An Interval Valued Triangular Fuzzy Soft Sets and Its Application in Decision-Making Process Using New Aggregation Operator

D. Nagarajan, M. Lathamaheswari, Said Broumi, Florentin Smarandache and J. Kavikumar

Abstract In decision-making process, aggregation plays a vital role as it is a process of combining various numerical scores with respect to all the criteria using aggregation operators. Hence, in this paper, new aggregation operators have been proposed, namely Interval Valued Triangular Fuzzy Soft Weighted Arithmetic (IVTFSWA) operator and Interval Valued Triangular Fuzzy Soft Weighted Geometric (IVTFSWG) operator for Interval Valued Triangular Fuzzy Soft Numbers (IVTFSNs) using Frank triangular norms and applied them in a decision-making problem to identify the patient who has more illness. This application shows the soundness of the presented operators.

Keywords Fuzzy soft set · Aggregation operator · Decision-making process · Type-2 fuzzy model · Triangular norms · Interval valued triangular soft fuzzy number

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1 Introduction

Triangular inequalities were protracted by the theoretical concept of triangular norms which are introduced from the scope of prospect metric [1]. Many of the realistic optimization problems are described by flexibility of the constraint which can be used for further decision between correcting the impartial function and fulfilling the constraints [2]. Fuzzy set theory contributes a methodology of representing and handling flexible or soft constraints [3]. SS theory applications normally solve the problems using rough and FSSs [5]. T-norms and t-conorms are the general additional operations in the interval [0, 1] ever present in the theory and applications of Type-2 FS (T1FSs) [7]. Till date, there are many operators which have been introduced to aggregate the information [8]. In DM process, information is collected from a number of experts by preparing survey that involves imprecise words and linguistic terms which have immeasurable domain, and the conclusions of T1 fuzziness are uncertain boundaries of FSs. This can be solved by T2 fuzziness, where the FSs have degrees of membership that themselves fuzzy [9]. In T2FSs, there are two membership functions (MFs) available, namely primary and secondary. For all the value of the primary variable x on the universal set X, the function will be the membership rather than a characteristic value. For secondary MFs, domain is the set of primary membership value. The secondary MF provides three-dimensional T2FS, where the third dimension affords a certain degree of freedom to deal with uncertainties [10]. The soft set theory can be utilized as a common mathematical tool to handle uncertainty [13]. Representation of SSs is nothing but soft matrices which have many advantages like it is easy to collect and apply matrices. Construction of decision making using SSs is more realistic and applied in a fruitful manner to various problems which have uncertainties [18]. The mathematical functions which are applied to integrate the information are called aggregation operators. DM process using fuzzy logic is a sizzling area of the research community [19]. It contains the method of choosing the best alternatives from possible alternatives in consideration of many attributes, where the information of the decision generally with fuzziness is contributed by a number of experts in fuzzy setting. Uncertainty of the real world with its related problems can be dealt well with SS theory. Since there is an increasing difficulty of the real world environment and the inadequate data of the problem, decision-makers may give their preferences about alternatives in the form of Interval Valued Fuzzy Numbers. Matrices are playing a crucial role in the wide area of engineering and science. The conventional theory of matrices could not solve the problems which contain impreciseness [20–22].

2 Review of Literature

The authors of [1] examined a method of DM problem for many attributes using triangular interval type-2 fuzzy numbers (TIT2FNs). Rajarajeswari and Dhanalakshmi [2] defined IVIFS matrix, its types with illustrations, few operators using weights, and their properties. Tripathy and Sooraj [3] conferred few operations of IVIFSS and its application in DM process. Khalil et al. [4] corrected some points on Interval Valued Hesitant FSSs. Sudharsan and Ezhilmaran [5] presented weighted arithmetic averaging (WAA) operator using IVIF values and applied in MCDM in choosing the best investment. Mohd and Abdullah [6] surveyed about the application of aggregation methods in GDM process for the period of 10 years. Shenbagavalli et al. [7] applied FSSs and its matrices in MAGDM process with the partial information of the data. Selvachandran and Sallah [8] proposed IVCFSSs by combining T2 complex FSs and SSs. Hermandez [9] proposed and examined the expansion of some types of FNs, namely T1, T2, and generalized one. Castillo et al. [10] reviewed the applications of T2 fuzzy system in image processing. Dinagar and Rajesh [11] discussed a methodology for a MCDM problem using the AOs of IVFSM. Qin et al. [12] established a new method for decision making which has the capability of solving some complicated problems. Nazra et al. [13] extended hesitant IFSSs to a generalized case and obtained an algebraic structure. Alcantud and Torrecillas [14] combined the aspects of SSs and FSSs and solved a decision-making problem. Selvachandran and Singh [15] handled complex data with the help of the properties of IVCFSs. Dayan and Zulqarnain [16] proposed the notions of generalized IVFSMs and discussed a few of its types, operational laws, and similarity measure. Mahmooda et al. [17] proposed the notion of lattice-ordered IFSS and its operational laws and applied in DM process. Arora and Garg [18] proposed aggregation operators using IFSSs and applied in DM process. Arora and Garg [19] did a comparative analysis of two IF soft numbers and presented weighted averaging and geometric AOs with their different properties. This literature study is the motivation of the present study. Nagarajan et al. [20] analyzed traffic control management using aggregation operators under interval type-2 fuzzy and interval neutrosophic environments. Nagarajan et al. [21] presented edge detection method using triangular norms under type-2 fuzzy environment. Nagarajan et al. [22] introduced image extraction methodology for DICOM image based on the concept of type-2 fuzzy.

2.1 Preliminaries

The following basic result has been given as preliminaries for better understanding of the paper.

2.2 Interval Valued Triangular Fuzzy Weighted Averaging/ Geometric (IVTFWA/IVTFWG) Operator [Qin and Liu 2014]

Let $F_i = \left\langle \left[l_{F_i}^-, l_{F_i}^+ \right], m_{F_i}, \left[r_{F_i}^-, r_{F_i}^+ \right] \right\rangle, i = 1, 2, 3, \ldots, n$ be a set of Interval Valued Triangular Fuzzy Numbers (IVTFNs) and consider TIT2FWA : $\Delta^n \rightarrow \Delta$, if

$$\text{IVTFWA}_\rho(F_1, F_2, \ldots, F_n) = \rho_1 \cdot F_1 \oplus \rho_2 \cdot F_2 \oplus \ldots \oplus \rho_n \cdot F_n (\text{IVTFWA}) \quad (1)$$

$$\text{IVTFWG}_\rho(F_1, F_2, \ldots, F_n) = F_1^{\rho_1} \otimes F_2^{\rho_2} \otimes, \ldots, \otimes F_n^{\rho_n} (\text{IVTFWG}) \quad (2)$$

where $\rho = (\rho_1, \rho_2, \ldots, \rho_n)^T$ is the weight vector (WV) F_i, $\rho_i \geq 0$. If $\rho = (1/n, 1/n, \ldots, 1/n)^T$ then IVTFWA/IVTFWG operator will become Interval Valued Triangular Fuzzy Arithmetic/Geometric Averaging (IVTFAA/IVTFGA) operator.

2.3 Frank Triangular Norms [Qin and Liu 2014]

Frank triangular norms include Frank sum and product which are the examples of t-norm and t-conorm, respectively. Here t-norm is Frank product, and t-conorm is Frank sum, and is defined by

$$\begin{aligned} m \oplus n &= 1 - \log_\beta \left(1 + \frac{\left(\beta^{1-m} - 1\right)\left(\beta^{1-n} - 1\right)}{\beta - 1} \right), \\ m \otimes n &= \log_\beta \left(1 + \frac{(\beta^m - 1)(\beta^n - 1)}{\beta - 1} \right) \\ &\beta > 1, \forall (m, n) \in [0, 1]^2 \end{aligned} \quad (3)$$

2.4 Ranking Formula (Score Function) for Interval Valued Triangular Fuzzy Numbers [Qin and Liu 2014]

For the type-2 triangular fuzzy number, the ranking formula is defined by

$$S(\chi) = \quad \frac{a_{ij}^- + c_{ij}^+}{2} + 1 \Bigg) \times \frac{a_{ij}^- + a_{ij}^+ + c_{ij}^- + c_{ij}^+ + 4b_{ij}}{8} \quad (4)$$

2.5 Aggregation Operators for Interval Valued Triangular Fuzzy Soft Numbers (IVTFSNs)

In this section, two aggregation operators, IVTFSWA, have been proposed.

2.5.1 Interval Valued Triangular Fuzzy Soft Weighted Average (IVTFSWA) Operator

Let $F_{e_{ij}} = \left\langle \left(a_{ij}^{-}, a_{ij}^{+}\right), b_{ij}, \left(c_{ij}^{-}, c_{ij}^{+}\right)\right\rangle, i = 1, 2, \ldots, n \& j = 1, 2, \ldots, m$ be an IVTFSNs and φ_j, ρ_i be the weight vectors for the parameters e_j's and experts x_i, respectively; then

$$\text{IVTFSWA}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{nm}}) = \overset{m}{\underset{j=1}{\oplus}} \varphi_j \left(\overset{n}{\underset{i=1}{\oplus}} \rho_i F_{e_{ij}} \right) \quad (5)$$

and satisfying the following conditions: $\sum_{j=1}^{m} \varphi_j = 1$ and $\sum_{i=1}^{n} \rho_i = 1$, $\varphi_j > 0, \rho_i > 0$.

2.6 Theorem

Let $F_{e_{ij}} = \left\langle \left(a_{ij}^{-}, a_{ij}^{+}\right), b_{ij}, \left(c_{ij}^{-}, c_{ij}^{+}\right)\right\rangle$ be an IVTFSNs, the aggregated value by IVTFSWA operator is also an IVTFSN and is given by

$$\begin{aligned} \text{IVTFSWA}(F_{e_{11}}, F_{e_{12},\ldots,}F_{e_{nm}}) = \Bigg\langle \Bigg[\Bigg(& 1 - \log_\beta \left(1 + \wp_j \left(\mathbb{P}_i \left(\beta^{1-a_{ij}^{-}} - 1\right)^{\rho_i}\right)^{\varphi_j}\right), \\ & 1 - \log_\beta \left(1 + \wp_j \left(\mathbb{P}_i \left(\beta^{1-a_{ij}^{+}} - 1\right)^{\rho_i}\right)^{\varphi_j}\right)\Bigg)\Bigg], \\ & 1 - \log_\beta \left(1 + \wp_j \left(\mathbb{P}_i \left(\beta^{1-b_{ij}} - 1\right)^{\rho_i}\right)^{\varphi_j}\right), \\ & \Bigg(1 - \log_\beta \left(1 + \wp_j \left(\mathbb{P}_i \left(\beta^{1-c_{ij}^{-}} - 1\right)^{\rho_i}\right)^{\varphi_j}\right), \\ & 1 - \log_\beta \left(1 + \wp_j \left(\mathbb{P}_i \left(\beta^{1-c_{ij}^{+}} - 1\right)^{\rho_i}\right)^{\varphi_j}\right)\Bigg)\Bigg]\Bigg\rangle \end{aligned} \quad (6)$$

Proof For $n = 1, \rho_1 = 1$:

$$\begin{aligned}
\text{IVTFSWA}\left(F_{e_{11}}, F_{e_{12,\ldots,}} F_{e_{1m}}\right) &= \overset{m}{\underset{j=1}{\oplus}} \varphi_j F_{e_{1j}} \\
&= \Big\langle \Big[\Big(1 - \log_\beta \Big(1 + \wp_j \big(\beta^{1-a_{1j}^-} - 1 \big)^{\varphi_j} \Big), \\
&\quad 1 - \log_\beta \Big(1 + \wp_j \Big(\beta^{1-a_{1j}^+} - 1 \Big)^{\varphi_j} \Big) \Big) \Big], \\
&\quad 1 - \log_\beta \Big(1 + \wp_j \big(\beta^{1-b_{1j}} - 1 \big)^{\varphi_j} \Big), \\
&\quad \Big(1 - \log_\beta \Big(1 + \wp_j \big(\beta^{1-c_{1j}^-} - 1 \big)^{\varphi_j} \Big), \\
&\quad 1 - \log_\beta \Big(1 + \wp_j \Big(\beta^{1-c_{1j}^+} - 1 \Big)^{\varphi_j} \Big) \Big) \Big] \Big\rangle
\end{aligned}$$

For $m = 1, \varphi_1 = 1$

$$\begin{aligned}
\text{IVTFSWA}\left(F_{e_{11}}, F_{e_{21,\ldots,}} F_{e_{n1}}\right) &= \overset{n}{\underset{i=1}{\oplus}} \rho_i F_{e_{i1}} \\
&= \Big\langle \Big[\Big(1 - \log_\beta \Big(1 + \mathbb{P}_i \big(\beta^{1-a_{i1}^-} - 1 \big)^{\rho_i} \Big), \\
&\quad 1 - \log_\beta \Big(1 + \mathbb{P}_i \Big(\beta^{1-a_{i1}^+} - 1 \Big)^{\rho_i} \Big) \Big) \Big], \\
&\quad 1 - \log_\beta \Big(1 + \mathbb{P}_i \big(\beta^{1-b_{i1}} - 1 \big)^{\rho_i} \Big), \\
&\quad \Big(1 - \log_\beta \Big(1 + \mathbb{P}_i \big(\beta^{1-c_{i1}^-} - 1 \big)^{\rho_i} \Big), \\
&\quad 1 - \log_\beta \Big(1 + \mathbb{P}_i \Big(\beta^{1-c_{i1}^+} - 1 \Big)^{\rho_i} \Big) \Big) \Big] \Big\rangle
\end{aligned}$$

Hence, the result is hold for $n = 1, m = 1$.
Consider the result is hold $m = k_1 + 1, n = k_2$ and $m = k_1, n = k_2 + 1$

$$\begin{aligned}
\overset{k_1+1}{\underset{j=1}{\oplus}} \varphi_j \left(\overset{k_2}{\underset{i=1}{\oplus}} \rho_i F_{e_{ij}} \right) &= \Big\langle \Big[\Big(1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \big(\beta^{1-a_{ij}^-} - 1 \big)^{\rho_i} \Big)^{\varphi_j} \Big), \\
&\quad 1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \Big(\beta^{1-a_{ij}^+} - 1 \Big)^{\rho_i} \Big)^{\varphi_j} \Big) \Big) \Big], \\
&\quad 1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \big(\beta^{1-b_{ij}} - 1 \big)^{\rho_i} \Big)^{\varphi_j} \Big), \\
&\quad \Big(1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \big(\beta^{1-c_{ij}^-} - 1 \big)^{\rho_i} \Big)^{\varphi_j} \Big), \\
&\quad 1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \Big(\beta^{1-c_{ij}^+} - 1 \Big)^{\rho_i} \Big)^{\varphi_j} \Big) \Big) \Big] \Big\rangle
\end{aligned}$$

And

$$\overset{k_1}{\underset{j=1}{\oplus}} \varphi_j \left(\overset{k_2+1}{\underset{i=1}{\oplus}} \rho_i F_{e_{ij}} \right) = \Big\langle \Big[\Big(1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \big(\beta^{1-a_{ij}^-} - 1 \big)^{\rho_i} \Big)^{\varphi_j} \Big), \\ 1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \Big(\beta^{1-a_{ij}^+} - 1 \Big)^{\rho_i} \Big)^{\varphi_j} \Big) \Big) \Big], \\ 1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \big(\beta^{1-b_{ij}} - 1 \big)^{\rho_i} \Big)^{\varphi_j} \Big), \\ \Big(1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \big(\beta^{1-c_{ij}^-} - 1 \big)^{\rho_i} \Big)^{\varphi_j} \Big), \\ 1 - \log_\beta \Big(1 + \wp_j \Big(\mathbb{P}_i \Big(\beta^{1-c_{ij}^+} - 1 \Big)^{\rho_i} \Big)^{\varphi_j} \Big) \Big) \Big] \Big\rangle$$

Therefore, it is hold for $m = k_1 + 1, n = k_2 + 1$. Hence, by method of induction, the result is true for all the values of $m, n \geq 1$.

2.7 Proposed Methodology for Decision-Making Process

Step 1. Gather the information associated with all the alternatives under various criterions/parameters in the form of Interval Valued Triangular Fuzzy Soft Matrix (IVTFSM) is SD and defined

by $\mathrm{SD} = \left\langle \left(a_{ij}^-, a_{ij}^+ \right), b_{ij}, \left(c_{ij}^-, c_{ij}^+ \right) \right\rangle_{n \times m}$

Step 2. Normalize the matrix SD by converting the grade values into benefit type of the cost parameters by applying the following formula.

$$R_{ij} = \begin{cases} F_{e_{ij}}, \text{for benefit type parameters} \\ F_{e_{ij}}^c, \text{for cost type parameters} \end{cases} \tag{7}$$

where $F_{e_{ij}}^c = \left\langle \left(1 - c_{ij}^-, 1 - c_{ij}^+ \right), 1 - b_{ij}, \left(1 - a_{ij}^-, 1 - a_{ij}^+ \right) \right\rangle$ is the complement of $F_{e_{ij}}$.

If all the parameters are of the same type, then normalization is not necessary.

Step 3. Aggregate the IVTFSNs $F_{e_{ij}}$ for all the alternatives $A_k (k = 1, 2, \ldots, u)$ into collective decision matrix ξ_k using proposed IVTFSWA operator.

Step 4. Calculate the score value of all the alternatives $A_k (k = 1, 2, \ldots, u)$.

Step 5. Using score value of the alternatives, select the best one.

Step 6. End.

2.7.1 Decision-Making Problem

By considering a practical example from [20], the above decision-making process has been illustrated for medical diagnosis. The board of four doctors d_1, d_2, d_3, d_4 whose weight vector is $\rho = (0.3, 0.2, 0.1, 0.4)^T$ will award their preference values for four patients A_1, A_2, A_3, A_4 under some criterions/parameters E = {"Temperature (e_1), Headache (e_2), Stomach Pain (e_3), Chest Pain (e_4)} with the weight vector $\varphi = (0.35, 0.25, 0.25, 0.15)^T$. The result obtained using the methodology is as follows.

Step 1. The four doctors d_i will measure the illness of the four patients in terms of IVTFSNs. Parameters and their values of rating are summarized in Tables 1, 2, 3 and 4.

Step 2. Here every parameter is of the same type; hence, it is not necessary for normalization.

Step 3. The different opinions of the doctors for every patient $A_k, k = 1, 2, 3, 4$ are aggregated by using the Eq. (6). The aggregated values for the four patients are

$$\xi_1 = \langle (0.4354, 0.5392), 0.6247, (0.6951, 0.8049) \rangle$$
$$\xi_2 = \langle (0.3974, 0.5010), 0.5989, (0.6940, 0.8059) \rangle$$
$$\xi_3 = \langle (0.3439, 0.4477), 0.5400, (0.6415, 0.7534) \rangle$$
$$\xi_4 = \langle (0.4421, 0.5455), 0.5972, (0.6500, 0.7494) \rangle$$

Step 4. The score values are $SV(\xi_1) = 1.0073$, $SV(\xi_2) = 0.9597$, $SV(\xi_3) = 0.8414$, $SV(\xi_4) = 0.9527$ and the ranking is $SV(\xi_1) > SV(\xi_2) > SV(\xi_4) > SV(\xi_3)$.

Step 5. Therefore, the patient, A_1 has more illness.

Table 1 Interval valued triangular FSM for the patient A_1

$\mathbb{U}$	e_1	e_2	e_3	e_4
d_1	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$
d_2	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.6, 0.7) \rangle$	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$
d_3	$\langle (0.3, 0.4), 0.5, (0.5, 0.6) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$
d_4	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.6, 0.7), 0.7, (0.7, 0.8) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$

Table 2 Interval valued triangular FSM for the patient A_2

$\mathbb{U}$	e_1	e_2	e_3	e_4
d_1	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$
d_2	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$
d_3	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$
d_4	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$

Table 3 Interval valued triangular FSM for the patient A_3

$\mathbb{U}$	e_1	e_2	e_3	e_4
d_1	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$
d_2	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$
d_3	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$
d_4	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$

Table 4 Interval valued triangular FSM for the patient A_4

$\mathbb{U}$	e_1	e_2	e_3	e_4
d_1	$\langle (0.3, 0.4), 0.5, (0.5, 0.6) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$
d_2	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.5, (0.5, 0.6) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.8) \rangle$
d_3	$\langle (0.4, 0.5), 0.6, (0.6, 0.7) \rangle$	$\langle (0.7, 0.8), 0.8, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$
d_4	$\langle (0.5, 0.6), 0.6, (0.7, 0.8) \rangle$	$\langle (0.6, 0.7), 0.7, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.6, (0.7, 0.8) \rangle$	$\langle (0.3, 0.4), 0.5, (0.5, 0.7) \rangle$

2.7.2 Comparative Analysis

In [19], decision making is done to find the patient with more illness using intuitionistic fuzzy soft weighted arithmetic operator. As the expert's decision may be in terms of interval values, the existing method is not able to deal with interval data. The methodology proposed in this paper is able to sort out this issue, and hence, the proposed method can be applied and deal more uncertainties in the decision-making process.

2.8 Conclusion

Decision making is an essential task in Science and Engineering. Since most of the real-world problems have uncertainty in nature, making decision is challengeable one for the decision-makers. At this junction, fuzzy sets provide mathematical tool which handles the uncertainty in an efficient way. In dealing with impreciseness/ uncertainty, Interval Valued FSs perform well than Type-1 FSs. Due to this reason, IVTFSS has been considered in this paper and proposed aggregation operator IVTFSWA for IVTFSNs using Frank triangular norms. The proposed operator is applied in decision-making process to identify the patient who has more illness as the medical diagnosis which proves the validity of the proposed operator. Also comparative analysis has been done with existing method. In future, more aggregation operators can be obtained under various fuzzy, soft fuzzy environments.

Here FSM is Fuzzy Soft Matrix.

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