = \sum_{u_p \in N^* - w_p} CONN_{T_r(\c{G})}(u_p, w_p) + \sum_{u_p \in N^* - w_p} CONN_{I_n(\c{G})}(u_p, w_p) + \sum_{u_p \in N^* - w_p} CONN_{F_i(\c{G})}(u_p, w_p)$. *Then,*

1. w_p *is a NCEN if* $l < (2/(\kappa - 2))CI_N(\c{G} - w_p)$
2. w_p *is a NCRN if* $l > (2/(\kappa - 2))CI_N(\c{G} - w_p)$
3. w_p *is a neutrosophic neutral node if* $l = (2/(\kappa - 2))CI_N(\c{G} - w_p)$

Proof. Supoose w_p be a neutrosophic neutral node. Then, $ACI_N(\c{G} - w_p) = ACI_N(\c{G})$.

We see that

$$CI_N(\c{G}) = CI_N(\c{G} - w_p) + \sum_{u \in N^* - w} CONN_{T_r(\c{G})}(u_p, w_p) + \sum_{u_p \in N^* - w_p} CONN_{I(\c{G})}(u_p, w_p) + \sum_{u_p \in N^* - w_p} CONN_{F_i(\c{G})}(u_p, w_p) \tag{5.10}$$

$$CI_N(\c{G}) = CI_N(\c{G} - w_p) + l, \tag{5.11}$$

$$l = CI_N(\c{G}) - CI_N(\c{G} - w_p) \tag{5.12}$$

$$\frac{1}{\binom{\kappa}{2}} l = \frac{1}{\binom{\kappa}{2}} CI_N(\c{G}) - \frac{1}{\binom{\kappa}{2}} CI_N(\c{G} - w_p) \tag{5.13}$$

$$= \frac{1}{\binom{\kappa-1}{2}} CI_N(\c{G} - w_p) - \frac{1}{\binom{\kappa}{2}} CI_N(\c{G} - w_p) \tag{5.14}$$

$$= CI_N(\c{G} - w_p) \left[\frac{1}{\binom{\kappa-1}{2}} - \frac{1}{\binom{\kappa}{2}} \right], \tag{5.15}$$

$$l = CI_N(\c{G} - w_p) \left[\frac{\binom{\kappa}{2}}{\binom{\kappa-1}{2}} - 1 \right] \tag{5.16}$$

$$= \frac{2}{\kappa - 2} CI_N(\c{G} - w_p). \tag{5.17}$$

□

6 Application of Neutrosophic graph with Transport Network Flow

Take a neutrosophic directed network $\c{G}$, with traffic flow, as seen in Fig 9 Assume $T_r^N(v_p), I_n^N(v_p), F_i^N(v_p)) = (0.8, 0.8, 0.2)$ for all $v_p \in V(\c{G})$. Undirected neutrosophic graph and directed neutrosophic graph have identical connectivity. Both the directed and undirected neutrosophic graphs have comparable connectedness. Therefore, these notions may be expanded to directed neutrosophic graphs. The junctions at the vertices include correct, indeterminate and incorrect metrics for vehicles. The weights of the edges, which stand for the roadways that connect two junctions, represent the amount of vehicles carrying correct, indeterminant and incorrect information. Now, the authors will talk about certain network flow connection aspects.

First, the corresponding T_r connectivity matrix will be identified; $TM(\c{G})$ is the directed neutrosophic graphs.

$$TM(\c{G}) = \begin{bmatrix} 0 & 0.4 & 0.7 & 0.7 & 0.7 \\ 0.5 & 0 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.6 & 0 & 0.5 & 0.5 \\ 0.5 & 0.6 & 0.7 & 0 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.5 & 0 \end{bmatrix}$$

The matrix above is not symmetric since the graph is directed. We must thus add up every element of the matrix. Thus, $T_rCI_N(\c{G}) = 6.784, AT_rCI_N(\c{G}) = 0.6784$.

Now, the associated I_n-connectivity matrix $IM(\c{G})$ is given as follows

$$IM(\c{G}) = \begin{bmatrix} 0 & 0.4 & 0.5 & 0.4 & 0.7 \\ 0.7 & 0 & 0.4 & 0.6 & 0.7 \\ 0.7 & 0.7 & 0 & 0.6 & 0.7 \\ 0.5 & 0.5 & 0.5 & 0 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.6 & 0 \end{bmatrix}$$

cumulative each component of the matrix. Thus, $I_nCI_N(\c{G}) = 6.976, AI_nCI_N(\c{G}) = 0.6976$.

The corresponding $FM(\c{G})$ matrix for F_i-connectivity is now stated.

$$FM(\c{G}) = \begin{bmatrix} 0 & 0.6 & 0.4 & 0.6 & 0.6 \\ 0.2 & 0 & 0.6 & 0.4 & 0.6 \\ 0.2 & 0.2 & 0 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0 \end{bmatrix}$$

By summing up all of $FM(\c{G})$ entries, we get $F_iCI_N(\c{G}) = 3.600, AF_iCI_N(\c{G}) = 0.360$. Thus, $ACI_N(\c{G}) = AT_rCI_N(\c{G}) + AI_nCI_N(\c{G}) + AF_iCI_N(\c{G}) = 0.6784 + 0.6976 + 0.360 = 1.736$

Consider $\c{G} - v_{p_5}$. Moreover, it is a directed neutrosophic graph.

The matrices $TM(\c{G} - v_{p_5}), IM(\c{G} - v_{p_5})$ and $FM(\c{G} - v_{p_5})$ are given by

$$TM(\c{G} - v_{p_5}) = \begin{bmatrix} 0 & 0.6 & 0.7 & 0.7 \\ 0.5 & 0 & 0.5 & 0.5 \\ 0.5 & 0.6 & 0 & 0.5 \\ 0.5 & 0.6 & 0.7 & 0 \end{bmatrix}$$

$$IM(\c{G} - v_{p_5}) = \begin{bmatrix} 0 & 0.4 & 0.4 & 0.4 \\ 0.7 & 0 & 0.4 & 0.4 \\ 0.7 & 0.7 & 0 & 0.4 \\ 0.5 & 0.5 & 0.5 & 0 \end{bmatrix}$$

$$FM(\c{G} - v_{p_5}) = \begin{bmatrix} 0 & 0.6 & 0.6 & 0.6 \\ 0.2 & 0 & 0.6 & 0.6 \\ 0.2 & 0.2 & 0 & 0.6 \\ 0.4 & 0.4 & 0.4 & 0 \end{bmatrix}$$

by calculation we have

$$T_rCI_N(\c{G}-v_{p_5}) = 4.416, AT_rCI_N(\c{G}-v_{p_5}) = 4.416/6 = 0.736 \tag{6.1}$$

$$I_nCI_N(\c{G}-v_{p_5}) = 3.84, AI_nCI_N(\c{G}-v_{p_5}) = 3.84/6 = 0.464 \tag{6.2}$$

$$F_iCI_N(\c{G}-v_{p_5}) = 0.216, AF_iCI_N(\c{G}-v_{p_5}) = 0.216/6 = 0.036. \tag{6.3}$$

Thus

$$ACI_N(\c{G}-v_{p_5}) = AT_rCI_N(\c{G}-v_{p_5}) + AI_nCI_N(\c{G}-v_{p_5}) + AF_iCI_N(\c{G}-v_{p_5}) \tag{6.4}$$

$$= 0.736 + 0.464 + 0.036 = 1.412. \tag{6.5}$$

As $ACI_N(\c{G}-v_{p_5}) < ACI_N(\c{G})$, which implies that v_{p_5} is $NCRN$. After that, we consider $\c{G}-v_{p_5}$.

The matrices $TM(\c{G}-v_{p_1}), IM(\c{G}-v_{p_1})$ and $FM(\c{G}-v_{p_1})$ its given by

$$TM(\c{G}-v_{p_1}) = \begin{bmatrix} 0 & 0.3 & 0.3 & 0.3 \\ 0.6 & 0 & 0.3 & 0.3 \\ 0.6 & 0.7 & 0 & 0.3 \\ 0.5 & 0.5 & 0.5 & 0 \end{bmatrix}$$

$$IM(\c{G}-v_{p_1}) = \begin{bmatrix} 0 & 0.3 & 0.3 & 0.3 \\ 0.7 & 0 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0 & 0.3 \\ 0.5 & 0.5 & 0.6 & 0 \end{bmatrix}$$

$$FM(\c{G}-v_{p_1}) = \begin{bmatrix} 0 & 0.6 & 0.6 & 0.6 \\ 0.2 & 0 & 0.6 & 0.6 \\ 0.4 & 0.4 & 0 & 0.6 \\ 0.4 & 0.4 & 0.4 & 0 \end{bmatrix}$$

we have

$$T_rCI_N(\c{G}-v_{p_1}) = 3.328, AT_rCI_N(\c{G}-v_{p_1}) = 3.328/6 = 0.555 \tag{6.6}$$

$$I_nCI_N(\c{G}-v_{p_1}) = 3.264, AI_nCI_N(\c{G}-v_{p_1}) = 3.264/6 = 0.544 \tag{6.7}$$

$$F_iCI_N(\c{G}-v_{p_1}) = 2.32, AF_iCI_N(\c{G}-v_{p_1}) = 2.32/6 = 0.3867. \tag{6.8}$$

Thus

$$ACI_N(\c{G}-v_{p_1}) = AT_rCI_N(\c{G}-v_{p_1}) + AI_nCI_N(\c{G}-v_{p_1}) + AF_iCI_N(\c{G}-v_{p_1}) \tag{6.9}$$

$$= 0.555 + 0.544 + 0.3867 = 1.4857. \tag{6.10}$$

As $ACI_N(\c{G}-v_{p_1}) < ACI_N(\c{G})$, which implies that v_{p_1} is $NCRN$.

After that, we consider $\c{G}-v_{p_2}$.

The matrices $TM(\c{G}-v_{p_2}), IM(\c{G}-v_{p_2})$ and $FM(\c{G}-v_{p_2})$ its given by

$$TM(\c{G}-v_{p_2})=\begin{bmatrix}0 & 0.7 & 0.7 & 0.7\\ 0 & 0 & 0 & 0\\ 0 & 0.7 & 0 & 0\\ 0 & 0.5 & 0.5 & 0\end{bmatrix}$$

$$IM(\c{G}-v_{p_2})=\begin{bmatrix}0 & 0.4 & 0.6 & 0.7\\ 0 & 0 & 0 & 0\\ 0 & 0.5 & 0 & 0\\ 0.5 & 0.5 & 0.6 & 0\end{bmatrix}$$

$$FM(\c{G}-v_{p_2})=\begin{bmatrix}0 & 0.6 & 0.6 & 0.4\\ 0 & 0 & 0 & 0\\ 0 & 0.4 & 0 & 0\\ 0 & 0.4 & 0.4 & 0\end{bmatrix}$$

we have

$$T_rCI_N(\c{G}-v_{p_2})=2.432, AT_rCI_N(\c{G}-v_{p_2})=2.432/6=0.4053 \quad (6.11)$$

$$I_nCI_N(\c{G}-v_{p_2})=2.112, AI_nCI_N(\c{G}-v_{p_2})=2.112/6=0.352 \quad (6.12)$$

$$F_iCI_N(\c{G}-v_{p_2})=1.12, AF_iCI_N(\c{G}-v_{p_2})=1.12/6=0.1867. \quad (6.13)$$

Thus

$$ACI_N(\c{G}-v_{p_2})=AT_rCI_N(\c{G}-v_{p_2})+AI_nCI_N(\c{G}-v_{p_2})+AF_iCI_N(\c{G}-v_{p_2}) \quad (6.14)$$

$$=0.4053+0.352+0.1867=0.9444. \quad (6.15)$$

As $ACI_N(\c{G}-v_{p_2})<ACI_N(\c{G})$, which implies that v_{p_2} is $NCRN$.

Now we consider $\c{G}-v_{p_3}$.

The matrices $TM(\c{G}-v_{p_3}), IM(\c{G}-v_{p_3})andFM(\c{G}-v_{p_3})$its given by

$$TM(\c{G}-v_{p_3})=\begin{bmatrix}0 & 0 & 0.7 & 0.7\\ 0.5 & 0 & 0.5 & 0.5\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0.5 & 0\end{bmatrix}$$

$$IM(\c{G}-v_{p_3})=\begin{bmatrix}0 & 0 & 0.4 & 0.7\\ 0 & 0 & 0.6 & 0.7\\ 0 & 0 & 0 & 0\\ 0.5 & 0 & 0.6 & 0\end{bmatrix}$$

$$FM(\c{G}-v_{p_3})=\begin{bmatrix}0&0&0.6&0.4\\0.2&0&0.4&0.4\\0&0&0&0\\0&0&0.4&0\end{bmatrix}$$

we have

$$T_rCI_N(\c{G}-v_{p_3})=2.176, AT_rCI_N(\c{G}-v_{p_3})=2.1762/6=0.3627 \tag{6.16}$$

$$I_nCI_N(\c{G}-v_{p_3})=2.368, AI_nCI_N(\c{G}-v_{p_3})=2.368/6=0.3947 \tag{6.17}$$

$$F_iCI_N(\c{G}-v_{p_3})=0.096, AF_iCI_N(\c{G}-v_{p_3})=0.096/6=0.016. \tag{6.18}$$

Thus

$$ACI_N(\c{G}-v_{p_3})=AT_rCI_N(\c{G}-v_{p_3})+AI_nCI_N(\c{G}-v_{p_3})+AF_iCI_N(\c{G}-v_{p_3}) \tag{6.19}$$

$$=0.3627+0.3947+0.016=0.7734. \tag{6.20}$$

As $ACI_N(\c{G}-v_{p_3})<ACI_N(\c{G})$, which implies that v_{p_3} is $NCRN$.

Now we consider $\c{G}-v_{p_4}$.

The matrices $TM(\c{G}-v_{p_4}), IM(\c{G}-v_{p_4})$ and $FM(\c{G}-v_{p_4})$ its given by

$$TM(\c{G}-v_{p_4})=\begin{bmatrix}0&0.4&0.4&0.7\\0.5&0&0.4&0.5\\0.5&0.6&0&0.5\\0.4&0.4&0.4&0\end{bmatrix}$$

$$IM(\c{G}-v_{p_4})=\begin{bmatrix}0&0.4&0.4&0.7\\0.7&0&0.4&0.7\\0.7&0.7&0&0.7\\0.7&0.4&0.4&0\end{bmatrix}$$

$$FM(\c{G}-v_{p_4})=\begin{bmatrix}0&0.6&0.6&0.4\\0.2&0&0.6&0.4\\0.2&0.2&0&0.4\\0.6&0.6&0.6&0\end{bmatrix}$$

we have

$$T_rCI_N(\c{G}-v_{p_4})=3.648, AT_rCI_N(\c{G}-v_{p_4})=3.648/6=0.608 \tag{6.21}$$

$$I_nCI_N(\c{G}-v_{p_4})=4.416, AI_nCI_N(\c{G}-v_{p_4})=4.416/6=0.736 \tag{6.22}$$

$$F_iCI_N(\c{G}-v_{p_4})=0.216, AF_iCI_N(\c{G}-v_{p_4})=0.216/6=.036. \tag{6.23}$$

Thus

$$ACI_N(\c{G}-v_{p_4}) = AT_rCI_N(\c{G}-v_{p_4}) + AI_nCI_N(\c{G}-v_{p_4}) + AF_iCI_N(\c{G}-v_{p_4}) \tag{6.24}$$

$$= 0.608 + 0.736 + 0.036 = 1.38. \tag{6.25}$$

As $ACI_N(\c{G}-v_{p_4}) < ACI_N(\c{G})$, which implies that v_{p_4} is $NCRN$. So, the removal of junction v_{p_1} increases the average connec-

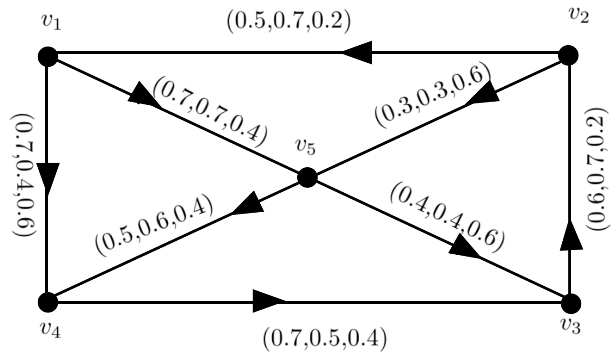

Figure 9. various connection indexes when a vertex is removed.

$\c{G}-v_{pi}$	$ACI_N(\c{G}-v_{pi})$	$\lvert ACI_N(\c{G})-ACI_N(\c{G}-v_{pi})\rvert$
$\c{G}-v_{p_1}$	1.4857	0.2503
$\c{G}-v_{p_2}$	0.944	0.792
$\c{G}-v_{p_3}$	0.7734	0.9626
$\c{G}-v_{p_4}$	1.38	0.356

Table 2. Various connection indexes when a vertex is removed.

tivity amongst the other junctions. Table 2 show that there is a small difference between $ACI_N(\c{G})$ and $ACI_N(\c{G}-v_{p_1})$. So, the removal of v_{p_1} has too much effect on the network.

It is also seen that the difference between $ACI_N(\c{G})$ and $ACI_N(\c{G}-v_{p_3})$ higher than the other differences, so the removal of v_{p_3} has maximum negative effects on the connectivity. The removal of $ACI_N(\c{G}-v_{p_2})$ or $ACI_N(\c{G}-v_{p_4})$ may have indeterminant effects on the traffic network flow.

7 Real Life Application

7.1 Highway System

Accident rates are rising daily as a result of the heavy traffic on the roadways. The government should make considerable efforts to reduce the number of traffic accidents in order to reduce these incidents. To address this issue, a visual representation of neutrosophic graphs is offered here. The average connection indices between each pair of vertices in neutrosophic networks may be calculated to achieve this. The roads with the highest average connection index carry the most traffic and are the main sites of traffic accidents. To reduce accidents on certain roads, the government can build speed bumps, speed breakers, and deploy additional traffic wardens.

7.2 Computer Network

The exchange of data between computers in a network of several computers. The top performing computer or computers in a network that share the most data with all other computers in the network must be identified. The average connection indices between each pair of computers in a network may be computed to do this. The necessary computers for sending the most data to all the other computers in a network will be the pair of computers with the highest average connection indices.

7.3 Advantages

As a result of our examination, the following are the main benefits and traits:

1. Due to the fact that neutrosophic graphs manage uncertain information with three membership graphs, the primary goal of our work is to define the idea of $CI_N s$ in this setting.

2. Neutrosophic graphs are described with the aid of three forms of components: membership, indeterminacy membership, and non-membership, whilst neutrosophic graphs are characterised by simplest one issue.

3. The authors have generalised the findings of $CI_N s$ in neutrosophic graphs in their locating. For instance, if the second feature is ignored, the outcome of $CI_N s$ in neutrosophic graphs appears as a particular case of their results in neutrosophic graphs.

4. In contrast, the investigation proves that neutrosophic graphs might have less facts compared with fuzzy and intutionistic fuzzy graphs.

8 Conclusion

The authors have developed some $CI_N s$ in the neutrosophic framework uncertainty and ambiguity based on three levels of membership. Some important results of this study are:

1. The authors have developed results on CI_N and proposed the neutrosophic graphs classification for $CI_N s$. Additionally, examples are provided to help CI.

2. Also included is CI_N for edge and vertex-delete neutrosophic graphs along with an example.

3. The neutrosophic graph's average connectivity indices are defined.

4. Several connection node types are delivered, including $NCER, NCRN$, and neutrosophic neural nodes, as well as their results.

5. Applications in a network of transport flows have been suggested.

6. Also included two type of real time applications discussed.

Future Work: Future study by the authors will include image neutrosophic fuzzy planer graphs as well as complete neutrosophic fuzzy graphs. The authors also wish to add a few more connection indices to neutrosophic graphs and discuss their possible applications.

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