# Neutrosophic Generalized Regular Star Compact, Connected, Regular and Normal Topological Spaces

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Abstract: Real-life structures always include indeterminacy. The Mathematical tool which is well-known in dealing with indeterminacy is Neutrosophic. Smarandache proposed the approach of Neutrosophic sets. Neutrosophic sets deal with uncertain data. The notion of Neutrosophic set is generally referred to as the generalization of intuitionistic fuzzy set. In 2022, S. Pious Missier, A. Anusuya and J. Martina Jency introduced and studied the concepts of Neutrosophic generalized regular star closed $\left(N_{eu}gr^*\text{-closed}\right)$ sets and Neutrosic generalized regular star $\left(N_{eu}gr^*\text{-open}\right)$ sets in Neutrosophic topological spaces and studied some of their properties and characterizations as well as analyzed the relationships between these newly introduced sets and the already existing neutrosophic sets. In this paper, we introduce the notions of $N_{eu}gr^*$-compact spaces, $N_{eu}gr^*$-Lindelof space, countably $N_{eu}gr^*$-compact spaces, $N_{eu}gr^*$-connected spaces, $N_{eu}gr^*$-separated sets, $N_{eu}$-Super-$gr^*$-connected spaces, $N_{eu}$-Extremely-$gr^*$-disconnected spaces, and $N_{eu}$-Strongly-$gr^*$-connected spaces, $N_{eu}gr^*$-Regular spaces, strongly $N_{eu}gr^*$-Regular spaces, $N_{eu}gr^*$-Normal spaces, and strongly $N_{eu}gr^*$-Normal spaces by using $N_{eu}gr^*$-open sets and $N_{eu}gr^*$-closed sets in Neutrosophic topological spaces. We study the basic properties and fundamental characteristics of these spaces in Neutrosophic topological spaces.

Keywords: $N_{eu}gr^*$-closed set, $N_{eu}gr^*$-open set, $N_{eu}gr^*$-compact space, $N_{eu}gr^*$-Lindelof space, Countably $N_{eu}gr^*$-compact space, $N_{eu}gr^*$-connected space, $N_{eu}gr^*$-separated set, $N_{eu}$-Super-$gr^*$-connected space, $N_{eu}$-Extremely-$gr^*$-disconnected space, $N_{eu}$-Strongly-$gr^*$-connected space, $N_{eu}gr^*$-Regular space, Strongly $N_{eu}gr^*$-Regular space, $N_{eu}gr^*$-Normal space, Strongly $N_{eu}gr^*$-Normal space

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## 1. Introduction

Many real-life problems in Business, Finance, Medical Sciences, Engineering, and Social Sciences deal with uncertainties. There are difficulties in solving the uncertainties in data by traditional mathematical models. There are approaches such as fuzzy sets, intuitionistic fuzzy sets, vague sets, and rough sets, Nano sets, micro sets which can be treated as mathematical tools to avert obstacles in dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminant and inconsistent data due to the inadequacy of parameterization tools. Molodtsov introduced the soft set theory. Smarandache studies neutrosophic sets as an approach to solving issues that cover unreliable, indeterminacy, and persistent data. Applications of neutrosophic topology depend upon the properties of neutrosophic closed sets, neutrosophic interior and closure operators, and neutrosophic open sets. In 2022, S. Pious Missier, A. Anusuya and J. Martina Jency introduced and studied the concepts of Neutrosophic generalized regular star closed $\left(N_{eu}gr^*\text{-closed}\right)$ sets and Neutrosic generalized regular star $\left(N_{eu}gr^*\text{-open}\right)$ sets in Neutrosophic topological spaces and studied some of their properties and characterizations as well as analyzed the relationships between these newly introduced sets and the already existing neutrosophic sets. In this paper, we introduce the notions of $N_{eu}gr^*$-compact spaces, $N_{eu}gr^*$-Lindelof space, countably $N_{eu}gr^*$-compact spaces, $N_{eu}gr^*$-connected spaces, $N_{eu}gr^*$-separated sets, $N_{eu}$-Super-$gr^*$-connected spaces, $N_{eu}$-Extremely-$gr^*$-disconnected spaces, and $N_{eu}$-Strongly-$gr^*$-connected spaces, $N_{eu}gr^*$-Regular spaces, strongly $N_{eu}gr^*$-Regular spaces, $N_{eu}gr^*$-Normal spaces, and strongly $N_{eu}gr^*$-Normal spaces by using $N_{eu}gr^*$-open sets and $N_{eu}gr^*$-closed sets in Neutrosophic topological spaces.

We study the basic properties and fundaments characteristics of these spaces in Neutrosophic topological spaces.

## 2. Preliminaries

**Definition 2.1.** Let $X$ be a non-empty fixed set. A neutrosophic set (briefly $N_{eu}$–set) $P$ is an object having a form $P=\{\langle x,\mu_P(x),\sigma_P(x),\gamma_P(x)\rangle : x\in X\}$, *where* $\mu_P(x)$- represents the degree of membership, $\sigma_P(x)$- represents the degree of indeterminacy, and $\gamma_P(x)$- represents the degree of non-membership.

**Definition 2.2.** A neutrosophic topology on a non-empty set $X$ is a family $T_N$ of neutrosophic subsets of $X$ satisfying $(i)$ $0_{Neu}, 1_{Neu}\in T_N$. $(ii)$ $G\bigcap H\in T_N$ for every $G, H\in T_N$, $(iii)$ $\bigcup_{j\in J} G_j\in T_N$ for every $\{G_j : j\in J\}\subseteq T_N$. Then the pair $(X,T_N)$ is called a neutrosophic topological space (briefly, $N_{eu}$–Top–Space). The elements of $T_N$ are called neutrosophic open (briefly $N_{eu}$–open) sets in $X$. A $N_{eu}$–set $A$ in $X$ is called a neutrosophic closed (briefly $N_{eu}$–closed) set if and only if its complement $A^C$ is a $N_{eu}$–open set.

**Definition 2.3.** Let $(X,T_N)$ be a $N_{eu}$–Top–Space and $A$ be a $N_{eu}$–set. Then

(i) The neutrosophic interior of $A$, denoted by $N_{eu}Int(A)$ is the union of all $N_{eu}$–open subsets of $A$.

(ii) The neutrosophic closure of $A$ denoted by $N_{eu}Cl(A)$ is the intersection of all $N_{eu}$–closed sets containing A.

**Definition 2.4.** Let $A$ be a $N_{eu}$–subset of a $N_{eu}$–Top–Space $(X,T_N)$. Then $A$ is said to be a (i) neutrosophic regular (briefly $N_{eu}$–Regular) open set if $A=N_{eu}Int[N_{eu}Cl(A)]$. (ii) neutrosophic regular (briefly $N_{eu}$–Regular) closed set if $A=N_{eu}Cl[N_{eu}Int(A)]$. Clearly $N_{eu}$–Regular open sets and $N_{eu}$–Regular closed sets in $(X,T_N)$ are complements of each other.

**Definition 2.5.** Let $(X,T_N)$ be a $N_{eu}$–Top–Space and $A$ be a $N_{eu}$–set of $X$. Then $A$ is said to be a neutrosophic generalized closed (briefly $N_{eu}g$–closed) set if $N_{eu}Cl(A)\subseteq G$ whenever $A\subseteq G$ and $G$ is $N_{eu}$–open set in $(X,T_N)$. The complement of a $N_{eu}g$–closed set is called a $N_{eu}g$–open set in $(X,T_N)$.

**Definition 2.6.** Let $(X,T_N)$ be a $N_{eu}$–Top–Space and $A$ be a $N_{eu}$–set of $X$. Then $A$ is said to be a neutrosophic generalized regular star closed (resp. $N_{eu}gr^*$–closed) set if $N_{eu}reg$–$Cl(A)\subseteq G$ whenever $A\subseteq G$ and $G$ is $N_{eu}g$–open set in $(X,T_N)$.

The complement of a $N_{eu}gr^*$–closed set is called a neutrosophic generalized regular star open (briefly $N_{eu}gr^*$–open) set in $(X,T_N)$.

The family of all $N_{eu}gr^*$–open (resp. $N_{eu}gr^*$–closed) in a $N_{eu}$–Top–Space $(X,T_N)$ is denoted by $N_{eu}gr^*$–$O(X,T_N)$ (resp. $N_{eu}gr^*$–$C(X,T_N)$).

**Definition 2.7.** Let $(X,T_N)$ be a $N_{eu}$–Top–Space and $A$ be a $N_{eu}$–set of $X$. Then $(i)$ the neutrosophic generalized regular star closure of $A$ is denoted and defined by: $N_{eu}gr^*$–$Cl(A)=\bigcap\{F\in N_{eu}gr^*$–$C(X,T_N)$ & $A\subseteq F\}$.

$(ii)$ the neutrosophic generalized regular star interior of $A$ is denoted and defined by: $N_{eu}gr^*$–$Int(A)=\bigcup\{G\in N_{eu}gr^*$–$O(X,T_N)$ & $G\subseteq A\}$.

**Theorem 2.8.** Every $N_{eu}$–closed (resp. $N_{eu}$–open) set in a $N_{eu}$–Top–Space $(X,T_N)$ is a $N_{eu}gr^*$–closed (resp. $N_{eu}gr^*$–open) set in $(X,T_N)$.

**Theorem 2.9.** In a $N_{eu}$–Top–Space $(X,T_N)$ we have the following conditions.

(i) $0_{N_{eu}}$ and $1_{N_{eu}}$ are $N_{eu}gr^*$–open sets in $(X,T_N)$.

(ii) The intersection of any two $N_{eu}gr^*$–closed sets is $N_{eu}gr^*$–closed set in $(X,T_N)$.

(iii) The union of any two $N_{eu}gr^*$–open sets is $N_{eu}gr^*$–open set in $(X,T_N)$.

**Theorem 2.10.** Let $(X,T_N)$ be a $N_{eu}$–Top–Space. Then for any $N_{eu}$–subsets $A$ and $B$ of $X$, we have

(i) $N_{eu}gr^*$–$Int(A)\subseteq A\subseteq N_{eu}gr^*$–$Cl(A)$

(ii) $A$ is $N_{eu}gr^*$–open set in $X$ if and only if $N_{eu}gr^*$–$Int(A)=A$.

(iii) $A$ is $N_{eu}gr^*$–closed set in $X$ if and only if $N_{eu}gr^*$–$Cl(A)=A$.

(iv) $N_{eu}gr^*$–$Int[N_{eu}gr^*$–$Int(A)]=N_{eu}gr^*$–$Int(A)$.

(v) $N_{eu}gr^*$–$Cl[N_{eu}gr^*$–$Cl(A)]=N_{eu}gr^*$–$Cl(A)$.

(vi) If $A\subseteq B$, then $N_{eu}gr^*$–$Int(A)\subseteq N_{eu}gr^*$–$Int(B)$

(vii) If $A \subseteq B$, then $N_{eu}gr^*-Cl(A) \subseteq N_{eu}gr^*-Cl(B)$

(viii) $(N_{eu}gr^*-Cl(A))^c = N_{eu}gr^*-Int(A^c)$

(iX) $(N_{eu}gr^*-Int(A))^c = N_{eu}gr^*-Cl(A^c)$

(X) $N_{eu}gr^*-Int(0_{N_{eu}}) = 0_{N_{eu}}$, $N_{eu}gr^*-Int(1_{N_{eu}}) = 1_{N_{eu}}$

(Xi) $N_{eu}gr^*-Cl(0_{N_{eu}}) = 0_{N_{eu}}$, $N_{eu}gr^*-Cl(1_{N_{eu}}) = 1_{N_{eu}}$

(Xii) $N_{eu}gr^*-Int(A \cap B) \subseteq N_{eu}gr^*-Int(A) \cap N_{eu}gr^*-Int(B)$

(Xiii) $N_{eu}gr^*-Cl(A) \cup N_{eu}gr^*-Cl(B) \subseteq N_{eu}gr^*-Cl(A \cup B)$

(Xiv) $N_{eu}gr^*-Int(A) \cup N_{eu}gr^*-Int(B) \subseteq N_{eu}gr^*-Int(A \cup B)$

(Xiii) $N_{eu}gr^*-Cl(A \cap B) \subseteq N_{eu}gr^*-Cl(A) \cap N_{eu}gr^*-Cl(B)$

**Definition 2.11.** A function $f:(X,T_N) \to (Y,\sigma_N)$ is called a $N_{eu}gr^*-$continuous function if $f^{-1}(B)$ is a $N_{eu}gr^*-$open (resp. $N_{eu}gr^*-$closed) set in $X$, for every $N_{eu}-$open (resp. $N_{eu}-$closed) set $B$ in $Y$.

**Definition 2.12.** A function $f:(X,T_N) \to (Y,\sigma_N)$ is called a $N_{eu}gr^*-$irresolute function if $f^{-1}(B)$ is a $N_{eu}gr^*-$open (resp. $N_{eu}gr^*-$closed) set in $X$, for every $N_{eu}gr^*-$open (resp. $N_{eu}gr^*-$closed) set $B$ in $Y$.

## 3 Neutrosophic Generalized Regular Star Compact Spaces

In this section, we introduce $N_{eu}gr^*-$compact space, $N_{eu}gr^*-$Lindelof space, and countably $N_{eu}gr^*-$compact space and investigate their basic properties and characterizations.

**Definition 3.1.** A collection $\{A_i : i \in I\}$ of $N_{eu}-$open (resp. $N_{eu}gr^*-$open) sets in a $N_{eu}-$Top$-$Space $(X,T_N)$ is called a $N_{eu}-$open (resp. $N_{eu}gr^*-$open) cover of a subset $B$ of $X$ if $B \subseteq \bigcup\{A_i : i \in I\}$ holds.

**Definition 3.2.** A subset $B$ of a $N_{eu}-$Top$-$Space $(X,T_N)$ is said to be $N_{eu}-$compact (resp. $N_{eu}gr^*-$compact) relative to $(X,T_N)$, if for every collection $\{A_i : i \in I\}$ of $N_{eu}-$open (resp. $N_{eu}gr^*-$open) subsets of $(X,T_N)$ such that $B \subseteq \bigcup\{A_i : i \in I\}$ there exists a finite subset $I_0$ of $I$ such that $B \subseteq \bigcup\{A_i : i \in I_0\}$.

**Definition 3.3.** A subset $B$ of a $N_{eu}-$Top$-$Space $(X,T_N)$ is called $N_{eu}-$compact (resp $N_{eu}gr^*-$compact) if $B$ is $N_{eu}-$compact (resp. $N_{eu}gr^*-$compact) as a subspace of $X$.

**Theorem 3.4.** A $N_{eu}gr^*-$closed subset of a $N_{eu}gr^*-$compact $(X,T_N)$ is $N_{eu}gr^*-$compact relative to $(X,T_N)$.

**Proof.** Let $A$ be a $N_{eu}gr^*-$closed subset of a $N_{eu}gr^*-$compact space $(X,T_N)$. Then $A^C$ is $N_{eu}gr^*-$open in $(X,T_N)$. Let $S = \{A_i : i \in I\}$ be a $N_{eu}gr^*-$open cover of $A$ by $N_{eu}gr^*-$open subsets of $(X,T_N)$. Then $S^* = S \cup \{A^C\}$ is a $N_{eu}gr^*-$open cover of $(X,T_N)$. That is $X = \left(\bigcup_{i\in I} A_i\right) \cup A^C$. By hypothesis $(X,T_N)$ is $N_{eu}gr^*-$compact and hence $S^*$ is reducible to a finite subcover of $(X,T_N)$ say $X = A_{i_1} \cup A_{i_2} .......\cup A_{i_n} \cup A^C$, $A_{i_k} \in S \subseteq S^*$. Then $A = A_{i_1} \cup A_{i_2} .......\cup A_{i_n}$. Thus a $N_{eu}gr^*-$open cover S of $A$ contains a finite subcover. Hence $A$ is $N_{eu}gr^*-$compact relative to $(X,T_N)$.

**Theorem 3.5.** A $N_{eu}-$Top$-$Space $(X,T_N)$ is $N_{eu}gr^*-$compact if and only if every family of $N_{eu}gr^*-$closed sets of $(X,T_N)$ having finite intersection property has a non-empty intersection.

**Proof.** Suppose $(X,T_N)$ is $N_{eu}gr^*-$compact. Let $\{A_i : i \in I\}$ be a family of $N_{eu}gr^*-$closed sets with finite intersection property. Suppose $\bigcap_{i\in I} A_i = \phi$. Then $X - \bigcap_{i\in I} A_i = X$. This implies $\bigcup_{i\in I}(X - A_i) = X$. Thus the cover $\{X - A_i : i \in I\}$ is a $N_{eu}gr^*-$open cover of $(X,T_N)$. Then the $N_{eu}gr^*-$open cover $\{X - A_i : i \in I\}$ has a finite subcover say $\{X - A_i : i \in I_0\}$ for some finite subset $I_0$ of $I$. This implies $X = \bigcup_{i\in I_0}(X - A_i)$, which implies $X - \bigcup_{i\in I_0}(X - A_i) = \phi$ which implies $\bigcap_{i\in I_0} A_i = \phi$. This contradicts the assumption. Hence $\bigcap_{i\in I} A_i \neq \phi$. Conversely, suppose $(X,T_N)$ is not $N_{eu}gr^*-$compact. Then there exists a $N_{eu}gr^*-$open cover of $(X,T_N)$ say $\{G_i : i \in I\}$

having no finite subcover. This implies for any finite subfamily $\{G_i : i = 1,2,...,n\}$ of $\{G_i : i \in I\}$, we have $\bigcup_{i=1}^{n} G_i \neq X$, which implies $X - \bigcup_{i=1}^{n} G_i \neq X - X$, which implies $\bigcap_{i=1}^{n}(X - G_i) \neq \phi$. Then the family $\{X - G_i : i \in I\}$ of $N_{eu}gr^*$-closed sets has a finite intersection property. Also, by assumption $\bigcap_{i\in I}(X - G_i) \neq \phi$ which implies $X - \bigcup_{i\in I} G_i \neq \phi$ so that $\bigcup_{i\in I} G_i \neq X$. This implies $\{G_i : i \in I\}$ is not a cover for $(X, T_N)$. This contradicts the fact $\{G_i : i \in I\}$ is a cover for $(X, T_N)$. Therefore a $N_{eu}gr^*$-open cover $\{G_i : i \in I\}$ of $(X, T_N)$ has a finite subcover $\{G_i : i = 1,2,...,n\}$. Hence $(X, T_N)$ is a $N_{eu}gr^*$-compact.

**Theorem 3.6.** The image of a $N_{eu}gr^*$-compact space under a $N_{eu}gr^*$-irresolute mapping is $N_{eu}gr^*$-compact.

**Proof.** Let $f:(X, T_N) \to (Y, \sigma_N)$ be a $N_{eu}gr^*$-irresolute mapping from a $N_{eu}gr^*$-compact space $(X, T_N)$ onto a $N_{eu}gr^*$-Top-Space $(Y, \sigma_N)$. Let $\{A_i : i \in I\}$ be a $N_{eu}gr^*$-open cover of $(Y, \sigma_N)$. Then $\{f^{-1}(A_i) : i \in I\}$ is a $N_{eu}gr^*$-open cover of $(X, T_N)$, since $f$ is $N_{eu}gr^*$-irresolute. As $(X, T_N)$ is $N_{eu}gr^*$-compact, the $N_{eu}gr^*$-open cover $\{f^{-1}(A_i) : i \in I\}$ of $(X, T_N)$ has a finite subcover $\{f^{-1}(A_i) : i = 1,2,...,n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$. Then $f(X) = \bigcup_{i=1}^{n} A_i$, that is $Y = \bigcup_{i=1}^{n} A_i$. Then $\{A_1, A_2, ..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for $(Y, \sigma_N)$. Hence $Y$ is a $N_{eu}gr^*$-compact space.

**Definition 3.7.** A $N_{eu}$-Top-Space $(X, T_N)$ is countably $N_{eu}gr^*$-compact if every countable $N_{eu}gr^*$-open cover of $(X, T_N)$ has a finite subcover.

**Definition 3.8.** A $N_{eu}$-Top-Space $(X, T_N)$ is said to be $N_{eu}gr^*$-Hausdorff if whenever $x_{(\alpha,\beta,\gamma)}$ and $y_{(r,s,t)}$ are distinct points of $(X, T_N)$, there exist disjoint $N_{eu}gr^*$-open sets $A$ and $B$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in A$ and $y_{(r,s,t)} \in B$.

**Theorem 3.9.** Let $(X, T_N)$ be a $N_{eu}$-Top-Space and $(Y, \sigma_N)$ be a $N_{eu}gr^*$-Hausdorff space. If $f:(X, T_N) \to (Y, \sigma_N)$ is $N_{eu}gr^*$-irresolute injective mapping, then $(X, T_N)$ is $N_{eu}gr^*$-Hausdorff.

**Proof.** Let $x_{(\alpha,\beta,\gamma)}$ and $y_{(r,s,t)}$ be any two distinct $N_{eu}$-points of $(X, T_N)$. Then $f(x_{(\alpha,\beta,\gamma)})$ and $f(y_{(r,s,t)})$ are distinct $N_{eu}$-points of $(Y, \sigma_N)$, because $f$ is injective. Since $(Y, \sigma_N)$ is $N_{eu}gr^*$-Hausdorff, there are disjoint $N_{eu}gr^*$-open sets $G$ and $H$ in $(Y, \sigma_N)$ containing $f(x_{(\alpha,\beta,\gamma)})$ and $f(y_{(r,s,t)})$ respectively. Since $f$ is $N_{eu}gr^*$-irresolute and $G \cap H = \phi$, we have $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $N_{eu}gr^*$-open sets in $(X, T_N)$ such that $x_{(\alpha,\beta,\gamma)} \in f^{-1}(G)$ and $y_{(r,s,t)} \in f^{-1}(H)$. Hence $(X, T_N)$ is $N_{eu}gr^*$-Hausdorff.

**Theorem 3.10.** If $f:(X, T_N) \to (Y, \sigma_N)$ is $N_{eu}gr^*$-irresolute and bijective and if $X$ is $N_{eu}gr^*$-compact and $Y$ is $N_{eu}gr^*$-Hausdorff, then $f$ is a $N_{eu}gr^*$-homeomorphism.

**Proof.** We have to show that the inverse function $g$ of $f$ is $N_{eu}gr^*$-irresolute. For this we show that if $A$ is $N_{eu}gr^*$-open in $(X, T_N)$ then the pre-image $g^{-1}(A)$ is $N_{eu}gr^*$-open in $(Y, \sigma_N)$. Since the $N_{eu}gr^*$-open (or $N_{eu}gr^*$-closed) sets are just the complements of $N_{eu}gr^*$-closed (resp. $N_{eu}gr^*$-open) subsets, and $g^{-1}(X - A) = Y - g^{-1}(A)$. We see that the $N_{eu}gr^*$-irresolute mapping of $g$ is equivalent to: if $B$ is $N_{eu}gr^*$-closed in $(X, T_N)$ then the pre-image $g^{-1}(B)$ is $N_{eu}gr^*$-closed in $Y$. To prove this, let $B$ be a $N_{eu}gr^*$-closed subset of $X$. Since $g$ is the inverse of $f$, we have $g^{-1}(B) = f(B)$, hence we have to show that $f(B)$ is a $N_{eu}gr^*$-closed set in $Y$. By theorem 3.4, $B$ is $N_{eu}gr^*$-compact. By Theorem 3.6, implies that $f(B)$ is $N_{eu}gr^*$-copmpact. Since $Y$ is $N_{eu}gr^*$-Hausdorff space implies that $f(B)$ is $N_{eu}gr^*$-closed in $(Y, \sigma_N)$.

**Definition 3.11.** A $N_{eu}$-Top-Space $(X, T_N)$ is said to be $N_{eu}gr^*$-Lindelof space if every $N_{eu}gr^*$-open cover of $(X, T_N)$ has a countable subcover.

**Theorem 3.12.** Every $N_{eu}gr^*$-compact space is a $N_{eu}gr^*$-Lindelof space.

**Proof.** Let $(X, T_N)$ be $N_{eu}gr^*$-compact. Let $\{A_i : i \in I\}$ be a $N_{eu}gr^*$-open cover of $(X, T_N)$. Then $\{A_i : i \in I\}$ has a finite subcover $\{A_i : i = 1, 2, ..., n\}$, since $(X, T_N)$ is $N_{eu}gr^*$-copmpact. Since every finite subcover is always a countable subcover and therefore, $\{A_i : i = 1.2, ..., n\}$ is countable subcover of $\{A_i : i \in I\}$ for $(X, T_N)$. Hence $(X, T_N)$ is $N_{eu}gr^*$-Lindelof space.

**Theorem 3.13.** The image of a $N_{eu}gr^*$-Lindelof space under a $N_{eu}gr^*$-irresolute mapping is $N_{eu}gr^*$-Lindelof.

**Proof.** Let $f : (X, T_N) \to (Y, \sigma_N)$ be a $N_{eu}gr^*$-irresolute mapping from a $N_{eu}gr^*$-Lindelof space $(X, T_N)$ onto a $N_{eu}$-Top-Space $(Y, \sigma_N)$. Let $\{A_i : i \in I\}$ be a $N_{eu}gr^*$-open cover of $(Y, \sigma_N)$. Then $\{f^{-1}(A_i) : i \in I\}$ is a $N_{eu}gr^*$-open cover of $(X, T_N)$, since $f$ is $N_{eu}gr^*$-irresolute. As $(X, T_N)$ is $N_{eu}gr^*$-Lindelof, the $N_{eu}gr^*$-open cover $\{f^{-1}(A_i) : i \in I\}$ of $(X, T_N)$ has a countable subcover $\{f^{-1}(A_i) : i \in I_0\}$ for some countable subset $I_0$ of $I$. Therefore $X = \bigcup_{i \in I_0} f^{-1}(A_i)$ which implies $f(X) = Y = \bigcup_{i \in I_0} A_i$, that is $\{A_i : i \in I_0\}$ a countable subcover of $\{A_i : i \in I\}$ for $(Y, \sigma_N)$. Hence $(Y, \sigma_N)$ is $N_{eu}gr^*$-Lindelof space.

**Theorem 3.14.** Let $(X, T_N)$ be $N_{eu}gr^*$-Lindelof and countably $N_{eu}gr^*$-copmpact space. Then $(X, T_N)$ is $N_{eu}gr^*$-copmpact.

**Proof.** Let $\{A_i : i \in I\}$ be a $N_{eu}gr^*$-open cover of $(X, T_N)$. Since $(X, T_N)$ is $N_{eu}gr^*$-Lindelof space. Hence $\{A_i : i \in I\}$ has a countable subcover $\{A_{i_n} : n \in N\}$. Therefore, $\{A_{i_n} : n \in N\}$ is a countable subcover of $(X, T_N)$ and $\{A_{i_n} : n \in N\}$ is a subfamily of $\{A_i : i \in I\}$ and so $\{A_{i_n} : n \in N\}$ is a countable $N_{eu}gr^*$-open cover of $(X, T_N)$. Again since $(X, T_N)$ is countably $N_{eu}gr^*$-copmpact, $\{A_{i_n} : n \in N\}$ has a finite subcover $\{A_{i_k} : k = 1, 2, ..., n\}$. Therefore $\{A_{i_k} : k = 1, 2, ..., n\}$ is a finite subcover of $\{A_i : i \in I\}$ for $(X, T_N)$. Hence $(X, T_N)$ is $N_{eu}gr^*$-copmpact space.

**Theorem 3.15.** A $N_{eu}$-Top-Space $(X, T_N)$ is $N_{eu}gr^*$-copmpact if and only if every basic $N_{eu}gr^*$-open cover of $(X, T_N)$ has a finite subcover.

**Proof.** Let $(X, T_N)$ be $N_{eu}gr^*$-copmpact. Then every $N_{eu}gr^*$-open cover of $(X, T_N)$ has a finite subcover. Conversely, suppose that every basic $N_{eu}gr^*$-open cover of $(X, T_N)$ has a finite subcover and let $C = \{G_\lambda : \lambda \in \Lambda\}$ be any $N_{eu}gr^*$-open cover of $(X, T_N)$. If $B = \{D_\alpha : \alpha \in \Delta\}$ is any $N_{eu}gr^*$-open base for $(X, T_N)$, then each $G_\lambda$ is union of some members of $B$ and the totality of all such members of $B$ evidently a basic $N_{eu}gr^*$-open cover of $(X, T_N)$. By hypothesis this collection of members of $B$ has a finite subcover, $\{D_{\alpha_i} : i = 1, 2, ..., n\}$. For each $D_{\alpha_i}$ in this finite subcover, we can select a $G_{\lambda_i}$ from $C$ such that $D_{\alpha_i} \subseteq G_{\lambda_i}$. It follows that the finite subcollection $\{G_{\lambda_i} : i = 1, 2, ..., n\}$, which arises in this way is a subcover of $C$. Hence $(X, T_N)$ is $N_{eu}gr^*$-copmpact.

## 4. Neutrosophic Generalized Regular Star Connected Spaces

In this section, we introduce and study the notions of $N_{eu}gr^*$-connected spaces, $N_{eu}gr^*$-separated sets, $N_{eu}$-Super-$gr^*$-connected spaces, $N_{eu}$-Extremely-$gr^*$-disconnected spaces, and $N_{eu}$-Strongly-$gr^*$-connected spaces in $N_{eu}$-Top-Spaces.

**Definition 4.1.** A $N_{eu}$-Top-Space $(X, T_N)$ is $N_{eu}gr^*$-disconnected if there exist $N_{eu}gr^*$-open sets $A$, $B$ in $X$, $A \neq 0_{Neu}$, $B \neq 0_{Neu}$ such that $A \bigcup B = 1_{Neu}$ and $A \bigcap B = 0_{Neu}$. If $(X, T_N)$ is not $N_{eu}gr^*$-disconnected then it is said to be $N_{eu}gr^*$-connected.

**Theorem 4.2.** A $N_{eu}$-Top-Space $(X, T_N)$ is $N_{eu}gr^*$-connected space if and only if there exists no nonempty $N_{eu}gr^*$-open sets $U$ and $V$ in $(X, T_N)$ such that $U = V^C$.

**Proof. Necessity:** Let $U$ and $V$ be two $N_{eu}gr^*$-open sets in $(X, T_N)$ such that $U \neq 0_{Neu}$, $V \neq 0_{Neu}$ and $U = V^C$. Therefore $V^C$ is a $N_{eu}gr^*$-closed set. Since $U \neq 0_{Neu}$, $V \neq 1_{Neu}$. This implies $V$ is a proper $N_{eu}$-subset which is both $N_{eu}gr^*$-open set and $N_{eu}gr^*$-closed set in $X$. Hence $X$ is not a

$N_{eu}gr^*-$connected space. But this is a contradiction to our hypothesis. Thus, there exist no nonempty $N_{eu}gr^*-$open sets $U$ and $V$ in $X$, such that $U = V^C$.

**Sufficiency:** Let $U$ be both $N_{eu}gr^*-$open and $N_{eu}gr^*-$closed set of $X$ such that $U \neq 0_{Neu}$, $U \neq 1_{Neu}$. Now let $V = U^C$. Then $V$ is a $N_{eu}gr^*-$open set and $V \neq 1_{Neu}$. This implies $U^C = V \neq 0_{Neu}$, which is a contradiction to our hypothesis. Therefore $X$ is $N_{eu}gr^*-$connected space.

**Theorem 4.3.** A $N_{eu}-$Top$-$Space $(X, T_N)$ is $N_{eu}gr^*-$connected space if and only if there do not exist nonempty $N_{eu}-$subsets $U$ and $V$ in $X$ such that $U = V^C$, $V = [N_{eu}gr^*-Cl(U)]^C$ and $U = [N_{eu}gr^*-Cl(V)]^C$.

**Proof. Necessity:** Let $U$ and $V$ be two $N_{eu}-$subsets of $(X, T_N)$ such that $U \neq 0_{Neu}$, $V \neq 0_{Neu}$ and $U = V^C$, $V = [N_{eu}gr^*-Cl(U)]^C$ and $U = [N_{eu}gr^*-Cl(V)]^C$. Since $[N_{eu}gr^*-Cl(U)]^C$ and $[N_{eu}gr^*-Cl(V)]^C$ are $N_{eu}gr^*-$open sets in $X$, so $U$ and $V$ are $N_{eu}gr^*-$open sets in $X$. This implies $X$ is not a $N_{eu}gr^*-$connected space, which is a contradiction. Therefore, there exist no nonempty $N_{eu}gr^*-$open sets $U$ and $V$ in $X$, such that $U = V^C$, $V = [N_{eu}gr^*-Cl(U)]^C$ and $U = [N_{eu}gr^*-Cl(V)]^C$.

**Sufficiency:** Let $U$ be both $N_{eu}gr^*-$open and $N_{eu}gr^*-$closed set in $X$ such that $U \neq 0_{Neu}$, $U \neq 1_{Neu}$. Now by taking $V = U^C$ we obtain a contradiction to our hypothesis. Hence $X$ is $N_{eu}gr^*-$connected space.

**Theorem 4.4.** Let $f:(X, T_N) \to (Y, \sigma_N)$ be a $N_{eu}gr^*-$irresolure surjection and $X$ be $N_{eu}gr^*-$connected. Then $Y$ is $N_{eu}gr^*-$connected.

**Proof.** Assume that $Y$ is not $N_{eu}gr^*-$connected, then there exist nonempty $N_{eu}gr^*-$open sets $U$ and $V$ in $Y$ such that $U \bigcup V = 1_{Neu}$ and $U \bigcap V = 0_{Neu}$. Since $f$ is $N_{eu}gr^*-$irresolure mapping, $A = f^{-1}(U) \neq 0_N$, $B = f^{-1}(V) \neq 0_{Neu}$, which are $N_{eu}gr^*-$open sets in $X$ and $f^{-1}(U) \bigcup f^{-1}(V) = f^{-1}(1_{Neu}) = 1_{Neu}$, which implies $A \bigcup B = 1_{Neu}$. Also $f^{-1}(U) \bigcap f^{-1}(V) = f^{-1}(0_{Neu}) = 0_{Neu}$, which implies $A \bigcap B = 0_{Neu}$. Thus, $X$ is $N_{eu}gr^*-$disconnected, which is a contradiction to our hypothesis. Hence $Y$ is $N_{eu}gr^*-$connected.

**Definition 4.5.** Let $A$ and $B$ be nonempty $N_{eu}-$subsets in a $N_{eu}-$Top$-$Space $(X, T_N)$. Then $A$ and $B$ are said to be $N_{eu}gr^*-$separated if $[N_{eu}gr^*-Cl(A)] \bigcap B = A \bigcap [N_{eu}gr^*-Cl(B)] = 0_{Neu}$.

**Remark 4.6.** Any two disjoint non-empty $N_{eu}gr^*-$closed sets are $N_{eu}gr^*-$separated.

**Proof.** Suppose $A$ and $B$ are disjoint non-empty $N_{eu}gr^*-$closed sets. Then $[N_{eu}gr^*-Cl(A)] \bigcap B = A \bigcap [N_{eu}gr^*-Cl(B)] = A \bigcap B = 0_{Neu}$. This shows that $A$ and $B$ are $N_{eu}gr^*-$separated.

**Theorem 4.7.** (i) Let $A$ and $B$ be two $N_{eu}gr^*-$separated subsets of a $N_{eu}-$Top$-$Space $(X, T_N)$ and $C \subseteq A$, $D \subseteq B$. Then $C$ and $D$ are also $N_{eu}gr^*-$separated.

(ii) Let $A$ and $B$ be both $N_{eu}gr^*-$separated subsets of a $N_{eu}-$Top$-$Space $(X, T_N)$ and let $H = A \bigcap B^C$ and $G = B \bigcap A^C$. Then $H$ and $G$ are also $N_{eu}gr^*-$separated.

**Proof.** (i) Let $A$ and $B$ be two $N_{eu}gr^*-$separated sets in $N_{eu}-$Top$-$Space $(X, T_N)$. Then $[N_{eu}gr^*-Cl(A)] \bigcap B = 0_{Neu} = A \bigcap [N_{eu}gr^*-Cl(B)]$. Since $C \subseteq A$ and $D \subseteq B$, then $N_{eu}gr^*-Cl(C) \subseteq N_{eu}gr^*-Cl(A)$ and $N_{eu}gr^*-Cl(D) \subseteq N_{eu}gr^*-Cl(B)$. This implies that, $[N_{eu}gr^*-Cl(C)] \bigcap D \subseteq [N_{eu}gr^*-Cl(A)] \bigcap B = 0_{Neu}$ and hence $[N_{eu}gr^*-Cl(C)] \bigcap D = 0_{Neu}$. Similarly $[N_{eu}gr^*-Cl(D)] \bigcap C \subseteq [N_{eu}gr^*-Cl(B)] \bigcap A = 0_{Neu}$ and hence $[N_{eu}gr^*-Cl(D)] \bigcap C = 0_{Neu}$. Therefore $C$ and $D$ are $N_{eu}gr^*-$separated.

(ii) Let $A$ and $B$ be both $N_{eu}gr^*-$open subsets of $X$. Then $A^C$ and $B^C$ are $N_{eu}gr^*-$closed sets. Since $H \subseteq B^C$, then $N_{eu}gr^*-Cl(H) \subseteq N_{eu}gr^*-Cl(B^C) = B^C$ and so $N_{eu}gr^*-Cl(H) \bigcap B = 0_{Neu}$. Since $G \subseteq B$, then $[N_{eu}gr^*-Cl(H)] \bigcap G \subseteq [N_{eu}gr^*-Cl(H)] \bigcap B = 0_{Neu}$. Thus, we have $[N_{eu}gr^*-Cl(H)] \bigcap G = 0_{Neu}$. Similarly, $[N_{eu}gr^*-Cl(G)] \bigcap H = 0_{Neu}$. Hence $H$ and $G$ are $N_{eu}gr^*-$separated.

**Theorem 4.8.** Two $N_{eu}$-subsets $A$ and $B$ of a $N_{eu}$-Top-Space $(X, T_N)$ are $N_{eu}gr^*$-separated if and only if there exist $N_{eu}gr^*$-open sets $U$ and $V$ in $X$ such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = 0_{Neu}$ and $B \cap U = 0_{Neu}$.

**Proof.** Let $A$ and $B$ be $N_{eu}gr^*$-separated. Then $A \cap [N_{eu}gr^*\text{-}Cl(B)] = 0_{Neu} = B \cap [N_{eu}gr^*\text{-}Cl(A)]$. Let $V = (N_{eu}gr^*\text{-}Cl(A))^C$ and $U = (N_{eu}gr^*\text{-}Cl(B))^C$. Then $U$ and $V$ are $N_{eu}gr^*$-open sets in $X$ such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = 0_{Neu}$ and $B \cap U = 0_{Neu}$.
Conversely, let $U$ and $V$ be $N_{eu}gr^*$-open sets such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = 0_{Neu}$, $B \cap U = 0_{Neu}$. Then $A \subseteq V^C$ and $B \subseteq U^C$ and $V^C$ and $U^C$ are $N_{eu}gr^*$-closed. This implies $N_{eu}gr^*\text{-}Cl(A) \subseteq N_{eu}gr^*\text{-}Cl(V^C) = V^C \subseteq B^C$ and $N_{eu}gr^*\text{-}Cl(B) \subseteq N_{eu}gr^*\text{-}Cl(U^C) = U^C \subseteq A^C$. That is, $N_{eu}gr^*\text{-}Cl(A) \subseteq B^C$ and $N_{eu}gr^*\text{-}Cl(B) \subseteq A^C$. So $A \cap [N_{eu}gr^*\text{-}Cl(B)] = 0_{Neu} = [N_{eu}gr^*\text{-}Cl(A)] \cap B$. Hence $A$ and $B$ are $N_{eu}gr^*$-separated.

**Theorem 4.9.** Every two $N_{eu}gr^*$-separated sets are always disjoint.

**Proof.** Let $A$ and $B$ be $N_{eu}gr^*$-separated. Then $A \cap [N_{eu}gr^*\text{-}Cl(B)] = 0_{Neu} = [N_{eu}gr^*\text{-}Cl(A)] \cap B$. Now, $A \cap B \subseteq A \cap [N_{eu}gr^*\text{-}Cl(B)] = 0_{Neu}$. Therefore $A \cap B = 0_N$ and hence $A$ and $B$ are disjoint.

**Theorem 4.10.** A $N_{eu}$-Top-Space $(X, T_N)$ is $N_{eu}gr^*$-connected if and only if $A \cup B \neq 1_{Neu}$, where $A$ and $B$ are $N_{eu}gr^*$-separated sets.

**Proof.** Assume that $(X, T_N)$ is $N_{eu}gr^*$-connected space. Suppose $A \cup B = 1_{Neu}$, where $A$ and $B$ are $N_{eu}gr^*$-separated sets. Then $[N_{eu}gr^*\text{-}Cl(A)] \cap B = A \cap [N_{eu}gr^*\text{-}Cl(B)] = 0_{Neu}$. Since $A \subseteq N_{eu}gr^*\text{-}Cl(A)$, we have $A \cap B \subseteq [N_{eu}gr^*\text{-}Cl(A)] \cap B = 0_{Neu}$. Therefore $N_{eu}gr^*\text{-}Cl(A) \subseteq B^C = A$ and $N_{eu}gr^*\text{-}Cl(B) \subseteq A^C = B$. Hence $A = N_{eu}gr^*\text{-}Cl(A)$ and $B = N_{eu}gr^*\text{-}Cl(B)$. Therefore $A$ and $B$ are $N_{eu}gr^*$-closed sets and hence $A = B^C$ and $B = A^C$ are disjoint $N_{eu}gr^*$-open sets. Thus $A \neq 0_{Neu}$, $B \neq 0_{Neu}$ such that $A \cup B = 1_{Neu}$ and $A \cap B = 0_{Neu}$, $A$ and $B$ are $N_{eu}gr^*$-open sets. That is $X$ is not $N_{eu}gr^*$-connected, which is a contradiction to $X$ is a $N_{eu}gr^*$-connected space. Hence $1_{Neu}$ is not the union of any two $N_{eu}gr^*$-separated sets.
Conversely, assume that $1_{Neu}$ is not the union of any two $N_{eu}gr^*$-separated sets. Suppose $X$ is not $N_{eu}gr^*$-connected. Then $A \cup B = 1_{Neu}$, where $A \neq 0_{Neu}$, $B \neq 0_{Neu}$ such that $A \cap B = 1_{Neu}$, $A$ and $B$ are $N_{eu}gr^*$-open sets in $X$. Since $A \subseteq B^C$ and $B \subseteq A^C$, $[N_{eu}gr^*\text{-}Cl(A)] \cap B \subseteq B^C \cap B = 0_{Neu}$ and $A \cap [N_{eu}gr^*\text{-}Cl(B)] \subseteq A \cap A^C = 0_{Neu}$. That is $A$ and $B$ are $N_{eu}gr^*$-separated sets. This is a contradiction. Therefore $X$ is $N_{eu}gr^*$-connected.

**Definition 4.11.** A $N_{eu}$-Top-Space $(X, T_N)$ is $N_{eu}$-Super-$gr^*$-disconnected if there exists a $N_{eu}$-Regular-$gr^*$-open set $A$ in $X$ such that $A \neq 0_N$ and $A \neq 1_{Neu}$. A $N_{eu}$-Top-Spaces $(X, T_N)$ is called $N_{eu}$-Super-$gr^*$-connected if $X$ is not $N_{eu}$-Super-$gr^*$-disconnected.

**Theorem 4.12.** Let $(X, T_N)$ be a $N_{eu}$-Top-Space. Then following assertions are equivalent:

(i) $X$ is $N_{eu}$-Super-$gr^*$-connected.

(ii) For each $N_{eu}gr^*$-open set $U \neq 0_{Neu}$ in $X$, we have $N_{eu}gr^*\text{-}Cl(U) = 1_{Neu}$.

(iii) For each $N_{eu}gr^*$-closed set $U \neq 1_{Neu}$ in $X$, we have $N_{eu}gr^*\text{-}Int(U) = 0_{Neu}$.

(iv) There do not exist $N_{eu}gr^*$-open subsets $U$ and $V$ in $(X, T_N)$, such that $U \neq 0_{Neu}$, $V \neq 0_{Neu}$ and $U \subseteq V^C$.

(v) There do not exist $N_{eu}gr^*$-open subsets $U$ and $V$ in $(X, T_N)$, such that $U \neq 0_{Neu}$, $V \neq 0_{Neu}$, $V = (N_{eu}gr^*\text{-}Cl(U))^C$ and $U = (N_{eu}gr^*\text{-}Cl(V))^C$.

(vi) There do not exist $N_{eu}gr^*$-closed subsets $U$ and $V$ in $(X, T_N)$, such that $U \neq 1_{Neu}$, $V \neq 1_{Neu}$, $V = (N_{eu}gr^*\text{-}Int(U))^C$ and $U = (N_{eu}gr^*\text{-}Int(V))^C$.

**Proof.** (i)⇒(ii): Assume that there exists a $N_{eu}gr^*$-open set $A \neq 0_{Neu}$ such that $N_{eu}gr^*\text{-}Cl(A) \neq 1_{Neu}$. Now take $B = N_{eu}gr^*\text{-}Int[N_{eu}gr^*\text{-}Cl(A)]$. Then $B$ is a proper $N_{eu}$-Regular-$gr^*$-open set in $X$ which contradicts that $X$ is $N_{eu}$-Super-gr*-connected. Therefore $N_{eu}gr^*\text{-}Cl(A)=1_{Neu}$.

$(ii)\Rightarrow(iii)$: Let $A\neq 1_{Neu}$ be a $N_{eu}gr^*$-closed set in $X$. Then $A^C$ is $N_{eu}gr^*$-open set in $X$ and $A^C\neq 0_{Neu}$. Hence by hypothesis, $N_{eu}gr^*\text{-}Cl(A^C)=1_{Neu}$, and so $N_{eu}gr^*\text{-}Cl(A^C)=(N_{eu}gr^*\text{-}Int(A))^C=1_{Neu}$. This implies that $N_{eu}gr^*\text{-}Int(A)=0_{Neu}$.

$(iii)\Rightarrow(iv)$: Let $A$ and $B$ be $N_{eu}gr^*$-open sets in $X$ such that $A\neq 0_{Neu}\neq B$ and $A\subseteq B^C$. Since $B^C$ is $N_{eu}gr^*$-closed set in $X$ and $B\neq 0_{Neu}$ implies $B^C\neq 1_{Neu}$, we obtain $N_{eu}gr^*\text{-}Int(B^C)=0_N$. But, from $A\subseteq B^C$, $0_{Neu}\neq A=N_{eu}gr^*\text{-}Int(A)\subseteq N_{eu}gr^*\text{-}Int(B^C)=0_{Neu}$, which is a contradiction.

$(iv)\Rightarrow(i)$: Let $0_{Neu}\neq A\neq 1_{Neu}$ be $N_{eu}$-Regular-gr*-open set in $X$. Let $B=(N_{eu}gr^*\text{-}Cl(A))^C$. Since $N_{eu}gr^*\text{-}Int[N_{eu}gr^*\text{-}Cl(B)]=N_{eu}gr^*\text{-}Int\left[N_{eu}gr^*\text{-}Cl\left(N_{eu}gr^*\text{-}Cl(A)\right)^C\right]$ $=N_{eu}gr^*\text{-}Int\left[N_{eu}gr^*\text{-}Int\left(N_{eu}gr^*\text{-}Cl(A)\right)\right]^C=$ $N_{eu}gr^*\text{-}Int(A^C)=[N_{eu}gr^*\text{-}Cl(A)]^C=B$. Also we get $B\neq 0_{Neu}$, since otherwise, we have $B=0_{Neu}$ and this implies $(N_{eu}gr^*\text{-}Cl(A))^C=0_{Neu}$. That implies $N_{eu}gr^*\text{-}Cl(A)=1_{Neu}$. That shows that $A=N_{eu}gr^*\text{-}Int[N_{eu}gr^*\text{-}Cl(A)]=$ $N_{eu}gr^*\text{-}Int(1_{N_{eu}})=1_{N_{eu}}$. That is $A=1_{Neu}$, which is a contradiction. Therefore $B\neq 0_N$ and $A\subseteq B^C$. But this is a contradiction to $(iv)$. Therefore $(X,T_N)$ is $N_{eu}$-Super-gr*-connected space.

$(i)\Rightarrow(v)$: Let $A$ and $B$ be $N_{eu}gr^*$-open sets in $(X,T_N)$ such that $A\neq 0_{Neu}\neq B$, $B=[N_{eu}gsCl(A)]^C$, $A=[N_{eu}gr^*\text{-}Cl(B)]^C$. Now $N_{eu}gr^*\text{-}Int[N_{eu}gr^*\text{-}Cl(A)]=N_{eu}gr^*\text{-}Int(B^C)=$ $[N_{eu}gr^*\text{-}Cl(B)]^C=A$, $A\neq 0_{Neu}$ and $A\neq 1_{Neu}$, since if $A=1_{Neu}$, then $1_N=[N_{eu}gr^*\text{-}Cl(B)]^C$. This implies $N_{eu}gr^*\text{-}Cl(B)=0_{Neu}$. But $B\neq 0_{Neu}$. Therefore $A\neq 1_{Neu}$ implies that $A$ is proper $N_{eu}$-Regular-gr*-open set in $(X,T_N)$, which is a contradiction to $(i)$. Hence $(v)$ is true.

$(v)\Rightarrow(i)$: Let $A$ be $N_{eu}$-Regular-gr*-open set in $(X,T_N)$ such that $A=N_{eu}gr^*\text{-}Int[N_{eu}gr^*\text{-}Cl(A)]$ and $0_{Neu}\neq A\neq 1_{Neu}$. Now take $B=[N_{eu}gr^*\text{-}Cl(A)]^C$. In this case we get $B\neq 0_{Neu}$ and $B$ is $N_{eu}$-Regular-gr*-open set in $(X,T_N)$. $B=[N_{eu}gr^*\text{-}Cl(A)]^C$ and $[N_{eu}gr^*\text{-}Cl(B)]^C=$ $\left[N_{eu}gr^*\text{-}Cl\left(N_{eu}gr^*\text{-}Cl(A)\right)^C\right]^C=$ $N_{eu}gr^*\text{-}Int\left[\left(N_{eu}gr^*\text{-}Cl(A)\right)^C\right]^C=$ $N_{eu}gr^*\text{-}Int\left[\left(N_{eu}gr^*\text{-}Cl(A)\right)\right]=A$. But this is a contradiction. Therefore $(X,T_N)$ is $N_{eu}$-Super-gr*-connected space.

$(v)\Rightarrow(vi)$: Let $A$ and $B$ be two $N_{eu}$-Regular-gr*-closed sets in $(X,T_N)$ such that $A\neq 1_{Neu}\neq B$, $B=[N_{eu}gr^*\text{-}Int(A)]^C$, $A=[N_{eu}gr^*\text{-}Int(B)]^C$. Take $C=A^C$ and $D=B^C$, $C$ and $D$ become $N_{eu}$-Regular-gr*-open sets in $(X,T_N)$ with $C\neq 0_{Neu}\neq D$, $D=[N_{eu}gr^*\text{-}Int(C)]^C$, $C=[N_{eu}gr^*\text{-}Int(D)]^C$, which is a contradiction to $(v)$. Hence $(vi)$ is true.

$(vi)\Rightarrow(v)$: It can be easily proved by the similar way as in $(v)\Rightarrow(vi)$.

**Definition 4.13.** A $N_{eu}$-Top-Space $(X,T_N)$ is said to be $N_{eu}$-Extremely-gr*-disconnected if the $N_{eu}gr^*$-closure of every $N_{eu}gr^*$-open set in $(X,T_N)$ is $N_{eu}gr^*$-open set in $X$.

**Theorem 4.14.** Let $(X,T_N)$ be a $N_{eu}$-Top-Space. Then the following statements are equivalent.

(i) $X$ is $N_{eu}$-Extremely-gr*-disconnected space.

(ii) For each $N_{eu}gr^*$-closed set $A$, $N_{eu}gr^*\text{-}Int(A)$ is $N_{eu}gr^*$-closed set.

(iii) For each $N_{eu}gr^*$-open set $A$, $N_{eu}gr^*\text{-}Cl(A)=\left[N_{eu}gr^*\text{-}Cl\left(N_{eu}gr^*\text{-}Cl(A)\right)^C\right]^C$.

(iv) For each $N_{eu}gr^*$-open sets $A$ and $B$ with $N_{eu}gr^*-Cl(A)=B^C$,
$N_{eu}gr^*-Cl(A)=\left[N_{eu}gr^*-Cl(B)\right]^C$.

**Proof.** $(i)\Rightarrow(ii)$: Let $A$ be any $N_{eu}gr^*$-closed set in $(X,T_N)$. Then $A^C$ is $N_{eu}gr^*$-open set. So $(i)$ implies that $N_{eu}gr^*-Cl(A^C)=\left[N_{eu}gr^*-Int(A)\right]^C$ is $N_{eu}gr^*$-open set. Thus $N_{eu}gr^*-Int(A)$ is $N_{eu}gr^*$-closed set in $(X,T_N)$.

$(ii)\Rightarrow(iii)$: Let $A$ be $N_{eu}gr^*$-open set. Then we have $\left[N_{eu}gr^*-Cl\left(N_{eu}gr^*-Cl(A)\right)^C\right]^C=$ $\left[N_{eu}gr^*-Cl\left(N_{eu}gr^*-Int\left(A^C\right)\right)\right]^C$. Since $A$ is $N_{eu}gr^*$-open set. Then $A^C$ is $N_{eu}gr^*$-closed set. So, by $(ii)$ $N_{eu}gr^*-Int(A^C)$ is $N_{eu}gr^*$-closed set. That is $N_{eu}gr^*-Cl\left[N_{eu}gr^*-Int\left(A^C\right)\right]=$
$\left[N_{eu}gr^*-Cl\left(N_{eu}gr^*-Cl(A)\right)^C\right]^C=$
$\left[N_{eu}gr^*-Cl\left(N_{eu}gr^*-Int\left(A^C\right)\right)\right]^C=$
$\left[N_{eu}gr^*-Int\left(A^C\right)\right]^C=$ $N_{eu}gr^*-Int\left(A^C\right)$. Hence we obtain $N_{eu}gr^*-Cl(A)$ which implies that $N_{eu}gr^*-Cl(A)=\left[N_{eu}gr^*-Cl\left(N_{eu}gr^*-Cl(A)\right)^C\right]^C$.

$(iii)\Rightarrow(iv)$: Let $A$ and $B$ be any two $N_{eu}gr^*$-open sets in $(X,T_N)$ such that $N_{eu}gr^*-Cl(A)=B^C$. Then $(iii)\Rightarrow N_{eu}gr^*-Cl(A)=\left[N_{eu}gr^*-Cl\left(N_{eu}gr^*-Cl(A)\right)^C\right]^C$ $=\left[N_{eu}gr^*-Cl\left(B^C\right)^C\right]^C=\left[N_{eu}gr^*-Cl(B)\right]^C$.

$(iv)\Rightarrow(i)$: Let $A$ be any $N_{eu}gr^*$-open set in $(X,T_N)$. Let $B=\left[N_{eu}gr^*-Cl(A)\right]^C$. Then $N_{eu}gr^*-Cl(A)=B^C$. Then (iv) implies $N_{eu}gr^*-Cl(A)=\left[N_{eu}gr^*-Cl(B)\right]^C$. Since $N_{eu}gr^*-Cl(B)$ is $N_{eu}gr^*$-closed set, this implies that $N_{eu}gr^*-Cl(A)$ is $N_{eu}gr^*$-open set. This implies that $(X,T_N)$ is $N_{eu}$-Extremely-$gr^*$-disconnected space.

**Definition 4.15.** A $N_{eu}$-Top-Space. $(X,T_N)$ is $N_{eu}$-Strongly-$gr^*$-connected, if there does not exist any nonempty $N_{eu}gr^*$-closed sets $A$ and $B$ in $X$ such that $A\cap B=0_{Neu}$.

**Theorem 4.16.** Let $f:(X,T_N)\to(Y,\sigma_N)$ be a $N_{eu}gr^*$-irresolute surjection and $X$ be a $N_{eu}$-Strongly-$gr^*$-connected space. Then $Y$ is $N_{eu}$-Strongly-$N_{eu}gr^*$-connected.

**Proof.** Assume that $Y$ is not $N_{eu}$-Strongly-$N_{eu}gr^*$-connected, then there exist nonempty $N_{eu}gr^*$-closed sets $U$ and $V$ in $Y$ such that $U\neq 0_{Neu}$, $V\neq 0_{Neu}$, and $U\cap V=0_{Neu}$. Since $f$ is $N_{eu}gr^*$-irresolute mapping, $A=f^{-1}(U)\neq 0_{Neu}$, $B=f^{-1}(V)\neq 0_{Neu}$, which are $N_{eu}gr^*$-closed sets in $X$ and $f^{-1}(U)\cap f^{-1}(V)=f^{-1}(0_{Neu})=0_{Neu}$, which implies $A\cap B=0_{Neu}$. Thus, $X$ is not $N_{eu}$-Strongly-$gr^*$-connected, which is a $N_{eu}$-Strongly-$gr^*$-connected. Hence it follows that this is contradiction to our hypothesis. Consequently $Y$ is $N_{eu}$-Strongly-$gr^*$-connected.

## 5. Neutrosophic Generalized Regular Star Topological Spaces

In this section, we define $N_{eu}gr^*$-Regular spaces and Strongly-$N_{eu}gr^*$-Regular spaces by using $N_{eu}gr^*$-open sets and $N_{eu}gr^*$-closed sets in $N_{eu}$-Top-Spaces. We study their basic properties and characterizations.

**Definition 5.1.** A $N_{eu}$-Top-Space $(X,\tau_N)$ is said to be $N_{eu}gr^*$-Regular if for each $N_{eu}gr^*$-closed set $A$ and a $N_{eu}$-point $x_{(\alpha,\beta,\gamma)}\notin A$, there exist disjoint $N_{eu}gr^*$-open sets $U$ and $V$ such that $A\subseteq U$, $x_{(\alpha,\beta,\gamma)}\in V$.

**Theorem 5.2.** Let $(X,\tau_N)$ be a $N_{eu}$-Top-Space. Then the following statements are equivalent:
(i) $X$ is $N_{eu}gr^*$-Regular.
(ii) For every $x_{(\alpha,\beta,\gamma)}\in X$ and every $N_{eu}gr^*$-open set $G$ containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gr^*$-open set $U$ such that $x_{(\alpha,\beta,\gamma)}\in U\subseteq N_{eu}gr^*-Cl(U)\subseteq G$.
(iii) For every $N_{eu}gr^*$-closed set $F$, the intersection of all $N_{eu}gr^*$-closed $N_{eu}gr^*$-neighbourhoods of $F$ is exactly $F$.

$(\text{iv})$ For any $N_{eu}$ −set $A$ and a $N_{eu}gr^*$ −open set $B$ such that $A \cap B \neq 0_{Neu}$, there exists a $N_{eu}gr^*$ −open set $U$ such that $A \cap U \neq 0_{Neu}$ and $N_{eu}gr^* - Cl(U) \subseteq B$.

$(\text{v})$ For every non-empty $N_{eu}$ −set $A$ and $N_{eu}gr^*$ −closed set $B$ such that $A \cap B = 0_{Neu}$, there exist disjoint $N_{eu}gr^*$ −open sets $U$ and $V$ such that $A \cap U \neq 0_{Neu}$ and $B \subseteq V$.

**Proof.** $(\text{i}) \Rightarrow (\text{ii})$: Suppose $X$ is $N_{eu}gr^*$ −Regular. Let $x_{(\alpha,\beta,\gamma)} \in X$ and let $G$ be a $N_{eu}gr^*$ −open set containing $x_{(\alpha,\beta,\gamma)}$. Then $x_{(\alpha,\beta,\gamma)} \notin G^C$ and $G^C$ is $N_{eu}gr^*$ −closed. Since $X$ is $N_{eu}gr^*$ −Regular, there exist $N_{eu}gr^*$ −open sets $U$ and $V$ such that $U \cap V = 0_{Neu}$ and $x_{(\alpha,\beta,\gamma)} \in U$, $G^C \subseteq V$. It follows that $U \subseteq V^C \subseteq G$ and hence $N_{eu}gr^* - Cl(U) \subseteq N_{eu}gr^* - Cl(V^C) = V^C \subseteq G$. That is $x_{(\alpha,\beta,\gamma)} \in U \subseteq N_{eu}gr^* - Cl(U) \subseteq G$.

$(\text{ii}) \Rightarrow (\text{iii})$: Let $F$ be any $N_{eu}gr^*$ −closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Then $F^C$ is $N_{eu}gr^*$ −open and $x_{(\alpha,\beta,\gamma)} \in F^C$. By assumption, there exists a $N_{eu}gr^*$ −open set $U$ such that $x_{(\alpha,\beta,\gamma)} \in U \subseteq N_{eu}gr^* - Cl(U) \subseteq F^C$. Thus $F \subseteq (N_{eu}gr^* - Cl(U))^C \subseteq U^C$. Now $U^C$ is $N_{eu}gr^*$ −closed and $N_{eu}gr^*$ −neighbourhood of $F$ which does not contain $x_{(\alpha,\beta,\gamma)}$. So, we get the intersection of all $N_{eu}gr^*$ −closed $N_{eu}gr^*$ −neighbourhoods of $F$ to be exactly equal to $F$.

$(\text{iii}) \Rightarrow (\text{iv})$: Suppose $A \cap B \neq 0_{Neu}$ and $B$ is $N_{eu}gr^*$ −open set. Let $x_{(\alpha,\beta,\gamma)} \in A \cap B$. Since $B$ is $N_{eu}gr^*$ −open, $B^C$ is $N_{eu}gr^*$ −closed and $x_{(\alpha,\beta,\gamma)} \notin B^C$. By using $(\text{iii})$, there exists a $N_{eu}gr^*$ −closed, $N_{eu}gr^*$ −neighbourhood $V$ of $B^C$ such that $x_{(\alpha,\beta,\gamma)} \notin V$. Now for the $N_{eu}gr^*$ −neighbourhood $V$ of $B^C$, there exists a $N_{eu}gr^*$ −open set $G$ such that $B^C \subseteq G \subseteq V$. Take $U = V^C$. Thus $U$ is a $N_{eu}gr^*$ −open set containing $x_{(\alpha,\beta,\gamma)}$. Also $A \cap U \neq 0_{Neu}$ and $N_{eu}gr^* - (U) \subseteq G^C \subseteq B$.

$(\text{iv}) \Rightarrow (\text{v})$: Suppose $A$ is a non-empty set and $B$ is a $N_{eu}gr^*$ −closed set such that $A \cap B = 0_{Neu}$. Then $B^C$ is $N_{eu}gr^*$ −open set and $A \cap B^C \neq 0_{Neu}$. By our assumption, there exists a $N_{eu}gr^*$ −open $U$ such that $A \cap U \neq 0_{Neu}$ and $N_{eu}gr^* - Cl(U) \subseteq B^C$. Take $V = (N_{eu}gr^* - Cl(U))^C$. Since $N_{eu}gr^* - Cl(U)$ is $N_{eu}gr^*$ −closed, $V$ is $N_{eu}gr^*$ −open. Also $B \subseteq V$ and $U \cap V \subseteq N_{eu}gr^* - Cl(U) \cap (N_{eu}gr^* - Cl(U))^C = 0_{Neu}$.

$(\text{v}) \Rightarrow (\text{i})$: Let $S$ be $N_{eu}gr^*$ −closed set and $x_{(\alpha,\beta,\gamma)} \notin S$. Then $S \cap \{x_{(\alpha,\beta,\gamma)}\} = 0_{Neu}$. By $(\text{v})$, there exist disjoint $N_{eu}gr^*$ −open sets $U$ and $V$ such that $U \cap \{x_{(\alpha,\beta,\gamma)}\} \neq 0_{Neu}$ and $S \subseteq V$. That is $U$ and $V$ are disjoint $N_{eu}gr^*$ −open sets containing $x_{(\alpha,\beta,\gamma)}$ and $S$ respectively. This proves that $(X, \tau_N)$ is $N_{eu}gr^*$ −Regular.

**Corollary 5.3.** Let $(X, \tau_N)$ be a $N_{eu}$ −Top −Space. Then the following statements are equivalent:

$(\text{i})$ $X$ is $N_{eu}gr^*$ −Regular.

$(\text{ii})$ For every $x_{(\alpha,\beta,\gamma)} \in X$ and every $N_{eu}gr^*$ −open set $G$ containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gr^*$ −open set $U$ such that $x_{(\alpha,\beta,\gamma)} \in U \subseteq N_{eu}gr^* - Cl(U) \subseteq G$.

$(\text{iii})$ For every $N_{eu}gr^*$ −closed $F$, the intersection of all $N_{eu}$ −closed, $N_{eu}$ −neighbourhoods of $F$ is exactly $F$.

$(\text{iv})$ For any $N_{eu}$ −set $A$ and a $N_{eu}$ −open set $B$ such that $A \cap B \neq 0_{Neu}$, there exists a $N_{eu}gr^*$ −open set $U$ such that $A \cap U \neq 0_{Neu}$ and $N_{eu}gr^* - Cl(U) \subseteq B$.

$(\text{v})$ For every non-empty $N_{eu}$ −set $A$ and a $N_{eu}$ −closed set $B$ such that $A \cap B = 0_{Neu}$, there exist disjoint $N_{eu}gr^*$ −open sets $U$ and $V$ such that $A \cap U \neq 0_N$ and $B \subseteq V$.

**Proof.** Since every $N_{eu}$ −open set is $N_{eu}gr^*$ −open and follows from Theorem 5.2.

**Theorem 5.4.** A $N_{eu}$ −Top −Space $(X, \tau_N)$ is $N_{eu}gr^*$ −Regular if and only if every $x_{(\alpha,\beta,\gamma)} \in X$ and every $N_{eu}gr^*$ −neighbourhood $N$ containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gr^*$ −open set $V$ such that $x_{(\alpha,\beta,\gamma)} \in V \subseteq N_{eu}gr^* - Cl(V) \subseteq N$.

**Proof.** Let $X$ be a $N_{eu}gr^*$ −Regular space. Let $N$ be any $N_{eu}gr^*$ −neighbourhood of $x_{(\alpha,\beta,\gamma)}$. Then there exists a $N_{eu}gr^*$ −open set $G$ such that $x_{(\alpha,\beta,\gamma)} \in G \subseteq N$. Since $G^C$ is $N_{eu}gr^*$ −closed set and $x_{(\alpha,\beta,\gamma)} \notin G^C$, by definition there exist $N_{eu}gr^*$ −open sets $U$ and $V$ such $G^C \subseteq U$ and $x_{(\alpha,\beta,\gamma)} \in V$ and $U \cap V = 0_{Neu}$ so that

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$V \subseteq U^C$. It follows that $N_{eu}gr^*-Cl(V) \subseteq N_{eu}gr^*-Cl(U^C) = U^C$. Also $G^C \subseteq U$ implies $U^C \subseteq G \subseteq N$. Hence $x_{(\alpha,\beta,\gamma)} \in V \subseteq N_{eu}gr^*-(V) \subseteq N$. Conversely, suppose for every $x_{(\alpha,\beta,\gamma)} \in X$ and every $N_{eu}gr^*-$neighbourhood $N$ containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gr^*-$open set $V$ such that $x_{(\alpha,\beta,\gamma)} \in V \subseteq N_{eu}gr^*-Cl(V) \subseteq N$. Let $F$ be any $N_{eu}gr^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Then $x_{(\alpha,\beta,\gamma)} \in F^C$. Since $F^C$ is $N_{eu}gr^*-$open set, $F^C$ is $N_{eu}gr^*-$neighbourhood containing $x_{(\alpha,\beta,\gamma)}$. By hypothesis there exists a $N_{eu}gr^*-$open set $V$ such that $x_{(\alpha,\beta,\gamma)} \in V$ and $N_{eu}gr^*-Cl(V) \subseteq F^C$. This implies that $F \subseteq \left(N_{eu}gr^*-Cl(V)\right)^C$. Then $\left(N_{eu}gr^*-Cl(V)\right)^C$ is a $N_{eu}gr^*-$open set containing $F$. Also $V \cap \left(N_{eu}gr^*-Cl(V)\right)^C = 0_{Neu}$. Hence $(X, \tau_{Neu})$ is $N_{eu}gr^*-$Regular.

**Theorem 5.5.** A $N_{eu}-$Top$-$Space $(X, \tau_N)$ is $N_{eu}gr^*-$Regular if and only if for each $N_{eu}gr^*-$closed set $F$ of $X$ and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist $N_{eu}gr^*-$open sets $U$ and $V$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $\left[N_{eu}gr^*-Cl(U)\right] \cap \left[N_{eu}gr^*-Cl(V)\right] = 0_{Neu}$.

**Proof.** Suppose $(X, \tau_N)$ is $N_{eu}gr^*-$Regular. Let $F$ be a $N_{eu}gr^*-$closed set in $X$ and $x_{(\alpha,\beta,\gamma)} \notin F$. Then there exist $N_{eu}gr^*-$open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and $V$ such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$, $F \subseteq V$ and $U_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_{Neu}$. This implies that $U_{\left(x_{(\alpha,\beta,\gamma)}\right)} \cap \left[N_{eu}gr^*-Cl(V)\right] = 0_{Neu}$. Also $N_{eu}gr^*-Cl(V)$ is a $N_{eu}gr^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin N_{eu}gr^*-Cl(V)$. Since $(X, \tau_N)$ is $N_{eu}gr^*-$Regular, there exist $N_{eu}gr^*-$open sets $G$ and $H$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in G$, $N_{eu}gr^*-Cl(V) \subseteq H$ and $G \cap V = 0_N$. This implies $\left[N_{eu}gr^*-Cl(G)\right] \cap H \subseteq \left[N_{eu}gr^*-Cl(H^C)\right] \cap H = H^C \cap H = 0_{Neu}$. Take $U = G$. Now $U$ and $V$ are $N_{eu}gr^*-$open sets in $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. Also $\left[N_{eu}gr^*-Cl(U)\right] \cap \left[N_{eu}gr^*-Cl(V)\right] \subseteq \left[N_{eu}gr^*-Cl(G)\right] \cap H = 0_{Neu}$.

Conversely, suppose for each $N_{eu}gr^*-$closed set $F$ of $X$ and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist $N_{eu}gr^*-$open sets $U$ and $V$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $\left[N_{eu}gr^*-Cl(U)\right] \cap \left[N_{eu}gr^*-Cl(V)\right] = 0_{Neu}$. Now $U \cap V \subseteq \left[N_{eu}gr^*-Cl(U)\right] \cap \left[N_{eu}gr^*-Cl(V)\right] = 0_{Neu}$. Therefore $U \cap V = 0_{Neu}$. This proves that $(X, \tau_N)$ is $N_{eu}gr^*-$Regular.

**Theorem 5.6.** Let $f:(X, \tau_N) \rightarrow (Y, \sigma_N)$ be a bijective function. If $f$ is $N_{eu}gr^*-$irresolute, $N_{eu}gr^*-$open and $X$ is $N_{eu}gr^*-$Regular, then $Y$ is $N_{eu}gr^*-$Regular.

**Proof.** Suppose $(X, \tau_N)$ is $N_{eu}gr^*-$Regular. Let $S$ be any $N_{eu}gr^*-$closed set in $Y$ such that $y_{(r,t,s)} \notin S$. Since $f$ is $N_{eu}gr^*-$irresolute, $f^{-1}(S)$ is $N_{eu}gr^*-$closed set in $X$. Since $f$ is onto, there exists $x_{(\alpha,\beta,\gamma)} \in X$ such that $y_{(r,t,s)} = f\left(x_{(\alpha,\beta,\gamma)}\right)$. Now $f\left(x_{(\alpha,\beta,\gamma)}\right) = y_{(r,t,s)} \notin S$ implies that $x_{(\alpha,\beta,\gamma)} \notin f^{-1}(S)$. Since $X$ is $N_{eu}gr^*-$Regular, there exist $N_{eu}gr^*-$open sets $U$ and $V$ in $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$, $f^{-1}(S) \subseteq$ and $U \cap V = 0_{Neu}$. Now $x_{(\alpha,\beta,\gamma)} \in U$ implies that $f\left(x_{(\alpha,\beta,\gamma)}\right) \in f(U)$ and $f^{-1}(S) \subseteq V$ implies that $S \subseteq f(V)$. Also $U \cap V = 0_N$ implies that $f(U \cap V) = 0_{Neu}$ which implies that $f(U) \cap f(V) = 0_{Neu}$. Since $f$ is a $N_{eu}gr^*-$open mapping, $f(U)$ and $f(V)$ are disjoint $N_{eu}gr^*-$open sets in $Y$ containing $y_{(r,t,s)}$ and $S$ respectively. Thus $Y$ is $N_{eu}gr^*-$Regular.

**Theorem 5.7.** Let $(X, \tau_N)$ be a $N_{eu}gr^*-$Regular space. Then

(i) Every $N_{eu}gr^*-$open set in $X$ is a union of $N_{eu}gr^*-$closed sets.

(ii) Every $N_{eu}gr^*-$closed set in $X$ is an intersection of $N_{eu}gr^*-$open sets.

**Proof.** (i) Suppose $X$ is $N_{eu}gr^*-$Regular. Let $G$ be a $N_{eu}gr^*-$open set and $x_{(\alpha,\beta,\gamma)} \in G$. Then $F = G^C$ is $N_{eu}gr^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Since $X$ is

$\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular, there exist disjoint $\mathrm{N}_{eu}\mathrm{gr}^*-$open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and $V$ in $X$ such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$ and $F \subseteq V$. Since $U_{x_{(\alpha,\beta,\gamma)}} \cap F \subseteq U_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_{Neu}$, we have $U_{x_{(\alpha,\beta,\gamma)}} \subseteq F^C = G$. Take $V_{\left(x_{(\alpha,\beta,\gamma)}\right)} = \mathrm{N}_{eu}\mathrm{gr}^*-Cl\left(U_{x_{(\alpha,\beta,\gamma)}}\right)$. Then $V_{x_{(\alpha,\beta,\gamma)}}$ is $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set and $V_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_N$. Now $F \subseteq V$ implies that $V_{x_{(\alpha,\beta,\gamma)}} \cap F \subseteq V_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_{Neu}$. It follows that $x_{(\alpha,\beta,\gamma)} \in V_{x_{(\alpha,\beta,\gamma)}} \subseteq F^C = G$. This proves that $G = \bigcup\left\{V_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in G\right\}$. Thus $G$ is a union of $\mathrm{N}_{eu}\mathrm{gr}^*-$closed sets. $(\mathrm{ii})$ Follows from $(\mathrm{i})$ and set theoretic properties.

**Theorem 5.8.** Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a $\mathrm{N}_{eu}\mathrm{gr}^*-$continuous and $\mathrm{N}_{\mathrm{eu}}-$closed injection from a $\mathrm{N}_{\mathrm{eu}}-\mathrm{Top}-\mathrm{Space}$ $(X,\tau_N)$ into a $\mathrm{N}_{\mathrm{eu}}-\mathrm{R}$ egular space $(Y,\sigma_N)$. If every $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set in $X$ is $\mathrm{N}_{\mathrm{eu}}-$closed, then $X$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

**Proof.** Let $x_{(\alpha,\beta,\gamma)} \in X$ and $A$ be a $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set in $X$ such that $x_{(\alpha,\beta,\gamma)} \notin A$. Then by assumption, $A$ is $\mathrm{N}_{\mathrm{eu}}-$closed in $X$. Since $f$ is $\mathrm{N}_{\mathrm{eu}}-$closed, $f(A)$ is a $\mathrm{N}_{\mathrm{eu}}-$closed set in $Y$ such that $f\left(x_{(\alpha,\beta,\gamma)}\right) \notin f(A)$. Since $Y$ is $\mathrm{N}_{\mathrm{eu}}-\mathrm{R}$ egular, there exist disjoint $\mathrm{N}_{\mathrm{eu}}-$open sets $G$ and $H$ in $Y$ such that $f\left(x_{(\alpha,\beta,\gamma)}\right) \in G$ and $f(A) \subseteq H$. Since $f$ is $\mathrm{N}_{eu}\mathrm{gr}^*-$continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $\mathrm{N}_{eu}\mathrm{gr}^*-$open sets in $X$ containing $x_{(\alpha,\beta,\gamma)}$ and $A$ respectively. Hence $X$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

**Theorem 5.9.** Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a $\mathrm{N}_{\mathrm{eu}}-$continuous, $\mathrm{N}_{eu}\mathrm{gr}^*-$open bijection of a $\mathrm{N}_{\mathrm{eu}}-\mathrm{R}$ egular space $X$ into a $\mathrm{N}_{\mathrm{eu}}-$space $Y$ and if every $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set in $Y$ is $\mathrm{N}_{\mathrm{eu}}-$closed, then $Y$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

**Proof.** Let $y_{(r,t,s)} \in Y$ and $B$ be a $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set in $Y$ such that $y_{(r,t,s)} \notin B$. Since $f$ is a bijection. So there exists a unique point $x_{(\alpha,\beta,\gamma)} \in X$ such that $f\left(x_{(\alpha,\beta,\gamma)}\right) = y_{(r,t,s)}$. Then by assumption, $B$ is $\mathrm{N}_{\mathrm{eu}}-$closed in $Y$. Since $f$ is a $\mathrm{N}_{\mathrm{eu}}-$continuous bijection, $f^{-1}(B)$ is a $\mathrm{N}_{\mathrm{eu}}-$closed set in $X$ such that $x_{(\alpha,\beta,\gamma)} \notin f^{-1}(B)$. Since $X$ is $\mathrm{N}_{\mathrm{eu}}-\mathrm{R}$ egular, there exist disjoint $\mathrm{N}_{\mathrm{eu}}-$open sets $G$ and $H$ in $X$ such that $x_{(\alpha,\beta,\gamma)} \in G$ and $f^{-1}(B) \subseteq H$. Since $f$ is $\mathrm{N}_{eu}\mathrm{gr}^*-$open, $f(G)$ and $f(H)$ are disjoint $\mathrm{N}_{eu}\mathrm{gr}^*-$open sets in $Y$ such that $f\left(x_{(\alpha,\beta,\gamma)}\right) = y_{(r,t,s)} \in f(G)$ and $B \subseteq f(H)$. Hence $Y$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

**Definition 5.10.** A $\mathrm{N}_{\mathrm{eu}}-\mathrm{Top}-\mathrm{Space}$ $(X,\tau_N)$ is said to be strongly $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular if for each $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set $A$ and a point $x_{(\alpha,\beta,\gamma)} \notin A$, there exist disjoint $\mathrm{N}_{\mathrm{eu}}-$open sets $U$ and $V$ such that $A \subseteq U$ and $x_{(\alpha,\beta,\gamma)} \in V$.

**Proposition 5.11.** $(\mathrm{i})$ Every strongly $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular space is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

$(\mathrm{ii})$ Every strongly $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular space is strongly $\mathrm{N}_{\mathrm{eu}}-\mathrm{R}$ egular.

**Proof.** $(\mathrm{i})$ Suppose $(X,\tau_N)$ is strongly $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular. Let $F$ be a $\mathrm{N}_{eu}\mathrm{gr}^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Since $X$ is strongly $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular, there exist disjoint $\mathrm{N}_{\mathrm{eu}}-$open sets $U$ and $V$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. Since every $\mathrm{N}_{\mathrm{eu}}-$open set is $\mathrm{N}_{eu}\mathrm{gr}^*-$open, so $U$ and $V$ are $\mathrm{N}_{eu}\mathrm{gr}^*-$open sets. This implies that $X$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

$(\mathrm{ii})$ This can be proved similarly as $(\mathrm{i})$.

**Definition 5.12.** A $\mathrm{N}_{\mathrm{eu}}-\mathrm{Top}-\mathrm{Space}$ $(X,\tau_N)$ is said to be strongly* $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular. if for each $\mathrm{N}_{\mathrm{eu}}-$closed set $A$ and a point $x_{(\alpha,\beta,\gamma)} \notin A$, there exist disjoint $\mathrm{N}_{eu}\mathrm{gr}^*-$open sets $U$ and $V$ such that $A \subseteq U$, $x_{(\alpha,\beta,\gamma)} \in V$.

**Proposition 5.13.** Every $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular $\mathrm{N}_{\mathrm{eu}}-\mathrm{Top}-\mathrm{Space}$ $(X,\tau_N)$ is strongly* $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

**Proof.** Suppose $(X,\tau_N)$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular. Let $F$ be a $\mathrm{N}_{\mathrm{eu}}-$closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Then $F$ is $\mathrm{N}_{eu}\mathrm{gr}^*-$closed. Since $X$ is $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular, there exist disjoint $\mathrm{N}_{eu}\mathrm{gr}^*-$open sets $U$ and $V$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. This implies that $X$ is strongly* $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

**Theorem 5.14.** Let $(X,\tau_N)$ be a $\mathrm{N}_{\mathrm{eu}}-\mathrm{Top}-\mathrm{Space}$. Then the following statements are equivalent:

$(\mathrm{i})$ $X$ is strongly $\mathrm{N}_{eu}\mathrm{gr}^*-\mathrm{R}$ egular.

$(ii)$ For every $x_{(\alpha,\beta,\gamma)} \in X$ and every $\mathrm{N}_{eu}gr^*-$open set $G$ containing $x_{(\alpha,\beta,\gamma)}$, there exists a $\mathrm{N}_{eu}-$open set $U$ such that $x_{(\alpha,\beta,\gamma)} \in U \subseteq \mathrm{N}_{eu}Cl(U) \subseteq G.$

$(iii)$ For every $\mathrm{N}_{eu}gr^*-$closed set $F$, the intersection of all $\mathrm{N}_{eu}-$closed, $\mathrm{N}_{eu}-$neighbourhoods $F$ is exactly $F$.

$(iv)$ For any $\mathrm{N}_{eu}-$set $A$ and a $\mathrm{N}_{eu}gr^*-$open set $B$ such that $A \cap B \neq 0_{Neu}$, there exists a $\mathrm{N}_{eu}-$open set $U$ such that $A \cap U \neq 0_{Neu}$ and $\mathrm{N}_{eu}Cl(U) \subseteq B.$

$(v)$ For every non-empty $\mathrm{N}_{eu}-$set $A$ and $\mathrm{N}_{eu}gr^*-$closed set $B$ such that $A \cap B = 0_N$, there exist disjoint $\mathrm{N}_{eu}-$open sets $U$ and $V$ such that $A \cap U \neq 0_{Neu}$ and $B \subseteq V$.

**Proof.** $(i) \Rightarrow (ii)$: Suppose $X$ is strongly $\mathrm{N}_{eu}gr^*-$Regular. Let $x_{(\alpha,\beta,\gamma)} \in X$ and let $G$ be a $\mathrm{N}_{eu}gr^*-$open set containing $x_{(\alpha,\beta,\gamma)}$. Then $x_{(\alpha,\beta,\gamma)} \notin G^C$ and $G^C$ is $\mathrm{N}_{eu}gr^*-$closed. Since $X$ is $\mathrm{N}_{eu}gr^*-$Regular, there exist $\mathrm{N}_{eu}-$open sets $U$ and $V$ such that $U \cap V = 0_{Neu}$ and $x_{(\alpha,\beta,\gamma)} \in U$, $G^C \subseteq V$. It follows that $U \subseteq V^C \subseteq G$ and hence $\mathrm{N}_{eu}Cl(U) \subseteq \mathrm{N}_{eu}Cl(V^C) = V^C \subseteq G.$ That is $x_{(\alpha,\beta,\gamma)} \in U \subseteq \mathrm{N}_{eu}Cl(U) \subseteq G.$

$(ii) \Rightarrow (iii)$: Let $F$ be a $\mathrm{N}_{eu}gr^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin F.$ Then $F^C$ is $\mathrm{N}_{eu}gr^*-$open set and $x_{(\alpha,\beta,\gamma)} \in F^C$. By assumption, there exists a $\mathrm{N}_{eu}-$open set $U$ such that $x_{(\alpha,\beta,\gamma)} \in U \subseteq \mathrm{N}_{eu}Cl(U) \subseteq F^C.$ Thus $F \subseteq (\mathrm{N}_{eu}Cl(U))^C \subseteq U^C$. Now $U^C$ is $\mathrm{N}_{eu}-$closed, $\mathrm{N}_{eu}-$neighbourhood of $F$ which does not contain $x_{(\alpha,\beta,\gamma)}$. So, the intersection of all $\mathrm{N}_{eu}-$closed, $\mathrm{N}_{eu}-$neighbourhoods of $F$ is exactly $F$.

$(iii) \Rightarrow (iv)$: Suppose $A \cap B \neq 0_{Neu}$ and $B$ is $\mathrm{N}_{eu}gr^*-$open set. Let $x_{(\alpha,\beta,\gamma)} \in A \cap B.$ Since $B$ is $\mathrm{N}_{eu}gr^*-$open, $B^C$ is $\mathrm{N}_{eu}gr^*-$closed and $x_{(\alpha,\beta,\gamma)} \notin B^C.$ By using $(iii)$, there exists a $\mathrm{N}_{eu}-$closed, $\mathrm{N}_{eu}-$neighbourhood $V$ of $B^C$ such that $x_{(\alpha,\beta,\gamma)} \notin V.$ Now for the $\mathrm{N}_{eu}-$neighbourhood $V$ of $B^C$, there exists a $\mathrm{N}_{eu}-$open set $G$ such that $B^C \subseteq G \subseteq V.$ Take $U = V^C.$ Thus $U$ is a $\mathrm{N}_{eu}-$open set containing $x_{(\alpha,\beta,\gamma)}$. Also $A \cap U \neq 0_{Neu}$ and $\mathrm{N}_{eu}Cl(U) \subseteq G^C \subseteq B.$

$(iv) \Rightarrow (v)$: Suppose $A$ is a non-empty set and $B$ is $\mathrm{N}_{eu}gr^*-$closed set such that $A \cap B = 0_{Neu}$. Then $B^C$ is $\mathrm{N}_{eu}gr^*-$open set and $A \cap B^C \neq 0_{Neu}$. By our assumption, there exists a $\mathrm{N}_{eu}-$open set $U$ such that $A \cap U \neq 0_{Neu}$ and $\mathrm{N}_{eu}Cl(U) \subseteq B^C$. Take $V = (\mathrm{N}_{eu}Cl(U))^C$. Since $\mathrm{N}_{eu}Cl(U)$ is $\mathrm{N}_{eu}-$closed, $V$ is $\mathrm{N}_{eu}-$open. Also $B \subseteq V$ and $U \cap V \subseteq \mathrm{N}_{eu}Cl(U) \cap (\mathrm{N}_{eu}Cl(U))^C = 0_N.$

$(v) \Rightarrow (i)$: Let $S$ be $\mathrm{N}_{eu}gr^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin S.$ Then $S \cap \{x_{(\alpha,\beta,\gamma)}\} = 0_{Neu}$. By $(v)$, there exist disjoint $\mathrm{N}_{eu}-$open sets $U$ and $V$ such that $U \cap \{x_{(\alpha,\beta,\gamma)}\} \neq 0_{Neu}$ and $S \subseteq V$. That is $U$ and $V$ are disjoint $\mathrm{N}_{eu}-$open sets containing $x_{(\alpha,\beta,\gamma)}$ and $S$ respectively. This proves that $(X, \tau_N)$ is strongly $\mathrm{N}_{eu}gr^*-$Regular.

**Theorem 5.15.** A $\mathrm{N}_{eu}-$Top$-$Space. $(X, \tau_N)$ is $\mathrm{N}_{eu}gr^*-$Regular if and only if for each $\mathrm{N}_{eu}gr^*-$closed set $F$ of $X$ and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist $\mathrm{N}_{eu}-$open sets $U$ and $V$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) = 0_{Neu}$.

**Proof.** Suppose $(X, \tau_N)$ is strongly $\mathrm{N}_{eu}gr^*-$Regular. Let $F$ be a $\mathrm{N}_{eu}gr^*-$closed set in $X$ and $x_{(\alpha,\beta,\gamma)} \notin F.$ Then there exist $\mathrm{N}_{eu}-$open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and $V$ such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$, $F \subseteq V$ and $U_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_{Neu}$. This implies that $U_{(x_{(\alpha,\beta,\gamma)})} \cap \mathrm{N}_{eu}Cl(V) = 0_N$. Also $\mathrm{N}_{eu}Cl(V)$ is a $\mathrm{N}_{eu}-$closed set and $x_{(\alpha,\beta,\gamma)} \notin \mathrm{N}_{eu}Cl(V)$. Since $(X, \tau_N)$ is strongly $\mathrm{N}_{eu}gr^*-$Regular, there exist $\mathrm{N}_{eu}-$open sets $G$ and $H$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in G$, $\mathrm{N}_{eu}Cl(V) \subseteq H$ and $G \cap V = 0_{Neu}$. This implies $\mathrm{N}_{eu}Cl(G) \cap H \subseteq \mathrm{N}_{eu}Cl(H^C) \cap H = H^C \cap H = 0_{Neu}$.
Take $U = G.$ Now $U$ and $V$ are $\mathrm{N}_{eu}-$open sets in $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. Also $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) \subseteq \mathrm{N}_{eu}Cl(G) \cap H = 0_{Neu}$. Thus $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) = 0_{Neu}$.
Conversely, suppose for each $\mathrm{N}_{eu}gr^*-$closed set $F$ of $X$ and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist $\mathrm{N}_{eu}-$open sets $U$ and $V$ of $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) = 0_{Neu}$. Now

$U \cap V \subseteq N_{eu}Cl(U) \cap N_{eu}Cl(V) = 0_{Neu}$. Therefore $U \cap V = 0_{Neu}$. This proves that $(X, \tau_N)$ is strongly $N_{eu}$gr*−Regular.

**Theorem 5.16.** A $N_{eu}$−Top−Space. $(X, \tau_N)$ is strongly $N_{eu}$gr*−Regular if and only if every pair consisting of a $N_{eu}$−compact set and a disjoint $N_{eu}$gr*−closed set can be separated by $N_{eu}$−open sets.

**Proof.** Let $(X, \tau_N)$ be strongly $N_{eu}$gr*−Regular and let $A$ be a $N_{eu}$−compact set, and B be a $N_{eu}$gr*−closed set such that $A \cap B = 0_{Neu}$. Since $X$ is strongly $N_{eu}$gr*−Regular, for each $x_{(\alpha,\beta,\gamma)} \in A$, there exist disjoint $N_{eu}$−open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and $V_{x_{(\alpha,\beta,\gamma)}}$ such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$, $B \subseteq V_{x_{(\alpha,\beta,\gamma)}}$. Obviously, $\{U_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in A\}$ is a $N_{eu}$−open covering of $A$. Since $A$ is $N_{eu}$−compact, there exists a finite set $F \subseteq A$ such that $A \subseteq \bigcup\{U_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$ and $B \subseteq \bigcap\{V_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$. Put $U = \bigcup\{U_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$ and $V = \bigcap\{V_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$. Then $U$ and $V$ are $N_{eu}$−open sets in $X$. Also $U \cap V = 0_N$. Otherwise, if $x_{(\alpha,\beta,\gamma)} \in U \cap V$, then $x_{(\alpha,\beta,\gamma)} \in U_{(r,s,t)}$ for some $x_{(r,s,t)} \in F$ and $x_{(\alpha,\beta,\gamma)} \in V \subseteq V_{(r,s,t)}$. This implies that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(r,s,t)}} \cap V_{x_{(r,s,t)}}$, which is a contradiction to $U_{x_{(r,s,t)}} \cap V_{x_{(r,s,t)}} = \phi$. Thus $U$ and $V$ are disjoint $N_{eu}$−open sets containing $A$ and $B$ respectively.
Conversely, suppose every pair consisting of a $N_{eu}$−compact set and a disjoint $N_{eu}$gr*−closed set can be separated by $N_{eu}$−open sets. Let $F$ be a $N_{eu}$gr*−closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Then $\{x_{(\alpha,\beta,\gamma)}\}$ is $N_{eu}$−compact set of $X$ and $\{x_{(\alpha,\beta,\gamma)}\} \cap F = 0_{Neu}$. By our assumption, there exist disjoint $N_{eu}$−open sets $U$ and $V$ such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. This proves that $X$ is strongly $N_{eu}$gr*−Regular.

**Corollary 5.17.** If $X$ is a strongly $N_{eu}$gr*−Regular space, $A$ is a $N_{eu}$−compact subset of $X$ and $B$ is a $N_{eu}$gr*−open set containing $A$, then there exists a $N_{eu}$−Regular open set $V$ such that $A \subseteq V \subseteq N_{eu}Cl(V) \subseteq B$.

**Proof.** Let $X$ be strongly $N_{eu}$gr*−Regular and let $A$ be a $N_{eu}$−compact set, and $B$ be $N_{eu}$gr*−open set with $A \subseteq B$. Then $B^C$ is $N_{eu}$gr*−closed set such that $B^C \cap A = 0_{Neu}$. Since $X$ is a strongly $N_{eu}$gr*−Regular space, then there exist disjoint $N_{eu}$−open sets $G$ and $H$ such that $A \subseteq G$ and $B^C \subseteq H$. Take $V = N_{eu}Int[N_{eu}Cl(G)]$. Then $N_{eu}Cl(V) = N_{eu}Cl\lfloor N_{eu}Int(N_{eu}Cl(G))\rfloor \subseteq N_{eu}Cl\lfloor N_{eu}Cl(G)\rfloor = N_{eu}Cl(G)$. Since $G$ is a $N_{eu}$−open set and $G \subseteq N_{eu}Cl(G)$, we have $G = N_{eu}Int(G) \subseteq N_{eu}Int[N_{eu}Cl(G)] = V$. This implies that $N_{eu}Cl(G) \subseteq N_{eu}Cl(V)$. It follows that $N_{eu}Cl(V) = N_{eu}Cl(G)$ and $N_{eu}Int\lfloor N_{eu}Cl(V)\rfloor = N_{eu}Int\lfloor N_{eu}Cl(G)\rfloor = V$. Thus $V$ is $N_{eu}$−Regular open. Now $A \subseteq G = N_{eu}Int(G) \subseteq N_{eu}Int[(N_{eu}Cl(G))] = V$. This implies that $A \subseteq V$ and $N_{eu}Cl(V) = N_{eu}Cl(G) \subseteq H^C \subseteq B$ implies that $A \subseteq V \subseteq N_{eu}Cl(V) \subseteq B$.

**Theorem 5.18.** Let $f : (X, \tau_N) \to (Y, \sigma_N)$ be a bijective function. If $f$ is $N_{eu}$gr*−irresolute, $N_{eu}$−open and $X$ is strongly $N_{eu}$gr*−Regular, then $Y$ is strongly $N_{eu}$gr*−Regular.

**Proof.** Suppose $(X, \tau_N)$ is strongly $N_{eu}$gr*−Regular. Let $S$ be a $N_{eu}$gr*−closed set in $Y$ such that $y_{(r,t,s)} \notin S$. Since $f$ is $N_{eu}$gr*−irresolute, $f^{-1}(S)$ is $N_{eu}$gr*−closed set in $X$. Since $f$ is onto, there exists $x_{(\alpha,\beta,\gamma)} \in X$ such that $y_{(r,t,s)} = f(x_{(\alpha,\beta,\gamma)})$. Now $f(x_{(\alpha,\beta,\gamma)}) = y_{(r,t,s)} \notin S$ implies that $x_{(\alpha,\beta,\gamma)} \notin f^{-1}(S)$. Since $X$ is strongly $N_{eu}$gr*−Regular. there exist $N_{eu}$−open sets $U$ and $V$ in $X$ such that $x_{(\alpha,\beta,\gamma)} \in U$, $f^{-1}(S) \subseteq V$ and $U \cap V = 0_{Neu}$. Now $x_{(\alpha,\beta,\gamma)} \in U$ implies that $f(x_{(\alpha,\beta,\gamma)}) \in f(U)$ and $f^{-1}(S) \subseteq V$ implies that $S \subseteq f(V)$. Also $U \cap V = 0_{Neu}$ implies that $f(U \cap V) = 0_N$ which implies that $f(U) \cap f(V) = 0_{Neu}$. Since $f$ is a $N_{eu}$−open mapping, $f(U)$ and $f(V)$ are disjoint $N_{eu}$−open sets in $Y$ containing $y_{(r,t,s)}$ and $S$ respectively. Thus $Y$ is strongly $N_{eu}$gr*−Regular.

## 6. Neutrosophic Generalized Regular Star Normal Spaces

In this section, we introduce $N_{eu}gr^*-$Normal and strongly $N_{eu}gr^*-$Normal spaces and study their properties and characteristics.

**Definition 6.1.** A $N_{eu}-$Top$-$Space $(X, T_N)$ is said to be $N_{eu}gr^*-$Normal if for any two disjoint $N_{eu}gr^*-$closed sets $A$ and $B$, there exist disjoint $N_{eu}gr^*-$open sets $U$ and $V$ such that $A \subseteq U$ and $B \subseteq V$.

**Theorem 6.2.** Let $(X, T_N)$ be a $N_{eu}-$Top$-$Space. Then the following statements are equivalent:

$(a)$ $X$ is $N_{eu}gr^*-$Normal.

$(b)$ For every $N_{eu}gr^*-$closed set $A$ in $X$ and every $N_{eu}gr^*-$open set $U$ containing $A$, there exists a $N_{eu}gr^*-$open set $V$ containing $A$ such that $N_{eu}gr^*-Cl(V) \subseteq U$.

$(c)$ For each pair of disjoint $N_{eu}gr^*-$closed sets $A$ and $B$ in $X$, there exists a $N_{eu}gr^*-$open set $U$ containing $A$ such that $[N_{eu}gr^*-Cl(U)] \cap B = 0_{Neu}$.

$(d)$ For each pair of disjoint $N_{eu}gr^*-$closed sets $A$ and $B$ in $X$, there exist $N_{eu}gr^*-$open sets $U$ and $V$ containing $A$ and $B$ respectively such that $[N_{eu}gr^*-Cl(U)] \cap [N_{eu}gr^*-Cl(V)] = 0_{Neu}$.

**Proof.** $(a) \Rightarrow (b)$: Let $U$ be a $N_{eu}gr^*-$open set containing the $N_{eu}gr^*-$closed set $A$. Then $B = U^C$ is a $N_{eu}gr^*-$closed set disjoint from $A$. Since $X$ is $N_{eu}gr^*-$Normal, there exist disjoint $N_{eu}gr^*-$open sets $V$ and $W$ containing $A$ and $B$ respectively. Then $N_{eu}gr^*-Cl(V)$ is disjoint from $B$. Since if $y_{(r,t,s)} \in B$, the set $W$ is a $N_{eu}gr^*-$open set containing $y_{(r,t,s)} \in B$ disjoint from $V$. Hence $N_{eu}gr^*-Cl(V) \subseteq U$.

$(b) \Rightarrow (c)$: Let $A$ and $B$ be disjoint $N_{eu}gr^*-$closed sets in $X$. Then $B^C$ is a $N_{eu}gr^*-$open set containing $A$. By $(b)$, there exists a $N_{eu}gr^*-$open set $U$ containing $A$ such that $N_{eu}gr^*-Cl(U) \subseteq B^C$. Hence $[N_{eu}gr^*-Cl(U)] \cap B = 0_{Neu}$. This proves $(c)$.

$(c) \Rightarrow (d)$: Let $A$ and $B$ be disjoint $N_{eu}gr^*-$closed sets in $X$. Then by $(c)$, there exists a $N_{eu}gr^*-$open set $U$ containing $A$ such that $[N_{eu}gr^*-Cl(U)] \cap B = 0_N$. Since $N_{eu}gr^*-Cl(U)$ is $N_{eu}gr^*-$closed, $B$ and $N_{eu}gr^*-Cl(U)$ are disjoint $N_{eu}gr^*-$closed sets in $X$. Again by $(c)$, there exists a $N_{eu}gr^*-$open set $V$ containing $B$ such that $[N_{eu}gr^*-Cl(U)] \cap [N_{eu}gr^*-Cl(V)] = 0_{Neu}$ This proves $(d)$.

$(d) \Rightarrow (a)$: Let $A$ and $B$ be disjoint $N_{eu}gr^*-$closed sets in $X$. By $(d)$, there exist $N_{eu}gr^*-$open sets $U$ and $V$ containing $A$ and $B$ respectively such that $[N_{eu}gr^*-Cl(U)] \cap [N_{eu}gr^*-Cl(V)] = 0_{Neu}$. Since $U \cap V \subseteq [N_{eu}gr^*-Cl(U)] \cap [N_{eu}gr^*-Cl(V)]$, $U$ and $V$ are disjoint $N_{eu}gr^*-$open sets containing $A$ and $B$ respectively. Hence the result of $(a)$ follows.

**Theorem 6.3.** A $N_{eu}-$Top$-$Space $(X, T_N)$ is $N_{eu}gr^*-$Normal if and only if for every $N_{eu}gr^*-$closed set $F$ and $N_{eu}gr^*-$open set $W$ containing $F$, there exists a $N_{eu}gr^*-$open set $U$ such that $F \subseteq U \subseteq N_{eu}gr^*-Cl(U) \subseteq W$.

**Proof.** Let $(X, T_N)$ be $N_{eu}gr^*-$Normal. Let $F$ be a $N_{eu}gr^*-$closed set and let $W$ be a $N_{eu}gr^*-$open set containing $F$. Then $F$ and $W^C$ are disjoint $N_{eu}gr^*-$closed sets. Since $X$ is $N_{eu}gr^*-$Normal, there exist disjoint $N_{eu}gr^*-$open sets $U$ and $V$ such that $F \subseteq U$ and $W^C \subseteq V$. Thus $F \subseteq U \subseteq V^C \subseteq W$. Since $V^C$ is $N_{eu}gr^*-$closed, so $N_{eu}gr^*-Cl(U) \subseteq N_{eu}gr^*-Cl(V^C) = V^C \subseteq W$. Thus $F \subseteq U \subseteq N_{eu}gr^*-Cl(U) \subseteq W$.

Conversely, suppose the condition holds. Let $G$ and $H$ be two disjoint $N_{eu}gr^*-$closed sets in $X$. Then $H^C$ is a $N_{eu}gr^*-$open set containing $G$. By assumption, there exists a $N_{eu}gr^*-$open set $U$ such that $G \subseteq U \subseteq N_{eu}gr^*-Cl(U) \subseteq H^C$. Since $U$ is $N_{eu}gr^*-$open and $Cl(U)$ is $N_{eu}gr^*-$closed. Then $(N_{eu}gr^*-Cl(U))^C$ is $N_{eu}gr^*-$open. Now $N_{eu}gr^*-Cl(U) \subseteq H^C$ implies $H \subseteq (N_{eu}gr^*-Cl(U))^C$. Also we have $U \cap (N_{eu}gr^*-Cl(U))^C \subseteq N_{eu}gr^*-Cl(U) \cap (N_{eu}gr^*-Cl(U))^C = 0_{Neu}$. That is $U$ and $(N_{eu}gr^*-Cl(U))^C$ are disjoint $N_{eu}gr^*-$open sets containing $G$ and $H$ respectively. This shows that $(X, T_N)$ is $N_{eu}gr^*-$Normal.

**Theorem 6.4.** Let $(X, T_N)$ be a $N_{eu}$ −Top −Space. Then the following statements are equivalent:

$(a)$ $X$ is $N_{eu}gr^*$−Normal

$(b)$ For any two $N_{eu}gr^*$−open sets $U$ and $V$ whose union is $1_{Neu}$, there exist $N_{eu}gr^*$−closed subsets $A$ of $U$ and $B$ of $V$ such that $A \bigcup B = 1_{Neu}$.

**Proof.** $(a) \Rightarrow (b)$: Let $U$ and $V$ be two $N_{eu}gr^*$−open sets in a $N_{eu}gr^*$−Normal space $X$ such that $U \bigcup V = 1_N$. Then $U^C$ and $V^C$ are disjoint $N_{eu}gr^*$−closed sets. Since $X$ is $N_{eu}gr^*$−Normal, then there exist disjoint $N_{eu}gr^*$−open sets $G$ and $H$ such that $U^C \subseteq G$ and $V^C \subseteq H$. Let $A = G^C$ and $B = H^C$. Then $A$ and $B$ are $N_{eu}gr^*$−closed subsets of $U$ and $V$ respectively such that $A \bigcup B = 1_{Neu}$. This proves $(b)$.

$(b) \Rightarrow (a)$: Let $A$ and $B$ be disjoint $N_{eu}gr^*$−closed sets in $X$. Then $A^C$ and $B^C$ are $N_{eu}gr^*$−open sets whose union is $1_{Neu}$. By $(b)$, there exist $N_{eu}gr^*$−closed sets $E$ and $F$ such that $E \subseteq A^C$, $F \subseteq B^C$ and $E \bigcup F = 1_{Neu}$. Then $E^C$ and $F^C$ are disjoint $N_{eu}gr^*$−open sets containing $A$ and $B$ respectively. Therefore $X$ is $N_{eu}gr^*$−Normal

**Definition 6.5.** A $N_{eu}$ −Top −Space $(X, T_N)$ is said to be strongly $N_{eu}gr^*$−Normal if for every pair of disjoint $N_{eu}$ −closed sets $A$ and $B$ in $X$, there are disjoint $N_{eu}gr^*$−open sets $U$ and $V$ in $X$ containing $A$ and $B$ respectively.

**Theorem 6.6.** Every $N_{eu}gr^*$−Normal space is strongly $N_{eu}gr^*$−Normal

**Proof.** Suppose $X$ is $N_{eu}gr^*$−Normal Let $A$ and $B$ be disjoint $N_{eu}$ −closed sets in $X$. Then $A$ and $B$ are $N_{eu}gr^*$−closed in $X$. Since $X$ is $N_{eu}gr^*$−Normal, there exist disjoint $N_{eu}$ −open sets $U$ and $V$ containing $A$ and $B$ respectively. Since every $N_{eu}$ −open set is $N_{eu}gr^*$−open set. Therefore $U$ and $V$ are $N_{eu}gr^*$−open sets in $X$. This implies that $X$ is strongly $N_{eu}gr^*$−Normal

**Theorem 6.7.** Let $(X, T_N)$ be a $N_{eu}$ −Top −Space. Then the following statements are equivalent:

$(a)$ $X$ is strongly $N_{eu}gr^*$−Normal

$(b)$ For every $N_{eu}$ −closed set $F$ in $X$ and every $N_{eu}$ −open set $U$ containing $F$, there exists a $N_{eu}gr^*$−open set $V$ containing $F$ such that $N_{eu}gr^*\text{−}Cl(V) \subseteq U$. $(c)$ For each pair of disjoint $N_{eu}$ −closed sets $A$ and $B$ in $X$, there exists a $N_{eu}gr^*$−open set $U$ containing $A$ such that $N_{eu}gr^*\text{−}Cl(U) \bigcap B = 0_{Neu}$.

**Proof.** $(a) \Rightarrow (b)$: Let $U$ be a $N_{eu}$ −open set containing $N_{eu}$ −closed set $F$. Then $H = U^C$ is a $N_{eu}$ −closed set disjoint from $F$. Since $X$ is strongly $N_{eu}gr^*$−Normal, there exist disjoint $N_{eu}gr^*$−open sets $V$ and $W$ containing $F$ and $H$ respectively. Then $N_{eu}gr^*\text{−}Cl(V)$ is disjoint from $H$, since if $y_{(r,t,s)} \in H$, the set $W$ is a $N_{eu}gr^*$−open set containing $y_{(r,t,s)}$ disjoint from $V$. Hence $N_{eu}gr^*\text{−}Cl(V) \subseteq U$.

$(b) \Rightarrow (c)$: Let $A$ and $B$ be disjoint $N_{eu}$ −closed sets in $X$. Then $B^C$ is a $N_{eu}$ −open set containing $A$. By $(b)$, there exists a $N_{eu}gr^*$−open set $U$ containing $A$ such that $N_{eu}gr^*\text{−}Cl(U) \subseteq B^C$. Hence $N_{eu}gr^*\text{−}Cl(U) \bigcap B = 0_{Neu}$. This proves $(c)$.

$(c) \Rightarrow (a)$: Let $A$ and $B$ be disjoint $N_{eu}gr^*$−closed sets in $X$. By (c), there exists a $N_{eu}gr^*$−open set $U$ containing $A$ such that $\left[N_{eu}gr^*\text{−}Cl(U)\right] \bigcap B = 0_{Neu}$. Take $V = \left(N_{eu}gr^*\text{−}Cl(U)\right)^C$. Then $U$ and $V$ are disjoint $N_{eu}gr^*$−open sets containing $A$ and $B$ respectively. Thus $X$ is strongly $N_{eu}gr^*$−Normal.

**Theorem 6.8.** Let $(X, T_N)$ be a $N_{eu}$ −Top −Space. Then the following statements are equivalent:

$(a)$ $X$ is strongly $N_{eu}gr^*$−Normal.

$(b)$ For any two $N_{eu}$ −open sets $U$ and $V$ whose union is $1_N$, there exist $N_{eu}gr^*$−closed subsets $A$ of $U$ and $B$ of $V$ such that $A \bigcup B = 1_{Neu}$.

**Proof.** $(a) \Rightarrow (b)$: Let $U$ and $V$ be two $N_{eu}$ −open sets in a strongly $N_{eu}gr^*$−Normal space $X$ such that $U \bigcup V = 1_{Neu}$. Then $U^C$ and $V^C$ are disjoint $N_{eu}$ −closed sets. Since $X$ is strongly $N_{eu}gr^*$−Normal, then there exist disjoint $N_{eu}gr^*$−open sets $G$ and $H$ such that $U^C \subseteq G$ and $V^C \subseteq H$. Let $A = G^C$ and $B = H^C$. Then $A$ and $B$ are $N_{eu}gr^*$−closed subsets of $U$ and $V$ respectively such that $A \bigcup B = 1_{Neu}$.

$(b) \Rightarrow (a)$: Let $A$ and $B$ be disjoint $N_{eu}$ −closed sets in $X$. Then $A^C$ and $B^C$ are $N_{eu}$ −open sets such that $A^C \bigcup B^C = 1_{Neu}$. By $(b)$, there exist $N_{eu}gr^*$−closed sets

$G$ and $H$ such that $G \subseteq A^C$, $H \subseteq B^C$ and $G \bigcup H = 1_N$. Then $G^C$ and $H^C$ are disjoint $N_{eu}gr^*$-open sets containing $A$ and $B$ respectively. Therefore, $X$ is strongly $N_{eu}gr^*$-Normal.

## 7. Conclusion

Topology is an important and major area of mathematics, and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the $N_{eu}-set$ theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of $N_{eu}$-Top-Spaces. We introduced the idea of new types of $N_{eu}$-compactness, $N_{eu}$-cconnectedness, $N_{eu}$-Regular spaces, and $N_{eu}$-Normal spaces defined in terms of $N_{eu}gr^*$-open and $N_{eu}gr^*$-closed sets in a $N_{eu}$-Top-Space $(X, T_N)$ namely, $N_{eu}gr^*$-compact spaces, $N_{eu}gr^*$-Lindelof spaces, countably $N_{eu}gr^*$-compact spaces, $N_{eu}gr^*$-connected spaces, $N_{eu}gr^*$-cseparated sets, $N_{eu}$-Super-$gr^*$-connected spaces, $N_{eu}$-Extremely-$gr^*$-disconnected spaces, and $N_{eu}$-Strongly-$gr^*$-connected spaces, $N_{eu}gr^*$-Regular spaces, strongly $N_{eu}gr^*$-Regular spaces, $N_{eu}gr^*$-Normal spaces, and strongly $N_{eu}gr^*$-Normal spaces. Also, several of their topological properties are investigated. Finally, some effects of various kinds of $N_{eu}$-functions on them are studied. and have established several interesting properties. Because there exist compact connections between $N_{eu}$-sets and information systems, we can use the results deducted from the studies on $N_{eu}$-Top-Space to improve these kinds of connections. We see that this research work will help researchers enhance and promote further study on the $N_{eu}$-Topology to carry out a general framework for their applications in practical life.

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