

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/358582135>

FIXED POINT THEOREM FOR NEUTROSOPHIC EXTENDED METRIC-LIKE SPACES AND THEIR APPLICATION

Preprint · December 2021

CITATIONS

0

READS

64

6 authors, including:



Umar Ishtiaq

International Islamic University, Islamabad

6 PUBLICATIONS 15 CITATIONS

[SEE PROFILE](#)



Necip Simsek

Istanbul Commerce University

50 PUBLICATIONS 335 CITATIONS

[SEE PROFILE](#)



Khalil Javed

International Islamic University, Islamabad

13 PUBLICATIONS 50 CITATIONS

[SEE PROFILE](#)



Khaleel Ahmad

International Islamic University, Islamabad

3 PUBLICATIONS 0 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



<http://dergipark.gov.tr/fujma> [View project](#)



common fixed point results for single and multivalued mappings. [View project](#)

FIXED POINT THEOREM FOR NEUTROSOPHIC EXTENDED METRIC-LIKE SPACES AND THEIR APPLICATION

Umar Ishtiaq¹, Necip Simsek^{2,*}, Khalil Javed¹, Khalil Ahmed¹, Fahim Uddin¹, Murat Kirişçi³

ABSTRACT

In this manuscript, our objective is to introduce the notion of neutrosophic extended metric-like spaces. We establish some fixed point theorems in this setting. Neutrosophic extended metric-like spaces metric space uses the idea of continuous triangular norms and continuous triangular conorms in an extended intuitionistic fuzzy metric-like space. Triangular norms are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conorms are known as dual operations of triangular norms. Triangular norms and triangular conorm are very significant for fuzzy operation. The obtained results boost the approaches of existing ones in the literature and are supported by some examples and an application.

Keywords: Fixed point; Metric like space; Extended metric like space; Neutrosophic metric space; Integral equation.

INTRODUCTION

After being given the notion of fuzzy sets (FS) by Zadeh [12], many researchers provide abundant generalizations. In this continuation, Kramosil and Michalek [16] originated the approach of fuzzy metric spaces. George and Veeramani [8] initiated the approach of fuzzy metric spaces. Garbiec [9] gave the fuzzy interpretation of the Banach contraction principle in fuzzy metric spaces. For basic concepts, see [7, 10, 11, 14, 23]. Harandi [21] is credited with coining the term metric like spaces (MLS) which elegantly generalizes the idea of metric spaces. Shukla and Abbas [22] reformulated definition (MLS) in this context, resulting in a fuzzy metric-like spaces (FMLS).

Mehmood [13] originated the approach of a fuzzy extended b-metric space (FEBMS) by replacing coefficient $b \geq 1$ with a function $\alpha: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$. The approach of intuitionistic fuzzy metric spaces was introduced by Park in [1] and Konwar [4] who initiated an intuitionistic fuzzy b-metric space (IFBMS). Kirişçi and Simsek [17] generalized the approach of intuitionistic fuzzy metric space by presenting the approach of a neutrosophic metric space (NMS). Simsek and Kirişçi [19] and Sowndrarajan et al. [18] proved some fixed point results in the setting of an NMS.

In this manuscript, we use the concepts of fuzzy extended b-metric space, NMS, and FMLS to introduce the notion of a neutrosophic extended metric-like space (NEMLS). So the notion of NEMLS is a generalization of fuzzy extended b-metric space, FMLS, IFBMS, NMS, and other concepts in the existing literature. Also, some fixed point (FP) results with non-trivial examples and an application are provided. For related works, see [1, 2, 3, 5, 6, 15, 20, 24-34].

In the end, some notations which are important for the understanding of this manuscript are accommodated in the appendix section to avoid obscurity and vagueness.

In this article, CTM is used for a continuous triangular norm and CTCM for a continuous triangular co-norm.

PRELIMINARIES

In this section, some basic definitions are given that are helpful to understand the main results.

Definition 2.1 [13] A 4-tuple $(\mathfrak{B}, \Delta_\alpha, \circ, \alpha)$ is called a FEBMS if $\mathfrak{B}$ is a non-empty set, $\alpha: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$, $\circ$ is a CTM and Δ_α is an FS on $\mathfrak{B} \times \mathfrak{B} \times (0, \infty)$, meeting the below circumstances for all $\vartheta, \delta, \beta \in \mathfrak{B}$ and $\varsigma, \sigma > 0$:

$$(F1) \Delta_\alpha(\vartheta, \delta, 0) = 0;$$

$$(F2) \Delta_\alpha(\vartheta, \delta, \varsigma) = 1 \Leftrightarrow \vartheta = \delta;$$

$$(F3) \Delta_\alpha(\vartheta, \delta, \varsigma) = \Delta_\alpha(\delta, \vartheta, \varsigma);$$

$$(F4) \Delta_\alpha(\vartheta, \beta, \alpha(\vartheta, \beta)(\varsigma + \sigma)) \geq \Delta_\alpha(\vartheta, \delta, \varsigma) \circ \Delta_\alpha(\delta, \beta, \sigma);$$

$$(F5) \Delta_\alpha(\vartheta, \delta, \cdot): (0, \infty) \rightarrow [0, 1] \text{ is continuous.}$$

This paper was recommended for publication in revised form by Regional Editor Omid Mahian

¹ Department of Mechanical Engineering, Ozyegin University, Istanbul, Turkey

² Department of Materials Engineering, University of Technology, Baghdad, IRAQ

*E-mail address: layth.ismael@ozu.edu.tr, 130006@uotechnology.edu.iq

Orcid id: <https://orcid.org/0000-0002-5943-7763>

Manuscript Received 28 May 2017, Accepted 08 August 2017

Example 2.2 [13] Let $\mathfrak{B} = \{1,2,3\}$ and define Δ_α by

$$\Delta_\alpha(\vartheta, \delta, \varsigma) = \begin{cases} \frac{\varsigma}{\varsigma + d(\vartheta, \delta)} & \text{if } \varsigma > 0, \\ 0 & \text{if } \varsigma = 0 \end{cases}$$

Take $d(\vartheta, \delta) = (\vartheta - \delta)^2$ and given $\alpha: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$ as

$$\alpha(\vartheta, \delta) = 1 + \vartheta + \delta.$$

Also take the CTM: $a \circ b = a \wedge b = \min\{a, b\}$. Then $(\mathfrak{B}, \Delta_\alpha, \circ, \wedge)$ is an EFBMS.

Park introduced the concept of intuitionistic fuzzy metric spaces (IFMSs) and utilized this idea to investigate fixed point results. Park defined the notion of IFMSs as follows:

Definition 2.3 [1] Suppose $E \neq \emptyset$ is an arbitrary set, assume a five-tuple $(E, R, S, *, \Delta)$ where $*$ is a CTM, Δ is a CTCM, and R, S are FSs on $E \times E \times (0, \infty)$. If $(E, R, S, *, \Delta)$ meet the following circumstances for all $\beta, \delta, \vartheta \in E$ and $\pi, \lambda > 0$:

$$(B1) R(\beta, \delta, \lambda) + S(\beta, \delta, \lambda) \leq 1,$$

$$(B2) R(\beta, \delta, \lambda) > 0,$$

$$(B3) R(\beta, \delta, \lambda) = 1 \Leftrightarrow \beta = \delta,$$

$$(B4) R(\beta, \delta, \lambda) = R(\delta, \beta, \lambda),$$

$$(B5) R(\beta, \vartheta, (\lambda + \pi)) \geq R(\beta, \delta, \lambda) * R(\delta, \vartheta, \pi),$$

$$(B6) R(\beta, \delta, \cdot) \text{ is non decreasing (ND) function of } \mathbb{R}^+ \text{ and } \lim_{\lambda \rightarrow \infty} R(\beta, \delta, \lambda) = 1,$$

$$(B7) S(\beta, \delta, \lambda) > 0,$$

$$(B8) S(\beta, \delta, \lambda) = 0 \Leftrightarrow \beta = \delta,$$

$$(B9) S(\beta, \delta, \lambda) = S(\delta, \beta, \lambda),$$

$$(B10) S(\beta, \vartheta, (\lambda + \pi)) \leq S(\beta, \delta, \lambda) \Delta S(\delta, \vartheta, \pi),$$

$$(B11) S(\beta, \delta, \cdot) \text{ is non increasing (NI) function of } \mathbb{R}^+ \text{ and } \lim_{\lambda \rightarrow \infty} S(\beta, \delta, \lambda) = 0.$$

Then $(E, R, S, *, \Delta)$ is an IFMS.

The concept of neutrosophic metric spaces was discussed by Kirişçi and Simsek in his work and he defined the said concept as follows:

Definition 2.4 [17] Suppose $\mathfrak{B} \neq \emptyset$. Given a six tuple $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ where $*$ is a CTM, $\diamond$ is a CTCM, M_φ, N_φ and O_φ NS on $\mathfrak{B} \times \mathfrak{B} \times (0, \infty)$. If $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ meets the below circumstances for all $\vartheta, \delta, \beta \in \mathfrak{B}$ and $\varsigma, \sigma > 0$:

$$(I) M_\varphi(\vartheta, \delta, \varsigma) + N_\varphi(\vartheta, \delta, \varsigma) + O_\varphi(\vartheta, \delta, \varsigma) \leq 3,$$

$$(II) 0 \leq M_\varphi(\vartheta, \delta, \varsigma) \leq 1,$$

$$(III) M_\varphi(\vartheta, \delta, \varsigma) = 1 \Leftrightarrow \vartheta = \delta,$$

$$(IV) M_\varphi(\vartheta, \delta, \varsigma) = M_\varphi(\delta, \vartheta, \varsigma),$$

$$(V) M_\varphi(\vartheta, \beta, (\varsigma + \sigma)) \geq M_\varphi(\vartheta, \delta, \varsigma) * M_\varphi(\delta, \beta, \sigma),$$

$$(VI) M_\varphi(\vartheta, \delta, \cdot): [0, \infty) \rightarrow [0, 1] \text{ is continuous,}$$

$$(VII) \lim_{\varsigma \rightarrow \infty} M_\varphi(\vartheta, \delta, \varsigma) = 1,$$

$$(VIII) 0 \leq N_\varphi(\vartheta, \delta, \varsigma) \leq 1,$$

$$(IX) N_\varphi(\vartheta, \delta, \varsigma) = 0 \Leftrightarrow \vartheta = \delta,$$

$$(X) N_\varphi(\vartheta, \delta, \varsigma) = N_\varphi(\delta, \vartheta, \varsigma),$$

$$(XI) N_\varphi(\vartheta, \beta, (\varsigma + \sigma)) \leq N_\varphi(\vartheta, \delta, \varsigma) \diamond N_\varphi(\delta, \beta, \sigma),$$

$$(XII) N_\varphi(\vartheta, \delta, \cdot): [0, \infty) \rightarrow [0, 1] \text{ is continuous,}$$

$$(XIII) \lim_{\varsigma \rightarrow \infty} N_\varphi(\vartheta, \delta, \varsigma) = 0,$$

$$(XIV) 0 \leq O_\varphi(\vartheta, \delta, \varsigma) \leq 1,$$

$$(XV) O_\varphi(\vartheta, \delta, \varsigma) = 0 \Leftrightarrow \vartheta = \delta,$$

$$(XVI) O_\varphi(\vartheta, \delta, \varsigma) = O_\varphi(\delta, \vartheta, \varsigma),$$

$$(XVII) O_\varphi(\vartheta, \beta, (\varsigma + \sigma)) \leq O_\varphi(\vartheta, \delta, \varsigma) \diamond O_\varphi(\delta, \beta, \sigma),$$

- (XVIII) $O_\varphi(\vartheta, \delta, \cdot): [0, \infty) \rightarrow [0, 1]$ is a continuous,
 (XIX) $\lim_{\varsigma \rightarrow \infty} O_\varphi(\vartheta, \delta, \varsigma) = 0$,
 (XX) if $\varsigma \leq 0$ then $M_\varphi(\vartheta, \delta, \varsigma) = 0, N_\varphi(\vartheta, \delta, \varsigma) = 1, O_\varphi(\vartheta, \delta, \varsigma) = 1$,

Then $(M_\varphi, N_\varphi, O_\varphi)$ is a neutrosophic metric on and $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ is an NMS. The functions $M_\varphi(\vartheta, \delta, \varsigma)$, $N_\varphi(\vartheta, \delta, \varsigma)$ and $O_\varphi(\vartheta, \delta, \varsigma)$ represent the degree of nearness, naturalness, and non-nearness between ϑ and δ for ς , respectively.

MAIN RESULTS

Now, we introduce the notion of NEMLS and utilize this concept to investigate some fixed point results.

Definition 3.1 Suppose $\mathfrak{B} \neq \emptyset$. Given a six tuple $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ where $*$ is a CTM, $\diamond$ is a CTCM, $\varphi: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$, M_φ, N_φ and O_φ are NSs on $\mathfrak{B} \times \mathfrak{B} \times (0, \infty)$. If $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ meets the below circumstances for all $\vartheta, \delta, \beta \in \mathfrak{B}$ and $\varsigma, \sigma > 0$:

- (i) $M_\varphi(\vartheta, \delta, \varsigma) + N_\varphi(\vartheta, \delta, \varsigma) + O_\varphi(\vartheta, \delta, \varsigma) \leq 3$,
- (ii) $0 \leq M_\varphi(\vartheta, \delta, \varsigma) \leq 1$,
- (iii) $M_\varphi(\vartheta, \delta, \varsigma) = 1$ implies $\vartheta = \delta$,
- (iv) $M_\varphi(\vartheta, \delta, \varsigma) = M_\varphi(\delta, \vartheta, \varsigma)$,
- (v) $M_\varphi(\vartheta, \beta, \varphi(\vartheta, \beta)(\varsigma + \sigma)) \geq M_\varphi(\vartheta, \delta, \varsigma) * M_\varphi(\delta, \beta, \sigma)$,
- (vi) $M_\varphi(\vartheta, \delta, \cdot): [0, \infty) \rightarrow [0, 1]$ is continuous,
- (vii) $\lim_{\varsigma \rightarrow \infty} M_\varphi(\vartheta, \delta, \varsigma) = 1$,
- (viii) $0 \leq N_\varphi(\vartheta, \delta, \varsigma) \leq 1$,
- (ix) $N_\varphi(\vartheta, \delta, \varsigma) = 0$ implies $\vartheta = \delta$,
- (x) $N_\varphi(\vartheta, \delta, \varsigma) = N_\varphi(\delta, \vartheta, \varsigma)$,
- (xi) $N_\varphi(\vartheta, \beta, \varphi(\vartheta, \beta)(\varsigma + \sigma)) \leq N_\varphi(\vartheta, \delta, \varsigma) \diamond N_\varphi(\delta, \beta, \sigma)$,
- (xii) $N_\varphi(\vartheta, \delta, \cdot): [0, \infty) \rightarrow [0, 1]$ is continuous,
- (xiii) $\lim_{\varsigma \rightarrow \infty} N_\varphi(\vartheta, \delta, \varsigma) = 0$,
- (xiv) $0 \leq O_\varphi(\vartheta, \delta, \varsigma) \leq 1$,
- (xv) $O_\varphi(\vartheta, \delta, \varsigma) = 0$ implies $\vartheta = \delta$,
- (xvi) $O_\varphi(\vartheta, \delta, \varsigma) = O_\varphi(\delta, \vartheta, \varsigma)$,
- (xvii) $O_\varphi(\vartheta, \beta, \varphi(\vartheta, \beta)(\varsigma + \sigma)) \leq O_\varphi(\vartheta, \delta, \varsigma) \diamond O_\varphi(\delta, \beta, \sigma)$,
- (xviii) $O_\varphi(\vartheta, \delta, \cdot): [0, \infty) \rightarrow [0, 1]$ is a continuous,
- (xix) $\lim_{\varsigma \rightarrow \infty} O_\varphi(\vartheta, \delta, \varsigma) = 0$,
- (xx) if $\varsigma \leq 0$ then $M_\varphi(\vartheta, \delta, \varsigma) = 0, N_\varphi(\vartheta, \delta, \varsigma) = 1, O_\varphi(\vartheta, \delta, \varsigma) = 1$,

Then $(M_\varphi, N_\varphi, O_\varphi)$ is an extended neutrosophic metric-like (ENML) on and $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ is an NEMLS.

Example 3.2 Let $\mathfrak{B} = \mathbb{N}$. Define $M_\varphi, N_\varphi, O_\varphi: \mathfrak{B} \times \mathfrak{B} \times (0, \infty) \rightarrow [0, 1]$ by

$$M_\varphi(\vartheta, \delta, \varsigma) = \frac{\varsigma}{\varsigma + \max\{\vartheta, \delta\}^2}, \quad N_\varphi(\vartheta, \delta, \varsigma) = \frac{\max\{\vartheta, \delta\}^2}{\varsigma + \max\{\vartheta, \delta\}^2}$$

$$O_\varphi(\vartheta, \delta, \varsigma) = \frac{\max\{\vartheta, \delta\}^2}{\varsigma}.$$

for all $\vartheta, \delta \in \mathfrak{B}$ and $\varsigma > 0$. Define the CTN by $a \circ b = a \cdot b$ and CTCN " $\diamond$ " by $a \diamond b = \max\{a, b\}$ and define " φ " by

$$\varphi(\vartheta, \delta) = \begin{cases} 1 & \text{if } \vartheta = \delta, \\ \max\{\vartheta, \delta\} & \text{if } \vartheta \neq \delta \end{cases}$$

Then $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a NEMLS.

Proof: (i)–(iv), (vi) – (x), (xii) – (xvi) and (xviii) – (xx) are obvious. We shall prove (v), (xi) and (xix). We have $\max\{\vartheta, \beta\}^2 \leq \varphi(\vartheta, \beta)[\max\{\vartheta, \delta\}^2 + \max\{\delta, \beta\}^2]$.

Then

$$\varsigma \sigma \max\{\vartheta, \beta\}^2 \leq \varphi(\vartheta, \beta)[(\varsigma \sigma + \sigma^2) \max\{\vartheta, \delta\}^2 + (\varsigma \sigma + \varsigma^2) \max\{\delta, \beta\}^2].$$

That is

$$\varsigma \sigma \max\{\vartheta, \beta\}^2 \leq \varphi(\vartheta, \beta)[(\varsigma + \sigma) \sigma \max\{\vartheta, \delta\}^2 + (\varsigma + \sigma) \varsigma \max\{\delta, \beta\}^2].$$

Then

$$\varsigma \sigma \max\{\vartheta, \beta\}^2 \leq \varphi(\vartheta, \beta)[(\varsigma + \sigma) \sigma \max\{\vartheta, \delta\}^2 + (\varsigma + \sigma) \varsigma \max\{\delta, \beta\}^2 + (\varsigma + \sigma) \max\{\vartheta, \delta\}^2 \max\{\delta, \beta\}^2].$$

That is

$$\varsigma \sigma \max\{\vartheta, \beta\}^2 \leq \varphi(\vartheta, \beta)(\varsigma + \sigma)[\sigma \max\{\vartheta, \delta\}^2 + \varsigma \max\{\delta, \beta\}^2 + \max\{\vartheta, \delta\}^2 \max\{\delta, \beta\}^2].$$

Then

$$\begin{aligned} \varphi(\vartheta, \beta)(\varsigma + \sigma) \varsigma \sigma + \varsigma \sigma \max\{\vartheta, \beta\}^2 \\ \leq \varphi(\vartheta, \beta)(\varsigma + \sigma) \varsigma \sigma + \varphi(\vartheta, \beta)(\varsigma + \sigma)[\sigma \max\{\vartheta, \delta\}^2 + \varsigma \max\{\delta, \beta\}^2 + \max\{\vartheta, \delta\}^2 \max\{\delta, \beta\}^2]. \end{aligned}$$

Then

$$\varsigma \sigma [\varphi(\vartheta, \beta)(\varsigma + \sigma) + \max\{\vartheta, \beta\}^2] \leq \varphi(\vartheta, \beta)(\varsigma + \sigma)[\varsigma \sigma + \sigma \max\{\vartheta, \delta\}^2 + \varsigma \max\{\delta, \beta\}^2 + \max\{\vartheta, \delta\}^2 \max\{\delta, \beta\}^2].$$

That is

$$\varsigma \sigma [\varphi(\vartheta, \beta)(\varsigma + \sigma) + \max\{\vartheta, \beta\}^2] \leq \varphi(\vartheta, \beta)(\varsigma + \sigma)[\varsigma + \max\{\vartheta, \delta\}^2][\sigma + \max\{\delta, \beta\}^2].$$

Then

$$\frac{\varphi(\vartheta, \beta)(\varsigma + \sigma)}{\varphi(\vartheta, \beta)(\varsigma + \sigma) + \max\{\vartheta, \beta\}^2} \geq \frac{\varsigma \sigma}{[\varsigma + \max\{\vartheta, \delta\}^2][\sigma + \max\{\delta, \beta\}^2]}.$$

That is

$$\frac{\varphi(\vartheta, \beta)(\varsigma + \sigma)}{\varphi(\vartheta, \beta)(\varsigma + \sigma) + \max\{\vartheta, \beta\}^2} \geq \frac{\varsigma}{\varsigma + \max\{\vartheta, \delta\}^2} \cdot \frac{\sigma}{\sigma + \max\{\delta, \beta\}^2}.$$

Hence

$$\Rightarrow M_\varphi(\vartheta, \beta, \varphi(\vartheta, \beta)(\varsigma + \sigma)) \geq M_\varphi(\vartheta, \delta, \varsigma) \circ M_\varphi(\delta, \beta, \sigma).$$

(v) is satisfied.

$$\max\{\vartheta, \beta\}^2 = \max\{\vartheta, \beta\}^2 \max\{1, 1\}$$

Then

$$\max\{\vartheta, \beta\}^2 = \max\{\vartheta, \beta\}^2 \max\left\{\frac{\max\{\vartheta, \delta\}^2}{\max\{\vartheta, \delta\}^2}, \frac{\max\{\delta, \beta\}^2}{\max\{\delta, \beta\}^2}\right\}.$$

That is

$$\max\{\vartheta, \beta\}^2 \leq [\varphi(\vartheta, \beta)(\varsigma + \sigma) + \max\{\vartheta, \beta\}^2] \max\left\{\frac{\max\{\vartheta, \delta\}^2}{\varsigma + \max\{\vartheta, \delta\}^2}, \frac{\max\{\delta, \beta\}^2}{\sigma + \max\{\delta, \beta\}^2}\right\}.$$

Then

$$\frac{\max\{\vartheta, \beta\}^2}{\varphi(\vartheta, \beta)(\varsigma + \sigma) + \max\{\vartheta, \beta\}^2} \leq \max\left\{\frac{\max\{\vartheta, \delta\}^2}{\varsigma + \max\{\vartheta, \delta\}^2}, \frac{\max\{\delta, \beta\}^2}{\sigma + \max\{\delta, \beta\}^2}\right\}.$$

Hence

$$N_\varphi(\vartheta, \beta, \varphi(\vartheta, \beta)(\varsigma + \sigma)) \leq N_\varphi(\vartheta, \delta, \varsigma).$$

(xi) is satisfied.

$$\max\{\vartheta, \beta\}^2 \leq \varphi(\vartheta, \beta) \max\{\max\{\vartheta, \delta\}^2, \max\{\delta, \beta\}^2\}$$

Then

$$\frac{\max\{\vartheta, \beta\}^2}{\varsigma + \sigma} \leq \varphi(\vartheta, \beta) \max\left\{\frac{\max\{\vartheta, \delta\}^2}{\varsigma}, \frac{\max\{\delta, \beta\}^2}{\sigma}\right\}.$$

That is

$$\frac{\max\{\vartheta, \beta\}^2}{\varphi(\vartheta, \beta)(\varsigma + \sigma)} \leq \max\left\{\frac{\max\{\vartheta, \delta\}^2}{\varsigma}, \frac{\max\{\delta, \beta\}^2}{\sigma}\right\}.$$

Hence

$$\Rightarrow O_\varphi(\vartheta, \beta, \varphi(\vartheta, \beta)(\varsigma + \sigma)) \leq O_\varphi(\vartheta, \delta, \varsigma) \diamond O_\varphi(\delta, \beta, \sigma).$$

(xvii) is satisfied.

Remark 3.3 The above example is also satisfied for $a \circ b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$.

Definition 3.4 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a NEMLS and $\{\vartheta_n\}$ be a sequence in $\mathfrak{B}$. then $\{\vartheta_n\}$ is named to be:

(i) a convergent, if there exists $\vartheta \in \mathfrak{B}$ such that

$$\lim_{n \rightarrow \infty} M_\varphi(\vartheta_n, \vartheta, \varsigma) = M_\varphi(\vartheta, \vartheta, \varsigma), \quad \lim_{n \rightarrow \infty} N_\varphi(\vartheta_n, \vartheta, \varsigma) = N_\varphi(\vartheta, \vartheta, \varsigma), \text{ and } \lim_{n \rightarrow \infty} O_\varphi(\vartheta_n, \vartheta, \varsigma) = O_\varphi(\vartheta, \vartheta, \varsigma),$$

for all $\varsigma > 0$

- (ii) a Cauchy sequence (CS), if and only if for each $\varsigma > 0$, there exists $n_0 \in \mathbb{N}$ such that $\lim_{n \rightarrow \infty} M_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma)$, $\lim_{n \rightarrow \infty} N_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma)$ and $\lim_{n \rightarrow \infty} O_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma)$ exists and finite.
- (iii) If every Cauchy sequence convergent in $\mathfrak{B}$, then $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ is called complete NEMLS.

$$\begin{aligned}\lim_{n \rightarrow \infty} M_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} M_\varphi(\vartheta_n, \vartheta, \varsigma) = M_\varphi(\vartheta, \vartheta, \varsigma), \\ \lim_{n \rightarrow \infty} N_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} N_\varphi(\vartheta_n, \vartheta, \varsigma) = N_\varphi(\vartheta, \vartheta, \varsigma), \\ \lim_{n \rightarrow \infty} O_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} O_\varphi(\vartheta_n, \vartheta, \varsigma) = O_\varphi(\vartheta, \vartheta, \varsigma).\end{aligned}$$

At this time, we shall prove extended Neutrosophic like Banach contraction results.

Theorem 3.5 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a complete NEMLS in the company of $\varphi: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$ and suppose that

$$\lim_{\varsigma \rightarrow \infty} M_\varphi(\vartheta, \delta, \varsigma) = 1, \lim_{\varsigma \rightarrow \infty} N_\varphi(\vartheta, \delta, \varsigma) = 0 \text{ and } \lim_{\varsigma \rightarrow \infty} O_\varphi(\vartheta, \delta, \varsigma) = 0 \quad (1)$$

for all $\vartheta, \delta \in \mathfrak{B}$ and $\varsigma > 0$. Let $f: \mathfrak{B} \rightarrow \mathfrak{B}$ be a mapping satisfying

$$\begin{aligned}M_\varphi(f\vartheta, f\delta, k\varsigma) &\geq M_\varphi(\vartheta, \delta, \varsigma), \quad N_\varphi(f\vartheta, f\delta, k\varsigma) \leq N_\varphi(\vartheta, \delta, \varsigma), \\ O_\varphi(f\vartheta, f\delta, k\varsigma) &\leq O_\varphi(\vartheta, \delta, \varsigma)\end{aligned} \quad (2)$$

for all $\vartheta, \delta \in \mathfrak{B}$, $0 < k < 1$ and $\varsigma > 0$. Further, suppose that for an arbitrary $\vartheta_0 \in \mathfrak{B}$, and $n, q \in \mathbb{N}$, we have

$$\varphi(\vartheta_n, \vartheta_{n+q}) < \frac{1}{k},$$

where $\vartheta_n = f^n \vartheta_0 = f\vartheta_{n-1}$. Then f has a unique FP.

Proof: Let ϑ_0 be a random element of $\mathfrak{B}$ and take $\vartheta_n = f^n \vartheta_0 = f\vartheta_{n-1}$, $n \in \mathbb{N}$. By using (2) for all $\varsigma > 0$, we have

$$\begin{aligned}M_\varphi(\vartheta_n, \vartheta_{n+1}, k\varsigma) &= M_\varphi(f\vartheta_{n-1}, f\vartheta_n, k\varsigma) \geq M_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma) \geq M_\varphi\left(\vartheta_{n-2}, \vartheta_{n-1}, \frac{\varsigma}{k}\right) \\ &\geq M_\varphi\left(\vartheta_{n-3}, \vartheta_{n-2}, \frac{\varsigma}{k^2}\right) \geq \dots \geq M_\varphi\left(\vartheta_0, \vartheta_1, \frac{\varsigma}{k^n}\right), \\ N_\varphi(\vartheta_n, \vartheta_{n+1}, k\varsigma) &= N_\varphi(f\vartheta_{n-1}, f\vartheta_n, k\varsigma) \leq N_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma) \leq N_\varphi\left(\vartheta_{n-2}, \vartheta_{n-1}, \frac{\varsigma}{k}\right) \\ &\leq N_\varphi\left(\vartheta_{n-3}, \vartheta_{n-2}, \frac{\varsigma}{k^2}\right) \leq \dots \leq N_\varphi\left(\vartheta_0, \vartheta_1, \frac{\varsigma}{k^n}\right)\end{aligned}$$

and

$$\begin{aligned}O_\varphi(\vartheta_n, \vartheta_{n+1}, k\varsigma) &= O_\varphi(f\vartheta_{n-1}, f\vartheta_n, k\varsigma) \leq O_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma) \leq O_\varphi\left(\vartheta_{n-2}, \vartheta_{n-1}, \frac{\varsigma}{k}\right) \\ &\leq O_\varphi\left(\vartheta_{n-3}, \vartheta_{n-2}, \frac{\varsigma}{k^2}\right) \leq \dots \leq O_\varphi\left(\vartheta_0, \vartheta_1, \frac{\varsigma}{k^n}\right).\end{aligned}$$

We obtain

$$\begin{aligned}M_\varphi(\vartheta_n, \vartheta_{n+1}, k\varsigma) &\geq M_\varphi\left(\vartheta_0, \vartheta_1, \frac{\varsigma}{k^n}\right), \quad N_\varphi(\vartheta_n, \vartheta_{n+1}, k\varsigma) \leq N_\varphi\left(\vartheta_0, \vartheta_1, \frac{\varsigma}{k^n}\right) \text{ and} \\ O_\varphi(\vartheta_n, \vartheta_{n+1}, k\varsigma) &\leq O_\varphi\left(\vartheta_0, \vartheta_1, \frac{\varsigma}{k^n}\right).\end{aligned} \quad (3)$$

For any $q \in \mathbb{N}$, $\varsigma = \frac{q\varsigma}{q} = \frac{\varsigma}{q} + \frac{\varsigma}{q} + \dots + \frac{\varsigma}{q}$ and using definition of a NEMLS, we deduce

$$\begin{aligned}M_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &\geq M_\varphi\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))}\right) \circ M_\varphi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))}\right) \\ &\circ M_\varphi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))}\right) \circ \dots \circ \\ &M_\varphi\left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q})\dots\varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))}\right).\end{aligned}$$

Also,

$$\begin{aligned}N_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &\leq N_\varphi\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))}\right) \diamond N_\varphi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))}\right) \\ &\diamond N_\varphi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))}\right) \diamond \dots \diamond\end{aligned}$$

$$N_{\varphi} \left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}) \right)} \right)$$

and

$$\begin{aligned} O_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\leq O_{\varphi} \left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \right)} \right) \diamond O_{\varphi} \left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \right)} \right) \\ &\diamond O_{\varphi} \left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \right)} \right) \diamond \cdots \diamond \\ &O_{\varphi} \left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}) \right)} \right). \end{aligned}$$

Using (3) and the definition of an NEMLS, we deduce

$$\begin{aligned} M_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\geq M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \right) k^n} \right) \circ M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \right) k^{n+1}} \right) \\ &\circ M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \right) k^{n+2}} \right) \circ \cdots \circ \\ &M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}) \right) k^{n+q-1}} \right). \end{aligned}$$

Also,

$$\begin{aligned} N_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\leq N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \right) k^n} \right) \diamond N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \right) k^{n+2}} \right) \\ &\diamond N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \right) k^{n+3}} \right) \diamond \cdots \diamond \\ &N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}) \right) k^{n+q-1}} \right) \end{aligned}$$

and

$$\begin{aligned} O_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\leq O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \right) k^n} \right) \diamond O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \right) k^{n+2}} \right) \\ &\diamond O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \right) k^{n+3}} \right) \diamond \cdots \diamond \\ &O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q \left(\varphi(\vartheta_n, \vartheta_{n+q}) \varphi(\vartheta_{n+1}, \vartheta_{n+q}) \varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}) \right) k^{n+q-1}} \right). \end{aligned}$$

By hypothesis, for all $n, q \in \mathbb{N}$, we obtain $\varphi(\vartheta_n, \vartheta_{n+q})k < 1$, with $0 < k < 1$. Therefore, by (1) and for $n \rightarrow \infty$,

$$\begin{aligned} \lim_{n \rightarrow \infty} M_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &= 1 \circ 1 \circ \cdots \circ 1, \\ \lim_{n \rightarrow \infty} N_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &= 0 \diamond 0 \diamond \cdots \diamond 0 = 0 \end{aligned}$$

and

$$\lim_{n \rightarrow \infty} O_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) = 0 \diamond 0 \diamond \cdots \diamond 0 = 0.$$

That is, $\{\vartheta_n\}$ is a CS. Since $(\mathfrak{B}, M_{\varphi}, N_{\varphi}, O_{\varphi}, \circ, \diamond)$ is a complete NEMLS, let

$$\begin{aligned}\lim_{n \rightarrow \infty} M_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} M_\varphi(\vartheta_n, \vartheta, \varsigma) = M_\varphi(\vartheta, \vartheta, \varsigma), \\ \lim_{n \rightarrow \infty} N_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} N_\varphi(\vartheta_n, \vartheta, \varsigma) = N_\varphi(\vartheta, \vartheta, \varsigma), \\ \lim_{n \rightarrow \infty} O_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} O_\varphi(\vartheta_n, \vartheta, \varsigma) = O_\varphi(\vartheta, \vartheta, \varsigma).\end{aligned}$$

Now, we claim that ϑ is a FP of f . Using definition of the NEMLS and (1), we obtain

$$\begin{aligned}M_\varphi(f\vartheta, \vartheta, \varsigma) &\geq M_\varphi\left(fa, f\vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \circ M_\varphi\left(f\vartheta_n, a, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \\ &\geq M_\varphi\left(a, \vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))k}\right) \circ M_\varphi\left(\vartheta_{n+1}, \vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \rightarrow 1 \circ 1 = 1 \text{ as } n \rightarrow \infty.\end{aligned}$$

Also,

$$\begin{aligned}N_\varphi(f\vartheta, \vartheta, \varsigma) &\leq N_\varphi\left(fa, f\vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \diamond N_\varphi\left(f\vartheta_n, a, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \\ &\leq N_\varphi\left(a, \vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))k}\right) \diamond N_\varphi\left(\vartheta_{n+1}, \vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \rightarrow 0 \diamond 0 = 0 \text{ as } n \rightarrow \infty,\end{aligned}$$

and

$$\begin{aligned}O_\varphi(f\vartheta, \vartheta, \varsigma) &\leq O_\varphi\left(fa, f\vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \diamond O_\varphi\left(f\vartheta_n, a, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \\ &\leq O_\varphi\left(a, \vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))k}\right) \diamond O_\varphi\left(\vartheta_{n+1}, \vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta, \vartheta))}\right) \rightarrow 0 \diamond 0 = 0 \text{ as } n \rightarrow \infty.\end{aligned}$$

This implies that $f\vartheta = \vartheta$. To show its uniqueness, suppose that $fc = c$ for some $c \in \mathfrak{B}$, then

$$\begin{aligned}1 &\geq M_\varphi(c, \vartheta, \varsigma) = M_\varphi(fc, f\vartheta, \varsigma) \geq M_\varphi\left(c, \vartheta, \frac{\varsigma}{k}\right) = M_\varphi\left(fc, f\vartheta, \frac{\varsigma}{k}\right) \\ &\geq M_\varphi\left(c, \vartheta, \frac{\varsigma}{k^2}\right) \geq \dots \geq M_\varphi\left(c, \vartheta, \frac{\varsigma}{k^n}\right) \rightarrow 1 \text{ as } n \rightarrow \infty.\end{aligned}$$

Also,

$$\begin{aligned}0 &\leq N_\varphi(c, \vartheta, \varsigma) = N_\varphi(fc, f\vartheta, \varsigma) \leq N_\varphi\left(c, \vartheta, \frac{\varsigma}{k}\right) = N_\varphi\left(fc, f\vartheta, \frac{\varsigma}{k}\right) \\ &\leq N_\varphi\left(c, \vartheta, \frac{\varsigma}{k^2}\right) \leq \dots \leq N_\varphi\left(c, \vartheta, \frac{\varsigma}{k^n}\right) \rightarrow 0 \text{ as } n \rightarrow \infty,\end{aligned}$$

and

$$\begin{aligned}0 &\leq O_\varphi(c, \vartheta, \varsigma) = O_\varphi(fc, f\vartheta, \varsigma) \leq O_\varphi\left(c, \vartheta, \frac{\varsigma}{k}\right) = O_\varphi\left(fc, f\vartheta, \frac{\varsigma}{k}\right) \\ &\leq O_\varphi\left(c, \vartheta, \frac{\varsigma}{k^2}\right) \leq \dots \leq O_\varphi\left(c, \vartheta, \frac{\varsigma}{k^n}\right) \rightarrow 0 \text{ as } n \rightarrow \infty.\end{aligned}$$

By using the definition of a NEMLS, $\vartheta = c$.

Definition 3.6 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a NEMLS. A map $f: \mathfrak{B} \rightarrow \mathfrak{B}$ is an extended neutrosophic like contraction if there exists $0 < k < 1$ so that

$$\frac{1}{M_\varphi(f\vartheta, f\delta, \varsigma)} - 1 \leq k \left[\frac{1}{M_\varphi(\vartheta, \delta, \varsigma)} - 1 \right] \quad (4)$$

$$N_\varphi(f\vartheta, f\delta, \varsigma) \leq k N_\varphi(\vartheta, \delta, \varsigma), \quad (5)$$

and

$$O_\varphi(f\vartheta, f\delta, \varsigma) \leq k O_\varphi(\vartheta, \delta, \varsigma), \quad (6)$$

for all $\vartheta, \delta \in \mathfrak{B}$ and $\varsigma > 0$.

Now, we prove a result for the above extended neutrosophic-like contraction.

Theorem 3.7 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a complete NEMLS with $\varphi: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$ and suppose that

$$\lim_{\varsigma \rightarrow \infty} M_\varphi(\vartheta, \delta, \varsigma) = 1, \lim_{\varsigma \rightarrow \infty} N_\varphi(\vartheta, \delta, \varsigma) = 0 \text{ and } \lim_{\varsigma \rightarrow \infty} O_\varphi(\vartheta, \delta, \varsigma) = 0 \quad (7)$$

for all $\vartheta, \delta \in \mathfrak{B}$ and $\varsigma > 0$. Let $f: \mathfrak{B} \rightarrow \mathfrak{B}$ be an extended Neutrosophic like contraction mapping in definition 3.3.

Further, suppose that for an arbitrary $\vartheta_0 \in \mathfrak{B}$, and $n, q \in \mathbb{N}$, we have

$$\varphi(\vartheta_n, \vartheta_{n+q}) < \frac{1}{k},$$

where $\vartheta_n = f^n \vartheta_0 = fa_{n-1}$. Then f has a unique FP.

Proof: Let ϑ_0 be in $\mathfrak{B}$ and take $\vartheta_n = f^n \vartheta_0 = fa_{n-1}$, $n \in \mathbb{N}$. By using (4), (5) and (6) for all $\varsigma > 0$, $n > q$, we have

$$\begin{aligned} & \frac{1}{M_\varphi(\vartheta_n, \vartheta_{n+1}, \varsigma)} - 1 = \frac{1}{M_\varphi(f\vartheta_{n-1}, \vartheta_n, \varsigma)} - 1 \\ & \leq k \left[\frac{1}{M_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma)} - 1 \right] = \frac{k}{M_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma)} - k. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{1}{M_\varphi(\vartheta_n, \vartheta_{n+1}, \varsigma)} & \leq \frac{k}{M_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma)} + (1 - k) \\ & \leq \frac{k^2}{M_\varphi(\vartheta_{n-2}, \vartheta_{n-1}, \varsigma)} + k(1 - k) + (1 - k). \end{aligned}$$

Continuing in this way, we get

$$\begin{aligned} \frac{1}{M_\varphi(\vartheta_n, \vartheta_{n+1}, \varsigma)} & \leq \frac{k^n}{M_\varphi(\vartheta_0, \vartheta_1, \varsigma)} + k^{n-1}(1 - k) + k^{n-2}(1 - k) + \dots + k(1 - k) + (1 - k) \\ & \leq \frac{k^n}{M_\varphi(\vartheta_0, \vartheta_1, \varsigma)} + (k^{n-1} + k^{n-2} + \dots + 1)(1 - k) \leq \frac{k^n}{M_\varphi(\vartheta_0, \vartheta_1, \varsigma)} + (1 - k^n). \end{aligned}$$

We obtain

$$\frac{1}{\frac{k^n}{M_\varphi(\vartheta_0, \vartheta_1, \varsigma)} + (1 - k^n)} \leq M_\varphi(\vartheta_n, \vartheta_{n+1}, \varsigma). \quad (8)$$

Also,

$$\begin{aligned} N_\varphi(\vartheta_n, \vartheta_{n+1}, \varsigma) & = N_\varphi(f\vartheta_{n-1}, \vartheta_n, \varsigma) \leq k N_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma) = N_\varphi(f\vartheta_{n-2}, \vartheta_{n-1}, \varsigma) \\ & \leq k^2 N_\varphi(\vartheta_{n-2}, \vartheta_{n-1}, \varsigma) \leq \dots \leq k^n N_\varphi(\vartheta_0, \vartheta_1, \varsigma) \end{aligned} \quad (9)$$

and

$$\begin{aligned} O_\varphi(\vartheta_n, \vartheta_{n+1}, \varsigma) & = O_\varphi(f\vartheta_{n-1}, \vartheta_n, \varsigma) \leq k O_\varphi(\vartheta_{n-1}, \vartheta_n, \varsigma) = O_\varphi(f\vartheta_{n-2}, \vartheta_{n-1}, \varsigma) \\ & \leq k^2 O_\varphi(\vartheta_{n-2}, \vartheta_{n-1}, \varsigma) \leq \dots \leq k^n O_\varphi(\vartheta_0, \vartheta_1, \varsigma). \end{aligned} \quad (10)$$

For any $q \in \mathbb{N}$, $\varsigma = \frac{q\varsigma}{q} = \frac{\varsigma}{q} + \frac{\varsigma}{q} + \dots + \frac{\varsigma}{q}$ and using the definition of a NEMLS, we deduce

$$\begin{aligned} M_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) & \geq M_\varphi\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))}\right) \circ M_\varphi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))}\right) \\ & \quad \circ M_\varphi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))}\right) \circ \dots \circ \\ & \quad M_\varphi\left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}) \dots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))}\right). \end{aligned}$$

Also,

$$\begin{aligned} N_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) & \leq N_\varphi\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))}\right) \diamond N_\varphi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))}\right) \\ & \quad \diamond N_\varphi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))}\right) \diamond \dots \diamond \\ & \quad N_\varphi\left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}) \dots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))}\right) \end{aligned}$$

and

$$\begin{aligned} O_\varphi(\vartheta_n, \vartheta_{n+q}, \varsigma) & \leq O_\varphi\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))}\right) \diamond O_\varphi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))}\right) \\ & \quad \diamond O_\varphi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))}\right) \diamond \dots \diamond \end{aligned}$$

$$O_{\varphi} \left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))} \right).$$

One writes

$$\begin{aligned} M_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\geq \frac{1}{\frac{k^n}{M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))} \right)} + (1 - k^n)} \\ &\circ \frac{1}{\frac{k^{n+1}}{M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))} \right)} + (1 - k^{n+1})} \\ &\circ \frac{1}{\frac{k^{n+2}}{M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))} \right)} + (1 - k^{n+2})} \circ \dots \circ \\ &\frac{1}{\frac{k^{n+q-1}}{M_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))} \right)} + (1 - k^{n+q-1})}. \end{aligned}$$

Also,

$$\begin{aligned} N_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\leq k^n N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))} \right) \diamond k^{n+1} N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))} \right) \\ &\diamond k^{n+2} N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))} \right) \diamond \dots \diamond \\ &k^{n+q-1} N_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))} \right) \end{aligned}$$

and

$$\begin{aligned} O_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &\leq k^n O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q}))} \right) \diamond k^{n+1} O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q}))} \right) \\ &\diamond k^{n+2} O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}))} \right) \diamond \dots \diamond \\ &k^{n+q-1} O_{\varphi} \left(\vartheta_0, \vartheta_1, \frac{\varsigma}{q(\varphi(\vartheta_n, \vartheta_{n+q})\varphi(\vartheta_{n+1}, \vartheta_{n+q})\varphi(\vartheta_{n+2}, \vartheta_{n+q}) \cdots \varphi(\vartheta_{n+q-1}, \vartheta_{n+q}))} \right). \end{aligned}$$

By hypothesis for all $n, q \in \mathbb{N}$, we obtain $\varphi(\vartheta_n, \vartheta_{n+q})k < 1$. Therefore, from the definition of a NEMLS, (1) and for $n \rightarrow \infty$,

$$\begin{aligned} \lim_{n \rightarrow \infty} M_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &= 1 \circ 1 \circ \dots \circ 1 \\ \lim_{n \rightarrow \infty} N_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &= 0 \diamond 0 \diamond \dots \diamond 0 = 0 \end{aligned}$$

and

$$\lim_{n \rightarrow \infty} O_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) = 0 \diamond 0 \diamond \dots \diamond 0 = 0.$$

That is, $\{\vartheta_n\}$ is a CS. Since $(\mathfrak{B}, M_{\varphi}, N_{\varphi}, O_{\varphi}, \circ, \diamond)$ is complete, let

$$\lim_{n \rightarrow \infty} M_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) = \lim_{n \rightarrow \infty} M_{\varphi}(\vartheta_n, \vartheta, \varsigma) = M_{\varphi}(\vartheta, \vartheta, \varsigma),$$

$$\begin{aligned}\lim_{n \rightarrow \infty} N_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} N_{\varphi}(\vartheta_n, \vartheta, \varsigma) = N_{\varphi}(\vartheta, \vartheta, \varsigma), \\ \lim_{n \rightarrow \infty} O_{\varphi}(\vartheta_n, \vartheta_{n+q}, \varsigma) &= \lim_{n \rightarrow \infty} O_{\varphi}(\vartheta_n, \vartheta, \varsigma) = O_{\varphi}(\vartheta, \vartheta, \varsigma).\end{aligned}$$

Now, we investigate that ϑ is an FP of f . Using the definition of a NEMLS and (1), we obtain

$$\begin{aligned}\frac{1}{M_{\varphi}(f\vartheta_n, f\vartheta, \varsigma)} - 1 &\leq k \left[\frac{1}{M_{\varphi}(\vartheta_n, \vartheta, \varsigma)} - 1 \right] = \frac{k}{M_{\varphi}(\vartheta_n, \vartheta, \varsigma)} - k \\ &\Rightarrow \frac{1}{\frac{k}{M_{\varphi}(\vartheta_n, \vartheta, \varsigma)} + (1-k)} \leq M_{\varphi}(f\vartheta_n, f\vartheta, \varsigma).\end{aligned}$$

Using the above inequality, we obtain

$$\begin{aligned}M_{\varphi}(\vartheta, f\vartheta, \varsigma) &\geq M_{\varphi}\left(\vartheta, \vartheta_{n+1}, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \circ M_{\varphi}\left(f\vartheta_n, f\vartheta, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \\ &\geq M_{\varphi}\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \circ \frac{1}{\frac{k}{M_{\varphi}\left(\vartheta_n, \vartheta, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right)} + (1-k)} \rightarrow 1 \circ 1 = 1 \text{ as } n \rightarrow \infty.\end{aligned}$$

Also,

$$\begin{aligned}N_{\varphi}(\vartheta, f\vartheta, \varsigma) &\leq N_{\varphi}\left(\vartheta, \vartheta_{n+1}, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \diamond N_{\varphi}\left(f\vartheta_n, f\vartheta, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \\ &\leq N_{\varphi}\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \diamond k N_{\varphi}\left(\vartheta_n, \vartheta, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \rightarrow 0 \diamond 0 = 0 \text{ as } n \rightarrow \infty\end{aligned}$$

and

$$\begin{aligned}O_{\varphi}(\vartheta, f\vartheta, \varsigma) &\leq O_{\varphi}\left(\vartheta, \vartheta_{n+1}, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \diamond O_{\varphi}\left(f\vartheta_n, f\vartheta, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \\ &\leq O_{\varphi}\left(\vartheta_n, \vartheta_{n+1}, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \diamond k O_{\varphi}\left(\vartheta_n, \vartheta, \frac{\varsigma}{2\varphi(\vartheta, f\vartheta)}\right) \rightarrow 0 \diamond 0 = 0 \text{ as } n \rightarrow \infty.\end{aligned}$$

This implies that $f\vartheta = \vartheta$. Now, we show uniqueness. Suppose $fc = c$ for some $c \in \mathfrak{B}$, then

$$\begin{aligned}\frac{1}{M_{\varphi}(\vartheta, c, \varsigma)} - 1 &= \frac{1}{M_{\varphi}(f\vartheta, fc, \varsigma)} - 1 \\ &\leq k \left[\frac{1}{M_{\varphi}(\vartheta, c, \varsigma)} - 1 \right] < \frac{1}{M_{\varphi}(\vartheta, c, \varsigma)} - 1,\end{aligned}$$

which is a contradiction. Also,

$$N_{\varphi}(\vartheta, c, \varsigma) = N_{\varphi}(f\vartheta, fc, \varsigma) \leq k N_{\varphi}(\vartheta, c, \varsigma) < N_{\varphi}(\vartheta, c, \varsigma)$$

a contradiction, and

$$O_{\varphi}(\vartheta, c, \varsigma) = O_{\varphi}(f\vartheta, fc, \varsigma) \leq k O_{\varphi}(\vartheta, c, \varsigma) < O_{\varphi}(\vartheta, c, \varsigma).$$

It is also a contradiction. Therefore, we must have $M_{\varphi}(\vartheta, c, \varsigma) = 1$, $N_{\varphi}(\vartheta, c, \varsigma) = 0$ and $O_{\varphi}(\vartheta, c, \varsigma) = 0$, hence $\vartheta = c$.

Example 3.8 Let $\mathfrak{B} = \mathbb{N}$. Define

$$\varphi(\vartheta, \delta) = \begin{cases} 1 & \text{if } \vartheta = \delta, \\ \max\{\vartheta, \delta\} & \text{if } \vartheta \neq \delta \end{cases}$$

Also, take

$$M_{\varphi}(\vartheta, \delta, \varsigma) = \frac{\varsigma}{\varsigma + \max\{\vartheta, \delta\}^2}, \quad N_{\varphi}(\vartheta, \delta, \varsigma) = \frac{\max\{\vartheta, \delta\}^2}{\varsigma + \max\{\vartheta, \delta\}^2} \text{ and } O_{\varphi}(\vartheta, \delta, \varsigma) = \frac{\max\{\vartheta, \delta\}^2}{\varsigma}$$

with $a \circ b = a \cdot b$ and $a \diamond b = \max\{a, b\}$. Then $(\mathfrak{B}, M_{\varphi}, N_{\varphi}, O_{\varphi}, \circ, \diamond)$ is a complete NEMLS.

Define $f: \mathfrak{B} \rightarrow \mathfrak{B}$ by

$$f(\vartheta) = \begin{cases} 1 & \text{if } \vartheta \in \{1, 2\}, \\ \frac{\vartheta}{7} & \text{if otherwise} \end{cases}$$

Then we have four cases:

- (a) If $\vartheta, \delta \in \{1, 2\}$, then $f\vartheta = f\delta = 0$;
- (b) If $\vartheta \in \{1, 2\}$ and $\delta \notin \{1, 2\}$, then $f\vartheta = 0$ and $f\delta = \frac{\delta}{7}$;
- (c) If $\delta \in \{1, 2\}$ and $\vartheta \notin \{1, 2\}$, then $f\delta = 0$ and $f\vartheta = \frac{\vartheta}{7}$;
- (d) If $\vartheta, \delta \notin \{1, 2\}$ then $f\vartheta = \frac{\vartheta}{7}$ and $f\delta = \frac{\delta}{7}$.

In all above cases, one gets

$$M_\varphi(f\vartheta, f\delta, k\varsigma) \geq M_\varphi(\vartheta, \delta, \varsigma), N_\varphi(f\vartheta, f\delta, k\varsigma) \leq N_\varphi(\vartheta, \delta, \varsigma) \text{ and } O_\varphi(f\vartheta, f\delta, k\varsigma) \leq O_\varphi(\vartheta, \delta, \varsigma)$$

are satisfied for $k \in \left[\frac{1}{2}, 1\right)$. Then

$$\frac{1}{M_\varphi(f\vartheta, f\delta, \varsigma)} - 1 \leq k \left[\frac{1}{M_\varphi(\vartheta, \delta, \varsigma)} - 1 \right], N_\varphi(f\vartheta, f\delta, \varsigma) \leq k N_\varphi(\vartheta, \delta, \varsigma) \text{ and } O_\varphi(f\vartheta, f\delta, k\varsigma) \leq O_\varphi(\vartheta, \delta, \varsigma)$$

are satisfied for $k \in \left[\frac{1}{2}, 1\right)$.

Observe that all circumstances of Theorem 3.5 and Theorem 3.7 are fulfilled, and 1 is the unique FP of f .

AN APPLICATION TO INTEGRAL EQUATIONS

Let $\mathfrak{B} = \mathcal{C}([e, g], \mathbb{R})$ be the set of continuous functions defined on $[e, g]$. Consider the integral equation:

$$\vartheta(l) = f(j) + \beta \int_e^g F(l, j) \vartheta(l) dj \text{ for } l, j \in [e, g] \quad (11)$$

where $\beta > 0$, $f(j)$ is a fuzzy function of $j \in [e, g]$ and $F \in \mathfrak{B}$. Define M_φ, N_φ and O_φ by

$$M_\varphi(\vartheta(l), \delta(l), \varsigma) = \sup_{l \in [e, g]} \frac{\varsigma}{\varsigma + \max\{\vartheta(l), \delta(l)\}^2} \text{ for all } \vartheta, \delta \in \mathfrak{B} \text{ and } \varsigma > 0$$

$$N_\varphi(\vartheta(l), \delta(l), \varsigma) = \sup_{l \in [e, g]} \frac{\max\{\vartheta(l), \delta(l)\}^2}{\varsigma + \max\{\vartheta(l), \delta(l)\}^2} \text{ for all } \vartheta, \delta \in \mathfrak{B} \text{ and } \varsigma > 0$$

and

$$O_\varphi(\vartheta(l), \delta(l), \varsigma) = \sup_{l \in [e, g]} \frac{\max\{\vartheta(l), \delta(l)\}^2}{\varsigma} \text{ for all } \vartheta, \delta \in \mathfrak{B} \text{ and } \varsigma > 0$$

where the CTN and CTCN are defined by $a \circ \mathfrak{b} = a \cdot \mathfrak{b}$ and $a \diamond \mathfrak{b} = \max\{a, \mathfrak{b}\}$. Given $\varphi: \mathfrak{B} \times \mathfrak{B} \rightarrow [1, \infty)$ as

$$\varphi(\vartheta, \delta) = \begin{cases} 1 & \text{if } \vartheta = \delta; \\ \max\{\vartheta, \delta\} & \text{if otherwise.} \end{cases}$$

$(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ is a complete NEMLS.

Theorem 4.1 Suppose the below conditions hold:

- I. $\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\} \leq \max\{\vartheta(l), \delta(l)\}$ for $\vartheta, \delta \in \mathfrak{B}, k \in (0, 1)$ and $\forall l, j \in [e, g]$;
- II. $\beta \int_e^g dj \leq k < 1$.

Then integral equation (11) has a unique solution.

Proof: Define $f: \mathfrak{B} \rightarrow \mathfrak{B}$ by

$$f\vartheta(l) = f(j) + \beta \int_e^g F(l, j) \vartheta(l) dj \text{ for all } l, j \in [e, g].$$

For all $\vartheta, \delta \in \mathfrak{B}$, we have

$$\begin{aligned} M_\varphi(f\vartheta(l), f\delta(l), k\varsigma) &= \sup_{l \in [e, g]} \frac{k\varsigma}{k\varsigma + \max\{f\vartheta(l), f\delta(l)\}^2} \\ &= \sup_{l \in [e, g]} \frac{k\varsigma}{k\varsigma + \max\{f(j) + \beta \int_e^g F(l, j) \vartheta(l) dj, f(j) + \beta \int_e^g F(l, j) \delta(l) dj\}^2} \\ &= \sup_{l \in [e, g]} \frac{k\varsigma}{k\varsigma + \max\{\beta \int_e^g F(l, j) \vartheta(l) dj, \beta \int_e^g F(l, j) \delta(l) dj\}^2} \\ &= \sup_{l \in [e, g]} \frac{k\varsigma}{k\varsigma + \max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2 (\beta \int_e^g dj)^2} \\ &\geq \sup_{l \in [e, g]} \frac{\varsigma}{\varsigma + \max\{\vartheta(l), \delta(l)\}^2} \\ &\geq M_\varphi(\vartheta(l), \delta(l), \varsigma). \end{aligned}$$

Also,

$$\begin{aligned} N_\varphi(f\vartheta(l), f\delta(l), k\varsigma) &= \sup_{l \in [e, g]} \frac{\max\{f\vartheta(l), f\delta(l)\}^2}{k\varsigma + \max\{f\vartheta(l), f\delta(l)\}^2} \\ &= \sup_{l \in [e, g]} \frac{\max\{f(j) + \beta \int_e^g F(l, j) \vartheta(l) dj, f(j) + \beta \int_e^g F(l, j) \delta(l) dj\}^2}{k\varsigma + \max\{f(j) + \beta \int_e^g F(l, j) \vartheta(l) dj, f(j) + \beta \int_e^g F(l, j) \delta(l) dj\}^2} \\ &= \sup_{l \in [e, g]} \frac{\max\{\beta \int_e^g F(l, j) \vartheta(l) dj, \beta \int_e^g F(l, j) \delta(l) dj\}^2}{k\varsigma + \max\{\beta \int_e^g F(l, j) \vartheta(l) dj, \beta \int_e^g F(l, j) \delta(l) dj\}^2} \end{aligned}$$

$$\begin{aligned}
&= \sup_{l \in [e, g]} \frac{\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2 (\beta \int_e^g dj)^2}{k\zeta + \max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2 (\beta \int_e^g dj)^2} \\
&\leq \sup_{l \in [e, g]} \frac{\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2}{\zeta + \max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2} \\
&\leq \sup_{l \in [e, g]} \frac{\max\{\vartheta(l), \delta(l)\}^2}{\zeta + \max\{\vartheta(l), \delta(l)\}^2} \\
&\leq N_\varphi(\vartheta(l), \delta(l), \zeta).
\end{aligned}$$

Moreover,

$$\begin{aligned}
O_\varphi(f\vartheta(l), f\delta(l), k\zeta) &= \sup_{l \in [e, g]} \frac{\max\{f\vartheta(l), f\delta(l)\}^2}{k\zeta} \\
&= \sup_{l \in [e, g]} \frac{\max\{f(j) + \beta \int_e^g F(l, j)e(l)dj, f(j) + \beta \int_e^g F(l, j)e(l)dj\}^2}{k\zeta} \\
&= \sup_{l \in [e, g]} \frac{\max\{\beta \int_e^g F(l, j)e(l)dj, \beta \int_e^g F(l, j)e(l)dj\}^2}{k\zeta} \\
&= \sup_{l \in [e, g]} \frac{\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2 (\beta \int_e^g dj)^2}{k\zeta} \\
&\leq \sup_{l \in [e, g]} \frac{\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\}^2}{\zeta} \\
&\leq \sup_{l \in [e, g]} \frac{\max\{\vartheta(l), \delta(l)\}^2}{\zeta} \\
&= O_\varphi(\vartheta(l), \delta(l), \zeta).
\end{aligned}$$

Therefore, all circumstances of Theorem 3.5 are fulfilled. Hence, operator f has a single FP. This implies that integral equation (11) has a single solution.

CONCLUSION AND FUTURE WORK

This study aims to define a neutrosophic extended metric-like space and examine some properties. This work is the extended form of a fuzzy metric like space see [21, 22]. This new concept can also be studied to the fixed point theory, as in metric fixed metric theory and so it can construct the NEMLS fixed point theory. As is well known, in recent years, the study of metric fixed point theory has been widely researched because this theory has a fundamental role in various areas of mathematics, science, and economic studies. This work can easily extend in the structure of neutrosophic controlled metric like spaces, neutrosophic triple partial g-metric like space, and many others.

APPENDIX

Definition 6.1 [1] A binary operation (BO) $\circ : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous triangle norm if it meets the below assertions:

1. $a \circ b = b \circ a, (\forall) a, b \in [0, 1]$;
2. $\circ$ is continuous;
3. $a \circ 1 = a, (\forall) a \in [0, 1]$;
4. $(a \circ b) \circ c = a \circ (b \circ c), (\forall) a, b, c \in [0, 1]$;
5. If $a \leq c$ and $b \leq d$, with $a, b, c, d \in [0, 1]$, then $a \circ b \leq c \circ d$.

Definition 6.2 [1] A BO $\diamond : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous triangle conorm, if it meets the below assertions:

1. $a \diamond b = b \diamond a, (\forall) a, b \in [0, 1]$;
2. $\diamond$ is continuous;
3. $a \diamond 0 = 0, (\forall) a \in [0, 1]$;
4. $(a \diamond b) \diamond c = a \diamond (b \diamond c), (\forall) a, b, c \in [0, 1]$;
5. If $a \leq c$ and $b \leq d$, with $a, b, c, d \in [0, 1]$, then $a \diamond b \leq c \diamond d$.

REFERENCES

- [1] J. H. Park, "Intuitionistic fuzzy metric spaces," *Chaos, Solitons and Fractals*, 22 (2004), 1039–1046.
- [2] M. Rafi, M. S. M. Noorani, "Fixed theorems on intuitionistic fuzzy metric space," *Iranian Journal of Fuzzy Systems*, 3 (1) (2006), 23-29.
- [3] W. Sintunavarat, P. Kumam, "Fixed theorems for a generalized intuitionistic fuzzy contraction in intuitionistic Fuzzy metric spaces," *Thai Journal of Mathematics*, 10 (1) (2012), 123–135.
- [4] N. Konwar, "Extension of fixed results in intuitionistic fuzzy b-metric spaces," *Journal of Intelligent & Fuzzy Systems*, 39 (5) (2020) , 7831-7841.
- [5] C. Alaca, D. Turkoglu, C. Yildiz, "Fixed points in intuitionistic fuzzy metric spaces," *Chaos, Solitons and Fractals*, 29 (2006), 1073–1078.
- [6] A. Mohamad, "Fixed-point theorems in intuitionistic fuzzy metric spaces," *Chaos, Solitons and Fractals*, 34 (2007), 1689–1695.
- [7] D. Dey, M. Saha, "An extension of Banach fixed point theorem in fuzzy metric space," *Bol. Soc. Paran. Mat.* 32(2014), No. 1, 299-304.
- [8] A. George, P. Veeramani, "On some results in fuzzy metric spaces," *Fuzzy set and Systems*, 64(1994), 395-399.
- [9] M. Grabiec, "Fixed points in fuzzy metric spaces," *Fuzzy set and Systems*, 27(1988), 385-389.
- [10] S. Nadaban, "Fuzzy b-metric spaces," *Int. J. Comput. Commun. Control*, 11(2016), No. 2, 273-281.
- [11] B. Schweizer, A. Sklar, "Statistical metric spaces," *Pacific J. Math.*, 10(1960), 314-334.
- [12] L. A. Zadeh, "Fuzzy sets," *Information and Control*, 8(1965), 338-353.
- [13] F. Mehmood, R. Ali, C. Ionescu, T. Kamran, "Extended fuzzy b-metric spaces," *Journal of Mathematical Analysis*, Volume 8 Issue 6(2017), Pages 124-131.
- [14] V. Gregory, A. Sapena, "On fixed point theorems in fuzzy metric spaces," *Fuzzy set and Systems*, 125 (2002), 245-253.
- [15] R. Saadati, J. H. Park, "On the intuitionistic fuzzy topological spaces," *Chaos, Solitons and Fractals*, 27 (2006), 331-344.
- [16] I. Kramosil, J. Michlek, "Fuzzy metric and statistical metric spaces," *Kybernetika*, 11(1975), 336-344.
- [17] M. Kirişçi, N. Simsek, "Neutrosophic metric spaces," *Mathematical Sciences*, (2020) 14:241–248.
- [18] S. Sowndrarajan, M. Jeyarama, F. Smarandache, "Fixed Point Results for Contraction Theorems in Neutrosophic Metric Spaces," *Neutrosophic Sets and Systems* 36, 1 (2020).
- [19] N. Simsek, M. Kirişçi, "Fixed point theorems in Neutrosophic metric spaces," *Sigma J. Eng. Nat. Sci.* 10 (2), 2019, 221-230.
- [20] K. Javed, F. Uddin, H. Aydi, M. Arshad, U. Ishtiaq, H. Alsamir, "On Fuzzy b-Metric-Like Spaces," *Journal of Function Spaces*, Volume 2021, Article ID 6615976.
- [21] A. Harandi, "Metric-like paces, partial metric spaces and fixed point," *Fixed Point Theory Appl.* 2012, Article ID 204(2012).
- [22] S. Shukla, M. Abbas, "Fixed point results in fuzzy metric-like spaces," *Iranian Journal of Fuzzy Systems* Vol. 11, No. 5, (2014) pp. 81-92.
- [23] K. Javed, F. Uddin, H. Aydi, A. Mukheimer and M. Arshad, "Ordered-Theoretic Fixed Point Results in Fuzzy b-Metric Spaces with an Application," *Journal of Mathematics* Volume 2021, Article ID 6663707.
- [24] M. Kirişçi, N.Şimşek, M. Akyiğit, "Fixed point results for a new metric space," *Mathematical Methods in the Applied Sciences*, 2020(2020), 1-7.
- [25] F. Uddin, K. Javed, H. Aydi, U. Ishtiaq, M. Arshad, "Control fuzzy metric spaces via orthogonality with an application," *Journal of mathematics*, (2021).
- [26] K. Javed, H. Aydi, F. Uddin and M. Arshad, "On Orthogonal Partial b-Metric Spaces with an Application," *Journal of Mathematics*, Volume 2021, Article ID 6692063.
- [27] F. Uddin, C. Park, K. Javed, M. Arshad, and J. R. Lee; "Orthogonal m-metric spaces and an application to solve integral equations," *Adv. Differ. Equ.* 2021.
- [28] A. H. Ganie, N. A. Sheikh, "Generalized difference sequence spaces of fuzzy numbers," *New York J. Math.*, 19(2013), 431-438.
- [29] M. S. Sezen, "Controlled fuzzy metric spaces and related fixed point results," *Numerical methods for partial differential equations*, 18(2020).
- [30] T. A. Tarray, P. A. Naik, R. A. Najar, "Matrix representation of an all-inclusive Fibonacci sequence," *Asian journal of mathematics and statistics*, 11(2018), 18-26.

- [31] H. Huang, B. Caric, T. Dosenovic, D. Rakić, M. Brdar, "Fixed point theorems in fuzzy metric spaces via fuzzy F-contraction," *Mathematics*, 9(2021).
- [32] A. H. Ganie, N. A. Sheikh, M. A. Syed, "New generalized difference sequence spaces of fuzzy numbers," *Ann. Fuzzy Math. Inform.* 12(2016), 255-263.
- [33] A. H. Ganie, "Lacunary sequences with almost and statistical convergence," *Annals commu. Maths.*, 3(2020), 46-53.
- [34] S. Gupta, R. Bhardwaj, B. R. Wadkar, R. M. Sharaff, "Fixed point theorems in fuzzy metric spaces," *Materials Today*, 29(2020), 611-616.