



Dynamic interval valued neutrosophic set: Modeling decision making in dynamic environments

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ABSTRACT

Dynamic decision problems constrained by time are of highly-interest in many aspects of real life. This paper proposes a new concept called the Dynamic Interval-valued Neutrosophic Set (DIVNS) for such the dynamic decision-making applications. Firstly, we define the definitions and mathematical operations, properties and correlations of DIVNSs. Next, we develop a new TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method based on the proposed DIVNS theory. Finally, a practical application of the method for evaluating lecturers' performance at the University of Languages and International Studies, Vietnam National University, Hanoi (ULIS-VNU) is given to illustrate the efficiency of our approach.

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1. Introduction

Neutrosophic set (NS) [45] is able to handle indeterminacy information [51,52,58]. NS and its extensions have become widely applied in almost areas, such as decision-making [1,12,20,21,33,34,41,42,49,58–62], clustering analysis [56,59], image processing [27,28], etc. However, in some complex problems in real-life, data may be collected from different time intervals or multi-periods, which raises the need for dynamic decision making for such the situations. The term 'dynamic' can be regarded in term of criteria such as (a) a series of decisions required to reach a goal; (b) path dependent decision; (c) the state of decision. This research considers the 'dynamic' decision problems which are constrained by time, as seen, for example, in emergency management and patient care. Specifically, when the economic situation of a certain company is investigated, the economic growth level of product series should be investigated by changes of the trend of profit of all products through the periods. Another example can be found in medical diagnosis where clinicians have to exam patients by different time intervals.

Recently, Yan et al. [53] developed a dynamic multiple attribute decision making method with grey number (considering both attribute value aggregation of all periods and their fluctuation among periods) to calculate degree of every alternative. This model was also used in [32] to manage linguistic bipolar scales using transformation between bipolar and unipolar linguistic terms. Ye [57] proposed a dynamic neutrosophic multiset. For decision assistance in dynamic environments, some algorithms that used TOPSIS under neutrosophic linguistic environments were presented in [2,10,11,22,23,25,26,33–37,40,55]. There have been also some works that applied the Interval-Valued Neutrosophic Set (IVNS) with the TOPSIS method for decision making [6,11,29,33,49,54,62]. Other relevant decision making methods can be retrieved in [3–5,7–9,13–19]. However, the existing researches did not consider different time intervals as the objective of this research aims. To the best of our knowledge, fluctuation of alternative's attribute values within periods on NSs has not been examined. In many practical cases, there is not enough available information to judge complicated situations, indeed it often given approximate ranges.

In this paper, we propose a new TOPSIS method based on a new extension of NS called the Dynamic Interval-valued Neutrosophic Set (DIVNS) for dynamic decision-making problems. The main contribution includes:

- (a) We define definitions and mathematical operations, properties and correlations of DIVNSs.

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- (b) We develop a new TOPSIS method based on the proposed DIVNS theory.
- (c) A practical application of the method for evaluating lecturers' performance at the Vietnam National University, Hanoi (ULIS-VNU) is given to illustrate the efficiency of our approach.

Section 2 defines the new concept of Dynamic Interval-valued Neutrosophic Set (DIVNS). Section 3 presents the TOPSIS method for DIVNS. Section 4 illustrates the proposed method in a practical application. Finally, Section 5 summarizes the findings.

2. Dynamic interval-valued neutrosophic set

2.1. Set definition

Definition 2.1. [45]: Let U be a universe of discourse. A neutrosophic set is:

$$A = \{ \langle x : T_A(x), I_A(x), F_A(x) \rangle, x \in U \}$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ and $0 \leq \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \leq 3$.

Definition 2.2. [45]: A neutrosophic number is defined as $N = a + bI$, where a and b are real numbers, and I is the indeterminacy.

Definition 2.3. [57]: A Dynamic Single-Valued Neutrosophic Set (DSVNS) is: $A = \{x \in U; x(T_x(t), I_x(t), F_x(t))\}$ for all $x \in A$:

$$T_x, I_x, F_x : [0, \infty) \rightarrow [0, 1]$$

where T_x, I_x, F_x are continuous functions whose argument is time (t).

Based on the definition of DSVNS above, we formulate the new definition as below.

Definition 2.4. A Dynamic Interval-Valued Neutrosophic Set (DIVNS) is in the form below:

$$x \left(\left[T_x^L(t), T_x^U(t) \right], \left[I_x^L(t), I_x^U(t) \right], \left[F_x^L(t), F_x^U(t) \right] \right)$$

where $t \geq 0$,

$$T_x^L(t) < T_x^U(t), I_x^L(t) < I_x^U(t), F_x^L(t) < F_x^U(t)$$

And

$$\left[T_x^L(t), T_x^U(t) \right], \left[I_x^L(t), I_x^U(t) \right], \left[F_x^L(t), F_x^U(t) \right] \subseteq [0, 1]$$

In other words, a DIVNS is a neutrosophic set whose elements' neutrosophic components (membership, indeterminacy, non-membership) are all intervals changing with respect to time.

For a simplified notation, we denote:

$$T_x(t) = \left[T_x^L(t), T_x^U(t) \right], I_x(t) = \left[I_x^L(t), I_x^U(t) \right], F_x(t) = \left[F_x^L(t), F_x^U(t) \right]$$

where $T_x(t), I_x(t), F_x(t) : [0, \infty) \rightarrow P([0, 1])$ with $P([0, 1])$ been the power set of $[0, 1]$.

We can also use the notation $A(t)$ and $x(t)$, meaning that each element x in A depends on t . Or $T_x(t), I_x(t), F_x(t)$ are interval-valued functions (a particular case of neutrosophic function [1]).

The difference of the new definition against the existing one in [57]:

We have extended Ye's DSVNS [57] to DIVNS by considering a time sequence: $t = \{t_1, t_2, \dots, t_k\}$ then at each time $t_l, 1 \leq l \leq m$, the neutrosophic components of the generic element $x \in A$ change as follow:

$$x(\langle T_x(t_1), I_x(t_1), F_x(t_1) \rangle; \langle T_x(t_2), I_x(t_2), F_x(t_2) \rangle; \dots; \langle T_x(t_k), I_x(t_k), F_x(t_k) \rangle)$$

Example 2.1. A DIVNS in time sequence $t = \{t_1, t_2, t_3\}$ and a universal $NS = \{x_1, x_2, x_3\}$ is given:

$$A = \left\{ \begin{aligned} &\langle x_1, ([0.1, 0.25], [0.15, 0.2], [0.3, 0.6]), ([0.45, 0.5], [0.1, 0.3], [0.2, 0.4]), ([0.6, 0.7], [0.52, 0.6], [0.7, 0.9]) \rangle, \\ &\langle x_2, ([0.38, 0.4], [0.25, 0.4], [0.12, 0.3]), ([0.07, 0.1], [0.1, 0.2], [0.09, 0.1]), ([0.22, 0.3], [0.4, 0.5], [0.3, 0.43]) \rangle, \\ &\langle x_3, ([0.7, 0.9], [0.33, 0.45], [0.59, 0.6]), ([0.2, 0.22], [0.5, 0.6], [0.2, 0.3]), ([0.8, 0.9], [0.3, 0.41], [0.3, 0.33]) \rangle \end{aligned} \right\}$$

2.2. Set theoretic operations of DIVNS

Let $A(t)$ and $B(t)$ be two DIVNSs included in U ;

$$\begin{aligned} A(t) &= \left\{ \left(x(t), \langle T_x^A(t_l), I_x^A(t_l), F_x^A(t_l) \rangle \right), \forall t_l \in t, x \in U \right\}, B(t) \\ &= \left\{ \left(x(t), \langle T_x^B(t_l), I_x^B(t_l), F_x^B(t_l) \rangle \right), \forall t_l \in t, x \in U \right\} \end{aligned}$$

Definition 2.5. : DIVNS Intersection

$$\begin{aligned} A(t) \cap B(t) &= \left\{ \left(x(t), \langle T_x^A(t_l) \wedge T_x^B(t_l), I_x^A(t_l) \vee I_x^B(t_l), F_x^A(t_l) \vee F_x^B(t_l) \rangle \right), \right. \\ &\quad \left. \forall t_l \in t, x \in U \right\} \end{aligned}$$

Definition 2.6. DIVNS Union

$$\begin{aligned} A(t) \cup B(t) &= \left\{ \left(x(t), \langle T_x^A(t_l) \vee T_x^B(t_l), I_x^A(t_l) \wedge I_x^B(t_l), F_x^A(t_l) \wedge F_x^B(t_l) \rangle \right), \right. \\ &\quad \left. \forall t_l \in t, x \in U \right\} \end{aligned}$$

Definition 2.7. DIVNS Complement

$$A(t)^C = \left\{ \left(x(t), \langle F_x^A(t_l), 1 - I_x^A(t_l), T_x^A(t_l) \rangle \right), \forall t_l \in t, x \in U \right\}$$

Definition 2.8. DIVNS inclusion

$$A(t) \subseteq B(t) \sim T_x^A(t_l) \leq T_x^B(t_l), I_x^A(t_l) \geq I_x^B(t_l) \text{ and } F_x^A(t_l) \geq F_x^B(t_l).$$

Definition 2.9. DIVNS Equality

$$A(t) = B(t) \Leftrightarrow A(t) \subseteq B(t) \text{ and } A(t) \supseteq B(t).$$

In the above DIVNS aggregation operators by “ $\wedge$ ” we meant the “t-norm” and by “ $\vee$ ”

the t-conorm from the single-valued fuzzy sets

2.3. Operations on DIVNS numbers

Let us consider two DIVNS numbers:

$$\begin{aligned} a(t) &= \left\{ \left\langle T_x^A(t_1), I_x^A(t_1), F_x^A(t_1) \right\rangle, \dots, \left\langle T_x^A(t_k), I_x^A(t_k), F_x^A(t_k) \right\rangle \right\} \\ b(t) &= \left\{ \left\langle T_x^B(t_1), I_x^B(t_1), F_x^B(t_1) \right\rangle, \dots, \left\langle T_x^B(t_k), I_x^B(t_k), F_x^B(t_k) \right\rangle \right\}. \end{aligned}$$

Definition 2.14. Correlation coefficient of DIVNSs

Let

$$\begin{aligned} A(t) &= \left\{ \left\langle x(t), \left\langle T^A(x, t_l), I^A(x, t_l), F^A(x, t_l) \right\rangle \right\rangle, \forall t_l \in t, x \in U \right\}, \\ B(t) &= \left\{ \left\langle x(t), \left\langle T^B(x, t_l), I^B(x, t_l), F^B(x, t_l) \right\rangle \right\rangle, \forall t_l \in t, x \in U \right\} \end{aligned}$$

be two DIVNSs in $t = \{t_1, t_2, \dots, t_k\}$ and $U = \{x_1, x_2, \dots, x_n\}$.

A correlation coefficient is:

$$\rho(A(t), B(t)) = \frac{1}{k} \sum_{l=1}^k \frac{\left(\inf T^A(x_i, t_l) \times \inf T^B(x_i, t_l) + \sup T^A(x_i, t_l) \times \sup T^B(x_i, t_l) \right.}{\left(\inf I^A(t_l) \times \inf I^B(x_i, t_l) + \sup I^A(x_i, t_l) \times \sup I^B(x_i, t_l) \right.} \times \frac{\left(\inf F^A(x_i, t_l) \times \inf F^B(x_i, t_l) + \sup F^A(x_i, t_l) \times \sup F^B(x_i, t_l) \right)}{\left(\inf I^A(x_i, t_l) \times \inf I^B(x_i, t_l) + \sup I^A(x_i, t_l) \times \sup I^B(x_i, t_l) \right)} \quad (5)$$

Definition 2.10. Addition of DIVNS numbers

$$a(t) \oplus b(t) = \left\{ \left\langle \begin{aligned} &T_x^A(t_1) + T_x^B(t_1) - T_x^A(t_1) \times T_x^B(t_1), \\ &I_x^A(t_1) + I_x^B(t_1) - I_x^A(t_1) \times I_x^B(t_1) \\ &F_x^A(t_1) + F_x^B(t_1) - F_x^A(t_1) \times F_x^B(t_1) \end{aligned} \right\rangle, \dots, \left\langle \begin{aligned} &T_x^A(t_k) + T_x^B(t_k) - T_x^A(t_k) \times T_x^B(t_k), \\ &I_x^A(t_k) + I_x^B(t_k) - I_x^A(t_k) \times I_x^B(t_k) \\ &F_x^A(t_k) + F_x^B(t_k) - F_x^A(t_k) \times F_x^B(t_k) \end{aligned} \right\rangle \right\} \quad (1)$$

Definition 2.11. Multiplication of DIVNS numbers

$$\begin{aligned} a(t) \otimes b(t) &= \left\{ \left\langle \begin{aligned} &T_x^A(t_1) \times T_x^B(t_1) - I_x^A(t_1) \times I_x^B(t_1) \\ &I_x^A(t_1) \times I_x^B(t_1) - F_x^A(t_1) \times F_x^B(t_1) \end{aligned} \right\rangle, \dots, \left\langle \begin{aligned} &T_x^A(t_k) \times T_x^B(t_k) - I_x^A(t_k) \times I_x^B(t_k) \\ &I_x^A(t_k) \times I_x^B(t_k) - F_x^A(t_k) \times F_x^B(t_k) \end{aligned} \right\rangle \right\} \quad (2) \end{aligned}$$

Definition 2.12. Scalar Multiplication of DIVNS numbers

$$\alpha \times a(t) = \left\{ \left\langle 1 - \left(1 - T_x^A(t_1) \right)^\alpha, I_x^A(t_1)^\alpha, F_x^A(t_1)^\alpha \right\rangle, \dots, \left\langle 1 - \left(1 - T_x^A(t_k) \right)^\alpha, I_x^A(t_k)^\alpha, F_x^A(t_k)^\alpha \right\rangle \right\} \quad (3)$$

Definition 2.13. Power of the DIVNS numbers

$$a(t)^\alpha = \left\{ \left\langle T_x^A(t_1)^\alpha, 1 - \left(1 - I_x^A(t_1) \right)^\alpha, 1 - \left(1 - F_x^A(t_1) \right)^\alpha \right\rangle, \dots, \left\langle T_x^A(t_k)^\alpha, 1 - \left(1 - I_x^A(t_k) \right)^\alpha, 1 - \left(1 - F_x^A(t_k) \right)^\alpha \right\rangle \right\} \quad (4)$$

Theorem 2.1. The correlation coefficient between A and B satisfies:

- (Pr1) $0 \leq \rho(A(t), B(t)) \leq 1$;
- (Pr2) $\rho(A(t), B(t)) = 1$ if $A(t) = B(t)$;
- (Pr3) $\rho(A(t), B(t)) = \rho(B(t), A(t))$

Proof.

(Pr1) It is obvious that $\rho(A(t), B(t)) \geq 0$. From Cauchy–Schwarz inequality, we have

$$\begin{aligned} \sum_{i=1}^n \left(\inf T^A(x_i, t_l) \times \inf T^B(x_i, t_l) + \sup T^A(x_i, t_l) \times \sup T^B(x_i, t_l) \right. \\ \left. + \inf I^A(t_l) \times \inf I^B(x_i, t_l) + \sup I^A(x_i, t_l) \times \sup I^B(x_i, t_l) \right. \\ \left. + \inf F^A(x_i, t_l) \times \inf F^B(x_i, t_l) + \sup F^A(x_i, t_l) \times \sup F^B(x_i, t_l) \right) \leq \\ \left(\sum_{i=1}^n \left[\left(\inf T^A(x_i, t_l) \right)^2 + \left(\sup T^A(x_i, t_l) \right)^2 + \left(\inf I^A(x_i, t_l) \right)^2 \right] \right. \\ \left. + \left(\sup I^A(x_i, t_l) \right)^2 + \left(\inf F^A(x_i, t_l) \right)^2 + \left(\sup F^A(x_i, t_l) \right)^2 \right) \\ \times \left(\sum_{i=1}^n \left[\left(\inf T^B(x_i, t_l) \right)^2 + \left(\sup T^B(x_i, t_l) \right)^2 + \left(\inf I^B(x_i, t_l) \right)^2 \right] \right. \\ \left. + \left(\sup I^B(x_i, t_l) \right)^2 + \left(\inf F^B(x_i, t_l) \right)^2 + \left(\sup F^B(x_i, t_l) \right)^2 \right) \end{aligned}$$

for each $l \in \{1, 2, \dots, k\}$. Thus, $0 \leq \rho(A(t), B(t)) \leq 1$.

(Pr2) $A(t) = B(t)$. $\forall l \in \{1, 2, \dots, k\}$. We have $\inf T^A(x_i, t_l) = \inf T^B(x_i, t_l)$; $\sup T^A(x_i, t_l) = \sup T^B(x_i, t_l)$; $\inf I^A(x_i, t_l) = \inf I^B(x_i, t_l)$; $\sup I^A(x_i, t_l) = \sup I^B(x_i, t_l)$; $\inf F^A(x_i, t_l) = \inf F^B(x_i, t_l)$; $\sup F^A(x_i, t_l) = \sup F^B(x_i, t_l)$; $\inf T^A(x_i, t_l) = \inf T^B(x_i, t_l) \Rightarrow \rho(A(t), B(t)) = 1$

(Pr3) It is easily observed.

Definition 2.15. Weighted Correlation Coefficient of DIVNSs

Different weights for $x_i (i = 1, \dots, n)$ and $t_l (l = 1, \dots, k)$ are integrated as follows.

$$\rho_W(A(t), B(t)) = \frac{1}{k} \sum_{l=1}^k \omega_l \times \frac{\sum_{i=1}^n w_i \times \left(\inf T^A(x_i, t_l) \times \inf T^B(x_i, t_l) + \sup T^A(x_i, t_l) \times \sup T^B(x_i, t_l) \right.}{\left. + \inf I^A(x_i, t_l) \times \inf I^B(x_i, t_l) + \sup I^A(x_i, t_l) \times \sup I^B(x_i, t_l) \right.}{\left. + \inf F^A(x_i, t_l) \times \inf F^B(x_i, t_l) + \sup F^A(x_i, t_l) \times \sup F^B(x_i, t_l) \right)} \quad (6)$$

$$\left(\sqrt{\sum_{i=1}^n w(x_i) \times \left(\left(\inf T^A(x_i, t_l) \right)^2 + \left(\sup T^A(x_i, t_l) \right)^2 + \left(\inf I^A(x_i, t_l) \right)^2 \right.} \right. \\ \left. \left. + \left(\sup I^A(x_i, t_l) \right)^2 + \left(\inf F^A(x_i, t_l) \right)^2 + \left(\sup F^A(x_i, t_l) \right)^2 \right)} \right) \\ \times \left(\sqrt{\sum_{i=1}^n w(x_i) \times \left(\left(\inf T^B(x_i, t_l) \right)^2 + \left(\sup T^B(x_i, t_l) \right)^2 + \left(\inf I^B(x_i, t_l) \right)^2 \right.} \right. \\ \left. \left. + \left(\sup I^B(x_i, t_l) \right)^2 + \left(\inf F^B(x_i, t_l) \right)^2 + \left(\sup F^B(x_i, t_l) \right)^2 \right)} \right)$$

where $w = (w_1, w_2, \dots, w_n)^T$ and $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ are weighting vectors of $x_i (i = 1, \dots, n)$ and $t_l (l = 1, \dots, k)$ with $\sum_{i=1}^n w_i = 1$ and $\sum_{l=1}^k \omega_l = 1$.

When $w_i = 1/n; i = 1, \dots, n$ and $\omega_l = 1/k; l = 1, \dots, k$, Eq. (6) turns to (5).

The weighted correlation coefficient between A and B also satisfies the properties as in Theorem 2.1.

3. A topsis method for divns

Assume $A = \{A_1, A_2, \dots, A_n\}$ and $C = \{C_1, C_2, \dots, C_n\}$ and $D = \{D_1, D_2, \dots, D_h\}$ are sets of alternatives, attributes, and decision makers. For a decision maker $D_q; q = 1, \dots, h$, the evaluation characteristic of an alternatives $A_m; m = 1, \dots, n$, on an attribute $C_p; p = 1, \dots, n$, in time sequence $t = \{t_1, t_2, \dots, t_k\}$ is represented by the decision matrix $D^q(t_l) = (d_{mp}^q(t))_{n \times n}; l = 1, 2, \dots, k$. where

$$d_{mp}^q(t) = \langle x_{d_{mp}}^q(t), (T^q(d_{mp}, t), I^q(d_{mp}, t), F^q(d_{mp}, t)) \rangle; t = \{t_1, t_2, \dots, t_k\}$$

$$\overline{I_{mp}(x)} = \left[\left(\sum_{q=1}^h I_{pmq}^L(x_{t_l}) \right)^{\frac{1}{h * k}}, \left(\sum_{q=1}^h I_{pmq}^U(x_{t_l}) \right)^{\frac{1}{h * k}} \right]$$

$$\overline{F_{mp}(x)} = \left[\left(\sum_{q=1}^h F_{pmq}^L(x_{t_l}) \right)^{\frac{1}{h * k}}, \left(\sum_{q=1}^h F_{pmq}^U(x_{t_l}) \right)^{\frac{1}{h * k}} \right]$$

3.2. Importance weight aggregation

Let $x_{pq}(t_l) = \{ [T_{pq}^L(x_{t_l}), T_{pq}^U(x_{t_l})], [I_{pq}^L(x_{t_l}), I_{pq}^U(x_{t_l})], [F_{pq}^L(x_{t_l}), F_{pq}^U(x_{t_l})] \}$ be weight of D_q to criterion C_p in time sequence t_l , where: $p = 1, \dots, n; q = 1, \dots, h; l = 1, \dots, k$. The average weight $\overline{w_p} = \{ [T_p^L(x), T_p^U(x)], [I_p^L(x), I_p^U(x)], [F_p^L(x), F_p^U(x)] \}$ can be evaluated as:

$$\overline{w_p} = \frac{1}{h * k} \otimes \left\langle \left\{ [T_{p1}^L(x_{t_1}), T_{p1}^U(x_{t_1})], [I_{p1}^L(x_{t_1}), I_{p1}^U(x_{t_1})], [F_{p1}^L(x_{t_1}), F_{p1}^U(x_{t_1})] \right\} + \dots + \right. \\ \left. \left\{ [T_{ph}^L(x_{t_h}), T_{ph}^U(x_{t_h})], [I_{ph}^L(x_{t_h}), I_{ph}^U(x_{t_h})], [F_{ph}^L(x_{t_h}), F_{ph}^U(x_{t_h})] \right\} \right\rangle, \quad (8)$$

taken by DIVNSs evaluated by decision maker D_q .

3.1. Aggregate ratings

Let $x_{mpq}(t_l) = \{ [T_{mpq}^L(x_{t_l}), T_{mpq}^U(x_{t_l})], [I_{mpq}^L(x_{t_l}), I_{mpq}^U(x_{t_l})], [F_{mpq}^L(x_{t_l}), F_{mpq}^U(x_{t_l})] \}$ be the suitability rating of alternative A_m for criterion C_p by decision-maker D_q in time sequence t_l , where: $m = 1, \dots, n; p = 1, \dots, n; q = 1, \dots, h; l = 1, \dots, k$. The averaged suitability rating $\overline{x_{mp}} = \{ [T_{mp}^L(x), T_{mp}^U(x)], [I_{mp}^L(x), I_{mp}^U(x)], [F_{mp}^L(x), F_{mp}^U(x)] \}$ can be evaluated as:

$$\overline{x_{mp}} = \frac{1}{h * k} \otimes \left\langle \left\{ [T_{mpq}^L(x_{t_1}), T_{mpq}^U(x_{t_1})], [I_{mpq}^L(x_{t_1}), I_{mpq}^U(x_{t_1})], [F_{mpq}^L(x_{t_1}), F_{mpq}^U(x_{t_1})] \right\} + \dots + \right. \\ \left. \left\{ [T_{mpq}^L(x_{t_k}), T_{mpq}^U(x_{t_k})], [I_{mpq}^L(x_{t_k}), I_{mpq}^U(x_{t_k})], [F_{mpq}^L(x_{t_k}), F_{mpq}^U(x_{t_k})] \right\} \right\rangle, \quad (7)$$

where,

$$\overline{T_{mp}(x)} = \left[\left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^h T_{pmq}^L(x_{t_l}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}}, \left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^h T_{pmq}^U(x_{t_l}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}} \right\rangle \right]$$

where,

$$\begin{aligned}\overline{T_p(x)} &= \left[\left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^h T_{pq}^L(x_{t_i}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}}, \left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^h T_{pq}^U(x_{t_i}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}} \right\rangle \right] \\ \overline{I_p(x)} &= \left[\left(\sum_{q=1}^h I_{pq}^L(x_{t_i}) \right)^{\frac{1}{h * k}}, \left(\sum_{q=1}^h I_{pq}^U(x_{t_i}) \right)^{\frac{1}{h * k}} \right] \\ \overline{F_p(x)} &= \left[\left(\sum_{q=1}^h F_{pq}^L(x_{t_i}) \right)^{\frac{1}{h * k}}, \left(\sum_{q=1}^h F_{pq}^U(x_{t_i}) \right)^{\frac{1}{h * k}} \right]\end{aligned}$$

3.3. Compute the average weighted ratings

Average weighted ratings of alternatives in t_i , are:

$$\overline{G_m} = \frac{1}{n} \sum_{p=1}^n \overline{x_{mp}} * \overline{w_p}; m = 1, \dots, v; p = 1, \dots, n; \quad (9)$$

3.4. Determination of A^+ , A^- , d_i^+ and d_i^-

Interval neutrosophic positive and negative ideal solutions namely (PIS, A^+) and (NIS, A^-) are:

$$A^+ = \{x, ([1, 1], [0, 0], [0, 0])\} \quad (10)$$

$$A^- = \{x, ([0, 0], [1, 1], [1, 1])\} \quad (11)$$

The distances of each alternative A_m , $m = 1, \dots, t$ from A^+ and A^- in time sequence t_i , are calculated as:

$$\overline{d_m^+} = \sqrt{(\overline{G_m} - A^+)^2} \quad (12)$$

$$\overline{d_m^-} = \sqrt{(\overline{G_m} - A^-)^2} \quad (13)$$

where d_m^+ and d_m^- represents the shortest and farthest distances of A_m .

3.4. Obtain best coefficient

The best coefficient in time sequence t_i , is shown below where high value indicates closer to interval neutrosophic PIS and farther from interval neutrosophic NIS:

$$\overline{CC_m} = \frac{\overline{d_m^-}}{\overline{d_m^+} + \overline{d_m^-}} \quad (14)$$

4. Applications

This section applies the proposed method to evaluate lecturers' performance in the case study of ULIS-VNU having 11 Faculties, 11 Departments, 09 Functional departments, 05 Centers and 01 Foreign Language Specializing High School with over 700 lecturers and 8000 high school, undergraduate and graduate students. Assume that ULIS-VNU needs to evaluate the lecturers' performance. After preliminary screening, five lecturers, i.e. $A_1, \dots, A_5$,

and three decision makers, i.e. $D_1, \dots, D_3$, are chosen. Ratings of five lecturers are done by criteria as total of publications (C_1), teaching student evaluations (C_2), personality characteristics (C_3), professional society (C_4), teaching experience (C_5), fluency of foreign language (C_6).

4.1. Aggregate ratings

Suitability ratings $S = \{Ve_Po, Po, Me, Go, Ve_Go\}$ in $t = \{t_1, t_2, t_3\}$ is,
 $Ve_Po = \text{Very_Poor} = ([0.1, 0.2], [0.6, 0.7], [0.7, 0.8])$,
 $Po = \text{Poor} = ([0.2, 0.3], [0.5, 0.6], [0.6, 0.7])$,
 $Me = \text{Medium} = ([0.3, 0.5], [0.4, 0.6], [0.4, 0.5])$,
 $Go = \text{Good} = ([0.5, 0.6], [0.4, 0.5], [0.3, 0.4])$,
 $Ve_Go = \text{Very_Good} = ([0.6, 0.7], [0.2, 0.3], [0.2, 0.3])$,

Table 1 presents suitability ratings where the aggregated ratings of lecturers versus criteria are shown at the last column of Table 1.

4.2. Importance weight aggregation

The importance $V = \{U_IPA, O_IPA, IPA, V_IPA, A_IPA\}$ in $t = \{t_1, t_2, t_3\}$ is:

$U_IPA = ([0.1, 0.2], [0.4, 0.5], [0.6, 0.7]) = \text{Unimportant}$,
 $O_IPA = ([0.2, 0.4], [0.5, 0.6], [0.4, 0.5]) = \text{Ordinary_Important}$,
 $IPA = ([0.4, 0.6], [0.4, 0.5], [0.3, 0.4]) = \text{Important}$,
 $V_IPA = ([0.6, 0.8], [0.3, 0.4], [0.2, 0.3]) = \text{Very_Important}$,
 $A_IPA = ([0.7, 0.9], [0.2, 0.3], [0.1, 0.2]) = \text{Absolutely_Important}$ (Tables 2–4),

4.3. Weighted ratings

A^+, A^-, d_i^+ and d_i^-

4.4. Determine the lecturer

Table 5 shows the ranking order is $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$. Thus, the best lecturer is A_2 .

5. Comparison

This section compares the proposed TOPSIS method for DIVSN with the similarity measures between INs proposed by Ye [62] to illustrate the advantages and applicability of the proposed method. Using Ye's [62] method and the data in Table 3, the score function, the accuracy function and the certainty function of the lecturers are shown in Table 6.

Table 1
Aggregated ratings.

Criteria	Lecturers	Decision makers									Aggregated ratings
		t_1			t_2			t_3			
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	
C_1	A_1	Me	Go	Go	Go	Go	Go	Go	Ve_Go	Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_2	Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	([0.558, 0.659], [0.272, 0.4], [0.239, 0.3])
	A_3	Me	Go	Go	Go	Go	Go	Go	Go	Ve_Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_4	Go	Me	Go	Go	Go	Go	Go	Go	Go	([0.481, 0.590], [0.400, 0.5], [0.310, 0.4])
	A_5	Me	Go	Me	Go	Go	Me	Go	Go	Go	([0.441, 0.569], [0.400, 0.5], [0.330, 0.4])
C_2	A_1	Go	Go	Go	Ve_Go	Go	Go	Go	Go	Go	([0.512, 0.613], [0.370, 0.5], [0.287, 0.4])
	A_2	Ve_Go	Go	Ve_Go	Me	Go	Go	Ve_Go	Go	Go	([0.518, 0.627], [0.317, 0.4], [0.271, 0.4])
	A_3	Ve_Go	Go	Go	Go	Me	Go	Go	Me	Go	([0.474, 0.593], [0.370, 0.5], [0.306, 0.4])
	A_4	Go	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_5	Ve_Go	Go	Go	Go	Ve_Go	Go	Go	Go	Me	([0.506, 0.615], [0.343, 0.5], [0.283, 0.4])
C_3	A_1	Ve_Go	Ve_Go	Go	Go	Ve_Go	Go	Go	Me	Go	([0.518, 0.627], [0.317, 0.4], [0.271, 0.4])
	A_2	Go	Ve_Go	Go	Ve_Go	Go	Ve_Go	Go	Go	Ve_Go	([0.547, 0.648], [0.294, 0.4], [0.251, 0.4])
	A_3	Go	Ve_Go	Ve_Go	Go	Go	Go	Go	Ve_Go	Go	([0.536, 0.637], [0.317, 0.4], [0.262, 0.4])
	A_4	Go	Go	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_5	Ve_Go	Go	Go	Go	Ve_Go	Go	Go	Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
C_4	A_1	Me	Go	Me	Go	Go	Me	Me	Go	Me	([0.397, 0.547], [0.400, 0.6], [0.352, 0.5])
	A_2	Go	Me	Go	Go	Me	Go	Go	Me	Go	([0.441, 0.569], [0.400, 0.5], [0.330, 0.4])
	A_3	Go	Go	Go	Go	Go	Me	Go	Go	Ve_Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_4	Me	Po	Me	Go	Me	Me	Go	Go	Me	([0.365, 0.518], [0.410, 0.6], [0.380, 0.5])
	A_5	Me	Me	Po	Me	Me	Me	Me	Go	Me	([0.316, 0.494], [0.410, 0.6], [0.405, 0.5])
C_5	A_1	Me	Go	Me	Me	Go	Go	Go	Me	Go	([0.419, 0.558], [0.400, 0.5], [0.341, 0.4])
	A_2	Go	Ve_Go	Go	Ve_Go	Go	Go	Go	V_G	Go	([0.536, 0.637], [0.317, 0.4], [0.262, 0.4])
	A_3	Go	Go	Me	Go	Go	Go	Go	Ve_Go	Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_4	Ve_Go	Go	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	([0.536, 0.637], [0.317, 0.4], [0.262, 0.4])
	A_5	Go	Go	Go	Go	Go	Go	Go	Ve_Go	Go	([0.512, 0.613], [0.370, 0.5], [0.287, 0.4])
C_6	A_1	Ve_Go	Go	Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	([0.558, 0.659], [0.272, 0.4], [0.239, 0.3])
	A_2	Go	Go	Go	Go	Ve_Go	Ge	Go	Go	Ve_Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_3	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	([0.569, 0.670], [0.252, 0.4], [0.229, 0.3])
	A_4	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_5	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])

Table 2
Aggregated weights.

Criteria	Decision-makers									Aggregated weights
	t ₁			t ₂			t ₃			
	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	
C ₁	IPA	IPA	IPA	IPA	V_IPA	IPA	V_IPA	IPA	V_IPA	([0.476, 0.683], [0.363, 0.5], [0.262, 0.4])
C ₂	V_IPA	V_IPA	IPA	V_IPA	V_IPA	V_IPA	A_IPA	V_IPA	V_IPA	([0.595, 0.800], [0.296, 0.4], [0.194, 0.3])
C ₃	IPA	IPA	V_IPA	IPA	IPA	V_IPA	V_IPA	IPA	V_IPA	([0.499, 0.706], [0.352, 0.5], [0.251, 0.4])
C ₄	IPA	V_IPA	IPA	IPA	O_IPA	IPA	IPA	IPA	IPA	([0.408, 0.613], [0.397, 0.5], [0.296, 0.4])
C ₅	IPA	IPA	IPA	V_IPA	IPA	V_IPA	IPA	IPA	IPA	([0.452, 0.657], [0.375, 0.5], [0.274, 0.4])
C ₆	V_IPA	V_IPA	IPA	IPA	IPA	IPA	V_IPA	V_IPA	IPA	([0.499, 0.706], [0.352, 0.5], [0.251, 0.4])

Table 3
Weighted ratings.

Lecturers	Aggregated weights
A_1	([0.170, 0.397], [0.648, 0.8], [0.545, 0.6])
A_2	([0.190, 0.436], [0.617, 0.7], [0.519, 0.6])
A_3	([0.187, 0.419], [0.642, 0.8], [0.535, 0.6])
A_4	([0.178, 0.400], [0.643, 0.8], [0.538, 0.6])
A_5	([0.173, 0.395], [0.649, 0.8], [0.549, 0.6])

Table 5
Closeness coefficient.

Lecturers	Closeness coefficient	Ranking
A_1	0.339	4
A_2	0.367	1
A_3	0.351	2
A_4	0.345	3
A_5	0.338	5

Table 4
The distance of each lecturer from A^+ and A^- .

Lecturers	d^+	d^-
A_1	0.346	0.675
A_2	0.375	0.647
A_3	0.359	0.662
A_4	0.352	0.668
A_5	0.345	0.676

Table 6
Modified score, accuracy and certainty function of each lecturer.

Lecturers	Score function	Accuracy function	Certainty function	Ranking
A_1	0.332	−0.297	0.283	4
A_2	0.361	−0.241	0.313	1
A_3	0.345	−0.267	0.303	2
A_4	0.339	−0.284	0.289	3
A_5	0.331	−0.300	0.284	5

Table 6 shows that the ranking order of the five lecturers is $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$. Thus, the best lecturer is A_2 . The result is the same as that of the proposed method. This means that our method in the simplest form can procedure the results of the best method for this problem. Moreover, it is more generalized and flexible than the Ye's [62] method in dynamic environments.

6. Conclusion

This paper proposed a new concept of Dynamic Interval Valued Neutrosophic Set (DIVNS) where all the factors in DIVNSs such as truth, indeterminacy and falsity degrees are in different ranges of time. Mathematical operations associated with DIVNSs and correlation coefficients have also been defined. In addition, we have proposed a new TOPSIS method based on the DIVNSs and their application to evaluate lecturers' performance in the ULIS-VNU. This shows the feasibility and applications of Neutrosophic Theory in Industry.

In the future, we will use DIVNSs as well as the TOPSIS method to express dynamic information, and develop additional extension theories for DIVNSs such as operators, similarity measure. In addition, we extended this method to predictive problems such as in [24, 30, 31, 38, 39, 43, 44, 46, 47, 48, 50, 63–92].

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