Certain Types of Domination in Nover Top Graphs

R. Narmada Devi, G. Muthumari, and Suresh Rasappan

Abstract Neutrosophicover set (Nover) was introduced by smarandache. Due to some real-time situation, decision-makers deal with uncertainty and inconsistency to identify the best result. Connectivity will be very important in neutrosophic graph. In this research study, we introduced the certain types of domination which are so-called perfect NOverTop-dominating set (perf Noverdom set), connected perfect NOverTop-dominating set (CONN perfNoverdom set), total perfect NOverTop-dominating set (Tot perf Noverdom set), connected total perfect NOverTop-dominating sets (CONN Tot perf Noverdom set), and also properties of domination numbers are established with necessary examples. Further, those relationship are discussed.

Keywords NOverTop-dom set · Perfect NOverTop-dom set · Connected perfect NOverTop-dom set · Total perfect NOverTop-dom set · Connected total perfect NOverTop-dom set and connected total perfect NOverTop-dom number

1 Introduction

Zadeh [1] introduced the concept of fuzzy set in 1965. Rosenfield added a fuzzy graph to the equation. The use of fuzzy graphs for obtaining answers has become increasingly popular in the areas of traffic congestion, decision-making, networking, privacy and security, and so on. Smaradche [2, 3] defined “neutrosophic logic” as a generalization of intuitionstic fuzzy logic. Also [4, 5], he defined neutrosophic over set. Broumi [2, 3] introduced numerous fascinating concepts, such as interval-valued neutrosophic graphs, single-valued neutrosophic graphs and their applications

R. N. Devi (✉) · G. Muthumari
Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Avadi, Chennai, Tamilnadu 600062, India
e-mail: narmadadevi23@gmail.com

S. Rasappan
Department of Mathematics, University of Technology and Applied Sciences-Ibri, Ibri 466, 516, Sultanate of Oman

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[6–8]. The notions of neutrosophic topological area are introduced. Ore [9] was the source of dominance. Many studies afterwards acknowledged domination and kinds of domination in fuzzy graphs, including Somasundaram [10], Nagoor Gani [11], Revathi [1, 12] and others. The notions of neutrosophic complex N-continuity and minimal domination via neutrosophic over graphs were presented by Narmada Devi [13–15]. Nover's top dominating set and its numerous types are discussed in this article. Along with the examples, there are also interesting characterizations of various types of Nover top graphs.

2 Preliminaries

Definition 2.1 A Nover graph is a pair $\psi = (A, B)$ of a crisp graph $\psi^* = (V, E)$ where A is Nvertex over set on V and B is a Nedge over set on E such that $\mathcal{T}_{\mathcal{B}}(xy) \leq (\mathcal{T}_{\mathcal{A}}(x) \wedge \mathcal{T}_{\mathcal{A}}(y)), \mathcal{I}_{\mathcal{B}}(xy) \leq (\mathcal{I}_{\mathcal{A}}(x) \wedge \mathcal{I}_{\mathcal{A}}(y)), \mathcal{F}_{\mathcal{B}}(xy) \geq (\mathcal{F}_{\mathcal{A}}(x) \vee \mathcal{F}_{\mathcal{A}}(y))$.

Definition 2.2 Let ψ be a Nover top graph. Let $x, y \in \mathcal{V}$. Then x dominates y in ψ if edge xy is effective edge $\mathcal{T}_{\mathcal{B}}(xy) = (\mathcal{T}_{\mathcal{A}}(x) \wedge \mathcal{T}_{\mathcal{A}}(y)), \mathcal{I}_{\mathcal{B}}(xy) = (\mathcal{I}_{\mathcal{A}}(x) \wedge \mathcal{I}_{\mathcal{A}}(y))$, $\mathcal{F}_{\mathcal{B}}(xy) = (\mathcal{F}_{\mathcal{A}}(x) \vee \mathcal{F}_{\mathcal{A}}(y))$.

A subset $\mathcal{D}_{\mathcal{N}}$ of $\mathcal{V}$ is said to be a Nover top dom set in ψ if each vertex $\mathcal{V} \notin \mathcal{D}_{\mathcal{N}}$, there exists $u \in \mathcal{D}_{\mathcal{N}}$ such that u dominates $\mathcal{V}$.

Definition 2.3 A dom set $\mathcal{D}_{\mathcal{N}}$ of Nover top graph is called a minimal Nover top dom set if no proper subset of $\mathcal{D}_{\mathcal{N}}$ is dom set.

Definition 2.4 Minimum cardinality of a Nover top dom set in a Nover top graph ψ is said to be Nover top dom number of ψ and is represented by $\gamma^{NOT}(\psi)$ (or) γ^{NOT}.

3 Types of Nover Top Domination Graphs

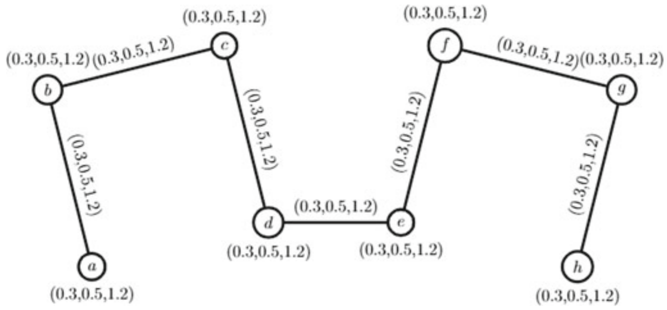
Definition 3.1 A subset $\mathcal{P}^{NOT}$ of $\mathcal{V}$ *is said to be perf. Nover top dom set of* ψ *if for each vertex* $v \notin \mathcal{P}^{NOT}$ *is dominating by* exactly one vertex u of $\mathcal{P}^{NOT}$.

Definition 3.2 A perfNover top dom set $\mathcal{P}^{NOT}$ of a Nover top graph ψ is called minimal perfNover top dom set for $\boldsymbol{v} \in \boldsymbol{\mathcal{P}^{NOT}}$, $\boldsymbol{\mathcal{P}^{NOT}} - \{\boldsymbol{v}\}$ is not a perfNover top dom set of ψ.

Definition 3.3 The minimal cardinality of a minimal perfNover top dom set of ψ is said to be the perfNover top dom number of ψ. It is represented by $\gamma_{pf}^{NOT}(\psi)$.

Definition 3.4 The maximal cardinality of a minimal perfNover top dom set of ψ is said to be the upper perfNover top dom number of ψ. It is represented as $\Gamma_{pf}^{NOT}(\psi)$.

Example 1



Let a, b, c, d, e, f, g and h denote the vertices and (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2) and (0.3, 0.5, 1.2) denote the edges which are labelled

$$\begin{aligned} f_u(0.3, 0.5, 1.2) &= (a, b), f_u(0.3, 0.5, 1.2) \\ &= (b, c), f_u(0.3, 0.5, 1.2) = (c, d), \\ f_u(0.3, 0.5, 1.2) &= (d, e), f_u(0.3, 0.5, 1.2) \\ &= (e, f), f_u(0.3, 0.5, 1.2) = (f, g), \\ f_u(0.3, 0.5, 1.2) &= (g, h). \end{aligned}$$

Let $\mathcal{X} = \{a, b, c, d, e, f, g, h$, (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2), (0.3, 0.5, 1.2)$\}$ be a topological space defined by the topology

$$\begin{aligned} \tau = \{&\emptyset, \mathcal{X}, \{a\}, \{b\}, \{c\}, \{d, e\}, \{f, g, h\}, \{a, b\}, \{a, c\}, \{a, d, e\}, \{a, f, g, h\}, \\ &\{b, c\}, \{b, d, e\}, \{b, f, g, h\}, \{c, d, e\}, \{c, f, g, h\}, \{d, e, f, g, h\}, \\ &\{a, b, c\}, \{a, b, d, e\}, \{a, b, f, g, h\}, \{b, c, d, e\}, \{b, c, f, g, h\}, \{c, d, e\}, \\ &\{c, f, g, h\}, \{d, e, f, g, h\}, \{a, b, c, d, e\}, \{a, b, c, f, g, h\}, \\ &\{a, b, d, e, f, g, h\}, \{a, b, c, e, f, g, h\}, \{a, b, c, d, f, g, h\}, \\ &\{a, b, c, d, e, g, h\}, \{a, b, c, d, e, f, h\}, \{a, b, c, d, e, f, g\}, \\ &\{a, b, c, d, g, h\}, \{a, b, c, d, f, h\}, \{a, b, c, d, e, f, g\}, \{a, b, c, d, g, h\}, \\ &\{a, b, c, d, f, h\}, \{a, b, c, d, f, g\}, \{a, b, c, d, e, h\}, \{a, b, c, d, e, g\} \\ &\{a, b, c, e, f, g, h\}, \{a, b, c, d, f, g, h\}, \{a, b, c, d, e, g, h\}, \{a, b, c, d, e, f, h\}, \} \end{aligned}$$

Here for every $x \in \mathcal{X}$, $\{x\}$ is open or closed.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a) = \{b\}$, $\partial(b) = \{a, c\}$, $\partial(c) = \{b, d\}$, $\partial(d) = \{c, e\}$, $\partial(e) = \{d, f\}$, $\partial(f) = \{e, g\}$, $\partial(g) = \{f, h\}$,

$\partial(h) = \{g\}$ with $\partial(a_i) = 2$ where $i = 1, 2, \cdots, 8$. Hence, this graph is Nover top graph.

Now, consider a perfect effective dominating Nover top graph $\psi = (A, B)$ where $\mathcal{V} = \{a, b, c, d, e, f, g, h\}$ and $\mathcal{T}_{\mathcal{A}}$,$\mathcal{I}_{\mathcal{A}}$,$\mathcal{F}_{\mathcal{A}}$ are given by $\mathcal{T}_{\mathcal{A}} : \mathcal{V} \rightarrow [0, \Omega]$, $\mathcal{I}_{\mathcal{A}} : \mathcal{V} \rightarrow [0, \Omega]$ and $\mathcal{F}_{\mathcal{A}} : \mathcal{V} \rightarrow [0, \Omega]$ where

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(a) &= \min[\mathcal{T}_{\mathcal{B}}(a, b)] = \min[0.3] = 0.3\\
\mathcal{I}_{\mathcal{A}}(a) &= \min[\mathcal{I}_{\mathcal{B}}(a, b)] = \min[0.5] = 0.5\\
\mathcal{F}_{\mathcal{A}}(a) &= \max[\mathcal{F}_{\mathcal{B}}(a, b)] = \max[1.2] = 1.2\\
\mathcal{T}_{\mathcal{A}}(b) &= \min[\mathcal{T}_{\mathcal{B}}(b, c), \mathcal{T}_{\mathcal{B}}(b, a)] = \min[0.3, 0.3] = 0.3\\
\mathcal{I}_{\mathcal{A}}(b) &= \min[\mathcal{I}_{\mathcal{B}}(b, c), \mathcal{I}_{\mathcal{B}}(b, a)] = \min[0.5, 0.5] = 0.5\\
\mathcal{F}_{\mathcal{A}}(b) &= \max[\mathcal{F}_{\mathcal{B}}(b, c), \mathcal{F}_{\mathcal{B}}(b, a)] = \max[1.2, 1.2] = 1.2
\end{aligned}$$

Similarly,

$$\begin{gathered}
\mathcal{T}_{\mathcal{A}}(c) = 0.3, \mathcal{I}_{\mathcal{A}}(c) = 0.5, \mathcal{F}_{\mathcal{A}}(c) = 1.2\\
\mathcal{T}_{\mathcal{A}}(d) = 0.3, \mathcal{I}_{\mathcal{A}}(d) = 0.5, \mathcal{F}_{\mathcal{A}}(d) = 1.2\\
\mathcal{T}_{\mathcal{A}}(e) = 0.3, \mathcal{I}_{\mathcal{A}}(e) = 0.5, \mathcal{F}_{\mathcal{A}}(e) = 1.2\\
\mathcal{T}_{\mathcal{A}}(f) = 0.3, \mathcal{I}_{\mathcal{A}}(f) = 0.5, \mathcal{F}_{\mathcal{A}}(f) = 1.2\\
\mathcal{T}_{\mathcal{A}}(g) = 0.3, \mathcal{I}_{\mathcal{A}}(g) = 0.5, \mathcal{F}_{\mathcal{A}}(g) = 1.2\\
\mathcal{T}_{\mathcal{A}}(h) = 0.3, \mathcal{I}_{\mathcal{A}}(h) = 0.5, \mathcal{F}_{\mathcal{A}}(h) = 1.2
\end{gathered}$$

Here a dominates b because

$$\begin{aligned}
\mathcal{T}_{\mathcal{B}}(a, b) &\le \mathcal{T}_{\mathcal{A}}(a) \wedge \mathcal{T}_{\mathcal{A}}(b), 0.3 \le 0.3 \wedge 0.3\\
\mathcal{I}_{\mathcal{A}}(a, b) &\le \mathcal{I}_{\mathcal{A}}(a) \wedge \mathcal{T}_{\mathcal{A}}(b), 0.5 \le 0.5 \wedge 0.5\\
\mathcal{F}_{\mathcal{A}}(a, b) &\ge \mathcal{F}_{\mathcal{A}}(a) \vee \mathcal{T}_{\mathcal{A}}(b), 1.2 \ge 1.2 \vee 1.2
\end{aligned}$$

Here b dominates c because

$$\begin{aligned}
\mathcal{T}_{\mathcal{B}}(b, c) &\le \mathcal{T}_{\mathcal{A}}(b) \wedge \mathcal{T}_{\mathcal{A}}(c), 0.3 \le 0.3 \wedge 0.3\\
\mathcal{I}_{\mathcal{B}}(b, c) &\le \mathcal{I}_{\mathcal{A}}(b) \wedge \mathcal{I}_{\mathcal{A}}(c), 0.5 \le 0.5 \wedge 0.5\\
\mathcal{F}_{\mathcal{B}}(b, c) &\ge \mathcal{F}_{\mathcal{A}}(b) \vee \mathcal{F}_{\mathcal{A}}(c), 1.2 \ge 1.2 \vee 1.2
\end{aligned}$$

Similarly,

c dominates d, d dominates e, e dominates f,
f dominates g, g dominates h

Here perfNover top dom sets are $\{b, e, h\}$, $\{a, d, g\}$, $\{b, d, g\}$ and $\{b, e, g\}$.
Minimal perfNover top domset $\mathcal{P}^{NOT} = \{b, e, h\}$.
PerfNover top domnumber $\gamma_{pf}^{NOT}(\psi) = 1, 2$.

Upper perfNover top domnumber $\Gamma_{pf}^{NOT}(\psi) = 1, 2$.

Definition 3.5 A perfNover top dom set $\mathcal{P}_c^{NOT}$ is said to be CONN perfNover top dom set if the Nover top *induced subgraph* $\mathcal{P}_c^{NOT}$ is CONN.

Definition 3.6 A perfNover top dom set $\mathcal{P}_c^{NOT}$ of a Nover top graph $\mathcal{G}$ is called a minimal CONN perfNover top dom set if for each vertex v in $\mathcal{P}_c^{NOT}$, $\mathcal{P}_c^{NOT} - \{v\}$ is not a CONN perfNover top dom set of ψ.

Definition 3.7 The minimum cardinality of a CONN perf Nover top dom set of a Nover top graph ψ is called the CONN perf Nover top dom number of ψ and is represented as $\gamma_{cpf}^{NOT}(\psi)$.

Definition 3.8 The maximum cardinality of a minimal CONN perf Nover top dom set ψ is called the upper CONN perf Nover top dom number of ψ and is represented by $\Gamma_{cpf}^{NOT}(\psi)$.

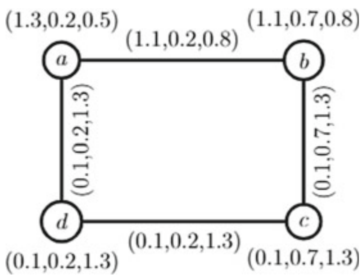
Definition 3.9 A perf Nover top dom set $\mathcal{P}_T^{NOT}$ of Nover top graph ψ is said to be Tot perfNover top dom set if each vertex $v \in \mathcal{V}$ in ψ is dominated to at least one vertex in $\mathcal{P}_T^{NOT}$.

Definition 3.10 The minimum cardinality of a Tot perfNover top dom set is Tot perfNover top dom number and is represented as $\gamma_{TPF}^{NOT}(\psi)$.

Definition 3.11 A Tot perfNover top dom set $\mathcal{P}_T^{NOT}$ of Nover top graph ψ is called a minimal Tot perfNover top dom set if for each vertex v in $\mathcal{P}_T^{NOT}$, $\mathcal{P}_T^{NOT} - \{v\}$ is not a Nover top dom set of ψ.

Definition 3.12 The maximum cardinality of a minimal Tot perfNover top dom set of ψ is called the upper Tot perfNover top number of Nover top graph ψ and is represented by $\Gamma_{TPF}^{NOT}(\psi)$.

Example 2



By the previous example, similarly we can find the Nover top conditions.

Hence, this graph is Nover top graph.
Here Nover top dom sets are $\{a, d\}$, $\{b, c\}$.
Tot perfNover top domset $= \{a, d\}$.
Minimal Tot perfNover top dom number $\gamma_{cpf}^{NOT} = 1$.
Upper Tot CONN perfNover top dom number $\Gamma_{pf}^{NOT}(\psi) = 1$.

Theorem 3.1 *Every CONN perf Nover top dom set is a Tot perfNover top dom set. Assume that* ψ *is a Nover top graph with no isolated vertices.*

By using Definition 3.8 CONN perfNover top dom set, if the induced subgraph $\mathcal{P}_{C}^{NOT}$ is CONN in the CONN perf Nover top dom set of ψ. It is clear that, if each vertex in ψ is dominated to atleast one vertex in $\mathcal{P}_{C}^{NOT}$ which is the definition of Tot perfNover top dom set of ψ. Therefore, every CONN perf Nover top dom set is a Tot perfNover top dom set of ψ.

Remark 1 The converse of the theorem is not true.

Example 3
In this Example 2, the collection of the sets $\{a, d\}$, $\{c, d\}$, $\{b, d\}$, $\{a, c\}$ are Tot perf Nover top dom sets, but they are not CONN perf Nover top dom sets. The set $\{a, b, c\}$ is CONN Perf Nover top set and Tot perfNover top dom set respectively.

CONN Tot perf Nover top dom set in Nover top graph

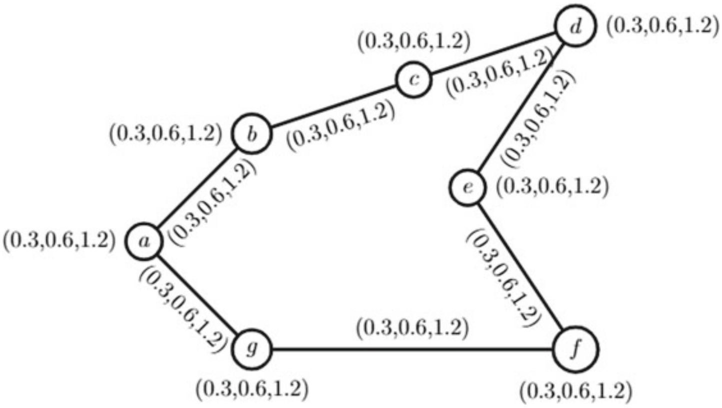
Definition 3.13 A Tot perfNover top dom set $\mathcal{P}_{ct}^{NOT}$ is said to be a CONN TotperfNover top dom set if the *induced subgraph* $\mathcal{P}_{ct}^{NOT}$ is CONN.

Definition 3.14 A perfNover top dom set $\mathcal{P}_{ct}^{NOT}$ of a Nover top graph ψ is said to be minimal CONN Tot perfNover top dom set if for each vertex v in $\mathcal{P}_{ct}^{NOT}$, $\mathcal{P}_{ct}^{NOT} - \{v\}$ is not a CONN Tot perfNover top dom set of ψ.

Definition 3.15 The minimum cardinality of a CONN Tot perfNover top dom set of a Nover top graph ψ is called the CONN Tot perfNover top dom number of ψ and is represented as $\gamma_{ctp}^{NOT}(\psi)$.

Definition 3.16 The maximum cardinality of a minimal CONN Tot perfNover top dom set of ψ is said to be the upper CONN Tot perfNover top dom number of ψ and is represented by $\Gamma_{ctp}^{NOT}(\psi)$.

Example 4



By the previous example, similarly we can find the Nover top condition.
CONN Tot perfNover top dom sets are $\{a, b, c, d, e, f\}$, $\{b, c, d, e, f, g\}$.
Minimal CONN Tot perfNover top domset $= \{a, b, c, d, e, f\}$.
CONN Tot perfNover top number $= 2.1$.
Upper CONN Tot perfNover top number $= 2.1$.

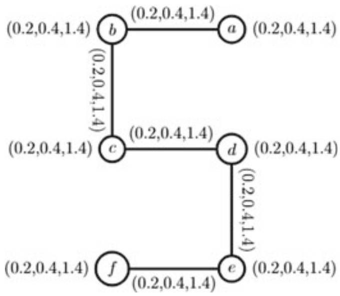
Theorem 3.2 *Every CONN Tot perfNover top dom set is a Tot perf Nover top dom set of* ψ.

Suppose ψ be a Nover top graph without isolated vertices. By using Definition 2.17.

CONN Tot perfNover top set, if the *induced subgraph* $\mathcal{P}_{ct}^{NOT}$ is CONN in the CONN Tot perfNover top dom set of a Nover top graph ψ. It is clear that, if each vertex in ψ is dominated to at least one vertex in $\mathcal{P}_{ct}^{NOT}$ which is the definition of Tot perfNover top dom set of ψ. Therefore, every CONN Tot perfNover top dom set is a Tot perfNover top dom set of ψ.

Theorem 3.3 *For any CONNNover top graph* $\mathcal{G}$, *then* $\gamma_{p}^{NOT}(\psi) \leq \gamma_{tp}^{NOT}(\psi) \leq \gamma_{ctp}^{NOT}(\psi)$.

Example 5

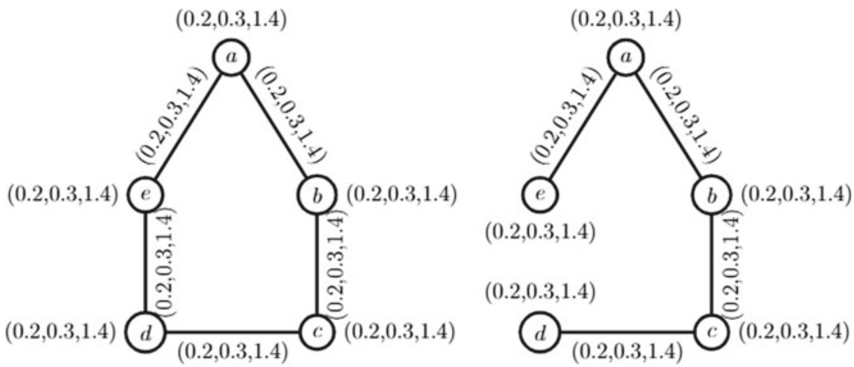


CONN Tot perfNover top dom set = $\{b, c, d, e\}$.
CONN Tot perfNover top domnumber $\gamma_{ctp}^{NOT}(\psi) = 0.4$.
Tot perfNover top domset = $\{b, c, d, e\}$.
Tot perfNover top domnumber $\gamma_{tp}^{NOT}(\psi) = 0.4$.
Perf Nover top dom set = $\{b, d, f\}$.
Perf Nover top dom number = .3.
Therefore, $\gamma_{pf}^{NOT}(\psi) \leq \gamma_{tp}^{NOT}(\psi) \leq \gamma_{ctp}^{NOT}(\psi)$.

Theorem 3.4 *If H is a CONN spanning subgraph of a Nover top graph $\mathcal{G}$, then* $\gamma_{ctp}^{NOT}(\psi) \leq \gamma_{ctp}^{NOT}(H)$.

However, every CONN Tot perfNover top dom set of H is also CONN Tot perfNover top dom set of ψ.

Example 6



By the previous example, similarly we can find the Nover top condition.
Hence, this graph is Nover top graph. CONN Tot perfNover top dom set $\{a, b, c\}$

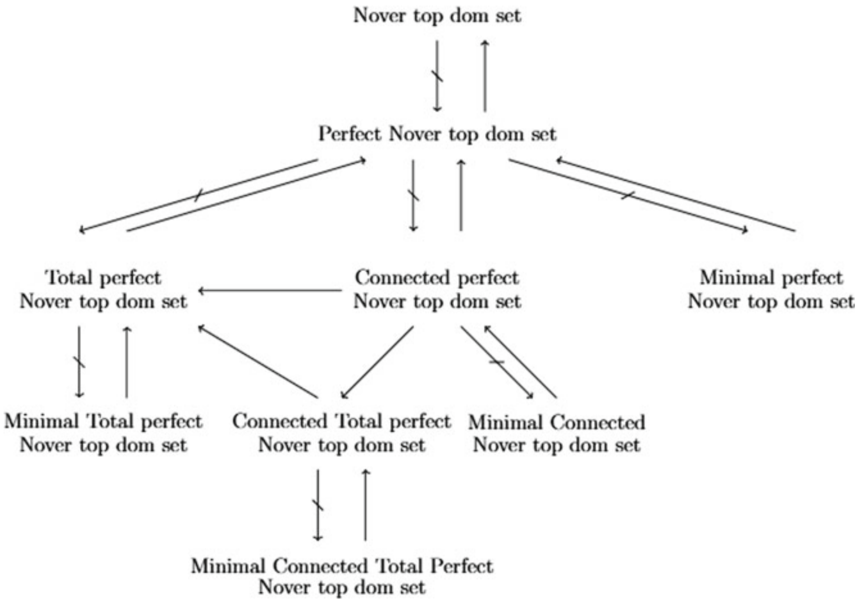
$$\gamma_{ctp}^{NOT}(\psi) = 0.15$$

Similarly, we can find out Nover top *spanning subgraph* (H).
$\therefore$ CONN Tot perfNover top dom set $\{a, b, c\}$

$$\gamma_{ctp}^{NOT}(H) = 0.15$$

Therefore, $\gamma_{ctp}^{NOT}(\psi) \leq \gamma_{ctp}^{NOT}(H)$.

Remark 2 The interrelation among Nover top dominating sets is



References

1. Revathi, S., P.J. Jayalakshmi, and C.V.R. Harinarayanan. 2013. Perfect dominating sets in fuzzy graphs. *IOSR Journal of Mathematics* 8 (3): 43–47.
2. Broumi, S., M. Talea, A. Bakali, and F. Smarandache. 2016. Single valued neutrosophic graphs. *Journal of New Theory* 10: 86–101.
3. Smarandache, F., 2016. Neutrosophic over set, neutrosophic under set, neutrosophic off set. *Pons Editions, Brussels.*
4. Smarandache, F., 1999. *A Unifying Field in Logics: Neutrosophic Logic*. American Research Press, pp.1–141.
5. Somasundaram, A., and S. Somasundaram. 1998. Domination in fuzzy graphs–I. *Pattern Recognition Letters* 19 (9): 787–791.
6. Salama, A.A., 2013. Neutrosophic crisp points and neutrosophic crisp ideals. *Neutrosophic Sets and Systems* 50–53.

7. Salama, A.A., F. Smarandache, and V. Kroumov. 2014. Neutrosophic crisp sets and neutrosophic crisp topological spaces. *Neutrosophic Sets and Systems* 25–30.
8. Salama, A.A., F. Smarandache, and V. Kromov, 2014. Neutrosophic closed set and neutrosophic continuous functions. *Neutrosophic Sets and Systems* 4–8.
9. Ore, O., 1962. Theory of graphs. *American Mathematical Society Colloquium Publications* 38.
10. Zadeh, L.A., 1965. Fuzzy sets. *Information and Control* 8 (3): 338–353.
11. Nagoorgani, A., and V.T. Chandrasekaran, 2006. Domination in fuzzy graph. *Advances in Fuzzy Sets and System* 17–26.
12. Revathi, S., C.V.R. Harinarayanan, and R. Muthuraj. 2015. Connected perfect domination in fuzzy graph. *Golden Research thoughts* 5: 1–5.
13. Devi, R.N., N. Kalaivani, S. Broumi, and K.A. Venkatesan, 2018. Characterizations of strong and balanced neutrosophic complex graphs. *Infinite Study*.
14. Devi, R.N., 2017. Neutrosophic complex N-continuity. *Infinite Study*.
15. Devi, R.N., 2020. Minimal domination via neutrosophic over graphs. *AIP Conference Proceedings, November* 2277: 100019.