Neutrosophic Soft e-Open Maps, Neutrosophic Soft e-Closed Maps and Neutrosophic Soft e-Homeomorphisms in Neutrosophic Soft Topological Spaces

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Abstract In this article, the concepts of N_sSe-open and N_sSe-closed mappings in neutrosophic soft topological spaces are introduced and their related properties are studied. Also, the work is developed to N_sS homeomorphism, N_sSe-homeomorphism, N_sSe-C homeomorphism and $N_sSeT_{\frac{1}{2}}$-space and some of their characteristics are discussed.

Keywords N_sSe-open map · N_sSe-closed map · N_sSe-homeomorphism · $N_sSeT_{\frac{1}{2}}$-space · N_sSe-C homeomorphism

1 Introduction

In Mathematics, the concept of fuzzy set was first introduced by Zadeh [1] and its topological structure was undertaken by Chang [2]. Atanassov [3–5] introduced intuitionistic fuzzy set in 1983 and its topological structure was introduced by Coker [6]. Molodstov [7] initiated the soft set theory as a new mathematical tool in 1999. Shabir and Naz [8] presented soft topological spaces in soft sets.

Smarandache [9] introduced the concepts of neutrosophy and neutrosophic set and its topological structure was given by Salama and Alblowi [10] in 2012. Maji [11] defined the Neutrosophic soft sets and the same was modified by Deli and

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Broumi [12]. Its topological structures were presented by Bera [13]. δ-open sets were defined by Saha [14] in fuzzy topological spaces and Vadivel et al. [15] in neutrosophic topological spaces. In 2019, Ahu Acikgoz and Ferhat Esenbel [16] defined neutrosophic soft δ-topology.

The notion of e-open sets were introduced by Ekici [17] in a general topology, Seenivasan et. al. [18] in fuzzy topological spaces, Chandrasekar et al. [19] in intuitionistic fuzzy topological spaces, Vadivel et al. [20] in neutrosophic topological spaces and recently, Revathi et al. [21] in neutrosophic soft topological spaces. In 2021, Vadivel et al. [22, 23] developed the concepts of neutrosophic e-Continuity, e-Irresolute maps, e-Open maps, e-Closed maps and e-Homeomorphisms in neutrosophic topological spaces. Recently, Revathi et al. [24] developed the concepts of neutrosophic soft e-Continuity and e-Irresolute maps.

The aim of this article is to introduce neutrosophic soft e-open and neutrosophic soft e-closed mappings in neutrosophic soft topological spaces. Moreover, neutrosophic soft e-homeomorphism, neutrosophic soft e-C homeomorphism and neutrosophic soft $eT_{\frac{1}{2}}$-space are introduced and their basic properties are obtained.

2 Preliminaries

The basic definitions and the properties of neutrosophic soft topological spaces are discussed in this section.

Definition 1 ([12]) Let Y be an initial universe, Q be a set of parameters. Let $P(Y)$ denote the set of all neutrosophic sets of Y. Then a neutrosophic soft set $(\tilde{H}, Q)$ over Y (in short, N_sSs) is defined by $(\tilde{H}, Q) = \{(q, \langle y, \mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y)\rangle : y \in Y) : q \in Q\}$, where $\mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y) \in [0, 1]$ are respectively called the degree of membership function, the degree of indeterminacy function and the degree of non-membership function of $\tilde{H}(q)$. Since the supremum of each μ, σ, ν is 1, the inequality $0 \leq \mu_{\tilde{H}(q)}(y) + \sigma_{\tilde{H}(q)}(y) + \nu_{\tilde{H}(q)}(y) \leq 3$ is obvious.

Definition 2 ([11, 13]) Let Y be an initial universe & the N_sSs's $(\tilde{H}, Q)$ & $(\tilde{G}, Q)$ are in the form $(\tilde{H}, Q) = \{(q, \langle y, \mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y)\rangle : y \in Y) : q \in Q\}$ & $(\tilde{G}, Q) = \{(q, \langle y, \mu_{\tilde{G}(q)}(y), \sigma_{\tilde{G}(q)}(y), \nu_{\tilde{G}(q)}(y)\rangle : y \in Y) : q \in Q\}$, then

- (i) $0_{(Y,Q)} = \{(q, \langle y, 0, 0, 1\rangle : y \in Y) : q \in Q\}$ and $1_{(Y,Q)} = \{(q, \langle y, 1, 1, 0\rangle : y \in Y) : q \in Q\}$
- (ii) $(\tilde{H}, Q) \subseteq (\tilde{G}, Q)$ iff $\mu_{\tilde{H}(q)}(y) \leq \mu_{\tilde{G}(q)}(y)$, $\sigma_{\tilde{H}(q)}(y) \leq \sigma_{\tilde{G}(q)}(y)$ & $\nu_{\tilde{H}(q)}(y) \geq \nu_{\tilde{G}(q)}(y) : y \in Y : q \in Q$.
- (iii) $(\tilde{H}, Q) = (\tilde{G}, Q)$ iff $(\tilde{H}, Q) \subseteq (\tilde{G}, Q)$ and $(\tilde{G}, Q) \subseteq (\tilde{H}, Q)$.
- (iv) $(\tilde{H}, Q)^c = \{(q, \langle y, \nu_{\tilde{H}(q)}(y), 1 - \sigma_{\tilde{H}(q)}(y), \mu_{\tilde{H}(q)}(y)\rangle : y \in Y) : q \in Q\}$.
- (v) $(\tilde{H}, Q) \cup (\tilde{G}, Q) = \{(q, \langle y, max(\mu_{\tilde{H}(q)}(y), \mu_{\tilde{G}(q)}(y)), max(\sigma_{\tilde{H}(q)}(y), \sigma_{\tilde{G}(q)}(y)), min(\nu_{\tilde{H}(q)}(y), \nu_{\tilde{G}(q)}(y))\rangle : y \in Y) : q \in Q\}$.

(vi) $(\tilde{H}, Q) \cap (\tilde{G}, Q) = \{(q, \langle y, min(\mu_{\tilde{H}(q)}(y), \mu_{\tilde{G}(q)}(y)), min(\sigma_{\tilde{H}(q)}(y), \sigma_{\tilde{G}(q)}(y)), max(\nu_{\tilde{H}(q)}(y), \nu_{\tilde{G}(q)}(y))\rangle : y \in Y) : q \in Q\}$.

Definition 3 ([13]) A neutrosophic soft topology (in short, N_sSt) on an initial universe Y is a family τ of neutrosophic soft subsets $(\tilde{H}, Q)$ of Y where Q is a set of parameters, satisfying

(i) $0_{(Y,Q)}, 1_{(Y,Q)} \in \tau$.
(ii) $[(\tilde{H}, Q) \cap (\tilde{G}, Q)] \in \tau$ for any $(\tilde{H}, Q), (\tilde{G}, Q) \in \tau$.
(iii) $\bigcup_{\rho \in A} (\tilde{H}, Q)_\rho \in \tau, \forall\ (\tilde{H}, Q)_\rho : \rho \in A \subseteq \tau$.

Then (Y, τ, Q) is known as a neutrosophic soft topological space (in short, N_sSts) and the τ elements are known as neutrosophic soft open sets (in short, N_sSos) in Y. A N_sSs $(\tilde{H}, Q)$ is known as a neutrosophic soft closed set (in short, N_sScs) if its complement $(\tilde{H}, Q)^c$ is N_sSos.

Definition 4 ([13]) Consider a N_sSts (Y, τ, Q) and a N_sSs $(\tilde{H}, Q)$ on Y. The neutrosophic soft interior of $(\tilde{H}, Q)$ (in short, $N_sSint(\tilde{H}, Q)$) and the neutrosophic soft closure of $(\tilde{H}, Q)$ (in short, $N_sScl(\tilde{H}, Q)$) are defined as

$$N_sSint(\tilde{H}, Q) = \bigcup\{(\tilde{G}, Q) : (\tilde{G}, Q) \subseteq (\tilde{H}, Q) \text{ and } (\tilde{G}, Q) \text{ is a } N_sSos \text{ in } Y\} \quad (1)$$

$$N_sScl(\tilde{H}, Q) = \bigcap\{(\tilde{G}, Q) : (\tilde{G}, Q) \supseteq (\tilde{H}, Q) \text{ and } (\tilde{G}, Q) \text{ is a } N_sScs \text{ in } Y\}. \quad (2)$$

Definition 5 ([13, 25]) Consider a N_sSts (Y, τ, Q) and a N_sSs $(\tilde{H}, Q)$ on Y. Then $(\tilde{H}, Q)$ is known as a neutrosophic soft *regular* (resp. *pre* & *semi*) open set (in short, N_sSros (resp. $N_sS\mathcal{P}os$, & N_sSSos)) if $(\tilde{H}, Q) = N_sSint(N_sScl(\tilde{H}, Q))$ (resp. $(\tilde{H}, Q) \subseteq N_sSint(N_sScl(\tilde{H}, Q))$ & $(\tilde{H}, Q) \subseteq N_sScl(N_sSint(\tilde{H}, Q))$). The complement of the respective open sets are their respective closed sets.

Definition 6 ([16]) A set $(\tilde{H}, Q)$ is known as a neutrosophic soft δ-open set (in short, $N_sS\delta os$) if $(\tilde{H}, Q) = N_sS\delta int(\tilde{H}, Q)$.

Definition 7 ([21]) A set $(\tilde{H}, Q)$ is known as a neutrosophic soft

(i) δ-pre open set (in short, $N_sS\delta\mathcal{P}os$) if $(\tilde{H}, Q) \subseteq N_sSint(N_sS\delta cl(\tilde{H}, Q))$.
(ii) δ-semi open set (in short, $N_sS\delta Sos$) if $(\tilde{H}, Q) \subseteq N_sScl(N_sS\delta int(\tilde{H}, Q))$.
(iii) e-open set (in short, N_sSeos) if $(\tilde{H}, Q) \subseteq N_sScl(N_sS\delta int(\tilde{H}, Q)) \cup N_sSint(N_sS\delta cl(\tilde{H}, Q))$.
(iv) e^*-open set (in short, N_sSe^*os) if $(\tilde{H}, Q) \subseteq N_sScl(N_sSint(N_sS\delta cl(\tilde{H}, Q)))$.

The complement of the respective open sets are their respective closed sets.

Definition 8 ([24]) Consider any two N_sSts's (Y, τ, Q) and (Z, σ, Q). A map $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ is called neutrosophic soft

(i) continuous (in short, N_sSCts) (resp. δ-continuous, $\delta\mathcal{S}$-continuous, $\delta\mathcal{P}$-continuous, e-continuous & e^*-continuous (in short, $N_sS\delta Cts$, $N_sS\delta\mathcal{S}Cts$, $N_sS\delta\mathcal{P}Cts$, N_sSeCts & N_sSe^*Cts)) if the inverse image of every N_sSos in (Z, σ, Q) is a N_sSos (resp. $N_sS\delta os$, $N_sS\delta\mathcal{S}os$, $N_sS\delta\mathcal{P}os$, N_sSeos & N_sSe^*os) in (Y, τ, Q).
(ii) e-irresolute (in short, N_sSeIrr) if the inverse image of every N_sSeos in (Z, σ, Q) is a N_sSeos in (Y, τ, Q).

3 Neutrosophic Soft *e*-Open Mapping

Definition 9 A mapping $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$, is neutrosophic soft *e-open* (resp. *open*, *δ open*, *δ-semi open*, *δ-pre open* & *e^*-open*) (in short, N_sSeO (resp. N_sSO, $N_sS\delta O$, $N_sS\delta\mathcal{S}O$, $N_sS\delta\mathcal{P}O$ & N_sSe^*O)) if the image of every N_sS open set of (Y, τ, Q) is N_sSeo (resp. N_sSo, $N_sS\delta o$, $N_sS\delta\mathcal{S}o$, $N_sS\delta\mathcal{P}o$ & N_sSe^*o) set in (Z, σ, Q).

Theorem 1 *The statements are hold but the converse need not be true. Every*

(i) $N_sS\delta O$ *mapping is a* N_sSO *mapping.*
(ii) N_sSO *mapping is a* $N_sS\delta\mathcal{S}O$ *mapping.*
(iii) N_sSO *mapping is a* $N_sS\delta\mathcal{P}O$ *mapping.*
(iv) $N_sS\delta\mathcal{S}O$ *mapping is a* N_sSeO *mapping.*
(v) $N_sS\delta\mathcal{P}O$ *mapping is a* N_sSeO *mapping.*
(vi) N_sSeO *mapping is a* N_sSe^*O *mapping.*

Example 1 Let $Y = \{y_1, y_2, y_3\} = \{z_1, z_2, z_3\} = Z$, $Q = \{q_1, q_2\}$ and N_sSs's $(\tilde{H}_1, Q)$ in Y and $(\tilde{G}_1, Q)$ & $(\tilde{G}_2, Q)$ in Z are defined as

$$(\tilde{H}_1, q_1) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.2, 0.5, 0.8)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_1, q_2) = \{\langle y_1, (0.3, 0.4, 0.7)\rangle, \langle y_2, (0.4, 0.4, 0.6)\rangle, \langle y_3, (0.4, 0.5, 0.5)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle z_1, (0.2, 0.5, 0.8)\rangle, \langle z_2, (0.2, 0.5, 0.8)\rangle, \langle z_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle z_1, (0.3, 0.4, 0.7)\rangle, \langle z_2, (0.4, 0.4, 0.6)\rangle, \langle z_3, (0.4, 0.5, 0.5)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle z_1, (0.4, 0.5, 0.6)\rangle, \langle z_2, (0.4, 0.5, 0.6)\rangle, \langle z_3, (0.5, 0.5, 0.5)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle z_1, (0.4, 0.5, 0.6)\rangle, \langle z_2, (0.5, 0.5, 0.6)\rangle, \langle z_3, (0.5, 0.5, 0.5)\rangle\}$$

Then we have $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, Q)\}$ and $\sigma = \{0_{(Z,Q)}, 1_{(Z,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q)\}$. Let $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ be an identity mapping. Then, $(\tilde{H}_1, Q)$ is N_sSO (resp. N_sSeO) mapping in Y but not $N_sS\delta O$ (resp. $N_sS\delta\mathcal{S}O$) mapping in Z.

Example 2 Let $Y = \{y_1, y_2, y_3\} = \{z_1, z_2, z_3\} = Z$, $Q = \{q_1, q_2\}$ and N_sSs's $(\tilde{H}_1, Q)$ in Y and $(\tilde{G}_1, Q)$, $(\tilde{G}_2, Q)$ & $(\tilde{G}_3, E)$ in Z are defined as

$$(\tilde{H}_1, q_1) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.4, 0.5, 0.6)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_1, q_2) = \{\langle y_1, (0.3, 0.4, 0.7)\rangle, \langle y_2, (0.5, 0.5, 0.7)\rangle, \langle y_3, (0.5, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle z_1, (0.2, 0.5, 0.8)\rangle, \langle z_2, (0.3, 0.5, 0.7)\rangle, \langle z_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle z_1, (0.3, 0.4, 0.7)\rangle, \langle z_2, (0.4, 0.5, 0.7)\rangle, \langle z_3, (0.5, 0.4, 0.6)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle z_1, (0.1, 0.5, 0.9)\rangle, \langle z_2, (0.1, 0.5, 0.9)\rangle, \langle z_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle z_1, (0.2, 0.3, 0.8)\rangle, \langle z_2, (0.3, 0.5, 0.8)\rangle, \langle z_3, (0.4, 0.4, 0.7)\rangle\}$$
$$(\tilde{G}_3, q_1) = \{\langle z_1, (0.2, 0.5, 0.8)\rangle, \langle z_2, (0.4, 0.5, 0.6)\rangle, \langle z_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_3, q_2) = \{\langle z_1, (0.3, 0.4, 0.7)\rangle, \langle z_2, (0.5, 0.5, 0.7)\rangle, \langle z_3, (0.5, 0.5, 0.6)\rangle\}$$

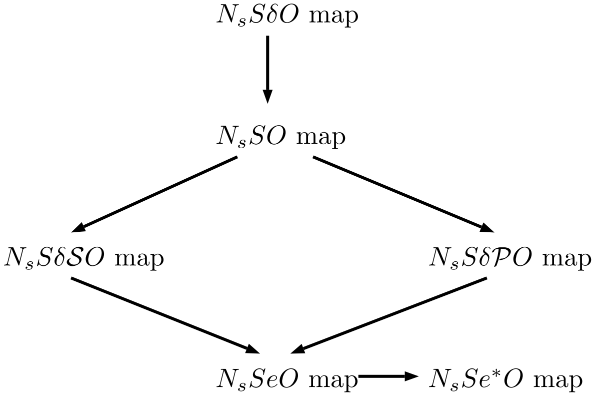
Then we have $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, Q)\}$ and $\sigma = \{0_{(Z,Q)}, 1_{(Z,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q)\}$. Let $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ be an identity mapping. Then, $(\tilde{H}_1, Q)$ is $N_sS\delta\mathcal{S}O$ (resp. $N_sS\delta\mathcal{P}O$ & N_sSeO) mapping in Y but not N_sSO (resp. N_sSO & $N_sS\delta\mathcal{P}O$) mapping in Z.

Example 3 Let $Y = \{y_1, y_2\} = \{z_1, z_2\} = Z$, $Q = \{q_1, q_2\}$ and N_sSs's $(\tilde{H}_1, Q)$ in Y and $(\tilde{G}_1, Q)$ & $(\tilde{G}_2, Q)$ in Z are defined as

$$(\tilde{H}_1, q_1) = \{\langle y_1, (0.3, 0.5, 0.7)\rangle, \langle y_2, (0.5, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_1, q_2) = \{\langle y_1, (0.4, 0.5, 0.6)\rangle, \langle y_2, (0.4, 0.4, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle z_1, (0.3, 0.5, 0.5)\rangle, \langle z_2, (0.2, 0.5, 0.5)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle z_1, (0.4, 0.4, 0.5)\rangle, \langle z_2, (0.3, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle z_1, (0.3, 0.5, 0.7)\rangle, \langle z_2, (0.5, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle z_1, (0.4, 0.5, 0.6)\rangle, \langle z_2, (0.4, 0.4, 0.6)\rangle\}$$

Then we have $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, Q)\}$ and $\sigma = \{0_{(Z,Q)}, 1_{(Z,Q)}, (\tilde{G}_1, Q)\}$. Let $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ be an identity mapping. Then, $(\tilde{H}_1, Q)$ is N_sSe^*O mapping in Y but not N_sSeO mapping in Z.

Remark 1 The diagram shows N_sSeO mapping's in N_sSts.



Theorem 2 *A mapping* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSeO *iff for every* N_sSs $(\tilde{H}, Q)$ *of* (Y, τ, Q), $\mathfrak{f}(N_sSint(\tilde{H}, Q)) \subseteq N_sSeint(\mathfrak{f}(\tilde{H}, Q))$.

Theorem 3 *Consider a* N_sSeO *mapping* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$. *Then,* $N_sSint(\mathfrak{f}^{-1}(\tilde{H}, Q)) \subseteq \mathfrak{f}^{-1}(N_sSeint(\tilde{H}, Q))$ *for every* N_sSs $(\tilde{H}, Q)$ *of* (Z, σ, Q).

Theorem 4 *A mapping* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSeO *iff for each* N_sSs $(\tilde{G}, Q)$ *of* (Z, σ, Q) *and for each* N_sScs $(\tilde{H}, Q)$ *of* (Y, τ, Q) *containing* $\mathfrak{f}^{-1}(\tilde{G}, Q)$, *there is a* N_sSecs $(\tilde{A}, Q)$ *of* (Z, σ, Q) *such that* $(\tilde{G}, Q) \subseteq (\tilde{H}, Q)$ *and* $\mathfrak{f}^{-1}(\tilde{A}, Q) \subseteq (\tilde{H}, Q)$.

Theorem 5 *A mapping* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSeO *iff* $\mathfrak{f}^{-1}(N_sS\,ecl(\tilde{G}, Q)) \subseteq N_sScl(\mathfrak{f}^{-1}(\tilde{G}, Q))$ *for every* N_sSs $(\tilde{G}, Q)$ *of* (Z, σ, Q).

Theorem 6 *Let* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *be two neutrosophic soft mappings and* $\mathfrak{g} \circ \mathfrak{f} : (Y, \tau, Q) \to (P, \rho, Q)$ *be* N_sSeO. *If* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *is* N_sSeIrr, *then* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSeO *mapping.*

Theorem 7 *If* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSO *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *is* N_sSeO *mappings, then* $\mathfrak{g} \circ \mathfrak{f} : (Y, \tau, Q) \to (P, \rho, Q)$ *is* N_sSeO.

4 Neutrosophic Soft *e*-Closed Mapping

Definition 10 A mapping $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ is neutrosophic soft *e-closed* (resp. *closed*, *δ closed*, *δ-semi closed*, *δ-pre closed* & *e**-*closed*) (in short, N_sSeC (resp.N_sSC, $N_sS\delta C$, $N_sS\delta SC$, $N_sS\delta\mathcal{P}C$ & N_sSe^*C)) if the image of every N_sS closed set of (Y, τ, Q) is N_sSec (resp. N_sSc, $N_sS\delta c$, $N_sS\delta Sc$, $N_sS\delta\mathcal{P}c$ & N_sSe^*c) set in (Z, σ, Q).

Theorem 8 *The statements are hold but the converse need not be true. Every*

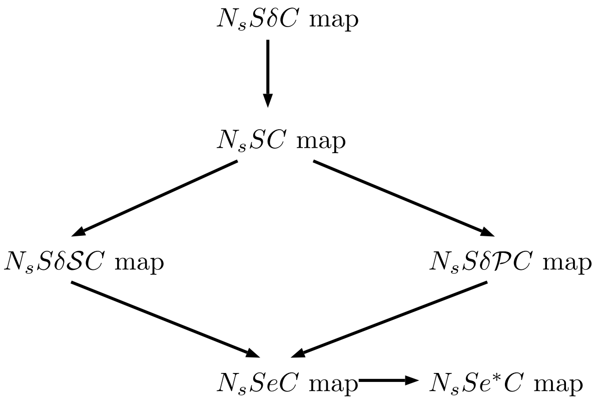
(i) $N_sS\delta C$ *mapping is a* N_sSC *mapping.*
(ii) N_sSC *mapping is a* $N_sS\delta\mathcal{S}C$ *mapping.*
(iii) N_sSC *mapping is a* $N_sS\delta\mathcal{P}C$ *mapping.*
(iv) $N_sS\delta\mathcal{S}C$ *mapping is a* N_sSeC *mapping.*
(v) $N_sS\delta\mathcal{P}C$ *mapping is a* N_sSeC *mapping.*
(vi) N_sSeC *mapping is a* N_sSe^*C *mapping.*

Example 4 In Example 1, $(\tilde{H}_1, Q)^c$ is N_sSC (resp. N_sSeC) mapping in Y but not $N_sS\delta C$ (resp. $N_sS\delta\mathcal{S}C$) mapping in Z.

Example 5 In Example 2, $(\tilde{H}_1, Q)^c$ is $N_sS\delta\mathcal{S}C$ (resp. $N_sS\delta\mathcal{P}C$ & N_sSeC) mapping in Y but not N_sSC (resp. N_sSC & $N_sS\delta\mathcal{P}C$) mapping in Z.

Example 6 In Example 3, $(\tilde{H}_1, Q)^c$ is N_sSe^*C mapping in Y but not N_sSeC mapping in Z.

Remark 2 The diagram shows N_sSeC mapping's in N_sSts.



Theorem 9 *A mapping* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSeC *iff for each* N_sSs $(\tilde{G}, Q)$ *of* (Z, σ, Q) *and for each* N_sSos $(\tilde{H}, Q)$ *of* (Y, τ, Q) *containing* $\mathfrak{f}^{-1}(\tilde{G}, Q)$, *there is a* N_sSeos $(\tilde{A}, Q)$ *of* (Z, σ, Q) *such that* $(\tilde{G}, Q) \subseteq (\tilde{A}, Q)$ *and* $\mathfrak{f}^{-1}(\tilde{A}, Q) \subseteq (\tilde{H}, Q)$.

Theorem 10 *If* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSC *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *is* N_sSeC. *Then* $\mathfrak{g} \circ \mathfrak{f} : (Y, \tau, Q) \to (P, \rho, Q)$ *is* N_sSeC.

Theorem 11 *If* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is* N_sSeC *map, then* $N_sSecl(\mathfrak{f}(\tilde{H}, Q)) \subseteq \mathfrak{f}(N_sScl(\tilde{H}, Q))$.

Theorem 12 *Let* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *are* N_sSeC *mappings. If every* N_sSecs *of* (Z, σ, Q) *is* N_sScs, *then* $\mathfrak{g} \circ \mathfrak{f} : (Y, \tau, Q) \to (P, \rho, Q)$ *is* N_sSeC.

Theorem 13 *Let $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ be a bijective mapping. Then the following statements are equivalent:*

(i) $\mathfrak{f}$ *is a* N_sSeO *mapping.*
(ii) $\mathfrak{f}$ *is a* N_sSeC *mapping.*
(iii) $\mathfrak{f}^{-1}$ *is* N_sSeCts *mapping.*

5 Neutrosophic Soft *e*-Homeomorphism

Definition 11 A bijection $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ is called a N_sS homeomorphism (in short N_sSHom) if $\mathfrak{f}$ and $\mathfrak{f}^{-1}$ are N_sSCts mappings.

Definition 12 A bijection $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ is called a N_sSe-homeomorphism (in short N_sSeHom) if $\mathfrak{f}$ and $\mathfrak{f}^{-1}$ are N_sSeCts.

Theorem 14 *Each* N_sSHom *is a* N_sSeHom*. But not conversely.*

Example 7 Let $Y = \{y_1, y_2, y_3\} = \{z_1, z_2, z_3\} = Z$, $Q = \{q_1, q_2\}$ and N_sSs's $(\tilde{H}_1, Q)$, $(\tilde{H}_2, Q)$ & $(\tilde{H}_3, Q)$ in Y and $(\tilde{G}_1, Q)$ in Z are defined as

$$(\tilde{H}_1, q_1) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.3, 0.5, 0.7)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_1, q_2) = \{\langle y_1, (0.3, 0.5, 0.7)\rangle, \langle y_2, (0.2, 0.5, 0.6)\rangle, \langle y_3, (0.4, 0.4, 0.6)\rangle\}$$
$$(\tilde{H}_2, q_1) = \{\langle y_1, (0.1, 0.5, 0.9)\rangle, \langle y_2, (0.1, 0.5, 0.9)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_2, q_2) = \{\langle y_1, (0.2, 0.4, 0.8)\rangle, \langle y_2, (0.2, 0.5, 0.7)\rangle, \langle y_3, (0.3, 0.4, 0.7)\rangle\}$$
$$(\tilde{H}_3, q_1) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.4, 0.5, 0.6)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_3, q_2) = \{\langle y_1, (0.3, 0.5, 0.6)\rangle, \langle y_2, (0.3, 0.5, 0.6)\rangle, \langle y_3, (0.5, 0.4, 0.5)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle z_1, (0.2, 0.5, 0.8)\rangle, \langle z_2, (0.4, 0.5, 0.6)\rangle, \langle z_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle z_1, (0.3, 0.5, 0.6)\rangle, \langle z_2, (0.3, 0.5, 0.6)\rangle, \langle z_3, (0.5, 0.4, 0.5)\rangle\}$$

Then we have $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, Q), (\tilde{H}_2, Q)\}$ and $\sigma = \{0_{(Z,Q)}, 1_{(Z,Q)}, (\tilde{G}_1, Q)\}$. Let $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ be an identity mapping. Then $\mathfrak{f}$ is N_sSeHom but not N_sSHom.

Theorem 15 *Consider a bijective mapping* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$*. If* $\mathfrak{f}$ *is* N_sSeCts*, then the following statements are equivalent:*

(i) $\mathfrak{f}$ *is a* N_sSeC *mapping.*
(ii) $\mathfrak{f}$ *is a* N_sSeO *mapping.*
(iii) $\mathfrak{f}^{-1}$ *is a* N_sSeHom*.*

Definition 13 A N_sSts (Y, τ, Q) is known as a neutrosophic soft $eT_{\frac{1}{2}}$ (in short, $N_sSeT_{\frac{1}{2}}$)-space if every N_sSecs is N_sSc in (Y, τ, Q).

Theorem 16 *Let* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *be a* N_sSeHom*. Then* $\mathfrak{f}$ *is a* N_sSHom *if* (Y, τ, Q) *and* (Z, σ, Q) *are* $N_sSeT_{\frac{1}{2}}$*-space.*

Theorem 17 *Let* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *be a* N_sSts*. If* (Z, σ, Q) *is a* $N_sSeT_{\frac{1}{2}}$*-space, Then the following statements are equivalent:*

(i) $\mathfrak{f}$ *is* N_sSeC *mapping.*
(ii) If $(\tilde{H}, Q)$ *is a* N_sSos *in* (Y, τ, Q)*, then* $\mathfrak{f}(\tilde{H}, Q)$ *is* N_sSeos *in* (Z, σ, Q)*.*
(iii) $\mathfrak{f}(N_sSint(\tilde{H}, Q)) \subseteq N_sScl(N_sSint(\mathfrak{f}(\tilde{H}, Q)))$ *for every* N_sSs $(\tilde{H}, Q)$ *in* (Y, τ, Q)*.*

Theorem 18 *Let* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *be* N_sSeC*, where* (Y, τ, Q) *and* (P, ρ, Q) *are two* N_sSts*'s and* (Z, σ, Q) *a* $N_sSeT_{\frac{1}{2}}$*-space, then the composition* $\mathfrak{g} \circ \mathfrak{f}$ *is* N_sSeC*.*

Theorem 19 *Let* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *be two* N_sSts*'s. Then the following are true:*

(i) If $\mathfrak{g} \circ \mathfrak{f}$ *is* N_sSeO *and* $\mathfrak{f}$ *is* N_sSCts*, then* $\mathfrak{g}$ *is* N_sSeO*.*
(ii) If $\mathfrak{g} \circ \mathfrak{f}$ *is* N_sSO *and* $\mathfrak{g}$ *is* N_sSeCts*, then* $\mathfrak{f}$ *is* N_sSeO*.*

6 Neutrosophic Soft *e*-C Homeomorphism

Definition 14 A bijection $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ is called a N_sSe-C homeomorphism (in short, $N_sSeCHom$) if $\mathfrak{f}$ and $\mathfrak{f}^{-1}$ are N_sSeIrr mappings.

Theorem 20 *Each* $N_sSeCHom$ *is a* N_sSeHom*. But not conversely.*

Example 8 Let $Y = \{y_1, y_2, y_3\} = \{z_1, z_2, z_3\} = Z$, $Q = \{q_1, q_2\}$ and N_sSs's $(\tilde{H}_1, Q)$, $(\tilde{H}_2, Q)$ & $(\tilde{H}_3, Q)$ in Y and $(\tilde{G}_1, Q)$ in Z are defined as

$$(\tilde{H}_1, q_1) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.3, 0.5, 0.7)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_1, q_2) = \{\langle y_1, (0.3, 0.5, 0.8)\rangle, \langle y_2, (0.2, 0.5, 0.8)\rangle, \langle y_3, (0.4, 0.5, 0.5)\rangle\}$$
$$(\tilde{H}_2, q_1) = \{\langle y_1, (0.1, 0.5, 0.9)\rangle, \langle y_2, (0.1, 0.5, 0.9)\rangle, \langle y_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_2, q_2) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.2, 0.5, 0.9)\rangle, \langle y_3, (0.3, 0.5, 0.7)\rangle\}$$
$$(\tilde{H}_3, q_1) = \{\langle y_1, (0.2, 0.5, 0.8)\rangle, \langle y_2, (0.2, 0.5, 0.6)\rangle, \langle y_3, (0.3, 0.5, 0.6)\rangle\}$$
$$(\tilde{H}_3, q_2) = \{\langle y_1, (0.1, 0.5, 0.9)\rangle, \langle y_2, (0.2, 0.5, 0.7)\rangle, \langle y_3, (0.3, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle z_1, (0.2, 0.5, 0.8)\rangle, \langle z_2, (0.2, 0.5, 0.8)\rangle, \langle z_3, (0.4, 0.5, 0.6)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle z_1, (0.3, 0.5, 0.8)\rangle, \langle z_2, (0.3, 0.5, 0.8)\rangle, \langle z_3, (0.4, 0.5, 0.5)\rangle\}$$

Then we have $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, Q), (\tilde{H}_2, Q)\}$ and $\sigma = \{0_{(Z,Q)}, 1_{(Z,Q)}, (\tilde{G}_1, Q)\}$. Let $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ be a mapping defined as $\mathfrak{f}(y_1) = z_1, \mathfrak{f}(y_2) = z_1$ & $\mathfrak{f}(y_3) = z_3$. Then $\mathfrak{f}$ is N_sSeHom but not $N_sSeCHom$.

Theorem 21 *If* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *is a* $N_s SeCHom$*, then* $N_s Secl(\mathfrak{f}^{-1}(\tilde{G}, Q)) \subseteq \mathfrak{f}^{-1}(N_s Scl(\tilde{G}, Q))$ *(resp.* $N_s Secl(\mathfrak{f}^{-1}(\tilde{G}, Q)) = \mathfrak{f}^{-1}(N_s Secl(\tilde{G}, Q))$*) for each* $N_s Ss$ $(\tilde{G}, Q)$ *in* (Z, σ, Q)*.*

Theorem 22 *If* $\mathfrak{f} : (Y, \tau, Q) \to (Z, \sigma, Q)$ *and* $\mathfrak{g} : (Z, \sigma, Q) \to (P, \rho, Q)$ *are* $N_s SeCHom$*'s, then* $\mathfrak{g} \circ \mathfrak{f}$ *is a* $N_s SeCHom$*.*

7 Conclusion

In this paper, the concepts of $N_s SeO$ and $N_s SeC$ mappings in $N_s Sts$ were discussed. Furthermore, the work was extended to include $N_s SHom$, $N_s SeHom$ and $N_s SeT_{\frac{1}{2}}$-space. In addition, the study demonstrated $N_s SeCHom$ and derived some of its related characteristics. This work can be used to investigate neutrosophic soft e-compactness, neutrosophic soft e-connectedness and neutrosophic soft contra e-continuous functions in future.

References

1. Zadeh, L.A.: Fuzzy sets. Information and control. 8, 338–353 (1965).
2. Chang, C.L.: Fuzzy topological space. J. Math. Anal. Appl. 24, 182–190 (1968).
3. Atanassov, K., Stoeva, S.: Intuitionistic fuzzy sets, in : polish symp. on interval and fuzzy mathematics. Poznan. 23–26 (1983).
4. Atanassov, K.: Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87–96 (1986).
5. Atanassov, K.: Review and new results on intuitionistic fuzzy sets. Preprint IM-MFAIS. Sofia, 1–88 (1988).
6. Coker, D.: An introduction to intuitionistic topological spaces. Fuzzy Sets Syst. 88, 81–89 (1997).
7. Molodtsov, D.: Soft set theory-first results. Comput. Math. Appl. 37, 19–31 (1999).
8. Shabir, M., Naz, M.: On soft topological spaces. Comput. Math. Appl. 61, 1786–1799 (2011).
9. Smarandache, F.: Neutrosophic set: A generalization of the intuitionistic fuzzy sets. Inter. J. Pure Appl. Math. 24, 287–297 (2005).
10. Salama, A.A., Alblowi, S.A.: Neutrosophic Set and Neutrosophic Topological Spaces. IOSR Journal of Mathematics. 3 (4), 31–35 (2012).
11. Maji, P.K.: Neutrosophic soft set. Ann. Fuzzy Math. Inform. 5, 157–168 (2013).
12. Deli, I., Broumi, S.: Neutrosophic soft relations and some properties. Ann. Fuzzy Math. Inform. 9, 169–182 (2015).
13. Bera, T., Mahapatra, N.K.: Introduction to neutrosophic soft topological space. Opsearch. 54, 841–867 (2017).
14. Saha, S.: Fuzzy δ-continuous mappings. J. Math. Anal. Appl. 126, 130–142 (1987).
15. Vadivel, A., Seenivasan, M., John Sundar, C.: An introduction to δ-open sets in a neutrosophic topological spaces. J. Phys. Conf. Ser. 1724, 012011 (2021).
16. Acikgoz, A., Esenbel, F.: Neutrosophic soft δ-topology and neutrosophic soft compactness. AIP Conf. Proc. 2183, 030002 (2019).
17. Ekici, E.: On e-open sets, $\mathcal{DP}^{\star}$-sets and $\mathcal{DP}\bullet^{\star}$-sets and decomposition of continuity. Arab J Sci Eng. 33 (2A), 269–282 (2008).

18. Seenivasan, V., Kamala, K.: Fuzzy e-continuity and fuzzy e-open sets. Ann. Fuzzy Math. Inform. 8, 141–148 (2014).
19. Chandrasekar, V., Sobana, D., Vadivel, A.: On Fuzzy e-open Sets, fuzzy e-continuity and fuzzy e-compactness in intuitionistic fuzzy topological spaces. Sahand Communications in Mathematical Analysis (SCMA). 12 (1), 131–153 (2018).
20. Vadivel, A., John Sundar, C., Thangaraja, P.: Neutrosophic e-open sets in a neutrosophic topological spaces (Submitted).
21. Revathi, P., Chitirakala, K., Vadivel, A.: Neutrosophic Soft e-open sets in Neutrosophic Soft Topological Spaces (Submitted).
22. Vadivel, A., Thangaraja, P., John Sundar, C.: Neutrosophic e-Continuous Maps and Neutrosophic e-Irresolute Maps. TURCOMAT. 12 (1S), 369–375 (2021).
23. Vadivel, A., Thangaraja, P., John Sundar, C.: Neutrosophic e-Open Maps, Neutrosophic e-Closed Maps and Neutrosophic e-Homeomorphisms in Neutrosophic Topological Spaces (Submitted).
24. Revathi, P., Chitirakala, K., Vadivel, A.: Neutrosophic Soft e-Continuous Maps and Neutrosophic Soft e-Irresolute Maps (Submitted).
25. Ebenanjar, P.E., Immaculate, H.J., Sivaranjani, K.: Introduction to neutrosophic soft topological spatial region. Neutrosophic Sets and Systems. 31, 297–304 (2020).
26. Aras, C.G., Sadi Bayramov, S.: Neutrosophic Soft Continuity in Neutrosophic Soft Topological Spaces. Filomat. 34:10, 3495–3506 (2020).