Introduction to Neutrosophic BE-algebras

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Neutrosophic logic and its applications

Introduction to Neutrosophic BE-algebras

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Neutrosophic logic and its application
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با سخنرانی پدر علم نوتروسوفیک:
Professor Florentin Smarandache
On overview of Neutrosophic
Theories and Application
سخنرانان:
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Introduction to Neutrosophic BE−algebras
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تحلیل تصمیم در محیط نوتروسوفیک
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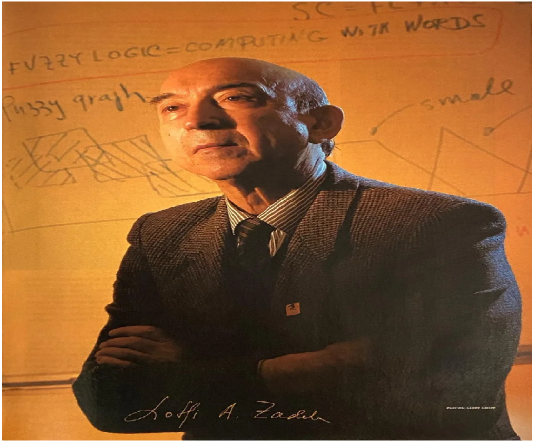

Figure : Prof. Lotfi Aliasker Zadeh

Introduction

The fuzzy set was introduced by L. Zadeh in 1965, where each element had a degree of membership [13]. In 1982, K. Atanassov [3] continued on tripartition and gave some generalizations of the fuzzy set (Intuitionistic fuzzy set, Intuitionistic L-fuzzy set, Interval-valued intuitionistic fuzzy set, Intuitionistic fuzzy set of second type, Temporal intuitionistic fuzzy set). The intuitionistic fuzzy sets consider both membership degree and non membership degree.

Figure : Prof. Florentin Smarandache

Introduction

Neutrosophy, introduced by F. Smarandache in 1998, is a new branch of philosophy in order to formally represent neutralities [10]. The nutrosophic set has not only a certain degree of truth and a certain degree of falsity but also an indeterminacy degree. Neutrosophic logic is an extension and generalization of fuzzy logic and intuitionistic fuzzy logic. In neutrosophic logic, each proposition is approximated to have the percentage of truth in the subset T, percentage of indeterminacy in the subset I and the percentage of falsity in the subset F where $T, I, F \in^{-}]0,1[^{+}$. For simplicity and practical applications, T, I, F are taken as single-valued numbers from the interval $[0,1]$ with $0 \leq T+I+F \leq 3$.

Neutrosophic Logic / Set / Probability / Statistics / Measure / Algebraic Structures etc. are all based on it. One of the most striking trends in the neutrosophic theory is the hybridization of neutrosophic set with other potential sets such as rough set, bipolar set, soft set, vague set, etc. [2, 4, 7-12]. The different hybrid structures such as rough neutrosophic set, single valued neutrosophic rough set, bipolar neutrosophic set, single valued neutrosophic vague set, etc. are proposed in the literature in a short period of time. Neutrosophic set has been a very important tool in all various areas of data mining, decision making, e-learning, engineering, computer science, graph theory, medical diagnosis, probability theory, topology, social science, etc.

Figure : Prof. Hee Sik Kim

H. S. Kim and Y. H. Kim introduced the notion of a BE-algebra as a generalization of a BCK-algebra [5]. A. Rezaei et al. investigated the relationship between Hilbert algebras and BE-algebras and showed that commutative self-distributive BE-algebras and Hilbert algebras are equivalent [6]. A classical algebra may be transformed into a NeutroAlgebra by a process called neutro-sophication, and into an AntiAlgebra by a process called anti-sophication. In this note, the concepts of Neutro-BE-algebras and Anti-BE-algebras are introduced, and some of related properties are investigated. We show that the class of Neutro-BE-algebra is an alternative of the class of BE-algebras.

Definition of classical BCI/BCK-algebras

An algebra $(X; *, 0)$ of type $(2, 0)$ is said to be a *BCI-algebra* if:
(L) The law $*$ is well-defined, i.e. $(\forall x, y \in X)(x * y \in X)$.
And the following axioms are totally true on X:

(BCI1) $((x * y) * (x * z)) * (z * y) = 0$,

(BCI2) $(x * (x * y)) * y = 0$,

(BCI3) $x * x = 0$,

(BCI4) $x * y = 0$ and $y * x = 0$ imply $x = y$.

for all $x, y, z \in X$. If a BCI-algebra $(X, *, 0)$ satisfies $0 * x = 0$ for all $x \in X$, then we say that X is a *BCK-algebra*.
We introduce a relation $\leq$ on X by $x \leq y$ if and only if $x * y = 0$.

Figure : Prof. Adesina AA Agboola

Let X be a nonempty set. A set $X(I) =< X, I >$ generated by X and I is called a neutrosophic set. The elements of $X(I)$ are of the form (x, yI) where x and y are elements of X.

Definition of neutrosophic BCI/BCK-algebras [1]

Let $(X, *, 0)$ be any BCI/BCK-algebra and let $X(I) =< X, I >$ be a set generated by X and I. The triple $(X(I), \star, (0, 0))$ is called a neutrosophic BCI/BCK-algebra. If (a, bI) and (c, dI) are any two elements of $X(I)$ with $a, b, c, d \in X$, we define

$$(a, bI) \star (c, dI) = (a * c, (a * d \wedge b * c \wedge b * d)I).$$

An element $x \in X$ is represented by $(x, 0) \in X(I)$ and $(0, 0)$ represents the constant element in $X(I)$.
For all $(x, 0), (y, 0) \in X(I)$, we define

$$(x, 0) \star (y, 0) = (x * y, 0) = (x \wedge \neg y, 0),$$

where $\neg y$ is the negation of y in X.

Example [1]

Let $X(I)$ be a neutrosophic set and let $A(I)$ and $B(I)$ be any two non-empty subsets of $X(I)$. Define

$$A(I) \star B(I) = A(I) \setminus B(I) = A(I) \cap B'(I).$$

Then $(X(I), \star, (\emptyset, \emptyset))$ is a neutrosophic BCK-algebra.

Definition of classical BE-algebras

An algebra $(X; *, 1)$ of type $(2, 0)$ is said to be a *BE-algebra* if:
(L) The law $*$ is well-defined, i.e. $(\forall x, y \in X)(x * y \in X)$.
And the following axioms are totally true on X:

(BE1) $x * x = 1$,

(BE2) $x * 1 = 1$,

(BE3) $1 * x = x$,

(BE4) $x * (y * z) = y * (x * z)$,

for all $x, y, z \in X$.
An algebra $(X; *, 1)$ of type $(2, 0)$ is said to be a *CI-algebra* if it satisfies (BE1), (BE3) and (BE4).
We introduce a relation $\leq$ on X by $x \leq y$ if and only if $x * y = 1$.

Definition of classical self distributive CI/BE-algebras

A CI/BE-algebra X is said to be *self-distributive* if for any $x, y, z \in X$,

$$x * (y * z) = (x * y) * (x * z).$$

Definition of classical commutative CI/BE-algebras

A CI/BE-algebra X is said to be *commutative* if for any $x, y \in X$,

$$(x * y) * y = (y * x) * x.$$

Example

Consider the interval $[0, 1]$ and define binary operation $*$ by

$$x * y = \begin{cases} 1 & \text{if } y \leq x, \\ \max\{1 - x, y\} & \text{otherwise.} \end{cases}$$

Then $([0, 1]; *, 1)$ is a BE-algebra.

Example

Let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and let $*$ be the binary operation on $\mathbb{N}_0$ defined by

$$x * y = \begin{cases} 0 & \text{if } y \leq x, \\ y - x & \text{if } x < y. \end{cases}$$

Then $(\mathbb{N}_0; *, 0)$ is a BE-algebra.

Definition of Neutro-sophications

Neutro-sophication of the law

The Neutro-sophication of the law (degree of well-defined, degree of indeterminacy, degree of outer-defined)
(NL) $(\exists x, y \in X)(x * y \in X)$ and
$(\exists x, y \in x)(x * y =$ indeterminate or $x * y \notin X)$.

The Neutro-sophication of the Axioms (degree of truth, degree of indeterminacy, degree of falsehood)

(NBE1) $(\exists x \in X)(x * x = 1)$ and
$(\exists x \in X)(x * x = \text{indeterminate or } x * x \neq 1)$,

(NBE2) $(\exists x \in X)(x * 1 = 1)$ and
$(\exists x \in X)(x * 1 = \text{indeterminate or } x * 1 \neq 1)$,

(NBE3) $(\exists x \in X)(1 * x = x)$ and
$(\exists x \in X)(1 * x = \text{indeterminate or } 1 * x \neq x)$,

(NBE4) $(\exists x, y, z \in X, \text{ with } x \neq y)(x * (y * z) = y * (x * z))$ and
$(\exists x, y, z \in X, \text{ with } x \neq y)(x * (y * z) = \text{indeterminate or } x * (y * z) \neq y * (x * z))$.

Definition of Anti-sophications

Anti-sophication of the law

The Anti-sophication of the law (totally outer-defined)
(AL) $(\forall x, y \in X)(x * y \notin X)$.

The Anti-sophication of the axioms (totally false)

(ABE1) $(\forall x \in X)(x * x \neq 1)$,

(ABE2) $(\forall x \in X)(x * 1 \neq 1)$,

(ABE3) $(\forall x \in X)(1 * x \neq x)$,

(ABE4) $(\forall x, y, z \in X, \text{ with } x \neq y)(x * (y * z) \neq y * (x * z))$.

Definition of Neutro-BE-algebras

A Neutro-BE-algebra is an alternative of BE-algebra that has at least a (NL) or at least one (NBEi), $i \in \{1, 2, 3, 4\}$, with no anti-law and no anti-axiom.

Example

Algebra $(\mathbb{N}; *, 1)$ is a Neutro-BE-algebra, where $*$ is defined by

$$x * y = \begin{cases} y & \text{if } x = 1, \\ \frac{1}{2} & \text{if } x \in \{3, 5, 7\}, \\ 1 & \text{otherwise.} \end{cases}$$

Since (NL) if $x \in \{3, 5, 7\}$, then $x * y = \frac{1}{2} \notin \mathbb{N}$, for all $y \in \mathbb{N}$, while if $x \notin \{3, 5, 7\}$ and $x \in \mathbb{N}$, then $x * y \in \{1, y\} \subseteq \mathbb{N}$, for all $y \in \mathbb{N}$.
(NBE1) $1 * 1 = 1 \in \mathbb{N}$ and $3 * 3 = \frac{1}{2} \notin \mathbb{N}$,
(NBE2) $5 * 1 = \frac{1}{2} \neq 1$ and if $x \notin \{3, 5, 7\}$, then $x * 1 = 1$,

(BE3) holds always since $1 * x = x$, for all $x \in \mathbb{N}$,
(NBE4) $5 * (3 * 4) = 5 * \frac{1}{2} =?$ (indeterminate) and
$3 * (5 * 4) = 3 * \frac{1}{2} =?$ (indeterminate).
Also, $2 * (3 * 4) = 2 * \frac{1}{2} =?$ (indeterminate), but
$3 * (2 * 4) = 3 * 1 = \frac{1}{2}$.
Further, $4 * (8 * 2) = 4 * 1 = 1 = 8 * (4 * 2)$.

Example

Let $\mathbb{R}$ be the set of all real numbers and $*$ be a binary operation on $\mathbb{R}$ defined by $x * y = |x - y|$. Then $(\mathbb{R}, *, 0)$ is a Neutro-BE-algebra.

Theorem

The total number of Neutro-BE-algebras is 31.

Definition of Anti-BE-algebras

An Anti-BE-algebra is an alternative of BE-algebra that has at least an (AL) or at least one (ABEi), $i \in \{1, 2, 3, 4\}$.

Example

Let $U = \{1, a, b, c, d\}$ be a universe of discourse, and a subset $S = \{1, c\}$ with the following Cayley table.

Table : Anti-BE-algebra $(S, *, 1)$

$*$	1	c
1	c	1
c	c	c

Then $(S, *, 1)$ is an Anti-BE-algebra, since (ABE1) is valid, because: $1 * 1 = c \neq 1$ and $c * c = c \neq 1$, and it is sufficient to have a single anti-axiom.

Example

Let $\mathbb{N}$ be the natural number set and $X := \mathbb{N} \cup \{0\}$. Define a binary operation $*$ on X by $x * y = x^2 + y^2 + 1$. Then $(X, *, 0)$ is not a BE-algebra, nor a Neutro-BE-algebra, but an Anti-BE-algebra.

Example

Let S be a nonempty set and $P(S)$ be the power set of S. Define the binary operation $\triangle$ (i.e. symmetric difference) by

$$A \triangle B = (A \cup B) \setminus (A \cap B)$$

for every $A, B \in P(S)$. Then $(P(S), \triangle, S)$ is not a BE-algebra, nor Neutro-BE-algebra, but it is an Anti-BE-algebra.

Theorem

The total number of Anti-BE-algebras is 211.

Theorem

As a particular case, for BE-algebras, we have:
1 (classical) BE-algebra
$+$ 31 Neutro-BE-algebras
$+$ 211 Anti-BE-algebras
= 243 algebras.
Where, $31 = 2^5 - 1$, and $211 = 3^5 - 2^5$.

Theorem

Let U be a nonempty finite or infinite universe of discourse, and S a nonempty finite or infinite subset of U. A classical Algebra is defined on S.
In general, for a given classical Algebra, having n operations (laws) and axioms altogether, for integer $n \geq 1$, there are total number of Algebra / NeutroAlgebras / AntiAlgebras as below:
1 (classical) Algebra,
$(2^n - 1)$ Neutro-Algebras, and
$(3^n - 2^n)$ Anti-Algebras.

Definition of NeutroSelf-distributive CI/BE-algebras

A CI/BE-algebra X is said to be *NeutroSelf-distributive* if for any $x, y, z \in X$,
$(\exists x, y, z \in X)(x * (y * z) = (x * y) * (x * z))$ and
$(\exists x, y, z \in X)(x * (y * z) \neq (x * y) * (x * z))$.

Definition of NeutroCommutative CI/BE-algebras

A CI/BE-algebra X is said to be *NeutroCommutative* if for any $x, y \in X$,
$(\exists x, y \in X \text{ with } x \neq y)(x * y) * y = (y * x) * x))$ and
$(\exists x, y \in X)(x * y) * y \neq (y * x) * x)$.

Definition of AntiSelf-distributive CI/BE-algebras

A CI/BE-algebra X is said to be *AntiSelf-distributive* if for any $x, y, z \in X$,
$(\forall x, y, z \in X)(x * (y * z) \neq (x * y) * (x * z))$.

Definition of AntiCommutative CI/BE-algebras

A CI/BE-algebra X is said to be *AntiCommutative* if for any $x, y \in X$,
$(\forall x, y \in X \text{ with } x \neq y)((x * y) * y \neq (y * x) * x)$.

Corollary

(i) Let X be an AntiSelf-distributive CI-algebra. Then X is not a BE-algebra.
(ii) There is no AntiSelf-distributive BE-algebra.
(iii) Let X be an AntiCommutative CI-algebra. Then X is not a BE-algebra.
(iv) There is no AntiCommutative BE-algebra.
(v) Every NeutroSelf-distributive CI-algebra is a Neutro-CI-algebra.
(vi) Every AntiSelf-distributive CI-algebra is an Anti-BE-algebra.
(vii) Every AntiCommutative CI-algebra is an Anti-BE-algebra.

References

1. A. A. A. Agboola and B. Davvaz, *Introduction to neutrosophic BCI/BCK-algebras*, International Journal of Mathematics and Mathematical Sciences, 2015, Article ID 37026.
2. M. Ali, L. Q. Dat, L. H. Son and F. Smarandache, *Interval complex neutrosophic set: formulation and applications in Decision-Making*, International of Journal Fuzzy Systems, 20(3) (2018), 986–999.
3. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20(1) (1986), 87–96.
4. M. Hamidi and F. Smarandache, *On Neutro-BCK-algebras*, International Journal of Neutrosophic Science (IJNS), 8(2) (2020), 110-117.

5. H. S. Kim and Y. H. Kim, *On BE-algebras*, Scientiae Mathematicae Japonicae, 66(1) (2007), 113-117.
6. A. Rezaei, A. Borumand Saeid, R. A. Borzooei, *Relation between Hilbert algebras and BE-algebras,* Applications and Applied Mathematics, 8(2) (2013), 573-584.
7. A. Rezaei, F. Smarandache, *The Neutrosophic Triplet of BI-algebras,* Neutrosophic Sets and Systems, 33 (2020), 313-321.
8. A. Rezaei, F. Smarandache, *On Neutro-BE-algebras and Anti-BE-algebras,* International Journal of Neutrosophic Science (IJNS), 1(1) (2020), 11-14.
9. A. Rezaei, F. Smarandache and S. Mirvakili, *Applications of (Neutro/Anti)sophications to Semihyper-groups,* Journal of Mathematics, 2021 (2021), 7 pages.

10. F. Smarandache, *Neutrosophy / Neutrosophic probability, set, and logic*, American Research Press, 1998. See also: http://gallup.unm.edu/ smarandache/NeutLog.txt.
11. F. Smarandache, *Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures, in Advances of Standard and Nonstandard Neutrosophic Theories,* Pons Publishing House Brussels, Belgium, Ch. 6, 240-265, 2019.
12. F. Smarandache, A. Rezaei and H. S. Kim, *A New Trend to Extensions of CI-algebras*, International Journal of Neutrosophic Science (IJNS), 5(1) (2020), 5–15
13. L. A. Zadeh, *Fuzzy sets*, Information and Control, 8 (1965), 338–353.

Figure : Shahzadeh Mahan Historical, Kerman, IRAN

THANK YOU FOR YOUR ATTENTION!