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ORIGINAL ARTICLE

Application of artificial bee colony algorithm on a green production inventory problem with preservation for deteriorating items in neutrosophic fuzzy environment

Puja Supakar[1] · Amalesh Kumar Manna[2,3] · Sanat Kumar Mahato[1] · Asoke Kumar Bhunia[2]

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Abstract Due to fluctuation of the market and several other reasons, most of the parameters related to the production inventory system, like, deterioration rate, demand rate and different costs are not always constants. These parameters involve some sorts of impreciseness which may be represented by stochastic, fuzzy, fuzzy-stochastic, interval, intuitionistic fuzzy, neutrosophic fuzzy, etc. approaches. In this work, a green production model is formulated for deteriorating items in neutrosophic uncertain environment. In this proposed model, customers' demand rates are influenced by their green level and the selling price of the manufacturing goods. Here, it is also considered that the deterioration rate is dependent on preservation investment whereas the production cost is dependent on the product's green level. We have designed, analyzed, and solved the proposed model in crisp, neutrosophic and crispified forms. To illustrate this model numerically, two examples are constructed and solved by an artificial bee colony algorithm. Finally, sensitivity analyses are conducted graphically with respect to some of the system parameters involved in the crispified model.

Keywords Manufacturing · Deterioration · Green product · Preservation technology · Neutrosophic fuzzy set

✉ Amalesh Kumar Manna
akmanna1987@gmail.com

Puja Supakar
supakarpuja@gmail.com

Sanat Kumar Mahato
sanatkmahato@gmail.com

Asoke Kumar Bhunia
akbhunia@math.buruniv.ac.in

[1] Department of Mathematics, Sidho-Kanho-Birsha University, Purulia 723104, India

[2] Department of Mathematics, The University of Burdwan, Burdwan 713104, India

[3] Department of Mathematics, C. V. Raman Global University, Bhubaneswar 752054, India

1 Introduction

In the modern age, human health is dependent on manmade products like as garments, food, cosmetics etc. which are essential in our life. An item is called green product if it is not harmful for human body or it does not pollutes the environment. In the manufacturing world, most of the manmade products are not fully green. Over the last few years, green product plays a significant role to control the pollution of environment and human disease. Presently, several researchers worked in the area of green product based production process. Laroche et al. (2001) did a survey on the behaviour of customers' who are interested to procure environment friendly products. Yan and Yazdanifard (2014) proposed the concept of green product purchase approach for sustainable development in the environment. Ghosh and Shah (2015) established a supply chain model considering green level and selling price sensitive customers' demand. Jamali and Rasti-Barzoki (2018) determined the optimal selling prices of green product and non-green product via game theory approach in a supply chain model. Dong et al. (2019) considered green investment policy to produce green product in a supply chain model (SCM). Sana (2020) presented a comparison study between non-green and green products marketing. He also assumed that Government collects minimum tax from the manufacturer of green product than the manufacturer of non-green product. In a SCM, Ghosh et al.

(2021) analyzed the impact of customers with green-sensitive demands.

Presevation facility is an important factor to maintain the quality of the produced deteriorated goods. Several researchers suggested to use presevation technology in their develoved inventory models for deteriorating goods. Hsu et al. (2010) proposed preservation investment in deteriorated inventory model to reduce the deterioration. Then, Dye and Hsieh (2012) investigated optimal replenishment strategyin a deteriorating inventory problem with preservation facility. Dye (2013) also proposed the preservation facility in a deteriorating inventory model. Zhang et al. (2014) determined the optimal pricing strategy in a deteriorating inventory model includeing preservation facility. Yang et al. (2015) formulated an inventory model with preservation technology for deteriorating items. Zhang et al. (2016) examined the impact of pricing, preservation investment and service policies for deteriorating goods under resource constraints. Mishra et al. (2017) suggested preservation facility in a deteriorated inventory problem with partial backlogging. Pal et al. (2018) developed a preservation facility based deteriorated inventory model. Also they assumed that the deterioration started at any random time. Then, Ullahet al. (2019) analyzed the impact of preservation investment in a SCM. Shaikh et al. (2019a) formulated an inventory model with preservation and trade credit facility for deteriorating goods. Das et al. (2020a) considered preservation investment in a deteriorating inventory problem with partial backlogged and price varying demand. Again, Das et al. (2021) investigated a deteriorating inventory problem under trade credit and preservation technology facility.

The first model of inventory control was developed by Harris (1913) in crisp environment. After that, due to fluctuation of different characteristics related to the inventory system, several developed inventory models in uncertain environment. In the uncertain environment, one or more than one model parameter is/are considered as random variable(s), fuzzy set(s)/number(s), interval valued, rough set(s), intuitionistic fuzzy set(s), nutrosophic fuzzy set(s) etc. Porteus (1986) developed a production inventory model under stochastic environment. He assumed that the production sytem produced defective product after a random time from the starting time. Later, Hung (2011), Ghosh et al. (2017), Mallick et al. (2018), Manna et al. (2018), Sana (2020) and many researchers established various types of inventory models in stochastic environment. Mandal et al. (1998) introduced fuzzy concept in the field of inventory control system. Then, Kao and Hsu (2002), Taleizadeh et al. (2009), Priyan and Uthayakumar(2016), Manna et al. (2017), Shaikh et al. (2018), Supakar and Mahato (2020), De et al. (2020a, b) applied the granular differentiability technique for solving optimization problem corresponding to fuzzy production inventory model. In a game-theoretic model of information security investment decisions under triangular fuzzy environments, Gupta et al. (2020) presented their findings with respect to firm investment decisions. According to Haseli et al. (2020), they extended fuzzy set theory-based criteria methods. Moghdani et al. (2020) developed a multi-item EPQ model with fuzzy uncertainty. Again, Gupta et al. (2007, 2009) introduced the interval-valued costs in the area of inventory control. After that, Bhunia et al. (2017), Shaikh et al. (2019a, b) , Rahman et al. (2020) and others developed inventory modelsin interval environment. Moheimani et al. (2021a, b) expanded the application of rough set and type-2 fuzzy in disaster management. In their study, Moheimani et al. (2021a, b) built guidelines for more preparedness to deal with COVID-19 based on the rough set theory. Souri et al. (2021) examined e-service acceptance and service preference by focusing on rough set theory. Zarafshan et al. (2021) discussed a theoretical framework for analyzing and understanding quality costs based on rough set theory for the ceramic industry. Mullai and Broumi (2018) introduced neutrosophic set in an inventory model. In the same year, Mullai and Surya (2018) developed a neutrosophic inventory model under the price break policy. Later, Mullai and Surya (2020), Das et al. (2020b) investigated inventory problem with considering neutrosophic fuzzy numbers.

In this work, a green production model for deteriorating items is formulated under both crisp and nutrosophic environments. The preservation investment dependent deterioration rate and green level dependent production cost are considered. In addition, consumers' demand is influenced by the selling price of the item and its greenness. Moreover, the deterioration rate, demand parameters. holding cost, production cost and setup cost are considered as pentagonal neutrosophic number. To investigate the validity of this model, two numerical examples under crisp and neutrosophic environments are considered and solved the same using artificial bee colony (ABC) algorithm. Finally, a sensitivity analysis is performed with respect to different model parameters.

2 Research gap and contributions

Several researchers developed different types of production inventory model considering various imprecise parameters, viz. inventory costs, demand parameters can be represented by stochastic, fuzzy, fuzzy stochastic, rough and interval approaches. To the best of our knowledge, nobody did not consider the deterioration rate, demand parameters, holding cost, production cost and setup cost as pentagonal neutrosophic number in a production inventory model. A comparative review of the related literature to the proposed model is represented in Table 1.

The main contributions of this study are as follows:

Table 1 Comparative study of the proposed model with the existing models

Author(s)	Customers' demand dependent on		Produduction/ supply chain model	Deteriorating item with preservation technology	Model development in neutrosophic fuzzy environment	Application of artificial bee colony algorithm
	Green Level	Selling price				
Karaboga and Basturk (2007)						√
Dye and Hsieh (2012)				√		
Ghosh and Shah (2015)	√		√			
Zhang et al. (2016)		√		√		
Mishra et al. (2017)		√		√		
Jamali and Rasti (2018)	√	√	√			
Pal and Bardhan (2018)				√		
Mullai and Broumi (2018)					√	
Supakar and Mahato (2018)					√	√
Chakraborty et al. (2019)					√	
Mullai and Surya (2020)					√	
De et al. (2020a, b)			√	√		
Sana (2020)	√	√	√			
Das et al. (2020a)		√		√		
This paper	√	√	√	√	√	√

- An imprecise production inventory model with preservation investment for deteriorating items is formulated in neutrosophic fuzzy environment.
- Selling price and green sensitive demand is considered where both are decision variable.
- Deterioration rate, demand parameters, holding cost, production cost and setup cost are considered as pentagonal neutrosophic number.
- The maximization problem related to the corresponding model (average profits) is solved by artificial bee colony algorithm.

3 Basic concepts of neutrosophic set

3.1 Neutrosophic set (NSS)

Let Y be the universe of discourse and let $y \in Y$. A neutrosophic set $\tilde{P}^N$ is described by the truth membership function $\sigma_{\tilde{P}^N}(y)$, non-membership function $\tau_{\tilde{P}^N}(y)$ and indeterminacy function $\omega_{\tilde{P}^N}(y)$ with the following expression:

$$\tilde{P}^N = \left\{(y, \sigma_{\tilde{P}^N}(y), \tau_{\tilde{P}^N}(y), \omega_{\tilde{P}^N}(y)) : y \in Y\right\}$$

where $\sigma_{\tilde{P}^N}(y) : Y \to (0^-, 1^+)$; $\tau_{\tilde{P}^N}(y) : Y \to (0^-, 1^+)$; $\omega_{\tilde{P}^N}(y) : Y \to (0^-, 1^+)$.

For the above functions,$\sigma_{\tilde{P}^N}(y)$, $\tau_{\tilde{P}^N}(y)$, $\omega_{\tilde{P}^N}(y)$ the inequality is satisfied

$$0^- \leq \sup\sigma_{\tilde{P}^N}(y) + \sup\tau_{\tilde{P}^N}(y) + \sup\omega_{\tilde{P}^N}(y) \leq 3^+.$$

From the definition, it is evident that a neutrosophic set $\tilde{P}^N$ takes real value in $(0^-, 1^+)$.

3.2 Single valued neutrosophic set (SVNS)

A single valued neutrosophic Set $\tilde{P}^N$ generated by the truth membership function $\sigma_{\tilde{P}^N}(y)$, non-membership function $\tau_{\tilde{P}^N}(y)$ and indeterminacy function $\omega_{\tilde{P}^N}(y)$ has the following expression:

$\tilde{P}^N = \left\{(y, \sigma_{\tilde{P}^N}(y), \tau_{\tilde{P}^N}(y), \omega_{\tilde{P}^N}(y)) : y \in Y\right\}$,

where, $\sigma_{\tilde{P}^N}(y) : Y \to [0,1]$; $\tau_{\tilde{P}^N}(y) : Y \to [0,1]$; $\omega_{\tilde{P}^N}(y) : Y \to [0,1]$.

The functions $\sigma_{\tilde{P}^N}(y)$, $\tau_{\tilde{P}^N}(y)$, $\omega_{\tilde{P}^N}(y)$ satisfy the following inequality

$$0 \leq \sigma_{\tilde{P}^N}(y) + \tau_{\tilde{P}^N}(y) + \omega_{\tilde{P}^N}(y) \leq 3 \; \forall y \in Y.$$

3.3 Single valued pentagonal neutrosophic number (SVPNN)

A single valued pentagonal neutrosophic number $\tilde{P}$ is stated as $\tilde{P} = \left(\left[\left(p_1, p_2, p_3, p_4, p_5\right);\rho\right], \left[\left(q_1, q_2, q_3, q_4, q_5\right);\lambda\right], \left[\left(r_1, r_2, r_3, r_4, r_5\right);\delta\right];\right)$, where $\rho, \lambda, \delta \in [0, 1]$.

The truth membership functions $(\sigma_{\tilde{P}}) : \mathbb{R} \to [0, \rho]$, the non-membership function $(\tau_{\tilde{P}}) : \mathbb{R} \to [\lambda, 1]$ and indeterminancy membership function $(\omega_{\tilde{P}}) : \mathbb{R} \to [\delta, 1]$ are given as:

$$\sigma_{\tilde{P}}(y) = \begin{cases} \sigma_{\tilde{P}l1}(y),\ p_1 \le y < p_2 \\ \sigma_{\tilde{P}l2}(y),\ p_2 \le y < p_3 \\ \sigma_{\tilde{P}l1}(y),\ p_3 \le y < p_4 \\ \rho,\ y = p_4 \\ \sigma_{\tilde{P}l1}(y), p_4 \le y < p_5 \\ 0,\ \text{Otherwise} \end{cases};$$

$$\tau_{\tilde{P}}(y) = \begin{cases} \tau_{\tilde{P}l1}(y),\ q_1 \le y < q_2 \\ \tau_{\tilde{P}l2}(y),\ q_2 \le y < q_3 \\ \tau_{\tilde{P}l1}(y),\ q_3 \le y < q_4 \\ \lambda,\ y = q_4 \\ \tau_{\tilde{P}l1}(y), q_4 \le y < q_5 \\ 1,\ \text{Otherwise} \end{cases};$$

$$\omega_{\tilde{P}}(y) = \begin{cases} \omega_{\tilde{P}l1}(y),\ r_1 \le y < r_2 \\ \omega_{\tilde{P}l2}(y),\ r_2 \le y < r_3 \\ \omega_{\tilde{P}l1}(y),\ r_3 \le y < r_4 \\ \delta,\ y = r_4 \\ \omega_{\tilde{P}l1}(y), r_4 \le y < r_5 \\ 1,\ \text{Otherwise} \end{cases}$$

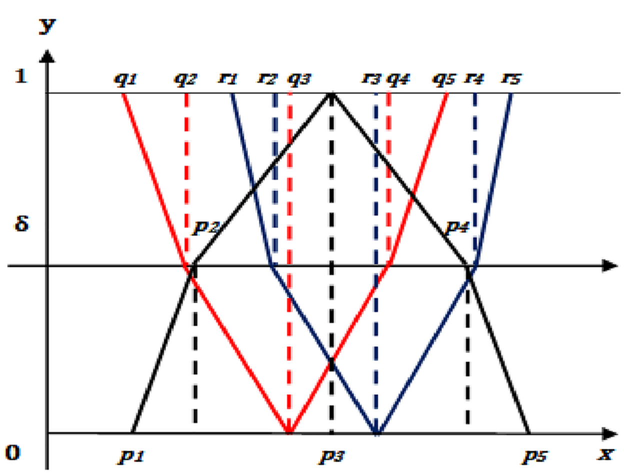

Fig. 1 Pictorial representation of linear pentagonal neutrosophic number

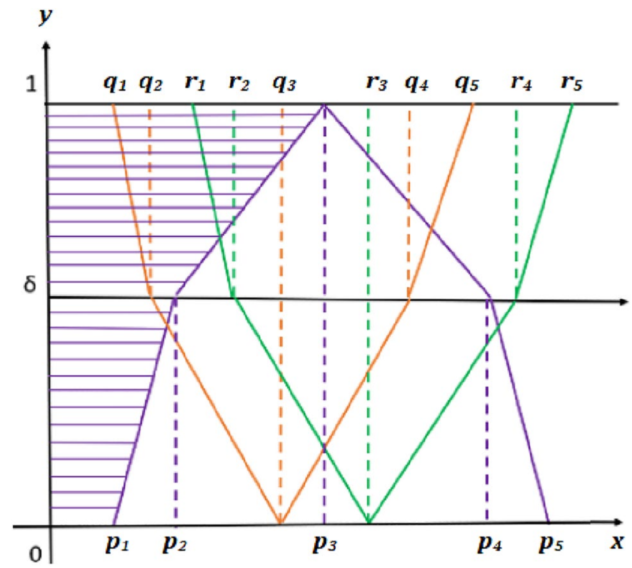

Fig. 2 Removal of area step 1

3.4 Crispification of linear neutrosophic pentagonal number

Let $\tilde{P}_{lpn} = (p_1, p_2, p_3, p_4, p_5; q_1, q_2, q_3, q_4, q_5; r_1, r_2, r_3, r_4, r_5)$ be a linear pentagonal neutrosophic number. Figure 1 shows the pictorial representation of linear pentagonal neutrosophic number. To display the truth membership function, non-membership function and indeterminacy membership function, we draw black, red and blue lined pentagonals respectively.

From Chakraborty et al. (2019) the are as of removalcan be computed as (Figs. 2, 3, 4, 5, 6, 7, 8),

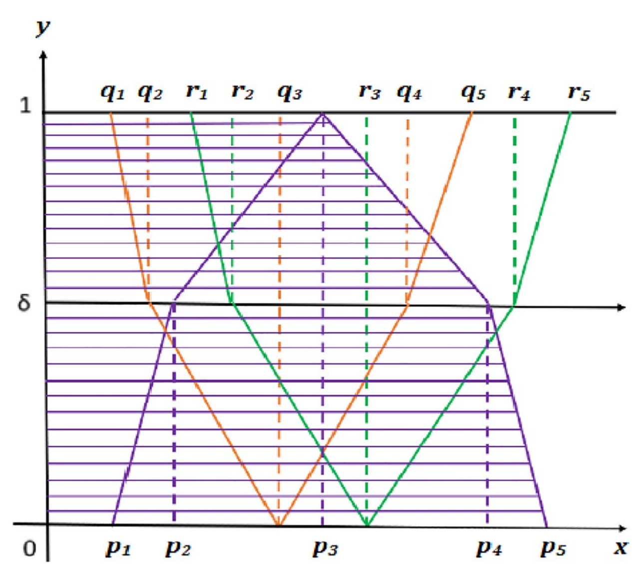

Fig. 3 Removal of area step 2

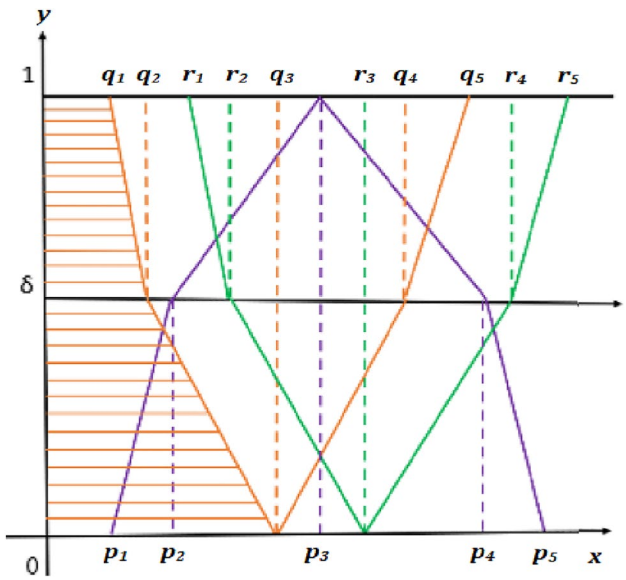

Fig. 4 Removal of area step 3

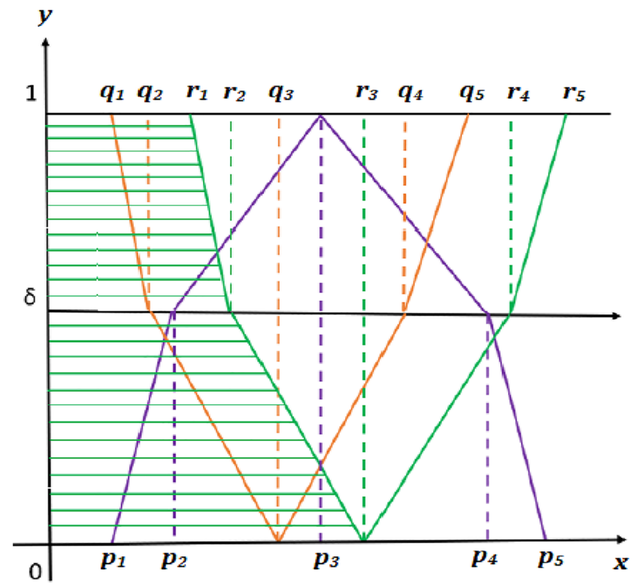

Fig. 6 Removal of area step 5

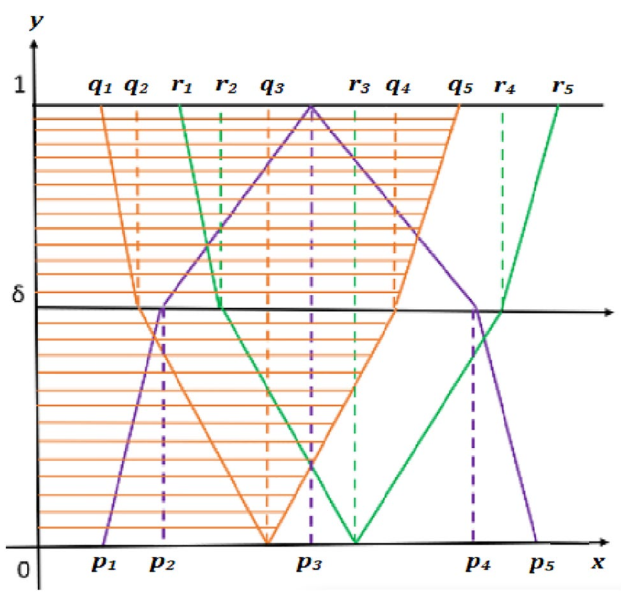

Fig. 5 Removal of area step 4

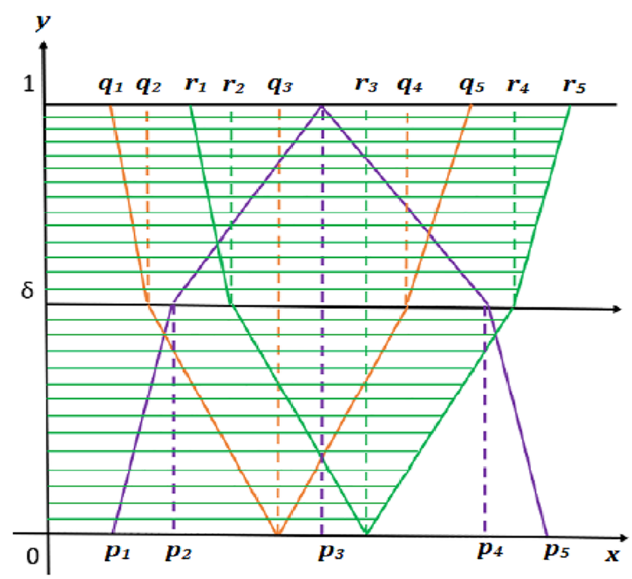

Fig. 7 Removal of area step 6

$$P_{Nu}(\tilde{A},p)=\frac{P_{Nu\,l}(\tilde{A},p)+P_{Nu\,r}(\tilde{A},p)}{2},$$
$$P_{Nu}(\tilde{B},p)=\frac{P_{Nu\,l}(\tilde{B},p)+P_{Nu\,r}(\tilde{B},p)}{2},$$
$$P_{Nu}(\tilde{C},p)=\frac{P_{Nu\,l}(\tilde{C},p)+P_{Nu\,r}(\tilde{C},p)}{2}.$$

Henceforth, the crispified value of a linear pentagonal neutrosophic number is hereunder,

$$P_{Nu}(\tilde{d}_{pn},p)=\frac{P_{Nu}(\tilde{A},p)+P_{Nu}(\tilde{B},p)+P_{Nu}(\tilde{C},p)}{3}$$

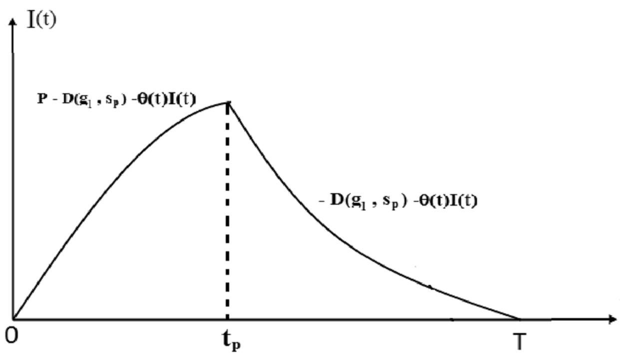

Fig. 8 Schematic diagram of the inventory level of produced product

$$P_{Nu}(\tilde{A},0)=\frac{P_{Nu\,l}(\tilde{A},0)+P_{Nu\,r}(\tilde{A},0)}{2},$$

$$P_{Nu}(\tilde{B},0)=\frac{P_{Nu\,l}(\tilde{B},0)+P_{Nu\,r}(\tilde{B},0)}{2},$$

$$P_{Nu}(\tilde{C},0)=\frac{P_{Nu\,l}(\tilde{C},0)+P_{Nu\,r}(\tilde{C},0)}{2}$$

Thus, $P_{Nu}(\tilde{d}_{pn},0)=\frac{P_{Nu}(\tilde{A},0)+P_{Nu}(\tilde{B},0)+P_{Nu}(\tilde{C},0)}{3}$

$$P_{Nu\,l}(\tilde{A},0)=\text{Area of Fig.2}=\frac{(p_1+p_2)\delta}{2}+\frac{(p_2+p_3)(1-\delta)}{2}$$

$$P_{Nu\,r}(\tilde{A},0)=\text{Area of Fig.3}=\frac{(p_4+p_5)\delta}{2}+\frac{(p_3+p_4)(1-\delta)}{2}$$

$$P_{Nu\,l}(\tilde{B},0)=\text{Area of Fig.4}=\frac{(q_1+q_2)(1-\delta)}{2}+\frac{(q_2+q_3)\delta}{2}$$

$$P_{Nu\,r}(\tilde{B},0)=\text{Area of Fig.5}=\frac{(q_4+q_5)(1-\delta)}{2}+\frac{(q_3+q_4)\delta}{2}$$

$$P_{Nu\,l}(\tilde{C},0)=\text{Area of Fig.6}=\frac{(r_1+r_2)(1-\delta)}{2}+\frac{(r_2+r_3)\delta}{2}$$

$$P_{Nu\,r}(\tilde{C},0)=\text{Area of Fig.7}=\frac{(r_4+r_5)(1-\delta)}{2}+\frac{(r_3+r_4)\delta}{2}$$

Then, $P_{Nu}(\tilde{A},0)=\frac{\frac{(p_1+p_2)\delta}{2}+\frac{(p_2+p_3)(1-\delta)}{2}+\frac{(p_4+p_5)\delta}{2}+\frac{(p_3+p_4)(1-\delta)}{2}}{2}$

$$P_{Nu}(\tilde{B},0)=\frac{\frac{(q_1+q_2)(1-\delta)}{2}+\frac{(q_2+q_3)\delta}{2}+\frac{(q_4+q_5)(1-\delta)}{2}+\frac{(q_3+q_4)\delta}{2}}{2}$$

$$P_{Nu}(\tilde{C},0)=\frac{\frac{(r_1+r_2)(1-\delta)}{2}+\frac{(r_2+r_3)\delta}{2}+\frac{(r_4+r_5)(1-\delta)}{2}+\frac{(r_3+r_4)\delta}{2}}{2}$$

$$P_{Nu}(\tilde{d}_{pn},0)=\frac{1}{12}\begin{bmatrix}(p_1+p_2+p_4+p_5+q_2+2q_3+q_4+r_2+2r_3+r_4)\delta+\\(p_2+2p_3+p_4+q_1+q_2+q_4+q_5+r_1+r_2+r_4+r_5)(1-\delta)\end{bmatrix}$$

4 Notation and assumptions

In constructing the mathematical model, different notation and fundamental assumptions are taken into account. These are stated here under:

5 Notation

$I(t)$	On-hand inventory level of the preserved goods.
P	Manufacturer's production rate.
t_p	Production period.
T	Business period.
u	Preservation investment level.
$\theta(u)$	Deteriorating rate.
$\tilde{\theta}_{lpn}(u)$	Neutrosophic deteriorating rate.
$P_{Nu}(\tilde{\theta}_{lpn}(u),0)$	Crispified deteriorating rate.
g_l	Product's green level.
s_p	Product's selling price.
$D(g_l,\ s_p)$	Customers' demand rate.
$\tilde{D}_{lpn}(g_l,\ s_p)$	Neutrosophic customers' demand rate.
$P_{Nu}(\tilde{D}_{lpn}(g_l,s_p),0)$	Crispified customers' demand rate.
h_c	Holding cost/unit/unit time.
$\tilde{h}_{clpn}$	Neutrosophic holding cost /unit/ unit time.
$P_{Nu}(\tilde{h}_{clpn},0)$	Crispified holding cost /unit time / unit.
$P_c(g_l)$	Production cost /unit product.
$\tilde{P}_{clpn}(g_l)$	Neutrosophic production cost /unit product.
$P_{Nu}(\tilde{P}_{clpn}(g_l),0)$	Crispified production cost /unit product.
A	Setup cost /cycle.
$\tilde{A}_{lpn}$	Neutrosophic setup cost /cycle.
$P_{Nu}(\tilde{A}_{lpn},0)$	Crispified setup cost /cycle.
$\pi(g_l,\ s_p,\ t_p)$	Manufacturer's average profit.
$\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p)$	Manufacturer's neutrosophic average profit.
$P_{Nu}(\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p),0)$	Manufacturer's crispified average profit.

5.1 Assumptions

(i) A manufacturing model over infinite time horizon is considered. The manufacturing firm produces single item at a constant rate P and meets up the customers' demand at the rate $D(s_p, g_l)$, i.e.,$P > D(s_p, g_l)$.

(ii) During the production period of the produced items, the product deteriorates continuously with time and the deterioration rate is preservation investment level dependent. The deterioration rate in crisp and neutrosophic environments are respectively

$$\theta(u) = \eta e^{-\xi u},\ u \in [0, \infty] \tag{1}$$

$$\text{and } \tilde{\theta}_{lpn}(u) = \tilde{\eta}_{lpn} e^{-\xi u},\ u \in [0, \infty] \tag{2}$$

(iii) The customers' demand rate is influenced by the greenness and selling price of the product. Also, it climbs with the greenness of the product and drops with its selling price. Hence the mathematical expression of customers' demand rate in crisp and neutrosophic environments are respectively

$$D(g_l, s_p) = \beta + \gamma g_l - \delta s_p \tag{3}$$

$$\tilde{D}_{lpn}(g_l, s_p) = \tilde{\beta}_{lpn} + \tilde{\gamma}_{lpn} g_l - \tilde{\delta}_{lpn} s_p \tag{4}$$

where β, γ and δ are positive constants in crisp nature, whereas $\tilde{\beta}_{lpn}$, $\tilde{\gamma}_{lpn}$and $\tilde{\delta}_{lpn}$ are neutrosophic numbers.

(iv) The unit production cost is dependent on the product green level which is expressed in crisp and neutrosophic nature are respectively as follows:

$$P_c(g_l) = c_f e^{\alpha u g_l} \tag{5}$$

$$\tilde{P}_{clpn}(g_l) = \tilde{c}_{flpn} e^{\tilde{\alpha}_{lpn} u g_l} \tag{6}$$

where c_f is the constant unit production cost in crisp nature and $\alpha(> 0)$ is the control parameter of the green level. Also, $\tilde{c}_{flpn}$ and $\tilde{\alpha}_{lpn}$ are fixed production cost and control parameter of the green level in neutrosophic nature respectively.

$$MHC = h_c \int_0^T I(t)dt$$

$$= \frac{h_c}{\{\theta(u)\}^2}\left[\left\{P - D(g_l, s_p)\right\}\left\{\theta(u)t_p + e^{-\theta(u)t_p} - 1\right\} + D(g_l, s_p)\left\{e^{\theta(u)(T-t_p)} - 1 - \theta(u)(T - t_p)\right\}\right]$$

6 Model description and formulation

Let us suppose that a manufacturer produces products during $[0, t_p]$ at a rate P. During the period $[0, t_p]$, produced products level increases continuously with the period $P - D(g_l, s_p) - \theta(u)I(t)$ per unit time. After production period i.e., during $[t_p, T]$, stock level of products is decreased due to both effects of demand $D(g_l, s_p)$ and deterioration $\theta(u)I(t)$ per unit time. The entire cycle is repeated.

6.1 Crisp model

In this model, all the parameters are crisp valued and the deterioration rate is dependent on preservation investment level whereas the production cost is green level dependent. Also, the customers' demand is selling price and green level dependent. The following differential equations representing the inventory level I(t) at any instant t are given by

$$\frac{dI(t)}{dt} = P - D(s_p, g_l) - \theta(u)I(t), \quad 0 < t \le t_p \tag{7}$$

$$\frac{dI(t)}{dt} = -D(s_p, g_l) - \theta(u)I(t), \quad t_p < t \le T \tag{8}$$

with $I(0) = 0 = I(T)$.

The solutions of (7) and (8) are

$$I(t) = \frac{1}{\theta(u)}\left[1 - e^{-\theta(u)t}\right]\left[P - D(s_p, g_l)\right], \quad 0 < t \le t_p \tag{9}$$

$$I(t) = \frac{1}{\theta(u)}\left[e^{(T-t)\theta(u)} - 1\right]D(s_p, g_l), \quad t_p < t \le T \tag{10}$$

The continuity of $I(t)$ at $t = t_p$ implies

$$T = t_p + \frac{1}{\theta(u)} \log\left[1 + \frac{P - D(g_l, s_p)}{D(g_l, s_p)}\left\{1 - e^{-\theta(u)t_p}\right\}\right] \tag{11}$$

The manufacturer's total production cost (MPC) is given by

$$\text{MPC} = P_c(g_l)\int_0^{t_p} P\,dt = P_c(g_l)Pt_p$$

The manufacturer's total holding cost (MHC) is given by

The deterioration cost (DC) is $DC = d_c\left\{Pt_p - D(g_l, s_p)T\right\}$.

The manufacturer's sales revenue (MSR) is $MSR = s_p \int_0^T D(g_l, s_p)\,dt = s_p D(g_l, s_p)T$.

The manufacturer's total profit (MTP) is given by

$$MTP(t_p,\ s_p,\ g_l) = s_p D(g_l,\ s_p)T - P_c(g_l)Pt_p - d_c\left\{Pt_p - D(g_l, s_p)T\right\} - A$$
$$-\frac{h_c}{\{\theta(u)\}^2}\left[\left\{P - D(s_p, s_p)\right\}\left\{\theta(u)t_p + e^{-\theta(u)t_p} - 1\right\} + D(g_l,\ s_p)\left\{e^{\theta(u)(T-t_p)} - 1 - \theta(u)(T-t_p)\right\}\right]$$

Henceforth, the average profit of the manufacturer is given by

$$\pi(t_p,\ \ s_p,\ g_l) = \frac{MTP(t_p,\ s_p,\ g_l)}{T} \tag{12}$$

Now the goal is to find the optimal values of g_l and t_p which maximize the manufacturer's average profit $\pi(t_p,\ \ s_p,\ g_l)$. Hence the otimization problem of this model is given by

$$\begin{aligned}&\textbf{Maximize}\ \pi(t_p,\ \ s_p,\ g_l)\\&\textbf{subject to}\ g_l > 0,\ \ s_p > 0,\ \ T > t_p > 0.\end{aligned} \tag{13}$$

6.2 Neutrosophic model

This section discusses production inventory model for deteriorating green product in neutrosophic environment. In real life situations, due to change of weather and season, labour problem, inadequate information, lack of evidence, fluctuating financial market, parameters like, the deteriorating rate, demand rate, holding cost, production cost, set up cost are uncertain. To represent their uncertainty, we have used linear pentagonal neutrosophic number as follows:

$$\begin{aligned}
&\text{Let}\ \tilde{\theta}_{lpn}(u) = (\kappa_{p1},\kappa_{p2},\kappa_{p3},\kappa_{p4},\kappa_{p5};\kappa_{q1},\kappa_{q2},\kappa_{q3},\kappa_{q4},\kappa_{q5};\kappa_{r1},\kappa_{r2},\kappa_{r3},\kappa_{r4},\kappa_{r5}),\\
&\tilde{D}_{lpn}(g_l,\ s_p) = (\Delta_{p1},\Delta_{p2},\Delta_{p3},\Delta_{p4},\Delta_{p5};\Delta_{q1},\Delta_{q2},\Delta_{q3},\Delta_{q4},\Delta_{q5};\Delta_{r1},\Delta_{r2},\Delta_{r3},\Delta_{r4},\Delta_{r5}),\\
&\tilde{h}_{clpn} = (\hbar_{p1},\hbar_{p2},\hbar_{p3},\hbar_{p4},\hbar_{p5};\hbar_{q1},\hbar_{q2},\hbar_{q3},\hbar_{q4},\hbar_{q5};\hbar_{r1},\hbar_{r2},\hbar_{r3},\hbar_{r4},\hbar_{r5}),\\
&\tilde{P}_{clpn}(g_l) = (p_{p1},p_{p2},p_{p3},p_{p4},p_{p5};p_{q1},p_{q2},p_{q3},p_{q4},p_{q5};p_{r1},p_{r2},p_{r3},p_{r4},p_{r5}),\\
&\tilde{A}_{lpn} = (a_{p1},a_{p2},a_{p3},a_{p4},a_{p5};a_{q1},a_{q2},a_{q3},a_{q4},a_{q5};a_{r1},a_{r2},a_{r3},a_{r4},a_{r5}).
\end{aligned}$$

The manufacturer's total neutrosophic profit (MTP) per inventory cycle can be evaluated as stated below:

$$\tilde{M}TP_{lpn}(t_p,\ g_l,\ s_p) = s_p\tilde{D}_{lpn}(g_l,\ s_p)T - \tilde{P}_{clpn}(g_l)Pt_p - d_c\left\{Pt_p - \tilde{D}_{lpn}(g_l,\ s_p)T\right\} - \tilde{A}_{lpn} - \frac{\tilde{h}_{clpn}}{\left\{\tilde{\theta}_{lpn}(u)\right\}^2}[\{P-$$
$$\tilde{D}_{lpn}(g_l,\ s_p)\}\left\{\tilde{\theta}_{lpn}(u)t_p + e^{-\tilde{\theta}_{lpn}(u)t_p} - 1\right\} + \tilde{D}_{lpn}(g_l,\ s_p)\left\{e^{(T-t_p)\tilde{\theta}_{lpn}(u)} - (T-t_p)\tilde{\theta}_{lpn}(u) - 1\right\}]$$

$$\begin{aligned}
&= s_p(\Delta_{p1},\Delta_{p2},\Delta_{p3},\Delta_{p4},\Delta_{p5};\Delta_{q1},\Delta_{q2},\Delta_{q3},\Delta_{q4},\Delta_{q5};\Delta_{r1},\Delta_{r2},\Delta_{r3},\Delta_{r4},\Delta_{r5})T-\\
&(p_{p1},p_{p2},p_{p3},p_{p4},p_{p5};p_{q1},p_{q2},p_{q3},p_{q4},p_{q5};p_{r1},p_{r2},p_{r3},p_{r4},p_{r5})Pt_p-\\
&d_c\left\{Pt_p - (\Delta_{p1},\Delta_{p2},\Delta_{p3},\Delta_{p4},\Delta_{p5};\Delta_{q1},\Delta_{q2},\Delta_{q3},\Delta_{q4},\Delta_{q5};\Delta_{r1},\Delta_{r2},\Delta_{r3},\Delta_{r4},\Delta_{r5})T\right\}-\\
&(a_{p1},a_{p2},a_{p3},a_{p4},a_{p5};a_{q1},a_{q2},a_{q3},a_{q4},a_{q5};a_{r1},a_{r2},a_{r3},a_{r4},a_{r5})-\\
&\frac{(\hbar_{p1},\hbar_{p2},\hbar_{p3},\hbar_{p4},\hbar_{p5};\hbar_{q1},\hbar_{q2},\hbar_{q3},\hbar_{q4},\hbar_{q5};\hbar_{r1},\hbar_{r2},\hbar_{r3},\hbar_{r4},\hbar_{r5})}{\left\{(\kappa_{p1},\kappa_{p2},\kappa_{p3},\kappa_{p4},\kappa_{p5};\kappa_{q1},\kappa_{q2},\kappa_{q3},\kappa_{q4},\kappa_{q5};\kappa_{r1},\kappa_{r2},\kappa_{r3},\kappa_{r4},\kappa_{r5})\right\}^2}\\
&\quad\left[\left\{P - (\Delta_{p1},\Delta_{p2},\Delta_{p3},\Delta_{p4},\Delta_{p5};\Delta_{q1},\Delta_{q2},\Delta_{q3},\Delta_{q4},\Delta_{q5};\Delta_{r1},\Delta_{r2},\Delta_{r3},\Delta_{r4},\Delta_{r5})\right\}\right.\\
&\left\{\begin{array}{l}(\kappa_{p1},\kappa_{p2},\kappa_{p3},\kappa_{p4},\kappa_{p5};\kappa_{q1},\kappa_{q2},\kappa_{q3},\kappa_{q4},\kappa_{q5};\kappa_{r1},\kappa_{r2},\kappa_{r3},\kappa_{r4},\kappa_{r5})t_p+\\ e^{-(\kappa_{p1},\kappa_{p2},\kappa_{p3},\kappa_{p4},\kappa_{p5};\kappa_{q1},\kappa_{q2},\kappa_{q3},\kappa_{q4},\kappa_{q5};\kappa_{r1},\kappa_{r2},\kappa_{r3},\kappa_{r4},\kappa_{r5})t_p} - 1\end{array}\right\}\\
&+(\Delta_{p1},\Delta_{p2},\Delta_{p3},\Delta_{p4},\Delta_{p5};\Delta_{q1},\Delta_{q2},\Delta_{q3},\Delta_{q4},\Delta_{q5};\Delta_{r1},\Delta_{r2},\Delta_{r3},\Delta_{r4},\Delta_{r5})\\
&\left.\left\{\begin{array}{l}e^{(T-t_p)(\kappa_{p1},\kappa_{p2},\kappa_{p3},\kappa_{p4},\kappa_{p5};\kappa_{q1},\kappa_{q2},\kappa_{q3},\kappa_{q4},\kappa_{q5};\kappa_{r1},\kappa_{r2},\kappa_{r3},\kappa_{r4},\kappa_{r5})}-\\ (T-t_p)(\kappa_{p1},\kappa_{p2},\kappa_{p3},\kappa_{p4},\kappa_{p5};\kappa_{q1},\kappa_{q2},\kappa_{q3},\kappa_{q4},\kappa_{q5};\kappa_{r1},\kappa_{r2},\kappa_{r3},\kappa_{r4},\kappa_{r5}) - 1\end{array}\right\}\right]
\end{aligned}$$

Hence, the neutrosophic average profit of the manufacturer is evaluated as stated below:$\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p) = \frac{\tilde{M}TP_{lpn}(t_p,\ g_l,s_p)}{T}$ (14).

Now the goal is to determine the optimal values of $t_p,\ g_l$ and s_p which maximize the manufacturer's neutrosophic average profit $\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p)$. Hence the corresponding maximization problem is as follows:

$$\begin{aligned}&\textbf{Maximize } \tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p)\\&\textbf{subject to } g_l > 0,\ \ s_p > 0,\ \ T > t_p > 0\end{aligned} \tag{15}$$

6.3 Crispified model

In this section, to simplify the computational process, conversion of neutrosophic model into crispified model is done. Now, in order to crispify the linear neutrosophic pentagonal numbergiven in (15), we have used the method discussed in Sect. 2.4.

$$P_{Nu}(\tilde{\theta}_{lpn}(u),0) = \frac{1}{12}\begin{bmatrix}(\kappa_{p1}+\kappa_{p2}+\kappa_{p4}+\kappa_{p5}+\kappa_{q2}+2\kappa_{q3}+\kappa_{q4}+\kappa_{r2}+2\kappa_{r3}+\kappa_{r4})\delta+\\(\kappa_{p2}+2\kappa_{p3}+\kappa_{q4}+\kappa_{q1}+\kappa_{q2}+\kappa_{q4}+\kappa_{q5}+\kappa_{r1}+\kappa_{r2}+\kappa_{r4}+\kappa_{r5})(1-\delta)\end{bmatrix}$$

$$P_{Nu}(\tilde{D}_{lpn}(g_l,s_p),0) = \frac{1}{12}\begin{bmatrix}(\Delta_{p1}+\Delta_{p2}+\Delta_{p4}+\Delta_{p5}+\Delta_{q2}+2\Delta_{q3}+\Delta_{q4}+\Delta_{r2}+2\Delta_{r3}+\Delta_{r4})\delta+\\(\Delta_{p2}+2\Delta_{p3}+\Delta_{q4}+\Delta_{q1}+\Delta_{q2}+\Delta_{q4}+\Delta_{q5}+\Delta_{r1}+\Delta_{r2}+\Delta_{r4}+\Delta_{r5})(1-\delta)\end{bmatrix}$$

$$P_{Nu}(\tilde{\hbar}_{clpn},0) = \frac{1}{12}\begin{bmatrix}(\hbar_{p1}+\hbar_{p2}+\hbar_{p4}+\hbar_{p5}+\hbar_{q2}+2\hbar_{q3}+\hbar_{q4}+\hbar_{r2}+2\hbar_{r3}+\hbar_{r4})\delta+\\(\hbar_{p2}+2\hbar_{p3}+\hbar_{q4}+\hbar_{q1}+\hbar_{q2}+\hbar_{q4}+\hbar_{q5}+\hbar_{r1}+\hbar_{r2}+\hbar_{r4}+\hbar_{r5})(1-\delta)\end{bmatrix}$$

$$P_{Nu}(\tilde{P}_{clpn}(g_l),0) = \frac{1}{12}\begin{bmatrix}(p_{p1}+p_{p2}+p_{p4}+p_{p5}+p_{q2}+2p_{q3}+p_{q4}+p_{r2}+2p_{r3}+p_{r4})\delta+\\(p_{p2}+2p_{p3}+p_{q4}+p_{q1}+p_{q2}+p_{q4}+p_{q5}+p_{r1}+p_{r2}+p_{r4}+p_{r5})(1-\delta)\end{bmatrix}$$

$$P_{Nu}(\tilde{A}_{lpn},0) = \frac{1}{12}\begin{bmatrix}(a_{p1}+a_{p2}+a_{p4}+a_{p5}+a_{q2}+2a_{q3}+a_{q4}+a_{r2}+2a_{r3}+a_{r4})\delta+\\(a_{p2}+2a_{p3}+a_{q4}+a_{q1}+a_{q2}+a_{q4}+a_{q5}+a_{r1}+a_{r2}+a_{r4}+a_{r5})(1-\delta)\end{bmatrix}$$

The manufacturer's total crispified profit (MTP) is given by

$$\begin{aligned}&P_{Nu}(\tilde{M}TP_{lpn}(t_p,g_l,s_p),0) = s_pP_{Nu}(\tilde{D}_{lpn}(g_l,s_p),0)T - P_{Nu}(\tilde{P}_{clpn}(g_l),0)Pt_p\\&-d_c\left\{Pt_p - P_{Nu}(\tilde{D}_{lpn}(g_l,s_p),0)T\right\} - P_{Nu}(\tilde{A}_{lpn},0)\\&-\frac{P_{Nu}(\tilde{\hbar}_{clpn},0)}{\left\{P_{Nu}(\tilde{\theta}_{lpn}(u),0)\right\}^2}[\left\{P - P_{Nu}(\tilde{D}_{lpn}(g_l,s_p),0)\right\}\left\{P_{Nu}(\tilde{\theta}_{lpn}(u),0)t_p + e^{-P_{Nu}(\tilde{\theta}_{lpn}(u),0)(u)t_p} - 1\right\}\\&+P_{Nu}(\tilde{D}_{lpn}(g_l,s_p),0)\left\{e^{(T-t_p)P_{Nu}(\tilde{\theta}_{lpn}(u),0)} - (T-t_p)P_{Nu}(\tilde{\theta}_{lpn}(u),0) - 1\right\}]\end{aligned}$$

Hence, the crispified average profit is

$$P_{Nu}(\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p),0) = \frac{P_{Nu}(\tilde{M}TP_{lpn}(t_p,\ g_l,\ s_p),0)}{T} \tag{16}$$

Now the goal is to determine the optimal values of $t_p,\ g_l$ and s_p which maximize the manufacturer's crispified average profit $P_{Nu}(\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p),0)$. Hence the corresponding maximization problem is as follows:

$$\begin{aligned}&\textbf{Maximize } P_{Nu}(\tilde{\pi}_{lpn}(t_p,\ g_l,\ s_p),0)\\&\textbf{subject to } g_l > 0,\ \ s_p > 0,\ \ T > t_p > 0\end{aligned} \tag{17}$$

7 Solution methodology

A swarm-based meta-heuristic algorithm, the artificial bee colony (ABC), was first proposed by Karaboga (2005). Though in the beginning, it was proposed to optimize numerical problems but can also be applied for combinatorial optimization problems (e.g. Pan et al. 2011). This technique is specifically dependent on the particular intelligent searching behaviour of honey bees (Karaboga et al. 2012). Supakar and Mahato (2018) developed an EOQ model of advance payment with uncertain environment using ABC algorithm. Initial applications of ABC algorithm were limited to unconstrained optimizations but now it can also be applied for constrained optimization problems (c.f. Mao et al. 2015). In ABC algorithm, there are three types of bees in the artificial hive.

(i) **Employed bees:** For a particular food sources, employed bees are specific and each food source is assigned to asingle employed bee. This means that there are exactly the same amounts of employed bees as well as solutions. For searching the food source, employed bees traverse the region at random, i.e., the ABC forms a population that is randomly distributed according to the size S_p.

(ii) **Onlooker bees:** Onlooker bees firstly watch the movement of those employed bees and then goes towards the search region.

(iii) **Scout bees:** Scout bees remain busy to explore the new food sources in the region. Also, it is possible that an employed bee turns into a scout bee when the food source is exhausted.

The usual procedure of ABC algorithm is described hereunder:

(a) Initialization stage: The bees hunt the region haphazardly to locate the food source in the first instance. Every solution Y_f, is a z-dimensional vector (i.e.,$Y_f = Y_{f,1}, Y_{f,2}, ..., Y_{f,z}$) which is to be optimized. So the position is initialized using the following equation (c.f. Xiang et al. 2014).

$$Y_{f,g} = Y_f^{\min} + rand(0,1)(Y_f^{\max} - Y_f^{\min}) \tag{18}$$

where, $f = 1, 2, ..., S_p$, $g = 1, 2, ..., z$;.

$Y_f^{\min}$ = Lower bound of food source location in the f-th dimension;

$Y_f^{\max}$ = Upper bound of food source location in the f-th dimension.

(b) Employed bees stage: In the nearby community of the previous food source in their memories, the employed bees move towards more enriched new food sources in the region using the expression given in Eq. (19) (c.f. Xiang et al. 2014)

$$v_{f,g} = Y_{f,g} + \psi_{f,g}(Y_{f,g} - Y_{h,g}) \tag{19}$$

where $h = 1, 2, ..., S_p$, $g = 1, 2, ..., z$; h and g are randomly selected and h and f must not be same;$\psi_{f,g}$ is anyarbitary number between $[-1, 1]$..

(c) Onlooker bees stage: By observing the employed bees, onlooker bees gather information about possible food sources in the nearby hive. Based on the values calculated by the Eq. (20) the onlooker bees select the food source.

$$p_f = \frac{0.9}{fit_{best}} fit_f + 0.1 \tag{20}$$

where fit_f indicates the fitness value of solution Y_f.

Using the following formula, we can easily calculate the fitness value of a solution

$$fit_f = \begin{cases} 1 + |fit(Y_f)|, & if\ fit(Y_f) < 0, \\ \dfrac{1}{1 + fit(Y_f)}, & if\ fit(Y_f) \geq 0, \end{cases} \tag{21}$$

where $fit(Y_f)$ indicates the objective function value of the decision vector Y_f.

Table 2 Pentagonal neutrosophic parameters and their crispified values (taking $\delta = 0.25$)

Parameter	Pentagonal neutrosophic value	Crispified value
$\tilde{\eta}_{lpn}$	(0.4,0.5,0.6,0.7,0.8;0.36,0.46,0.56,0.66,0.76;0.43,0.53,0.63,0.73,0.83)	0.596667
$\tilde{\alpha}_{lpn}$	(0.005,0.006,0.007,0.008, 0.009; 0.0045,0.0055, 0.0065, 0.0075,0.0085;0.01,0.03,0.05, 0.06,0.07)	0.019083
$\tilde{\beta}_{lpn}$	(300,350,400,450,500;275,325,375,425,475,315,365,415,465,515)	396.6667
$\tilde{\gamma}_{lpn}$	(15,20,25,30,35;10,16,22,28,34;23,29,35,41,47)	27.33333
$\tilde{h}_{clpn}$	(0.44,0.5,0.56,0.62,0.68;0.42,0.47,0.52,0.57,0.62;0.49,0.59,0.69,0.79,0.89)	0.59000
$\tilde{c}_{flpn}$	(10,14,18,22,26;5,9,13,17,21;15,25,35,45,55)	22.00000
$\tilde{A}_{lpn}$	(150,175,200,225,250;100,115,130,145,160;165,190,215,240,265)	181.6667

Table 3 Best found and worst found results of Example 1

Number of runs	Bestfound result					Worst found result					Average Profit
	π^*	S_p^*	gl^*	t_p^*	T^*	π^*	S_p^*	gl^*	t_p^*	T^*	
10	839.1240	33.837	0.7024	0.2881	1.731478	837.9878	33.292	0.5346	0.3042	1.823534	838.7052
20	839.1434	33.871	0.7102	0.2865	1.723606	837.7789	33.846	0.6313	0.3058	1.873236	838.6574
30	839.1472	33.845	0.6929	0.2843	1.715527	837.6614	34.455	0.8349	0.3047	1.858809	838.6441
40	839.1521	33.859	0.7021	0.2845	1.714265	837.5596	34.802	0.9420	0.2669	1.637091	838.5795
50	839.1527	33.865	0.7008	0.2840	1.712949	837.5806	34.540	0.9615	0.2874	1.707284	838.7247

*Indicates the optimal value

Table 4 Best found and worst found results of Example 2

Number of runs	Best found result					Worst found result					Average Profit
	π^*	S_p^*	gl^*	t_p^*	T^*	π^*	S_p^*	gl^*	t_p^*	T^*	
10	859.8232	34.0924	0.1542	0.2429	1.499851	856.9313	33.5893	0.1107	0.2957	1.754007	858.7436
20	859.9165	33.9290	0.1051	0.2396	1.478163	857.0586	33.7869	0.1686	0.2201	1.320502	858.9709
30	859.9500	33.9557	0.1456	0.2565	1.565127	858.2360	34.1907	0.1943	0.2220	1.371568	859.2270
40	860.0646	33.7292	0.1024	0.2546	1.540549	857.6962	34.2387	0.3066	0.2718	1.624875	859.2001
50	860.0887	33.8574	0.1140	0.2560	1.561843	857.4342	34.6115	0.3880	0.2373	1.440991	859.1278

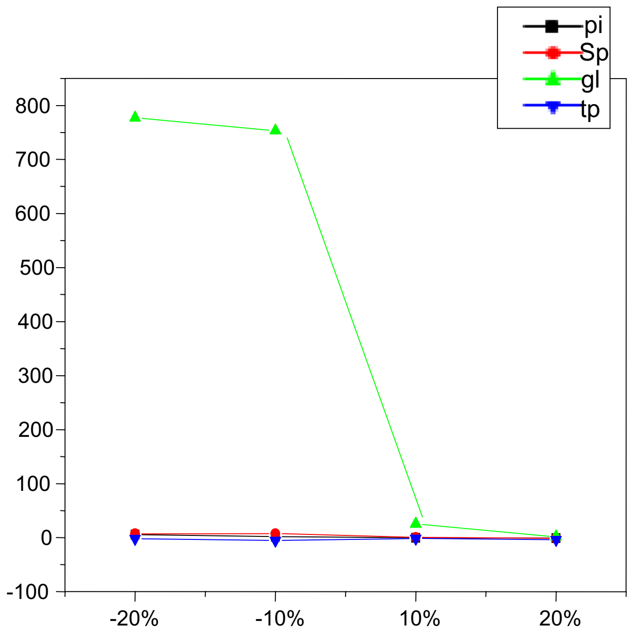

Fig. 9 Sensitivity of α on best found policy

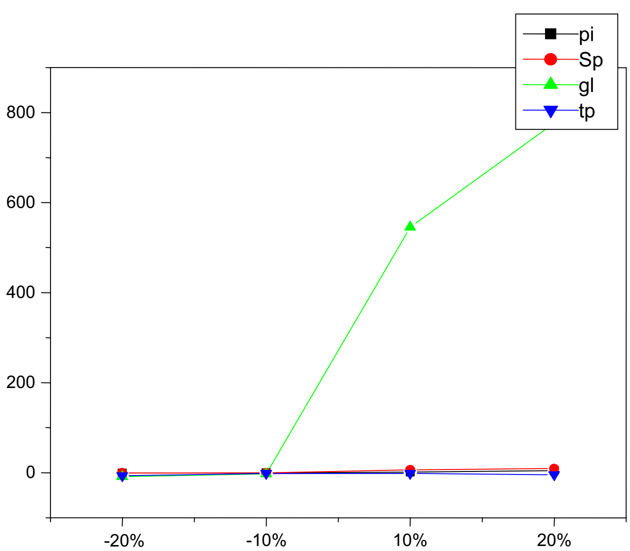

Fig. 10 Sensitivity of γ on best found policy

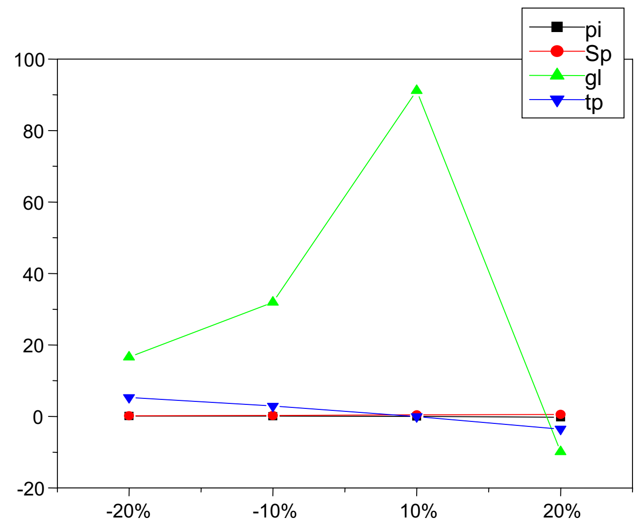

Fig. 11 Sensitivity of d_c on best found policy

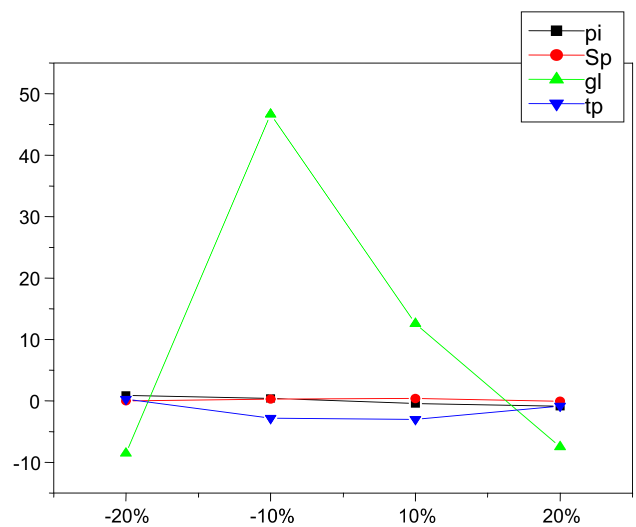

Fig. 12 Sensitivity of h_c on best found policy

(d) Scout bees stage: In this stage, some food sources are considered as abandon while searching. The employed bees of the neglected food sources turn into scouts. Thus these

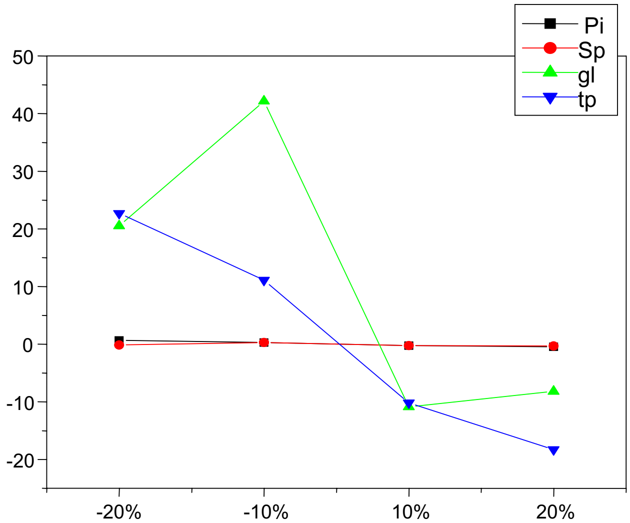

Fig. 13 Sensitivity of Pon best found policy

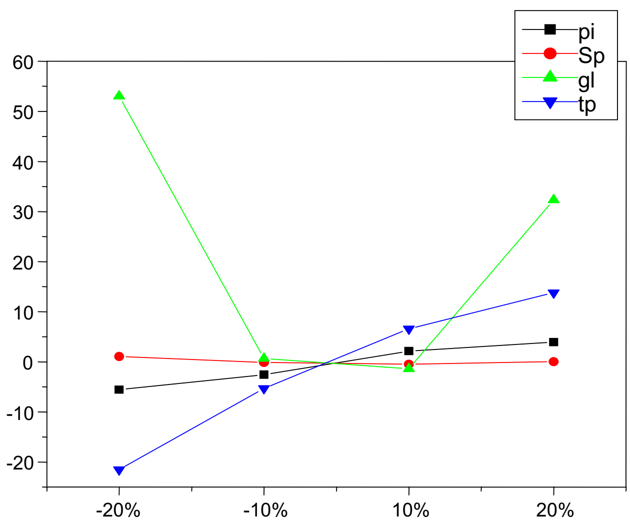

Fig. 14 Sensitivity of ξ on best found policy

scout bees have to look for a new food source using the following equation (Xiang et al. 2014).

$$Y_{f,g} = Y_f^{\min} + rand(0,1)(Y_g^{\max} - Y_g^{\min}) \tag{22}$$

where $g = 1, 2, ..., z$.

(e) **Termination criterion:** ABC algorithm terminates when the number of iterations reaches specified number of iterations.

The steps of ABC algorithm is stated here under:

1st Step: Set the size of the population S_p and the solutions $Y_{f,g}$ to the initial value using Eq. (18); where $f = 1, 2, ..., S_p$, $g = 1, 2, ..., z$, z is the dimension size

2nd Step: Determine the fitness value of $f-th$ solution with the help of Eq. (21)

3rd Step: Take iteration = 1

4th Step: A new solution v_f for the employed bees is derived by using Eq. (18)and then evaluate

$v_{f,g} = Y_{f,g} + \psi_{f,g}(Y_{f,g} - Y_{h,g})$ where $f, h = 1, 2, ..., S_p$, g $= 1, 2, ..., z$; h and g are randomly generated and h differs from $f, \psi_{f,g}$, is a random number within $[-1, 1]$.

5th Step: Employed honey bees should be selected using the greedy selection process

6th Step: If there is no improvement in the solution, assign trial = 1; else, assign trial = 0

7th Step: Determine the probability value of p_f for the solution Y_f

8th Step: Evaluate the latest solutions for the onlooker bees from the f-th solution, which is choosenbased on p_f

9th Step: Utilize greedy selection to select the onlooker bees

10th Step: If there is a discarded solution for the scout, restore it with a random solution using Eq. (18)

11th Step: Among all the solutions found so far, keep the best one

12th Step: If iteration < Maximum cycle number,continuing to 4th Step after setting iteration = iteration + 1

13th Step: Take the optimal solution to print

14th Step: End

8 Numerical experiments

Example 1 In this example, the model parameters are considered as crisp valued which aregiven below:

$c_f = 20$, $\alpha = 0.02$, $\beta = 380$, $\gamma = 29$, $\delta = 9$,$\eta = 0.65$, $\xi = 0.35$, $u = 7$, $M = 0.5$, $h_c = 0.65$, $A = 250$,$P = 600$, $d_c = 2.5$.

Example 2 In this example, some of the model parameters are considered as pentagonal neutrosophic nmber (which are given in Table 2) and the rest of the model parameters be the same as Example 1.

Solution: To obtain the best found solutions of Examples 1 and 2, ABC algorithm is used and the computed results of t_p, g_l, s_p, T as well as the average profit are shown in Tables 3 and 4.

From Tables 3 and 4, it is observed that the crispified input data give better output compare to crisp input data.

9 Sensitivity analysis

To analyse the effects of P_m, α_0, α_1, α_2, α_3, c_p, c_h and A_m on the best found (optimal) policy, sensitivity analyses are carried out considering single parameter at a time keeping others as their original values. The computational results of the sensitivity analysis are shown graphically in Figs. 9, 10, 11, 12, 13 and 14.

From Figs. 9, 10, 11, 12, 13 and 14, the following observations are made.

(i) Average profit of the manufacturer ($\pi(t_p,\ g_l,\ s_p)$) is less sensitive directly w.r.t. preservation investment (ξ) and it is insensitive w.r.t. $P,\ \alpha,\ \gamma,\ h_c$ and d_c.
(ii) Product's green level (g_l) is highly sensitive directly w.r.t. γ, whereas it is highly sensitive reversely w.r.t. P. Moreover, it has large impect w.r.t. $\alpha,\ \xi,\ h_c$ and d_c.
(iii) Selling price of the product (s_p) is insensitive w.r.t. $P,\ \alpha,\ \gamma,\ h_c,\ \xi$ and d_c.
(iv) The production period (t_p) is highly sensitive reversely w.r.t. P, whereas it is less sensitive reversely w.r.t. d_c. Also, it is equally sensitive directly w.r.t. ξ. Moreover, it is insensitive w.r.t. $\alpha,\ \gamma$ and h_c.

10 Managerial insights

In general, man-made products fall into two categories: green products and non-green products. Green products are always eco-friendly for human as well as environment but, these are costly than the non-green products. Although, many manufacturing companies wish to produce green products for fulfilling high market demands of environment conscious customers. On the other hand, different costs related to the production system, deterioration rate and customers' demand are not always fixed. In this connection, this work presents the importance of green products on the customers' demand and manufacturing system. Also, the customers' demand is dependent on the selling price and green level of the product. Moreover, it is assumed that deterioration rate, demand parameters, different costs related to the production system are uncertain in nature.

Based on the numerical experiment, the following managerial outcomes are drawn:

(i) The green product has the positive impact on both environment and economy.
(ii) Product's green level has a positive impact on manufacturer's average profit and customers' demand. So, the model can be applied in the green production industry, viz. garment industry, bag industry, cosmetics industry, etc.
(iii) Because of changing weather and seasons, labor problems, inadequate and insufficient information, lack of evidence, fluctuating financial markets, real-life conditions can create uncertainty concerning parameters such as deteriorating rate, demand parameters, holding cost, production cost, set up cost. For addressing the imprecision of the parameters, organiser can adapt the parameters as fuzzy, intuitionistic fuzzy, type—2 fuzzy, neutrosophic fuzzy, stochastic or combinations of these.
(iv) To simplify the computational process, the manager can convert the imprecise number into crispified one with the suitable crispification methods.

11 Conclusion and future scope of research

We have developed a food production inventory model involving deteriorating green products. In this work, the rate of deterioration is taken to be preservation investment level dependent and production cost is based on the greenness of the product. Also, the customer's demand rate is increased with the greenness of the product and decreased with the selling price to go through real life situations. Firstly, the crisp model is formulated, thereafter the related neutrosophic fuzzy model is also expressed with the linear pentagonal neutrosophic fuzzy parameters. To make the computational easier, we have converted the neutrosophic fuzzy model to a crispified model with the help of a suitable crispification method. After that we find out the solution of the crisp and crispified models with the ABC algorithm and sensitivities of different parameters of the crispified models are presented pictorially. For the first time, the linear pentagonal neutrosophic fuzzy number is used to construct the EPQ model for deteriorating green product with the demand dependent on selling price and green level of the product.

This study has the following limitations:

(i) In the proposed production model, manufacturer produces green products which are perfect. However, the concept of defective production is not considered.

(ii) Though there are so many soft computing techniques, we have only used the ABC algorithm to solve the model. The limitations of this method are as follows:

- Insufficient use of secondary information
- Requires new fitness test on new algorithm
- Higher number of objective function evaluation
- Slow sequential processing.

For future scope of research, one can developed more inventory models based on the ideas discussed in this work. This model can be enhanced by incorporating trade credit financing to generate higher demand rates. Additionally, we can extend this model to include Weibull distributed deterioration rate with two/ three parameters in stochastic environment. Researchers can use stochastic, intuitionistic fuzzy, fuzzy numbers or type-2 fuzzy to deal with the imprecision of their parameters with reference to real data.

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Declarations

Conflict of interest The authors declare that they have no conict of interest.

Ethical approval This article does not contain any data from any source and there is no involvement about animals.

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