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Intelligent algorithm for trapezoidal interval valued neutrosophic network analysis

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Abstract: The shortest path problem has been one of the most fundamental practical problems in network analysis. One of the good algorithms is Bellman-Ford, which has been applied in network, for the last some years. Due to complexity in the decision-making process, the decision makers face complications to express their view and judgment with an exact number for single valued membership degrees under neutrosophic environment. Though the interval number is a special situation of the neutrosophic, it did not solve the shortest path problems in an absolute manner. Hence, in this work, the authors have introduced the score function and accuracy function of trapezoidal interval valued neutrosophic numbers with their illustrative properties. These properties provide important theoretical base of the trapezoidal interval valued neutrosophic number. Also, they proposed an intelligent algorithm called trapezoidal interval valued neutrosophic version of Bellman's algorithm to solve neutrosophic shortest path problem in network analysis. Further, comparative analysis has been made with the existing algorithm.

1 Introduction

In wireless communication and digital electronics, the short distance is communicated using multifunctional sensors. This type of sensors consists of sensing and processing the data. Embedded systems, wireless communication, distributed processing, micro-electro-mechanical systems and applications using wireless sensors are the developments in the technology of sensors and these developments have contributed to large transformation in wireless sensor networks. Sensors help and boost work performed in the field of both industry and our daily life. Sensor network system is away from the actual phenomenon and could collect and process a huge number of data. These sensors are using sense perception. The sensor network and algorithm must possess self-organising capabilities.

Neighbourhood nodes are close to each other and the nodes are used for constant sensing. Multichip sensor networks are used to consume low power than other sensors. The topological information would be provided by every each node of the sensor network. Interconnection network can be used for parallel computing. Shortest path algorithms are used to message from any source to any destination. Bellman-Ford is mostly applied for a large network with a stable node. A set whose elements have degrees of membership called fuzzy set (FS) in 1965 [1] and mainly deals with numerous real-world situations, where the data possesses some sort of uncertainty.

The concept of FS deals with only the membership value of the elements, not the non-membership value. This issue was sorted out by intuitionistic FS (IFS) introduced by Atannasov in 1975 [2] which allows both the membership function (MF) and non-membership function. Since the real-world situations may contain indeterminacy in the data, FS and IFS could not deal with indeterminacy of the data. This problem was rectified by neutrosophic set (NS), which is the generalisation of FS and IFS, introduced by Smarandache [3].

NS is a set in which, all the elements have degree of membership, indeterminacy and non-membership and the sum of these MFs should be less than or equal to 3. All three MFs are independent of each other. Since the NSs are difficult to apply in real-world problems, Wang *et al.* introduced single valued NSs [4]. Uncertainty of the elements can be captured using fuzzy numbers and intuitionistic fuzzy numbers. In the same way, neutrosophic numbers are very useful in capturing uncertainty and indeterminacy of the elements. Hence, it is a special case of the NS which enhances the domain of real numbers to neutrosophic numbers. Fuzzy shortest path problems (FSPP) can be solved by considering the edge weights of the network as fuzzy or uncertain using Bellman dynamic programming approach and multi-objective linear programming technique [5].

Shortest path problem (SPP) has been solved by many researchers under fuzzy and intuitionistic fuzzy environments [6–8]. The concept of Bellman's algorithm has been applied in a fuzzy network [9] for solving SPP and it is not applied in neutrosophic network so far. Distance measure can be obtained using single and interval valued trapezoidal neutrosophic numbers in a multi-attribute decision-making problem [10]. Dijkstra algorithm is a very useful and optimised one to solve the SPP but incapable to handle negative weights, whereas Bellman can deal with negative weights. SPP also can be solved by using single valued neutrosophic graph. The information that a sender does convey in communication with a receiver is called the linguistic information involved with the nature of language and communication. This information is a correlation between the knowledge of the people about the old and new information and deviation in grammatical structure, reacting to this knowledge.

Finding a shortest path between two or more vertices is called SPP in which the sum of edge weights should be minimum. The practicability of a path is resolved its length under familiar measures such as distance and its corresponding nature. In the transportation system, with mode options, for a passenger to reach the destination from a source point, the selection of the mode and destination guide of the journey for an optimised route would be specified by linguistic terms. Hence, the linguistic information and SPPs are connected.

To deal with inconsistency, uncertainty, ambiguity, impreciseness and indeterminate, many methods have been recommended by the researchers under different environments, namely FS, IFS, interval valued IFS, triangular IFS, trapezoidal IFS and NS where the information can be represented in the form of triangles and trapezoid under all these environments. Also, the membership values lie in the real unit interval [0, 1]. Hence, trapezoidal interval valued neutrosophic number (TrIVNN) helps in real-world problems where the information is uncertain and indeterminate between some ranges of acceptable behaviour. Therefore, TrIVNN is the key to extract the MFs of truth, indeterminacy and falsity whose values depend on both trapezoidal neutrosophic numbers and the intervals [11–15].

In [16, 17], Broumi *et al.* made an overview of the SPP under various environments and solved SPP using single valued and triangular and trapezoidal interval valued neutrosophic environments. Aggregation operators for interval valued generalised single valued neutrosophic trapezoidal number have been derived and applied in decision-making problem [18]. An extension of FSs called type-2 FSs and its special cases called interval type-2 FSs have growing applications in control systems, edge detection in image processing and other medical fields. SPPs can be solved using triangular and trapezoidal interval neutrosophic environments as an extension of NSs. From the overview of solving SPP under various sets environments, one can understand the difference and capacity of handling uncertainty with various levels [19–25].

In [26], the authors considered the concept of (extended) derivable single-valued neutrosophic graph as the energy clustering of wireless sensor networks and applied this concept as a tool in wireless sensor (hyper) networks. In [27], Broumi *et al.* applied single-valued neutrosophic techniques for analysis of WIFI connection. Jan *et al.* [28] developed the concept of constant single valued neutrosophic graphs and applied it to a real-world problem of Wi-Fi system. For more information on the application of neutrosophic theory, we refer the readers to [29–36]. NSs are usually applied to model linguistic information. In our previous work, we solved SPP for a network with triangular and trapezoidal interval valued neutrosophic edge weights using an improved algorithm with the operational laws and new score function of interval valued neutrosophic numbers. Also, comparative analysis has been done with the existing methods.

In this paper, we are motivated to present neutrosophic version of Bellman's algorithm for solving neutrosophic SPP (NSPP). Therefore for the first time, we proposed trapezoidal interval valued neutrosophic version of Bellman's algorithm to solve NSPP in network analysis, where the edge weight is characterised by TrIVNN.

The rest of this paper is organised as follows. In Section 2, literature review is given with the existing work and application side. In Section 3, some concepts and theories are reviewed. Section 4 introduces the score function and accuracy function of TrIVNNs with their illustrative properties. In Section 5, an intelligent algorithm called trapezoidal interval valued neutrosophic version of Bellman-Ford algorithm is proposed with a numerical example as an application of our proposed algorithm. Section 6 gives the significance of the proposed algorithm. Section 7 gives the comparative analysis of the proposed algorithm with the existing algorithm to solve NSPP in network analysis. The last but not least, in Section 8 the conclusion is drawn with the advantages and limitations of the proposed work and some hints for further research is given.

2 Literature review

In this section, literature review on existing work and application side is given for solving SPP under fuzzy, intuitionistic fuzzy and neutrosophic environments.

2.1 Existing work

Zadeh [1] proposed FSs. Atanassov [2] introduced IFSs. Smarandache [3] proposed neutrosophic logic, set and probability. Wang *et al.* [4] invented single valued NSs. Bellman [13] proposed routing problem with functional equation approach. Bellman-Ford algorithm is explained in [14]. Lathamaheswari *et al.* [22] analysed the different applications of type-2 fuzzy in the field of bio-medicine. Lathamaheswari *et al.* [24] re-examined the usage of type-2 fuzzy controller in the area of control system.

2.2 Application side

De and Bhincher [5] described two different methods to solve SPP namely Bellman dynamic programming and multi-objective linear programming. Kumar *et al.* [6] introduced a new algorithm to solve SPP under interval valued intuitionistic trapezoidal fuzzy environment. Meenakshi and Kaliraja [7] determined shortest path for interval valued fuzzy network. Elizabeth and Sujatha [8] solved FSPP using interval valued fuzzy number matrices. Das and De [9] figured out SPP under intuitionistic fuzzy setting. Biswas *et al.* [10] introduced a new strategy for multi-attribute decision-making problem under interval trapezoidal neutrosophic environment. Broumi *et al.* [11] estimated a shortest path using single valued trapezoidal neutrosophic number as the edge weights. Broumi *et al.* [12] solved NSPP using Dijkstra algorithm. Broumi *et al.* [15] dealt with SPP using single valued neutrosophic graphs. Broumi *et al.* [16] made an analysis on SPP under various environments. Broumi *et al.* [17] solved SPP under interval valued trapezoidal and triangular neutrosophic setting. Deli [18] introduced interval valued generalised single valued neutrosophic trapezoidal number and its aggregation operators, also applied the proposed concept in decision-making problem. Giri *et al.* [19] solved a decision-making problem using TOPSIS method under interval trapezoidal neutrosophic environment. Deli *et al.* [20] determined a decision-making problem using single and interval valued trapezoidal and triangular neutrosophic numbers. Nagarajan *et al.* [21] introduced a new technique for edge detection on DICOM image under type-2 fuzzy environment. Nagarajan *et al.* [23] proposed a technique for image extraction on DICOM image under type-2 fuzzy environment. Sellappan *et al.* [25] evaluated risk priority number in design failure modes and used factor analysis for effects analysis. Mohammad and Arsham Borumand [26] introduced achievable single valued neutrosophic graphs and applied in wireless sensor networks. Broumi *et al.* [27] estimated information processing using mobile ad-hoc network with an example under neutrosophic environment. Jan *et al.* [28] introduced and studied the characteristics of constant single valued neutrosophic graph and applied in Wi-Fi network system. Harish [29] solved multi-attribute group decision-making problem using novel neutrality aggregation operators under single valued neutrosophic setting. Dimple and Harish [30] introduced some modified results of the subtraction and division operations on interval NSs. Harish and Nancy [31] solved multi-criteria decision-making (MCDM) problem using Frank Choquet Heronian mean operator under single valued neutrosophic setting. Nancy and Harish [32] introduced a novel divergence measure and used in TOPSIS method for MCDM problem under single-valued neutrosophic environment. Harish and Nancy [33] proposed some hybrid weighted aggregation operators and applied in MCDM problem under NS environment. Harish and Nancy [34] introduced new logarithmic operational laws for single-valued neutrosophic numbers and applied in multi-attribute decision-making problem. Harish and Nancy [35] proposed non-linear programming method under interval NS setting and applied in MCDM problem. Nancy and Harish [36] introduced an improved score function for the ranking proposed of NSs and applied in decision-making process.

3 Overview of trapezoidal interval valued neutrosophic number

In this section, we review some basic concepts regarding NSs, single valued NSs, trapezoidal NSs and some existing ranking functions for trapezoidal neutrosophic numbers which are the background of this study and will help us to further research.

3.1 Neutrosophic set [3]

Let ξ be points (objects) set and its generic elements denoted by x; we define the neutrosophic set A (NS A) as the form $\ddot{A} = \{\langle x{:}T_{\ddot{A}}(x), I_{\ddot{A}}(x), F_{\ddot{A}}(x)\rangle, x \in \xi\}$, where the functions T, I, F: $\xi \to]{-0},1+[$ are called the truth-MF, an indeterminacy-membership function, and a falsity-membership function, respectively, and they satisfy the following condition:

$$-0 \le T_{\ddot{A}}(x) + I_{\ddot{A}}(x) + F_{\ddot{A}}(x) \le 3+. \tag{1}$$

The values of these three MFs $T_{\ddot{A}}(x)$, $I_{\ddot{A}}(x)$ and $F_{\ddot{A}}(x)$ are real standard or non-standard subsets of $]{-0},1+[$. As we have difficulty in applying NSs to practical problems. Wang *et al.* [4] proposed the concept of a SVNS that represents the simplification of a NS and can be applied to real scientific and technical applications.

3.2 Single valued NS [4]

A single valued NS $\ddot{A}$ (SVNS $\ddot{A}$) in the universe set ξ is defined by the set

$$\ddot{A} = \{ < x{:}T_{\ddot{A}}(x), I_{\ddot{A}}(x), F_{\ddot{A}}(x) > , x \in \xi\} \tag{2}$$

where $T_{\ddot{A}}(x), I_{\ddot{A}}(x), F_{\ddot{A}}(x) \in [0, 1]$ satisfying the condition

$$0 \le T_{\ddot{A}}(x) + I_{\ddot{A}}(x) + F_{\ddot{A}}(x) \le 3 \tag{3}$$

3.3 Trapezoidal interval valued neutrosophic set [10]

Let x be TrIVNN. Then its truth, indeterminacy and falsity MFs are given by

$$T_x(z) = \begin{cases} \dfrac{(z-a)t_x}{(b-a)}, & a \le z < b, \\ t_x, & b \le z \le c \\ \dfrac{(d-z)t_x}{(d-c)}, & c \le z \le d \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Its indeterminacy MF is

$$I_x(z) = \begin{cases} \dfrac{(b-z)+(z-a)i_x}{(b-a)}, & a \le z < b \\ i_x, & b \le z \le c \\ \dfrac{z-c+(d-z)i_x}{d-c} & c < z \le d \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

Its falsity MF is

$$F_x(z) = \begin{cases} \dfrac{b-z+(z-a)f_x}{b-a}, & a \le z < b \\ f_x, & b \le z \le c \\ \dfrac{z-c+(d-z)f_x}{d-c}, & c < z \le d \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $0 \le T_x(z) \le 1$, $0 \le I_x(z) \le 1$ and $0 \le F_x(z) \le 1$, also t_x, i_x, f_x are subset of [0,1] and $0 \le a \le b \le c \le d \le 1$, $0 \le \sup(t_x) + \sup(i_x) + \sup(f_x) \le 3$; Then x is called an interval trapezoidal neutrosophic number $x = ([a, b, c, d]; t_x, i_x, f_x)$. We take $t_x = [\underline{t}, \overline{t}]$, $i_x = [\underline{i}, \overline{i}]$ and $f_x = [\underline{f}, \overline{f}]$.

3.4 Ranking technique [10]

Let $\tilde{a}$ and $\tilde{r}$ be two TrIVNNs, the ranking of $\tilde{a}$ and $\tilde{r}$ by score function and accuracy function is described as follows:

(i) if $s(\hat{r}^N) \prec s(\hat{s}^N)$ then $\hat{r}^N \prec \hat{s}^N$
(ii) if $s(\hat{r}^N) \simeq s(\hat{s}^N)$ and if

(a) $a(\hat{r}^N) \prec a(\hat{s}^N)$ then $\hat{r}^N \prec \hat{s}^N$
(b) $a(\hat{r}^N) \succ a(\hat{s}^N)$ then $\hat{r}^N \succ \hat{s}^N$
(c) $a(\hat{r}^N) \simeq a(\hat{s}^N)$ then $\hat{r}^N \simeq \hat{s}^N$

3.5 Bellman dynamic programming [13]

Given an acyclic directed connected graph $G=(V, E)$ with 'n' vertices where node '1' is the source node and 'n' is the destination node. The nodes of the given network are organised with the topological ordering (Eij: $i<j$). Now for the given network the shortest path can be obtained based on the formulation of Bellman dynamic programming by forward pass computation method.

The formulation of Bellman dynamic programming is described as follows:

$f(1)=0$

$$f(i) = \min_{i<j}\left\{ f(i) + d_{ij} \right\} \tag{7}$$

where d_{ij} is the weight of the directed edge E_{ij}, $f(i)$ is the length of the shortest path of ith node from the source node 1.

3.6 Advantages of trapezoidal interval valued neutrosophic number [17]

There are some advantages of using TrIVNN as follows:

(i) Interval trapezoidal neutrosophic number is a generalised form of single valued trapezoidal neutrosophic number.
(ii) In this number, the trapezoidal number is characterised by three independent membership degrees, which are in interval form.
(iii) The number can flexibly express neutrosophic information than the single valued neutrosophic trapezoidal number.

Therefore, the number can be employed to solve neutrosophic multiple attribute decision-making problem, where the preference values cannot be expressed in terms of single valued trapezoidal neutrosophic number.

4 Proposed concepts

Score function and accuracy function are measurement functions to rank fuzzy, intuitionistic and neutrosophic numbers. While solving NSPP, score function finds the aggregate value of each path and measures their accuracy and provides the relevant score that measures how well the path satisfies the requirement. Accuracy function gives the most intuitive performance measure. Here, the score and accuracy functions are introduced with their illustrative properties for trapezoidal interval neutrosophic numbers.

4.1 Score function of trapezoidal interval valued neutrosophic number

Let $x = ([a, b, c, d]; [\underline{t}, \overline{t}], [\underline{i}, \overline{i}], [\underline{f}, \overline{f}])$ be a TrIVNN then its score function is defined by

$$S(x) = \frac{1}{16}(a + b + c + d)(2 + \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f}) \quad (8)$$

and $S(x) \in [0, 1]$. Here we take $0 \le a \le b \le c \le d \le 1$, t_x, i_x, f_x are subset of $[0, 1]$ where $t_x = [\underline{t}, \overline{t}]$, $i_x = [\underline{i}, \overline{i}]$ and $f_x = [\underline{f}, \overline{f}]$.

4.1.1 Property: Score function is bounded on [0, 1].

Proof: Since, $0 \le a \le b \le c \le d \le 1$, we have

$$0 \le a + b + c + d \le 4 \quad (9)$$

Now

$$\begin{aligned} &-4 \le \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f} \le 2 \\ &=> 2 - 4 \le 2 + \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f} \le 4 \\ &=> -2 \le 2 + \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f} \le 4 \end{aligned} \quad (10)$$

Multiplying (9) and (10), we get

$$\begin{aligned} &0 \le (a + b + c + d)(2 + \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f}) \le 16 \\ &=> 0 \le \frac{1}{16}(a + b + c + d)(2 + \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f}) \le 1 \end{aligned}$$

Therefore, score function is bounded.
Example: Let a = ([0.1,0.2,0.3,0.4]; [0.1,0.2], [0.2,0.3], [0.4,0.5]) be a TrIVNN then its score value is $Sc(a) = \frac{1}{16}(0.1 + 0.2 + 0.3 + 0.4)(2 + .1 + .2 - .2 - .3 - .4 - .5) = 0.07875 \in [0, 1]$, hence the result. □

4.2 Accuracy function of trapezoidal interval valued neutrosophic number

Let $x = ([a, b, c, d]; [\underline{t}, \overline{t}], [\underline{i}, \overline{i}], [\underline{f}, \overline{f}])$ be a TrIVNN then its accuracy function is defined by

$$Ac(x) = \frac{1}{8}(c + d - a - b)(2 + \underline{t} + \overline{t} - \underline{f} - \overline{f}) \quad (11)$$

and $Ac(x) \in [0, 1]$. Here we take $0 \le a \le b \le c \le d \le 1$ and t_x, i_x, f_x are the subset of [0, 1] where $t_x = [\underline{t}, \overline{t}]$, $i_x = [\underline{i}, \overline{i}]$ and $f_x = [\underline{f}, \overline{f}]$.

4.2.1 Property: Accuracy function is bounded on [0, 1].

Proof: Since $0 \le a \le b \le c \le d \le 1$, we have

$$-2 \le c + d - a - b \le 2 \quad (12)$$

$$\begin{aligned} &\Rightarrow -2 \le \underline{t} + \overline{t} - \underline{f} - \overline{f} \le 2 \\ &=> 0 \le 2 + \underline{t} + \overline{t} - \underline{f} - \overline{f} \le 4 \end{aligned} \quad (13)$$

Multiplying (12) and (13), we get

$$\begin{aligned} &0 \le (c + d - a - b)(2 + \underline{t} + \overline{t} - \underline{f} - \overline{f}) \le 8 \\ &=> 0 \le \frac{1}{8}(a + b + c + d)(2 + \underline{t} + \overline{t} - \underline{f} - \overline{f}) \le 1 \end{aligned}$$

Therefore, accuracy function is bounded and is proved by a numerical illustration in Section 4.2.2. □

4.2.2 Numerical illustration: Let x =([0.1,0.2,0.3,0.4]; [0.1,0.2],[0.2,0.3],[0.4,0.5]) be the TrIVNN then its accuracy value is

$$\begin{aligned} Ac(x) &= \frac{1}{8}(0.1 + 0.2 + 0.3 + 0.4)(2 + .1 + .2 - .4 - .5) \\ &= 0.175 \in [0, 1] \end{aligned}$$

Hence the result.

5 Computation of shortest path based on TrIVNN

This section introduces an algorithmic approach to solve NSPP. Consider a network with 'n' nodes where the node '1' is the source node and the node 'n' is the destination node under trapezoidal interval valued neutrosophic environment. The neutrosophic distance between the nodes is denoted by d_{ij} (node 'i' to node 'j'). Here $M_{N(i)}$ denotes the set of all nodes having a relation with the node 'i'.

5.1 Revised version of trapezoidal interval valued neutrosophic Bellman-Ford algorithm

Applying the concept of Bellman's algorithm in neutrosophic environment, we get trapezoidal interval valued neutrosophic version of Bellman-Ford algorithm (Algorithm 1, see Fig. 1).

5.2 Illustrative example

The revised version of Bellman-Ford algorithm under trapezoidal interval valued neutrosophic environment is demonstrated by an illustrative example as follows for a better understanding.

For the illustrative purpose, a numerical problem from [11] is considered, to prove the inherent application of the proposed algorithm. It shows the clear procedure of the proposed algorithm.

Consider a network (Fig. 2) with six nodes and eight edges and the edge weights are characterised by TpIVNNs, where the first node is the source node and the sixth node is the destination node. Trapezoidal interval valued neutrosophic distance is given in Table 1.

In this situation, we need to evaluate the shortest distance from source node, i.e. node 1 to destination node, i.e. node 6. Table 1 represents the edges and their trapezoidal interval valued neutrosophic distance.

For all the edges, trapezoidal interval valued neutrosophic distance is reduced into crisp numbers using score function as a deneutrosophication process and is represented by Table 2.

According to the proposed neutrosophic Bellman-Ford algorithm in Section 5.1, the shortest path from node one to node six can be computed by Algorithm 2 (see Fig. 3).

Therefore, the path P: 1→2→5→6 is identified as the trapezoidal interval valued neutrosophic shortest path, and the crisp shortest path is 0, 85.

The neutrosophic shortest path can be obtained for the network with a large number of vertices and edges also.

6 Significance of the proposed work

The proposed trapezoidal interval valued neutrosophic version of Bellman-Ford algorithm has a potential significance as it has the following qualities:

(i) It deals with the network in which the edge weights are TrIVNNs and so that it characterises membership, indeterminacy and falsity of each edge.

```
nrank[s] ← 0
ndist[s] ← Empty neutrosophic number
Add s into Q
For each node i (except the s) in the neutrosophic graph G
rank[i] ← ∞
        Add iinto Q
End For
u←s
While(Q is not empty)
remove the vertex u from Q
For each adjacent vertex v of  vertex u
        relaxed←False
temp_ndist[v] ←  ndist[u]  ⊕ edge_weight(u,v)
//⊕ represents the addition of neutrosophic//
temp_nrank[v]
←rank_of_neutrosophic(temp_ndist[v])
Iftemp_nrank[v]<nrank[v]   then
ndist[v] ←temp_ndist[v]
nrank[v] ← temp_nrank[v]
prev[v] ←u
        End If
End For
If  relaxed equals False then
exit the loop
        End If
u ←Node in Q with minimum rank value
End While
  For each arc(u,v) in neutrosophic graph G do
If nrank[v]> rank_of_neutrosophic(ndist[u] ⊕
edge_weight(u,v))
return false
End If
End For
The neutrosophic number ndist[u] is a neutrosophic
number and it represents the shortest path between
source node s and end node u.
```

Fig. 1 *Algorithm 1: Trapezoidal interval valued neutrosophic Bellman-Ford algorithm for shortest path analysis of the network*

(ii) It proceeds with the concept of relaxation, whither approximations to the exact distance are replaced by better ones until they finally reach the solution.

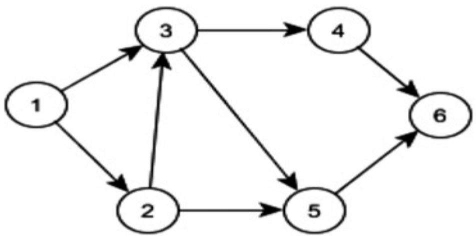

Fig. 2 *Network with six vertices and eight edges [Broumi et al. [11]]*

Table 1 Details of edge information in terms of TrIVNNs

Edges	Trapezoidal interval valued neutrosophic distance
1-2(e_1)	⟨(0.1, 0.2, 0.3, 0.4); [0.1, 0.2], [0.2, 0.3], [0.4, 0.5]⟩
1-3(e_2)	⟨(0.2, 0.5, 0.7, 0.8); [0.2, 0.4], [0.3, 0.5], [0.1, 0.2]⟩
2-3(e_3)	⟨(0.3, 0.7, 0.8, 0.9); [0.3, 0.4], [0.1, 0.2], [0.3, 0.5]⟩
2-5(e_4)	⟨(0.1, 0.5, 0.7, 0.9); [0.1, 0.3], [0.3, 0.4], [0.2, 0.3]⟩
3-4(e_5)	⟨(0.2, 0.4, 0.8, 0.9); [0.2, 0.3], [0.2, 0.5], [0.4, 0.5]⟩
3-5(e_6)	⟨(0.3, 0.4, 0.5, 1); [0.3, 0.6], [0.1, 0.2], [0.1, 0.4]⟩
4-6(e_7)	⟨(0.7, 0.8, 0.9, 1); [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]⟩
5-6(e_8)	⟨(0.2, 0.4, 0.5, 0.7); [0.2, 0.3], [0.3, 0.4], [0.1, 0.5]⟩

Table 2 Details of deneutrosophication value of edge (i, j)

Edges	Score function	Edges	Score function
e_{12}	0,05625	e_{34}	0,416875
e_{13}	0,48125	e_{35}	0,56375
e_{23}	0,6075	e_{46}	0,85
e_{25}	0,44	e_{56}	0,36

f(1)=0

f(2)=$\min_{i<2}\{f(1)+c_{12}\}$=$c^{*}_{12}$=0,05625

f(3)=$\min_{i<3}\{f(i)+c_{i3}\}$=$\min\{f(1)+c_{13}; f(2)+c_{23}\}$

={0+0,48125, 0,05625+0,6075}={0,48125; 0,66375}=0,48125

f(4)=$\min_{i<4}\{f(i)+c_{i4}\}$=$\min\{f(3)+c_{34}\}$={0,48125+0,416875}=0,89

f(5)=$\min_{i<5}\{f(i)+c_{i5}\}$=$\min\{f(2)+c_{25}; f(3)+c_{35}\}$

={0,05625+0,44; 0,48125+0,56375}={0,49, 1,045}=0,49

f(6)=$\min_{i<6}\{f(i)+c_{i6}\}$=$\min\{f(4)+c_{46}; f(5)+c_{56}\}$

={0,89+0,85;0,49+0,36}={1.74, 0,85}=0,85

Thus, f(6)=$f(5)+c_{56}$=$f(2)+c_{25}+c_{56}$=$f(1)+c_{12}+c_{25}+c_{56}$

=$c_{12}+c_{25}+c_{56}$.

Fig. 3 *Algorithm 2: Steps involved in finding trapezoidal interval valued neutrosophic shortest path*

(iii) This revised version of Bellman-Ford algorithm simply relaxes all the edges for $|V|-1$ times. In all these repetitions, the number of vertices with properly calculate distances become larger, from which it follows that, finally all vertices will get their exact distances. Here $|V|$ is the number of vertices in the trapezoidal interval valued neutrosophic network.

Hence, this proposed trapezoidal interval valued neutrosophic revised version of Bellman-Ford algorithm can be applied to a large number of inputs.

Table 3 Comparison of neutrosophic shortest path using the proposed method and existing method

Method	Neutrosophic shortest path length
in [18], Broumi *et al.* solved NSPP with triangular interval valued neutrosophic numbers and TrIVNNs as the edge weights of the network with six edges and eight edges	0.485 (using improved algorithm)
in this present work, we solved NSPP for the network with TrIVNNs as the edge weights	0.85 (using an intelligent algorithm called revised version of Bellman-Ford algorithm)

7 Comparative analysis

In [18], the authors solved NSPP under triangular and trapezoidal neutrosophic environment using an improved algorithm. However, NSPP is not solved using Bellman algorithm under trapezoidal interval neutrosophic environment to date. Hence, the comparative analysis is made in Table 3.

8 Conclusions

Hence in this work, the new definitions of score function and accuracy functions of trapezoidal interval neutrosophic numbers and their properties with numerical example are proposed. Also, the neutrosophic version of Bellman's algorithm based on the TrIVNN called an intelligent algorithm, which expresses the flexibility of the neutrosophic information absolutely under trapezoidal interval valued neutrosophic environment with a numerical example is proposed. In the future, the bipolar neutrosophic version of Bellman algorithm can be introduced.

8.1 Advantages of the proposed work

The proposed algorithm under trapezoidal interval valued neutrosophic environment has the following advantages:

(i) indeterminacy of the information can be dealt with efficiency.
(ii) cost of the neutrosophic shortest path can be minimised
(iii) the performance of the network can be maximised through the data have indeterminacy
(iv) Indeterminacy can be captured and shortest path can be obtained by splitting the various paths and hence performance of the system can be increased.

8.2 Limitations of the proposed work

The proposed revised version of Bellman-Ford algorithm has the following limitations:

(i) It runs only $O(|V|.|E|)$ times, where $|E|$ is the number of edges in the network.
(ii) It is unable to deal with the degree of contradiction of the edges.

9 References

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