Quasi-Degree in Neutrosophic Graphs

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Abstract

New setting is introduced to study quasi-degree and quasi-co-degree arising from co-neighborhood. quasi-degree and quasi-co-degree is about a vertex which are applied into the setting of neutrosophic graphs. . The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs and star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-multipartite-neutrosophic graphs are investigated in the terms of a vertex which is called either quasi-degree or quasi-co-degree. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form quasi-degree and quasi-co-degree. Quasi-degree is a value of a vertex which is maximum amid all values of vertices which are neighbors to a fixed vertex. Quasi-co-degree is a value of an edge which is maximum amid all values of edges which are neighbors to a fixed vertex but corresponded vertex is representative for this notion. Using different values which are related to a vertex inspire us to focus on edge and vertices which are corresponded to a fixed vertex. The notion of neighborhood is used to collect either vertices are titled neighbors or edges are incident to fixed vertex. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definitions are provided. Using fixed vertex has key role to have these notions in the form of vertex or edge. The value of an edge has eligibility to call quasi-co-degree but the value of a vertex has eligibility to call quasi-degree. Some results get more frameworks and perspective about these definitions. The way in that, two vertices have connection together, open the way to define neighborhood and co-neighborhood. The maximum values in neighborhood and co-neighborhood introduces quasi-degree and quasi-co-degree, respectively. New name is chosen from degree. Since amid all vertices with different degrees, one vertex is chosen. In other words, one vertex is fixed and its degree turns out quasi-degree where two degrees could be assigned to a vertex. Degree of edges and degree of vertices. The number of edges which are incident to the vertex and the number of vertices which are neighbors to the vertex. Degree and co-degree are the notions which are transformed to use in quasi-style. Two neutrosophic values introduce two neutrosophic vertices separately in each settings. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are

well-known, is an open way to purse this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Quasi-Co-Degree, Quasi-Degree, Vertex
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [16], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2, 14, 15], graphs and new notions on them in **Refs.** [5–12], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [13], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "Quasi-Degree", "Quasi-Co-Degree" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign neutrosophic quasi-degree and neutrosophic quasi-co-degree. Thus they're used to define new ideas which conclude to the structure neutrosophic quasi-degree and neutrosophic quasi-co-degree. The concept of having edge inspires us to study the behavior of edges in the way that, some types of numbers neutrosophic quasi-degree and neutrosophic quasi-co-degree are the cases of study in the settings of individuals. In both settings, a corresponded vertex concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of neutrosophic neighborhood, neutrosophic co-neighborhood, neutrosophic quasi-degree, and neutrosophic quasi-co-degree are highlighted, introduced and are clarified as individuals. In section "Preliminaries", general sets have the key role in this way. General results are obtained and also, the results about the basic notions of neutrosophic neighborhood, neutrosophic co-neighborhood, neutrosophic quasi-degree, and neutrosophic quasi-co-degree are elicited. Some classes of neutrosophic graphs are studied in the terms of neutrosophic quasi-Degree, in section "Setting of Neutrosophic Quasi-Degree," as individuals. In section "Setting of Neutrosophic Quasi-Co-Degree,", neutrosophic quasi-Degre is applied into individuals. As a concluding results, there are some statements, remarks, examples and clarfications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs and star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-multipartite-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Neutrosophic Quasi-Degree," and "Setting of Neutrosophic Quasi-Co-Degree," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for star-neutrosophic graphs and complete-multipartite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are complete as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e \in E} \Sigma_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and $x_{\mathcal{O}}$ is

$$\mu^{\infty}(x, y) = \bigwedge_{P : x_0, x_1, \cdots, x_{\mathcal{O}}} \bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1});$$

$(iv):$ a sequence of vertices $P: x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

$(v):$ it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

$(vi):$ t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2};

$(vii):$ complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2};

$(viii):$ a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2};

$(ix):$ it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

$(x):$ it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

The notions of neighbor and neighborhood are about some vertices which have one edge with a fixed vertex. These notions present vertices which are close to a fixed vertex as possible. Based on different types of edges, it's possible to define different neighborhood as follows.

Definition 1.5. (Neighborhood & Co-Neighborhood).
Let $NTG: (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

(i) **Neighborhood** of x is defined by

$$N(x) = \{y \in V \mid xy \in E\};$$

(ii) **co-neighborhood** of x is defined by

$$N_c(x) = \{e \in E \mid y \in V,\ xy \in E\}.$$

The main definition is presented in next section. The notions of neighborhood and co-neighborhood facilitate the ground to introduce new notions, quasi-degree and quasi-co-degree. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types.

Definition 1.6. (Quasi-Degree & Quasi-Co-Degree).
Let $NTG: (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

(i) **Quasi-degree** of x is defined by

$$\max_{y \in N(x)} \sigma(x)$$

and it's denoted by

$$QDR(x);$$

(ii) **quasi-co-degree** of x is defined by

$$\max_{e \in N_c(x)} \mu(e)$$

and it's denoted by

$$QCD(x).$$

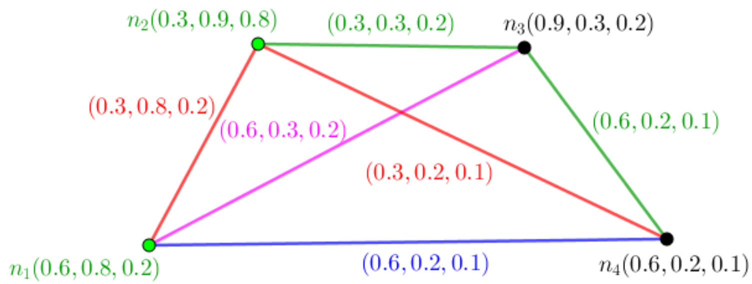

Figure 1. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree and its Neutrosophic Quasi-Co-Degree.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.7. In Figure (1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $N(n_1) = \{n_2, n_3, n_4\}$;

(ii) $N_c(n_1) = \{n_1n_2, n_1n_3, n_1n_4\}$;

(iii) $QDR(n_1) = \sigma(n_2) = (0.3, 0.9, 0.8)$;

(iv) $QCD(n_1) = \mu(n_1n_2) = (0.3, 0.8, 0.2)$;

(v) $N(n_3) = \{n_1, n_2, n_4\}$;

(vi) $N_c(n_3) = \{n_3n_1, n_3n_2, n_3n_4\}$;

(vii) $QDR(n_3) = \sigma(n_2) = (0.3, 0.9, 0.8)$;

(viii) $QCD(n_3) = \mu(n_3n_1) = (0.6, 0.3, 0.2)$;

2 Setting of Neutrosophic Quasi-Degree

In this section, the behaviors of some classes of neutrosophic graphs are studied where the concept of neutrosophic quasi-degree is applied. Parity of number of vertex set isn't considered when the classes are either paths or cycles. There are some efforts to obtain one neutrosophic number in the terms of neutrosophic quasi-degree.

An odd path is a path with leaves with odd indexes. If first leaf is assigned to first number, then the last leaf is also an odd number. Thus by every odd indexes are neighbors of even indexes, the set with minimum numbers which cover all vertices, is the set with vertices which have even indexes. In an even path, if one vertex indexed odd is leaf, then other vertex indexed even is another leaf. Thus odd indexes are as same as even indexes to form quasi-order. As optimal set, mentioned sets are only cases which are related. Other sets have more number of vertices. But these ideas don't work in the setting of neutrosophic quasi-degree. Two neighbors introduce one neighbor amid them to be neutrosophic quasi-degree for every given vertex.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be path-neutrosophic graph and $x \in V$. Then either*

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}$$

or

$$QDR(x) = \sigma(z).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be a path. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_{\mathcal{O}}$ where either $\mathcal{O}$ and 1 has same parity or different parity. There are two types of vertices. If x is a leaf, then there's one neighbor z which implies quasi-degree for x is $\sigma(z)$. It induces

$$QDR(x) = \sigma(z).$$

If x isn't a leaf, then there are two neighbors z, z' which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for two leaves, quasi-order is their unique neighbor but for other vertices there are two choices which the maximum value introduces quasi-degree for given vertex. For two leaves,

$$QDR(x) = \sigma(z).$$

For vertices excluding leaves,

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

□

In next part, one odd-path-neutrosophic graph is depicted. Quasi-degree and its corresponded set are computed. In next part, one even-path-neutrosophic graph is applied to compute its quasi-order and its corresponded set, too.

Example 2.2. There are two sections for clarifications.

(a) In Figure (2), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) $\max\{\sigma(n_2), \sigma(n_4)\}$ is quasi-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (ii) $\max\{\sigma(n_2), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (iii) $\sigma(n_2)$ is quasi-degree for n_3 since $\sigma(n_2) > \sigma(n_4)$;
- (iv) $\sigma(n_1)$ is quasi-degree for n_2 since $\sigma(n_1) > \sigma(n_3)$;
- (v) $\sigma(n_3)$ isn't quasi-degree for n_2 since $\sigma(n_3) \not> \sigma(n_1)$;
- (vi) $\sigma(n_2)$ is quasi-degree for leaf n_1 since $n_1n_i \in E$ implies $n_i = n_2$;
- (vii) $\sigma(n_4)$ is quasi-degree for leaf n_5 since $n_5n_i \in E$ implies $n_i = n_4$.

(b) In Figure (3), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) $\max\{\sigma(n_2), \sigma(n_4)\}$ is quasi-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (ii) $\max\{\sigma(n_2), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (iii) $\sigma(n_4)$ is quasi-degree for n_3 since $\sigma(n_4) > \sigma(n_2)$;

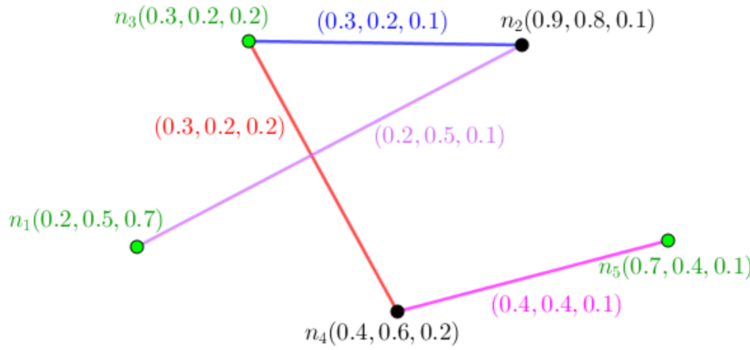

Figure 2. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree

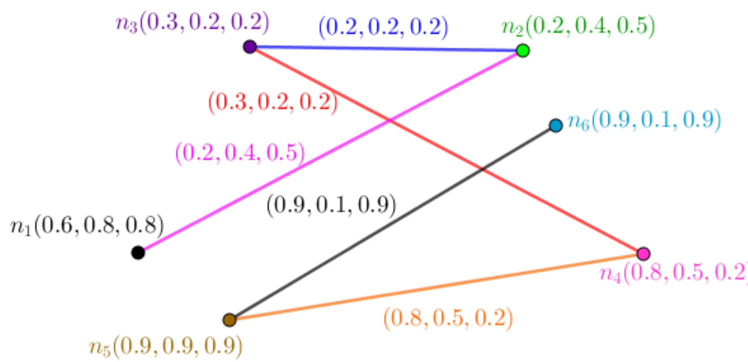

Figure 3. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

(*iv*) $\sigma(n_1)$ is quasi-degree for n_2 since $\sigma(n_1) > \sigma(n_3)$;

(*v*) $\sigma(n_3)$ isn't quasi-degree for n_2 since $\sigma(n_3) \not> \sigma(n_1)$;

(*vi*) $\sigma(n_2)$ is quasi-degree for leaf n_1 since $n_1n_i \in E$ implies $n_i = n_2$;

(*vii*) $\sigma(n_4)$ is quasi-degree for leaf n_6 since $n_6n_i \in E$ implies $n_i = n_5$.

Indexes in odd cycles imply first index and last index have same parity. In this case, vertices concerning odd indexes have more number of members than vertices concerning even indexes but both sets introduce quasi-order. Optimal set is a set of vertices having even indexes and this set points out a quasi-order which is minimum amid all quasi-order. Even cycle has vertices which could be assigned by indexes. In this case, the first vertex and last vertex has different parity. Thus a set of vertices containing even indexes has as same number of members as set of vertices containing odd indexes has. Thus these sets are optimal and they introduce optimal number titled quasi-order. But these ideas don't work in the setting of neutrosophic quasi-degree. Two neighbors introduce one neighbor amid them to be neutrosophic quasi-degree for every given vertex.

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be cycle-neutrosophic graph and $x \in V$. Then*

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be a cycle. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_{\mathcal{O}}, x_1$ where either $\mathcal{O}$ and 1 has same parity or different parity. There are two types of vertices. If x is a leaf, then there's one neighbor z which implies quasi-degree for x is $\sigma(z)$. It induces $QDR(x) = \sigma(z)$. But x isn't a leaf in any given cycle, then there are two neighbors z, z' which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for every given vertices, there are two choices which the maximum value introduces quasi-degree for given vertex. For every given vertex without any exception,

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.4. There are two sections for clarifications.

(a) In Figure (5), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) $\max\{\sigma(n_2), \sigma(n_4)\}$ is quasi-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (ii) $\max\{\sigma(n_2), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (iii) $\sigma(n_4)$ is quasi-degree for n_3 since $\sigma(n_4) > \sigma(n_2)$;
- (iv) $\sigma(n_3)$ is quasi-degree for n_2 since $\sigma(n_3) > \sigma(n_1)$;
- (v) $\sigma(n_1)$ isn't quasi-degree for n_2 since $\sigma(n_1) \not> \sigma(n_3)$;
- (vi) $\sigma(n_2)$ is quasi-degree for n_1 since $n_1n_i \in E$ implies $n_i = n_2, n_5$ and $\sigma(n_2) > \sigma(n_5)$;
- (vii) $\sigma(n_4)$ is quasi-degree for n_5 since $n_5n_i \in E$ implies $n_i = n_4, n_1$and $\sigma(n_4) > \sigma(n_1)$.

(b) In Figure (4), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) $\max\{\sigma(n_2), \sigma(n_4)\}$ is quasi-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (ii) $\max\{\sigma(n_2), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (iii) $\sigma(n_2)$ is quasi-degree for n_3 since $\sigma(n_2) > \sigma(n_4)$;
- (iv) $\sigma(n_3)$ is quasi-degree for n_2 since $\sigma(n_3) > \sigma(n_1)$;
- (v) $\sigma(n_1)$ isn't quasi-degree for n_2 since $\sigma(n_1) \not> \sigma(n_3)$;
- (vi) $\sigma(n_2)$ is quasi-degree for n_1 since $n_1n_i \in E$ implies $n_i = n_2, n_6$ and $\sigma(n_2) > \sigma(n_6)$;
- (vii) $\sigma(n_1)$ is quasi-degree for n_6 since $n_6n_i \in E$ implies $n_i = n_5, n_1$ and $\sigma(n_1) > \sigma(n_5)$;

A complete-neutrosophic graph is considered in next part. In complete-neutrosophic graph, all vertices have same numbers of neighbors. Thus finding one neighbor between all neighbors is difficult in the terms of quasi-degree.

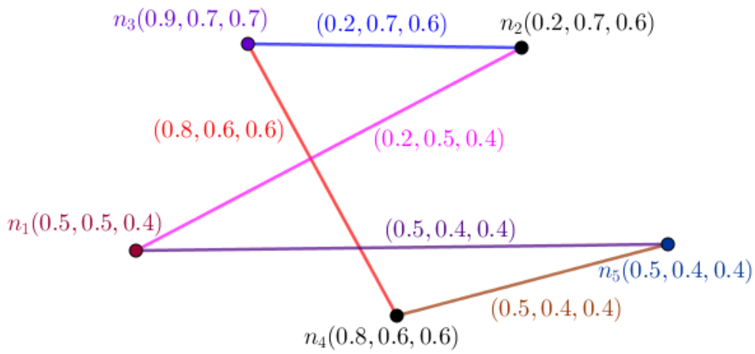

Figure 4. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree

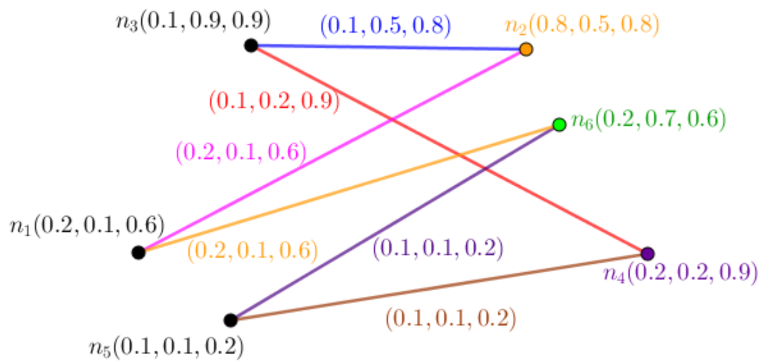

Figure 5. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

Proposition 2.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be complete-neutrosophic graph and* $x \in V$. *Then*

$$QDR(x) = \max\{\sigma(z_1), \cdots, \sigma(z_{\mathcal{O}-1})\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be complete-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are one type of vertices. If x is a leaf, then there's one neighbor z which implies quasi-degree for x is $\sigma(z)$ where $\mathcal{O} = 2$. It induces $QDR(x) = \sigma(z)$. If x isn't a leaf in a given complete-neutrosophic graph, then there are two neighbors z, z' which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$ where $\mathcal{O} = 3$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for every given vertices, there are $\mathcal{O} - 1$ choices which the maximum value introduces quasi-degree for given vertex. For every given vertex without any exception,

$$QDR(x) = \max\{\sigma(z_1), \cdots, \sigma(z_{\mathcal{O}-1})\}.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

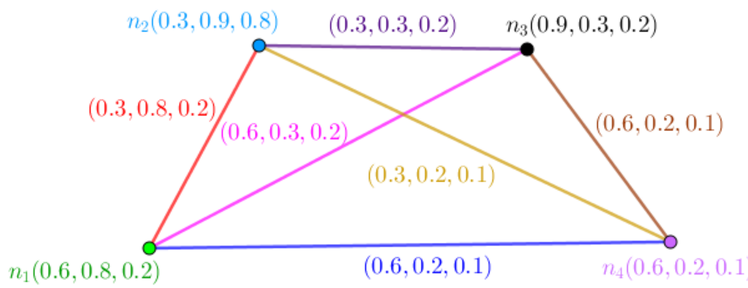

Figure 6. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

Example 2.6. There is one section for clarifications. In Figure (15), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $\max\{\sigma(n_1), \sigma(n_2), \sigma(n_4)\}$ is quasi-degree for n_3 since $n_3n_1, n_3n_2, n_3n_4 \in E$;

(ii) $\max\{\sigma(n_3), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_2 since $n_in_3, n_in_4 \in E,\ i \neq 2$ and $\max\{\sigma(n_i)\}_{i=1}^4 = \sigma(n_2)$;

(iii) $\sigma(n_2)$ is quasi-degree for n_3 since $\sigma(n_2) > \sigma(n_1), \sigma(n_4)$;

(iv) $\sigma(n_1)$ is quasi-degree for n_2 since $\sigma(n_1) > \sigma(n_3), \sigma(n_4)$;

(v) $\sigma(n_3)$ isn't quasi-degree for n_2 since $\sigma(n_3) \not> \sigma(n_1)$;

(vi) $\sigma(n_2)$ is quasi-degree for n_1 since $\max\{\sigma(n_i)\}_{i=1}^4 = \sigma(n_2)$;

(vii) $\sigma(n_4)$ isn't quasi-degree for n_3 since $\sigma(n_4) \not> \max\{\sigma(n_i)\}_{i=1}^4$.

A star, has a center which is connected to all other vertices. A center has common edge with every given vertex. Thus center has $\mathcal{O} - 1$ choices but leaves have only one choice.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be star-neutrosophic graph and $x \in V$. Then either*

$$QDR(x) = \max\{\mu(z_1), \cdots, \mu(z_{\mathcal{O}-1})\}$$

or

$$QDR(x) = \mu(z).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be star-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is a leaf, then there's one neighbor z which implies quasi-degree for x is $\sigma(z)$. It induces

$$QDR(x) = \sigma(z).$$

If x isn't a leaf in a given star-neutrosophic graph, then there are two neighbors z, z' which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$ where $\mathcal{O} = 3$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for every given center excluding leaves, there are $\mathcal{O} - 1$ choices which the maximum value introduces quasi-degree for given center. For every center without any exception,

$$QDR(x) = \max\{\sigma(z_1), \cdots, \sigma(z_{\mathcal{O}-1})\}.$$

□

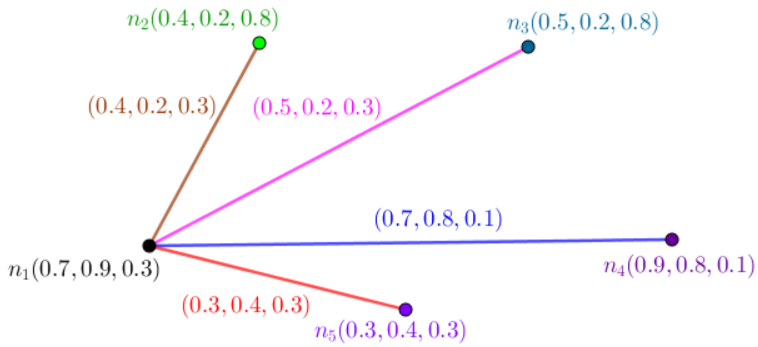

Figure 7. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.8. There is one section for clarifications. In Figure (20), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) $\max\{\sigma(n_2), \sigma(n_3), \sigma(n_4), \sigma(n_5)\}$ is quasi-degree for n_1 since $n_1n_2, n_1n_3, n_1n_4, n_1n_5 \in E$;

(*ii*) $\max\{\sigma(n_3), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_1 since $n_in_3, n_in_4 \notin E,\ i \neq 1$;

(*iii*) $\sigma(n_4)$ is quasi-degree for n_1 since $\max\{\sigma(n_i)\}_{i=2}^{5} = \sigma(n_4)$;

(*iv*) $\sigma(n_1)$ is quasi-degree for leaf n_2 since $n_in_2 \in E$ implies $i = 1$;

(*v*) $\sigma(n_3)$ isn't quasi-degree for leaf n_2 since $n_2n_3 \notin E$;

(*vi*) $\sigma(n_2)$ isn't quasi-degree for n_1 since $\sigma(n_2) \not> \sigma(n_3), \sigma(n_4), \sigma(n_5)$;

(*vii*) $\sigma(n_4)$ isn't quasi-degree for n_3 since $n_3n_4 \notin E$.

In a complete neutrosophic graph, one vertex has common edges with all given vertices. In complete-bipartite-neutrosophic graph, there are two parts and vertex set is partitioned into two sets which have complete connections with each other but inside, there's no connection. Thus the number of neighbors for every given vertex is exactly the number of vertices in other part. So there are some choices for quasi-degree, which are the number of vertices in another part.

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be complete-bipartite-neutrosophic graph and $x \in V$. Then*

$$QDR(x) = \max_{z_i \in V_1^t} \{\sigma(z_1), \cdots, \sigma(z_t)\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be complete-bipartite-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is in first part, then there's one neighbor z which implies quasi-degree for x is $\sigma(z)$. It induces

$$QDR(x) = \sigma(z).$$

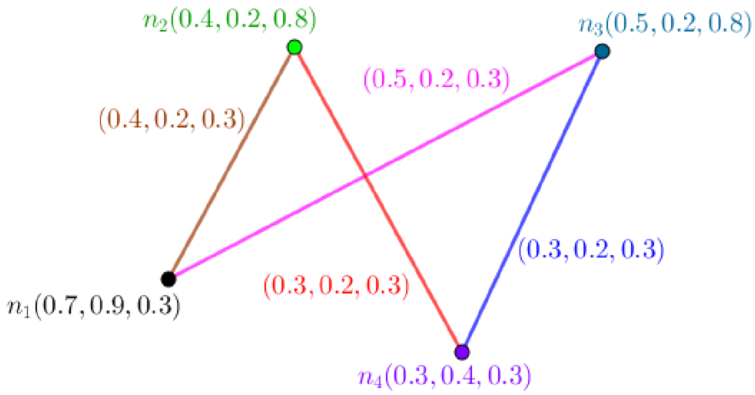

Figure 8. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

If x isn't a leaf in a given complete-bipartite-neutrosophic graph, then there are two neighbors z, z' which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$ where $\mathcal{O} = 3$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for every given vertex in second part, there are $t = |V_1^t|$ choices which the maximum value introduces quasi-degree. For every every given vertex in second part without any exception,

$$QDR(x) = \max_{z_i \in V_1^t} \{\mu(z_1), \cdots, \mu(z_t)\}.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $\max\{\sigma(n_2), \sigma(n_3)\}$ is quasi-degree for n_1 since $n_1n_2, n_1n_3 \in E$;

(ii) $\max\{\sigma(n_2), \sigma(n_4)\}$ isn't quasi-degree for a given vertex since either $n_1n_4 \notin E$ or $\sigma(n_2) \not> \sigma(n_3)$;

(iii) $\sigma(n_1)$ is quasi-degree for n_2 since $\max\{\sigma(n_i)\}_{i=1,4} = \sigma(n_1)$;

(iv) $\sigma(n_1)$ isn't quasi-degree for n_4 since $n_in_4 \in E$ implies $i = 2, 3$;

(v) $\sigma(n_3)$ isn't quasi-degree for n_2 since $n_2n_3 \notin E$;

(vi) $\sigma(n_2)$ isn't quasi-degree for n_1 since $\sigma(n_2) \not> \sigma(n_3)$;

(vii) $\sigma(n_4)$ isn't quasi-degree for n_1 since $n_1n_4 \notin E$.

In a complete neutrosophic graph, one vertex has common edges with all given vertices. In complete-multipartite-neutrosophic graph, there are some parts and vertex set is partitioned into some sets which have complete connections with each other but inside, there's no connection. Thus the number of neighbors for every given vertex is exactly the number of vertices in other parts. So there are some choices for quasi-degree, which are the number of vertices in another parts.

Proposition 2.11. *Let $NTG : (V, E, \sigma, \mu)$ be complete-multipartite-neutrosophic graph and $x \in V$. Then*

$$QDR(x) = \max_{z_i \in (V_1^{t_1} \cup V_2^{t_2} \cup \cdots \cup V_s^{t_{\mathcal{O}}}) - V_j^{t'}} \{\sigma(z_1), \cdots, \sigma(z_{(t_1+t_2+\cdots+t_{\mathcal{O}})-t'})\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be complete-multipartite-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is in first part, then there's one neighbor z which implies quasi-degree for x is $\sigma(z)$. It induces

$$QDR(x) = \sigma(z).$$

If x isn't a leaf in a given complete-multiartite-neutrosophic graph, then there are two neighbors z, z' which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$ where $\mathcal{O} = 3$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for every given vertex in second part, there are $t = |V_1^t|$ choices which the maximum value introduces quasi-degree. For every every given vertex in second part without any exception,

$$QDR(x) = \max_{z_i \in V_1^t} \{\mu(z_1), \cdots, \mu(z_t)\}.$$

Now consider, there are more parts. There are some parts and vertex set is partitioned into some sets which have complete connections with each other but inside, there's no connection. Thus the number of neighbors for every given vertex is exactly the number of vertices in other parts. So there are some choices for quasi-degree, which are the number of vertices in another parts. It induces

$$QDR(x) = \max_{z_i \in (V_1^{t_1} \cup V_2^{t_2} \cup \cdots \cup V_s^{t_{\mathcal{O}}}) - V_j^{t'}} \{\mu(z_1), \cdots, \mu(z_{(t_1+t_2+\cdots+t_{\mathcal{O}})-t'})\}.$$

□

The clarifications about results are in progress as follows. A complete-multipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-multipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.12. There is one section for clarifications. In Figure (9), a complete-multipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) $\max\{\sigma(n_2), \sigma(n_3), \sigma(n_5)\}$ is quasi-degree for n_1 since $n_1n_2, n_1n_3, n_1n_5 \in E$;

(*ii*) $\max\{\sigma(n_2), \sigma(n_4)\}$ isn't quasi-degree for a given vertex since either $n_1n_4 \notin E$ or $\sigma(n_2) \not> \sigma(n_3)$;

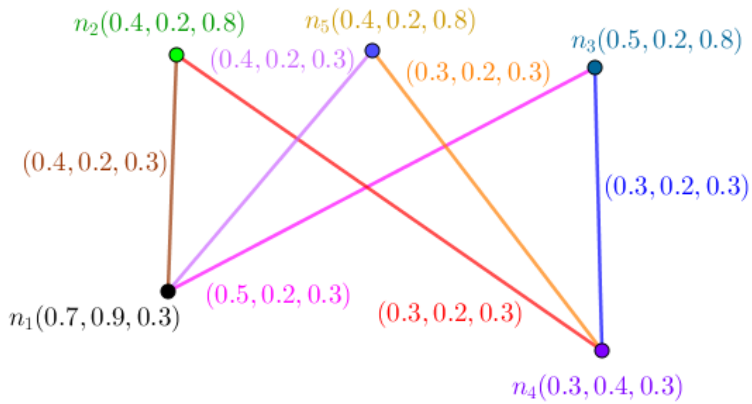

Figure 9. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

(*iii*) $\sigma(n_1)$ is quasi-degree for n_2 since $\max\{\sigma(n_i)\}_{i=1,4} = \sigma(n_1)$;

(*iv*) $\sigma(n_1)$ isn't quasi-degree for n_4 since $n_i n_4 \in E$ implies $i = 2, 3, 5$;

(*v*) $\sigma(n_3)$ isn't quasi-degree for n_2 since $n_2 n_3 \notin E$;

(*vi*) $\sigma(n_2)$ isn't quasi-degree for n_1 since $\sigma(n_2) \not> \sigma(n_3)$;

(*vii*) $\sigma(n_4)$ isn't quasi-degree for n_1 since $n_1 n_4 \notin E$.

A wheel-neutrosophic graph is a graph which consists of a path and a vertex joining to all vertices of path. Thus specific vertex is called center and it has $\mathcal{O}(NTG) - 1$ choices. But other vertices have three choices which one of them is the center.

Proposition 2.13. *Let $NTG : (V, E, \sigma, \mu)$ be wheel-neutrosophic graph and $x \in V$. Then either*

$$QDR(x) = \max\{\sigma(z_1), \cdots, \sigma(z_{\mathcal{O}(NTG)-1})\}$$

or

$$QDR(x) = \max\{\sigma(z), \sigma(z'), \sigma(z'')\}$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be wheel-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is in path, then there are three neighbors z, z', z'' which imply quasi-degree for x is one of $\sigma(z), \sigma(z')$ or $\sigma(z'')$. It induces

$$QDR(x) = \max\{\mu(z), \mu(z'), \mu(z'')\}$$

If x isn't in a path, then there are $\mathcal{O}(NTG) - 1$ neighbors $z_1, z_2, \cdots, z_{\mathcal{O}(NTG)-1}$ which imply quasi-degree for x is either $\sigma(z)$ or $\sigma(z')$ where $\mathcal{O}(NTG) = 3$. It induces

$$QDR(x) = \max\{\sigma(z), \sigma(z')\}.$$

To sum it up, for every given vertex in path, there are three choices which the maximum value introduces quasi-degree. For every every given vertex in path without any exception,

$$QDR(x) = \max\{\mu(z), \mu(z'), \mu(z'')\}$$

Now consider, there are more vertices. There are two parts and vertex set is partitioned into one set which is path. Thus the number of neighbors for center is exactly the

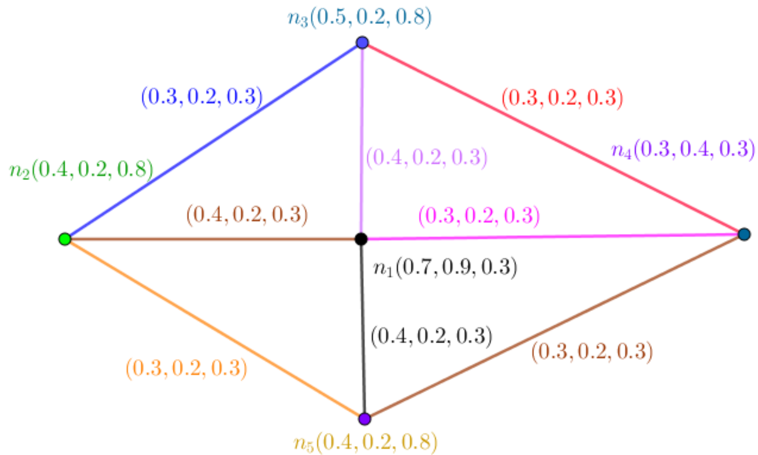

Figure 10. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

number of vertices minus one $\mathcal{O}(NTG) - 1$. So there are some choices for quasi-degree, which are the number of vertices in path. It induces

$$QDR(x) = \max\{\mu(z_1), \cdots, \mu(z_{\mathcal{O}(NTG)-1})\}$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.14. There is one section for clarifications. In Figure (10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $\max\{\sigma(n_2), \sigma(n_3), \sigma(n_4), \sigma(n_5)\}$ is quasi-degree for n_1 since $n_1n_2, n_1n_3, n_1n_4, n_1n_5 \in E$;

(ii) $\max\{\sigma(n_3), \sigma(n_4)\}$ isn't quasi-degree for a given vertex since either $n_2n_4 \notin E$ or $\sigma(n_3) \not> \sigma(n_1)$;

(iii) $\sigma(n_1)$ is quasi-degree for n_2 since $\max\{\sigma(n_i)\}_{i=2}^5 = \sigma(n_1)$;

(iv) $\sigma(n_2)$ isn't quasi-degree for n_4 since $n_in_4 \in E$ implies $i = 1, 3, 5$;

(v) $\sigma(n_3)$ isn't quasi-degree for n_5 since $n_3n_5 \notin E$;

(vi) $\sigma(n_2)$ isn't quasi-degree for n_1 since $\sigma(n_2) \not> \sigma(n_3)$;

(vii) $\sigma(n_1)$ is quasi-degree for every given vertex since $n_1n_i \in E,\ i = 1, 2, 3, 4, 5$ and $\sigma(n_1) = \max\{\sigma(n_2), \sigma(n_3), \sigma(n_4), \sigma(n_5)\}$.

3 Setting of Neutrosophic Quasi-Co-Degree

In this section, the behaviors of some classes of neutrosophic graphs are studied where the concept of neutrosophic quasi-co-degree is applied. Parity of number of vertex set isn't considered when the classes are paths or cycles. There are some efforts to obtain one neutrosophic number in the terms of neutrosophic quasi-co-degree.

An odd path is a path with leaves with odd indexes. If first leaf is assigned to first number, then the last leaf is also an odd number. Thus by every odd indexes are neighbors of even indexes, the set with minimum numbers which cover all vertices, is the set with vertices which have even indexes. In an even path, if one vertex indexed odd is leaf, then other vertex indexed even is another leaf. Thus odd indexes are as same as even indexes to form quasi-order. As optimal set, mentioned sets are only cases which are related. Other sets have more number of vertices. But these ideas don't work in the setting of neutrosophic quasi-degree. Two neighbors introduce one neighbor amid them to be neutrosophic quasi-degree for every given vertex.

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be path-neutrosophic graph and $x \in V$. Then either*

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}$$

or

$$QCD(x) = \mu(xz).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be a path. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_{\mathcal{O}}$ where either $\mathcal{O}$ and 1 has same parity or different parity. There are two types of vertices. If x is a leaf, then there's one neighbor z which implies quasi-co-degree for x is $\mu(xz)$. It induce

$$QCD(x) = \mu(xz).$$

If x isn't a leaf, then there's two neighbors z, z' which imply quasi-co-degree for x is either $\mu(xz)$ or $\mu(xz')$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for two leaves, quasi-co-degree is their unique edge but for other vertices there are two choices which the maximum value introduces quasi-co-degree for given vertex. For two leaves,

$$QCD(x) = \mu(xz).$$

For vertices excluding leaves,

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

□

In next part, one odd-path-neutrosophic graph is depicted. Quasi-co-degree and its corresponded set are computed. In next part, one even-path-neutrosophic graph is applied to compute its quasi-order and its corresponded set, too.

Example 3.2. There are two sections for clarifications.

(a) In Figure (11), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) $\max\{\mu(n_3n_2), \mu(n_3n_4)\}$ is quasi-co-degree for n_3 since $n_3n_2, n_3n_4 \in E$;

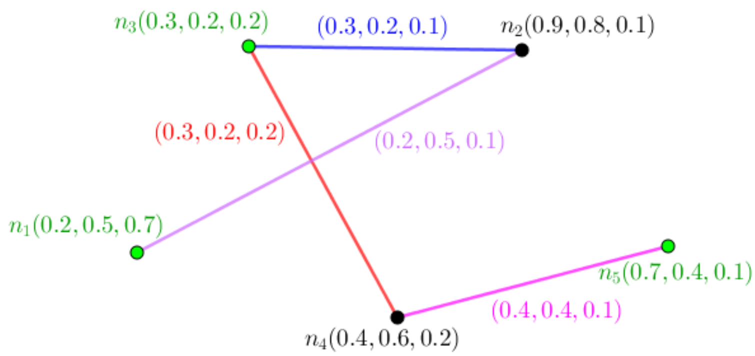

Figure 11. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree

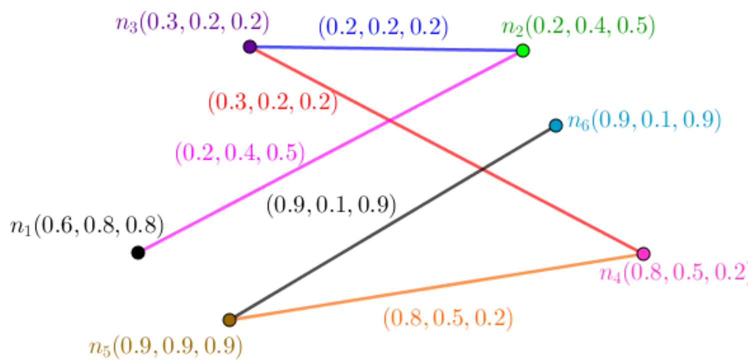

Figure 12. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

- (ii) $\max\{\mu(n_in_2), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (iii) $\mu(n_3n_2)$ is quasi-co-degree for n_3 since $\mu(n_3n_2) > \mu(n_3n_4)$;
- (iv) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\mu(n_2n_1) > \mu(n_2n_3)$;
- (v) $\mu(n_2n_3)$ isn't quasi-co-degree for n_2 since $\mu(n_2n_3) \not> \mu(n_2n_1)$;
- (vi) $\mu(n_1n_2)$ is quasi-co-degree for leaf n_1 since $n_1n_i \in E$ implies $n_i = n_2$;
- (vii) $\mu(n_5n_4)$ is quasi-co-degree for leaf n_5 since $n_5n_i \in E$ implies $n_i = n_4$.

(b) In Figure (12), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) $\max\{\mu(n_3n_2), \mu(n_3n_4)\}$ is quasi-co-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (ii) $\max\{\mu(n_in_2), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (iii) $\mu(n_3n_4)$ is quasi-co-degree for n_3 since $\mu(n_3n_4) > \mu(n_3n_2)$;
- (iv) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\mu(n_2n_1) > \mu(n_2n_3)$;
- (v) $\mu(n_2n_3)$ isn't quasi-co-degree for n_2 since $\mu(n_2n_3) \not> \mu(n_2n_1)$;
- (vi) $\mu(n_1n_2)$ is quasi-co-degree for leaf n_1 since $n_1n_i \in E$ implies $n_i = n_2$;
- (vii) $\mu(n_6n_4)$ is quasi-co-degree for leaf n_6 since $n_6n_i \in E$ implies $n_i = n_5$.

Indexes in odd cycles imply first index and last index have same parity. In this case, vertices concerning odd indexes have more number of members than vertices concerning even indexes but both sets introduce quasi-order. Optimal set is a set of vertices having even indexes and this set points out a quasi-order which is minimum amid all quasi-order. Even cycle has vertices which could be assigned by indexes. In this case, the first vertex and last vertex has different parity. Thus a set of vertices containing even indexes has as same number of members as set of vertices containing odd indexes has. Thus these sets are optimal and they introduce optimal number titled quasi-order. But these ideas don't work in the setting of neutrosophic quasi-co-degree. Two neighbors introduce one edge amid them to be neutrosophic quasi-co-degree for every given vertex.

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be cycle-neutrosophic graph and $x \in V$. Then*

$$QDR(x) = \max\{\mu(xz), \mu(xz')\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be a cycle. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_{\mathcal{O}}, x_1$ where either $\mathcal{O}$ and 1 has same parity or different parity. There are two types of vertices. If x is a leaf, then there's one neighbor z which implies quasi-co-degree for x is $\mu(xz)$. It induce $QCD(x) = \mu(xz)$. But x isn't a leaf in any given cycle, then there's two neighbors z, z' which imply quasi-co-degree for x is either $\mu(xz)$ or $\mu(xz')$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for every given vertices, there are two choices which the maximum value introduces quasi-co-degree for given vertex. For every given vertex without any exception,

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.4. There are two sections for clarifications.

(*a*) In Figure (14), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (*i*) $\max\{\mu(n_3n_2), \mu(n_3n_4)\}$ is quasi-co-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (*ii*) $\max\{\mu(n_in_2), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \notin E,\ i \neq 3$;
- (*iii*) $\mu(n_3n_4)$ is quasi-co-degree for n_3 since $\mu(n_3n_4) > \mu(n_3n_2)$;
- (*iv*) $\mu(n_2n_3)$ is quasi-co-degree for n_2 since $\mu(n_2n_3) > \mu(n_2n_1)$;
- (*v*) $\mu(n_2n_1)$ isn't quasi-co-degree for n_2 since $\mu(n_2n_1) \not> \mu(n_2n_3)$;
- (*vi*) $\mu(n_1n_5)$ is quasi-co-degree for n_1 since $n_1n_i \in E$ implies $n_i = n_2, n_5$ and $\mu(n_1n_5) > \mu(n_1n_2)$;
- (*vii*) $\mu(n_5n_4)$ is quasi-co-degree for n_5 since $n_5n_i \in E$ implies $n_i = n_4, n_1$ and $\mu(n_5n_4) = \mu(n_5n_1)$;

(*b*) In Figure (4), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

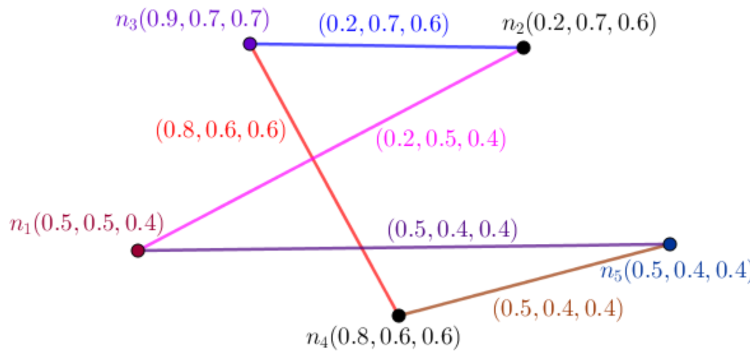

Figure 13. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree

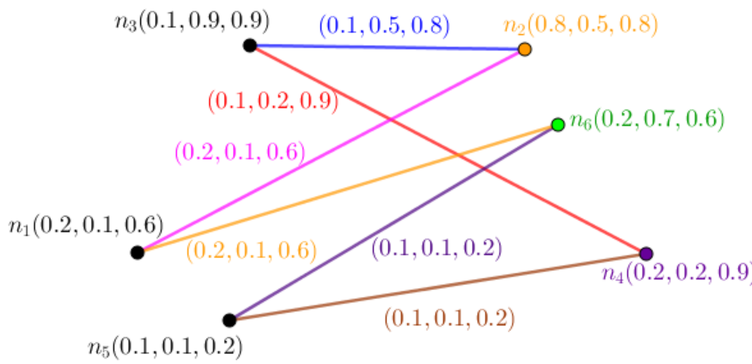

Figure 14. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

- (i) $\max\{\mu(n_3n_2), \mu(n_3n_4)\}$ is quasi-co-degree for n_3 since $n_3n_2, n_3n_4 \in E$;
- (ii) $\max\{\mu(n_in_2), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex excluding n_3 since $n_in_2, n_in_4 \in E,\ i \neq 3$;
- (iii) $\mu(n_3n_2)$ is quasi-co-degree for n_3 since $\mu(n_3n_2) > \mu(n_3n_4)$;
- (iv) $\mu(n_2n_3)$ is quasi-co-degree for n_2 since $\mu(n_2n_3) > \mu(n_2n_1)$;
- (v) $\mu(n_2n_1)$ isn't quasi-co-degree for n_2 since $\mu(n_2n_1) \not> \mu(n_2n_3)$;
- (vi) $\mu(n_1n_2)$ is quasi-co-degree for n_1 since $n_1n_i \in E$ implies $n_i = n_2, n_6$ and $\mu(n_1n_2) = \mu(n_1n_6)$;
- (vii) $\mu(n_6n_1)$ is quasi-co-degree for n_6 since $n_6n_i \in E$ implies $n_i = n_5, n_1$ and $\mu(n_6n_1) > \mu(n_6n_5)$;

A complete-neutrosophic graph is considered in next part. In complete-neutrosophic graph, all vertices have same numbers of neighbors. Thus finding one neighbor between all neighbors are difficult in the terms of quasi-co-degree.

Proposition 3.5. *Let $NTG : (V, E, \sigma, \mu)$ be complete-neutrosophic graph and $x \in V$. Then*

$$QCD(x) = \max\{\mu(xz_1), \cdots, \mu(xz_{\mathcal{O}-1})\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be complete-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are one type of vertices. If x is a leaf, then there's one neighbor z which implies quasi-degree for x is $\mu(xz)$ where $\mathcal{O} = 2$. It

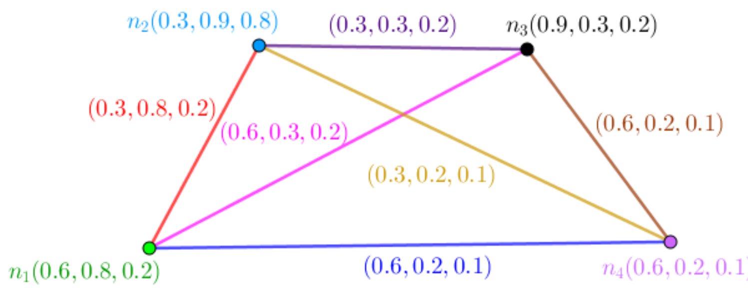

Figure 15. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

induces $QCD(x) = \mu(xz)$. If x isn't a leaf in a given complete-neutrosophic graph, then there's two neighbors z, z' which imply quasi-degree for x is either $\mu(xz)$ or $\mu(xz')$ where $\mathcal{O} = 3$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for every given vertices, there are $\mathcal{O} - 1$ choices which the maximum value introduces quasi-co-degree for given vertex. For every given vertex without any exception,

$$QCD(x) = \max\{\mu(xz_1), \cdots, \mu(xz_{\mathcal{O}-1})\}.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There is one section for clarifications. In Figure (15), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $\max\{\mu(n_3n_1), \mu(n_3n_2), \mu(n_3n_4)\}$ is quasi-co-degree for n_3 since $n_3n_1, n_3n_2, n_3n_4 \in E$;

(ii) $\max\{\mu(n_in_3), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex excluding n_2 since $n_in_3, n_in_4 \in E,\ i \neq 2$ and $\max\{\mu(n_in_j)\}_{i,j=1,i\neq j}^4 = \mu(n_2n_1)$;

(iii) $\mu(n_3n_1)$ is quasi-co-degree for n_3 since $\mu(n_3n_1) > \mu(n_3n_2), \mu(n_3n_4)$;

(iv) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\mu(n_2n_1) > \mu(n_2n_3), \mu(n_2n_4)$;

(v) $\mu(n_2n_3)$ isn't quasi-co-degree for n_2 since $\mu(n_2n_3) \not> \mu(n_2n_1)$;

(vi) $\mu(n_2)$ is quasi-co-degree for n_1 since $\max\{\mu(n_1n_i)\}_{i=2}^4 = \mu(n_1n_2)$;

(vii) $\mu(n_3n_2)$ isn't quasi-co-degree for n_3 since $\mu(n_3n_2) \not> \max\{\mu(n_3n_i)\}_{i=1,i\neq3}^4$.

A star, has a center which is connected to all other vertices. A center has common edge with every given vertex. Thus center has $n-1$ choices but leaves have only one choice.

Proposition 3.7. *Let $NTG : (V, E, \sigma, \mu)$ be star-neutrosophic graph and $x \in V$. Then either*

$$QCD(x) = \max\{\mu(xz_1), \cdots, \mu(xz_{\mathcal{O}-1})\}$$

or

$$QCD(x) = \mu(xz).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be star-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is a leaf, then there's one neighbor z which implies quasi-co-degree for x is $\mu(xz)$. It induces

$$QCD(x) = \mu(xz).$$

If x isn't a leaf in a given star-neutrosophic graph, then there's two neighbors z, z' which imply quasi-co-degree for x is either $\mu(xz)$ or $\mu(xz')$ where $\mathcal{O} = 3$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for every given center excluding leaves, there are $\mathcal{O} - 1$ choices which the maximum value introduces quasi-co-degree for given center. For every center without any exception,

$$QCD(x) = \max\{\mu(xz_1), \cdots, \mu(xz_{\mathcal{O}-1})\}.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) $\max\{\mu(n_1n_2), \mu(n_1n_3), \mu(n_1n_4), \mu(n_1n_5)\}$ is quasi-co-degree for n_1 since $n_1n_2, n_1n_3, n_1n_4, n_1n_5 \in E$;

(*ii*) $\max\{\mu(n_in_3), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex excluding n_1 since $n_in_3, n_in_4 \notin E,\ i \neq 1$;

(*iii*) $\mu(n_1n_4)$ is quasi-co-degree for n_1 since $\max\{\mu(n_1n_i)\}_{i=2}^{5} = \mu(n_1n_4)$;

(*iv*) $\mu(n_2n_1)$ is quasi-co-degree for leaf n_2 since $n_in_2 \in E$ implies $i = 1$;

(*v*) $\mu(n_2n_3)$ isn't quasi-co-degree for leaf n_2 since $n_2n_3 \notin E$;

(*vi*) $\mu(n_1n_2)$ isn't quasi-co-degree for n_1 since $\mu(n_1n_2) \not> \mu(n_1n_3), \mu(n_1n_4), \mu(n_1n_5)$;

(*vii*) $\mu(n_3n_4)$ isn't quasi-co-degree for n_3 since $n_3n_4 \notin E$.

In a complete neutrosophic graph, one vertex has common edges with all given vertices. In complete-bipartite-neutrosophic graph, there are two parts and vertex set is partitioned into two sets which have complete connections with each other but inside, there's no connection. Thus the number of neighbors for every given vertex is exactly the number of vertices in other part. So there are some choices for quasi-co-degree, which are the number of vertices in another part.

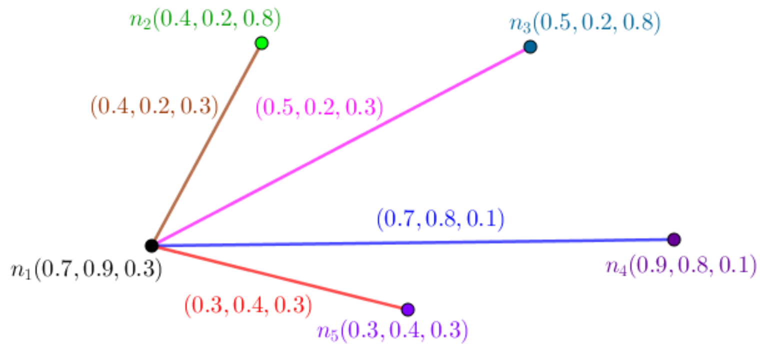

Figure 16. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be complete-bipartite-neutrosophic graph and $x \in V$. Then*

$$QCD(x) = \max_{z_i \in V_1^t} \{\mu(xz_1), \cdots, \mu(xz_t)\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be complete-bipartite-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is in first part, then there's one neighbor z which implies quasi-co-degree for x is $\mu(xz)$. It induces

$$QCD(x) = \mu(xz).$$

If x isn't a leaf in a given complete-bipartite-neutrosophic graph, then there's two neighbors z, z' which imply quasi-co-degree for x is either $\mu(xz)$ or $\mu(xz')$ where $\mathcal{O} = 3$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for every given vertex in second part, there are $t = |V_1^t|$ choices which the maximum value introduces quasi-co-degree. For every every given vertex in second part without any exception,

$$QCD(x) = \max_{z_i \in V_1^t} \{\mu(xz_1), \cdots, \mu(xz_t)\}.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $\max\{\mu(n_1n_2), \mu(n_1n_3)\}$ is quasi-co-degree for n_1 since $n_1n_2, n_1n_3 \in E$;

(ii) $\max\{\mu(n_1n_2), \mu(n_1n_4)\}$ isn't quasi-co-degree for a given vertex since either $n_1n_4 \notin E$ or $\mu(n_1n_2) \not> \mu(n_1n_3)$;

(iii) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\max\{\mu(n_2n_i)\}_{i=1,4} = \mu(n_2n_1)$;

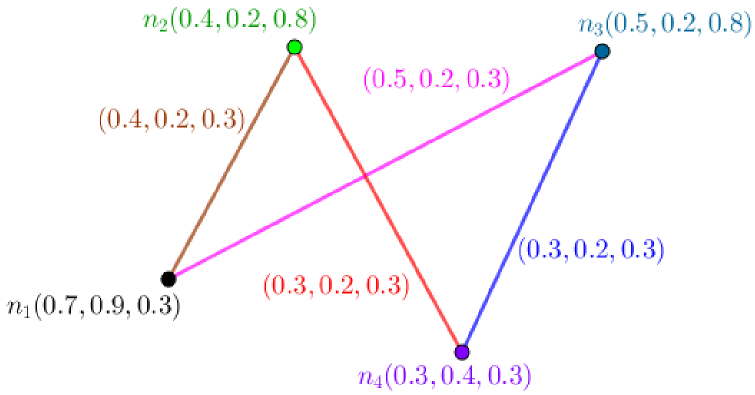

Figure 17. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

(iv) $\mu(n_4n_1)$ isn't quasi-co-degree for n_4 since $n_in_4 \in E$ implies $i = 2, 3$;

(v) $\mu(n_2n_3)$ isn't quasi-co-degree for n_2 since $n_2n_3 \notin E$;

(vi) $\mu(n_1n_2)$ isn't quasi-co-degree for n_1 since $\mu(n_1n_2) \not> \mu(n_1n_3)$;

(vii) $\mu(n_4)$ isn't quasi-co-degree for n_1 since $n_1n_4 \notin E$.

In a complete neutrosophic graph, one vertex has common edges with all given vertices. In complete-multipartite-neutrosophic graph, there are some parts and vertex set is partitioned into some sets which have complete connections with each other but inside, there's no connection. Thus the number of neighbors for every given vertex is exactly the number of vertices in other parts. So there are some choices for quasi-co-degree, which are the number of vertices in another parts.

Proposition 3.11. *Let $NTG : (V, E, \sigma, \mu)$ be complete-multipartite-neutrosophic graph and $x \in V$. Then*

$$QCD(x) = \max_{z_i \in (V_1^{t_1} \cup V_2^{t_2} \cup \cdots \cup V_s^{t_{\mathcal{O}}}) - V_j^{t'}} \{\mu(xz_1), \cdots, \mu(xz_{(t_1+t_2+\cdots+t_{\mathcal{O}})-t'})\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be complete-multipartite-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is in first part, then there's one neighbor z which implies quasi-degree for x is $\mu(xz)$. It induces

$$QCD(x) = \mu(xz).$$

If x isn't a leaf in a given complete-multiartite-neutrosophic graph, then there's two neighbors z, z' which imply quasi-co-degree for x is either $\mu(xz)$ or $\mu(xz')$ where $\mathcal{O} = 3$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for every given vertex in second part, there are $t = |V_1^t|$ choices which the maximum value introduces quasi-co-degree. For every every given vertex in second part without any exception,

$$QCD(x) = \max_{z_i \in V_1^t} \{\mu(xz_1), \cdots, \mu(xz_t)\}.$$

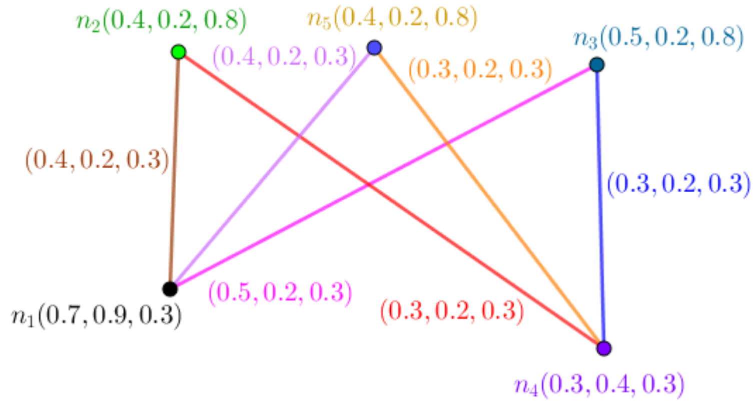

Figure 18. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

Now consider, there are more parts. There are some parts and vertex set is partitioned into some sets which have complete connections with each other but inside, there's no connection. Thus the number of neighbors for every given vertex is exactly the number of vertices in other parts. So there are some choices for quasi-co-degree, which are the number of vertices in another parts. It induces

$$QCD(x) = \max_{z_i \in (V_1^{t_1} \cup V_2^{t_2} \cup \cdots \cup V_s^{t_{\mathcal{O}}}) - V_j^{t'}} \{\mu(xz_1), \cdots, \mu(xz_{(t_1+t_2+\cdots+t_{\mathcal{O}})-t'})\}.$$

□

The clarifications about results are in progress as follows. A complete-multipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-multipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (21), a complete-multipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) $\max\{\mu(n_1n_2), \mu(n_1n_3), \mu(n_1n_5)\}$ is quasi-co-degree for n_1 since $n_1n_2, n_1n_3, n_1n_5 \in E$;

(ii) $\max\{\mu(n_in_3), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex since either $n_1n_4 \notin E$ or $\mu(n_1n_3) \not> \mu(n_1n_2)$;

(iii) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\max\{\mu(n_2n_i)\}_{i=1,4} = \mu(n_2n_1)$;

(iv) $\mu(n_4n_1)$ isn't quasi-co-degree for n_4 since $n_in_4 \in E$ implies $i = 2, 3, 5$;

(v) $\mu(n_2n_3)$ isn't quasi-co-degree for n_2 since $n_2n_3 \notin E$;

(vi) $\mu(n_1n_3)$ isn't quasi-co-degree for n_1 since $\mu(n_1n_3) \not> \mu(n_1n_5)$;

(vii) $\mu(n_1n_4)$ isn't quasi-co-degree for n_1 since $n_1n_4 \notin E$.

A wheel-neutrosophic graph is a graph which consists of a path and a vertex joining to all vertices of path. Thus specific vertex is called center and it has $\mathcal{O}(NTG) - 1$ choices. But other vertices have three choices which one of them is the center.

Proposition 3.13. *Let $NTG : (V, E, \sigma, \mu)$ be wheel-neutrosophic graph and $x \in V$. Then either*

$$QCD(x) = \max\{\mu(xz_1), \cdots, \mu(xz_{\mathcal{O}(NTG)-1})\}$$

or

$$QCD(x) = \max\{\mu(xz), \mu(xz'), \mu(xz'')\}$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be wheel-neutrosophic graph and $x \in V$. Thus $NTG : (V, E, \sigma, \mu)$ has vertex set $V = \{x_1, x_2, \cdots, x_{\mathcal{O}}\}$ where either $\mathcal{O}$ and 1 has same parity or different parity, it doesn't matter. There are two types of vertices. If x is in path, then there's three neighbors z, z', z'' which implies quasi-co-degree for x is one of $\mu(xz), \mu(xz')$ or $\mu(xz'')$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz'), \mu(xz'')\}$$

If x isn't in a path, then there are $\mathcal{O}(NTG) - 1$ neighbors $z_1, z_2, \cdots, z_{\mathcal{O}(NTG)-1}$ which imply quasi-co-degree for x is either $\mu(xz)$ or $\mu(xz')$ where $\mathcal{O}(NTG) = 3$. It induces

$$QCD(x) = \max\{\mu(xz), \mu(xz')\}.$$

To sum it up, for every given vertex in path, there are three choices which the maximum value introduces quasi-co-degree. For every every given vertex in path without any exception,

$$QCD(x) = \max\{\mu(xz), \mu(xz'), \mu(xz'')\}$$

Now consider, there are more vertices. There are two parts and vertex set is partitioned into one set which is path. Thus the number of neighbors for center is exactly the number of vertices minus one $\mathcal{O}(NTG) - 1$. So there are some choices for quasi-co-degree, which are the number of vertices in path. It induces

$$QCD(x) = \max\{\mu(xz_1), \cdots, \mu(xz_{\mathcal{O}(NTG)-1})\}$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.14. There is one section for clarifications. In Figure (19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) $\max\{\mu(n_1n_2), \mu(n_1n_3), \mu(n_1n_4), \mu(n_1n_5)\}$ is quasi-co-degree for n_1 since $n_1n_2, n_1n_3, n_1n_4, n_1n_5 \in E$;

(*ii*) $\max\{\mu(n_in_3), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex since either $n_2n_4 \notin E$ or $\mu(n_2n_3) \not> \mu(n_2n_1)$;

(*iii*) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\max\{\mu(n_in_j)\}_{i,j=2,i\neq j}^{5} = \mu(n_2n_1)$;

(*iv*) $\mu(n_4n_2)$ isn't quasi-co-degree for n_4 since $n_in_4 \in E$ implies $i = 1, 3, 5$;

(*v*) $\mu(n_5n_3)$ isn't quasi-co-degree for n_5 since $n_3n_5 \notin E$;

(*vi*) $\mu(n_1n_4)$ isn't quasi-co-degree for n_1 since $\mu(n_1n_4) \not> \mu(n_1n_3)$;

(*vii*) $\mu(n_1n_i)$ is quasi-co-degree for every given vertex since $n_1n_i \in E,\ i = 2, 3, 4, 5$ and $\mu(n_1n_i) = \max\{\mu(n_in_j)\}_{i,j=1,2,3,4,5,\ i\neq j}$.

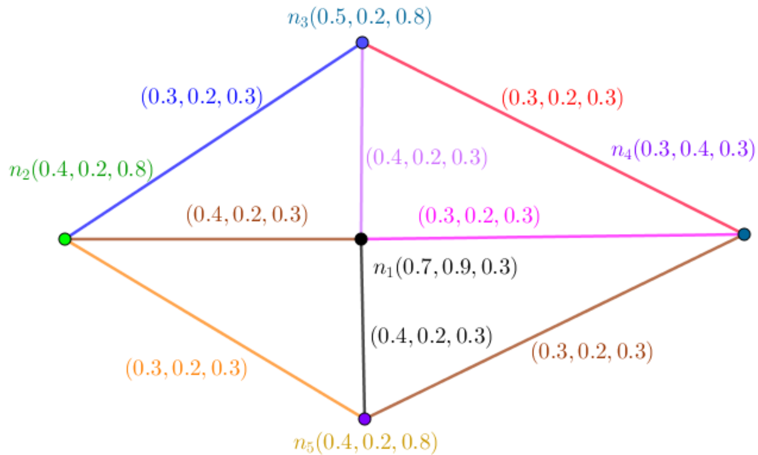

Figure 19. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_4
Values	$(0.6, 0.8, 0.2)$	$(0.3, 0.9, 0.8) \cdots$	$(0.6, 0.2, 0.1)$
Connections of NTG	E_1	$E_2 \cdots$	E_3
Values	$(0.3, 0.8, 0.2)$	$(0.6, 0.3, 0.2) \cdots$	$(0.6, 0.2, 0.1)$

4.1 Case 1: Star Model And Its Quasi-Degree

Step 4. (Solution) The neutrosophic graph and its quasi-degree as model, propose to use specific set. Every subject has connection with some subjects. Thus the

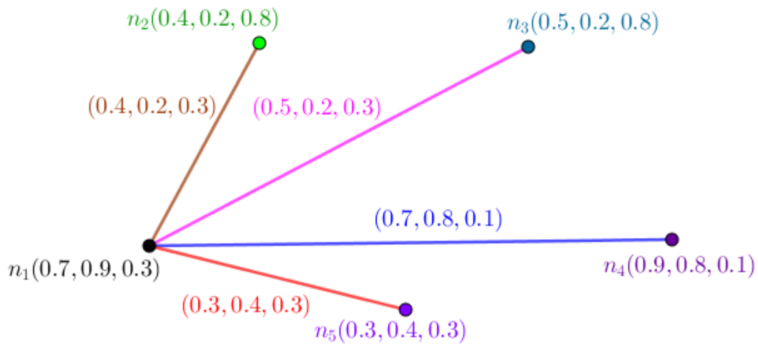

Figure 20. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Degree.

connection is applied as possible and the model demonstrates some connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the set is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of quasi-order when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (20). This model is strong. And the study proposes using specific vertex which is called quasi-degree. There are also some analyses on other sets in the way that, the clarification is gained about being special vertex or not. Also, in the last part, there is one neutrosophic numbers to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (20). In Figure (20), an star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) $\max\{\sigma(n_2), \sigma(n_3), \sigma(n_4), \sigma(n_5)\}$ is quasi-degree for n_1 since $n_1n_2, n_1n_3, n_1n_4, n_1n_5 \in E$;
- (ii) $\max\{\sigma(n_3), \sigma(n_4)\}$ isn't quasi-degree for a given vertex excluding n_1 since $n_in_3, n_in_4 \notin E,\ i \neq 1$;
- (iii) $\sigma(n_4)$ is quasi-degree for n_1 since $\max\{\sigma(n_i)\}_{i=2}^{5} = \sigma(n_4)$;
- (iv) $\sigma(n_1)$ is quasi-degree for leaf n_2 since $n_in_2 \in E$ implies $i = 1$;
- (v) $\sigma(n_3)$ isn't quasi-degree for leaf n_2 since $n_2n_3 \notin E$;
- (vi) $\sigma(n_2)$ isn't quasi-degree for n_1 since $\sigma(n_2) \not> \sigma(n_3), \sigma(n_4), \sigma(n_5)$;
- (vii) $\sigma(n_4)$ isn't quasi-degree for n_3 since $n_3n_4 \notin E$.

4.2 Case 2: Complete-Multipartite Model And Its Quasi-Co-Degree

Step 4. (Solution) The neutrosophic graph and its quasi-co-degree as model, propose to use specific set. Every subject has connection with every given subject in deemed way. Thus the connection is applied as possible and the model demonstrates full connections as possible between parts. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the vertex is

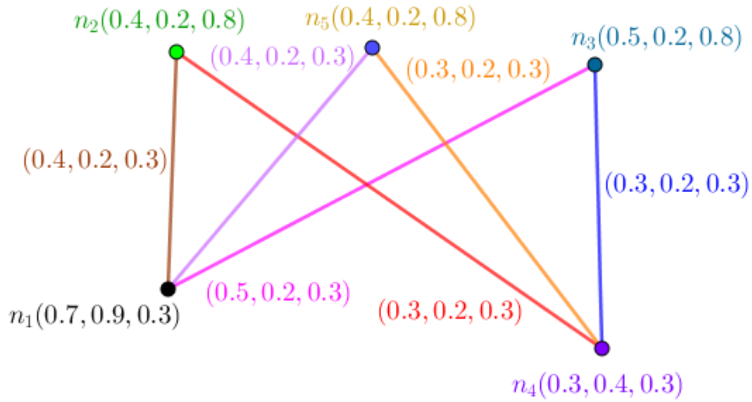

Figure 21. A Neutrosophic Graph in the Viewpoint of its Neutrosophic Quasi-Co-Degree.

different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of quasi-co-degree when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented in the formation of one model as Figure (21). This model is neutrosophic strong as individual. And the study proposes using specific vertex which is called quasi-co-degree for this model. There are also some analyses on other vertices in the way that, the clarification is gained about being special vertex or not. Also, in the last part, there are one neutrosophic number to assign to this model as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (21). There is one section for clarifications. In Figure (21), a complete-multipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) $\max\{\mu(n_1n_2), \mu(n_1n_3), \mu(n_1n_5)\}$ is quasi-co-degree for n_1 since $n_1n_2, n_1n_3, n_1n_5 \in E$;

(*ii*) $\max\{\mu(n_in_3), \mu(n_in_4)\}$ isn't quasi-co-degree for a given vertex since either $n_1n_4 \notin E$ or $\mu(n_1n_3) \not> \mu(n_1n_2)$;

(*iii*) $\mu(n_2n_1)$ is quasi-co-degree for n_2 since $\max\{\mu(n_2n_i)\}_{i=1,4} = \mu(n_2n_1)$;

(*iv*) $\mu(n_4n_1)$ isn't quasi-co-degree for n_4 since $n_in_4 \in E$ implies $i = 2, 3, 5$;

(*v*) $\mu(n_2n_3)$ isn't quasi-co-degree for n_2 since $n_2n_3 \notin E$;

(*vi*) $\mu(n_1n_3)$ isn't quasi-co-degree for n_1 since $\mu(n_1n_3) \not> \mu(n_1n_5)$;

(*vii*) $\mu(n_1n_4)$ isn't quasi-co-degree for n_1 since $n_1n_4 \notin E$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning quasi-degree and quasi-co-degree are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 5.1. *Is it possible to use other types quasi-degree and quasi-co-degree arising from different types of neighborhood to define new quasi-degree and quasi-co-degree?*

Question 5.2. *Are existed some connections amid different types of quasi-degree and quasi-co-degree in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definition concerning quasi-degree and quasi-co-degree arising neighborhood and co-neighborhood to study neutrosophic graphs. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid vertices and edges are done by using neutrosophic tool. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a vertex which its value is called either quasi-degree or

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Defining Quasi-Degree	1. General Results
2. Defining Quasi-Co-Degree	
3. Study on Classes	2. Study on Families
4. Using Neighborhood	
5. Using co-Neighborhood	3. Same Models in Family

quasi-co-degree. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

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